### Reasoning about Programs in Higher-Order Concurrent Separation Logic

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### Introduction

As PL researchers we study programs and PL's

An important part of this is proving our programs and PL's are correct

We use mathematical tools:

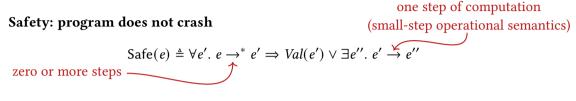
- Define the semantics of programs, e.g., operational semantics
- State theorems about programs and PL's in semantics terms, *e.g.*, safety, functional correctness, type safety, *etc*.
- Prove these properties using different tools and techniques

**Program logics** are important tools

- Provide a formal framework for stating and proving properties of programs
- In this talk: the Iris program logic

# Safety: program does not crash Safe(e) $\triangleq \forall e'. e \rightarrow^* e' \Rightarrow Val(e') \lor \exists e''. e' \rightarrow e''$ zero or more steps

- Example: Safe(letrec  $f x = f x \inf f 4$ )
- Counterexample: ¬Safe(if "a" then 2 else 3)

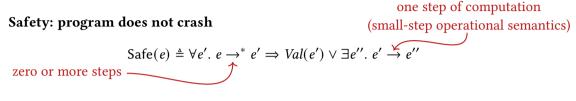


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Functional Correctness: safe, and upon termination postcondition holds

$$\operatorname{Correct}_{\phi}(e) \triangleq \operatorname{Safe}(e) \land \forall v. \, Val(v) \land e \to^{*} v \Rightarrow \phi(v)$$

- Example:  $Correct_{isEven}(3+5)$ 



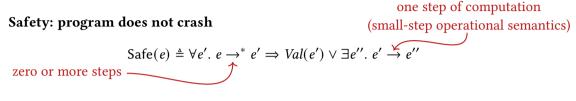
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Type safety: well-typed programs are safe



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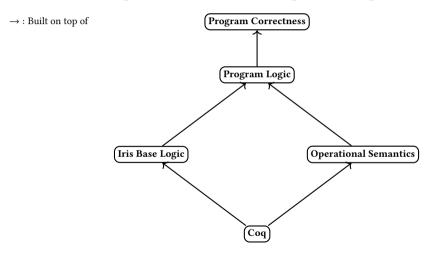
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Type safety: well-typed programs are safe

In this talk: Iris and how it helps us prove such properties

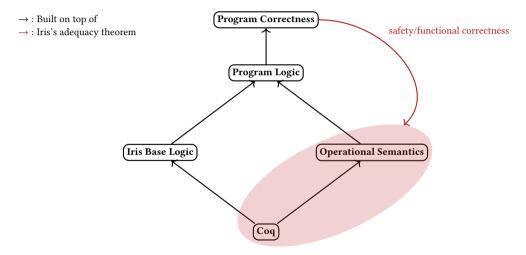
### What is Iris?

#### A Framework for Higher-order Concurrent Separation Logics



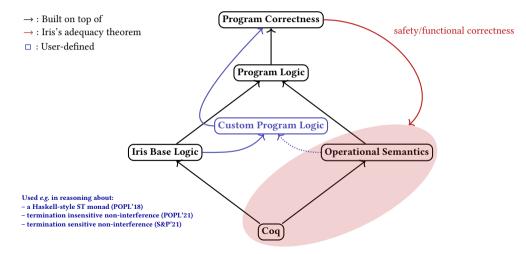
### What is Iris?

#### A Framework for Higher-order Concurrent Separation Logics



### What is Iris?

#### A Framework for Higher-order Concurrent Separation Logics



### Versatility of Iris

#### Iris have been used in many projects:

- Reasoning about session types (@CPP'21)
- Reasoning about capability machines (hardware language) (@POPL'21)
- Reasoning about non-interference (a security property) (@POPL'21)
- Reasoning about distributed systems (@POPL'21)
- Proving properties of gradual typing systems (@POPL'21)
- Reasoning about algebraic effect handlers (@POPL'21)
- Reasoning principles for weak memory
- Proving properties of DOT (core of Scala)
- Proving properties of the Rust programming language

etc.

#### This versatility is due to Iris's expressivity.

### Iris Base Logic

A logic with features designed for defining program logics:

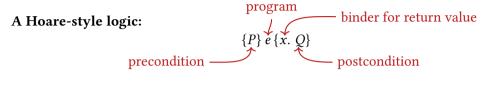
 $P ::= \text{True} \mid \text{False} \mid P \lor P \mid P \land P \mid P \rightarrow P \mid \forall x. P \mid \exists x. P \mid$  (higher-order logic)  $P * P \mid$  (separation logic)  $\begin{bmatrix} a \\ p \\ p \end{bmatrix} \mid \Rightarrow P \mid$  (user-defined resources)  $P \mid \mu r.P \mid$  (step indexing) P (invariants)

#### **Base logic inference rules:**

$\frac{\stackrel{\text{Lob-ind}}{\triangleright} P \vdash P}{\vdash P}$	$P \vdash P$	$\xrightarrow{P \mapsto Q} P \models Q$ *-TRUE $P \models P \models P$ $P \Rightarrow True \rightarrow P$	*-ELIM-L $P * Q \vdash P$	*-ELIM-R $P * Q \vdash Q$ $P_1 \vdash P_2$ $P_1 * Q_1 \vdash P_2$	$Q_1 \vdash Q_2$ $\frac{1}{2} * Q_2$	$\stackrel{*-\mathrm{comm}}{P*Q \vdash Q*P}$	*-ASSOC $(P * Q) * R \vdash P * (Q * R)$	$\frac{\stackrel{\wedge-\text{INTRO}}{P \vdash Q}  P \vdash}{P \vdash Q \land R}$	$\frac{R}{P \vdash P}$	$\frac{P \vdash Q}{P \vdash R}$	$\vdash R$ $\vdash$ -TRUE $P \vdash$ True	$\vdash$ -FALSO False $\vdash P$	$\wedge$ -true $P \wedge$ True ++ $P$
$\land$ -elim-L $P \land Q \vdash P$	$\land$ -ELIM-R $P \land Q \vdash Q$	$\frac{A - \text{MONO}}{P_1 \vdash P_2}$ $Q_1 \vdash Q_2$ $\overline{P_1 \land O_1 \vdash P_2 \land O_2}$	$\land$ -comm $P \land Q \vdash Q \land P$	$\wedge$ -ASSOC $(P \land Q) \land R \vdash P \land (Q$			$P \vdash R$ $P \vdash R$ $P \lor O \land R$ $V \vdash Palse$ $P \lor False$	$P \vdash R \qquad Q \vdash P \lor O \vdash R$	_	$Q_1 \vdash Q_2$ $Q_1 \vdash Q_2$ $Q_1 \vdash Q_2$	$\lor$ -comm $P \lor Q \vdash Q \lor P$	$\lor$ -ASSOC $(P \lor Q) \lor$	$R \vdash P \lor (Q \lor R)$

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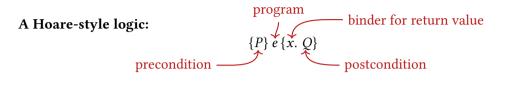
### Program Logic



**Examples:** 

 ${n \ge 0}$  fact  $n {x. x = n!}$  {True} letrec  $f x = f x \inf f 4 {x. False}$ 

### Program Logic



**Examples:**  $\{n \ge 0\}$  fact  $n\{x, x = n!\}$   $\{\text{True}\} \text{ letrec } f x = f x \text{ in } f 4\{x, \text{ False}\}$ 

### Theorem (Adequacy)

If we prove

 $\vdash$  {*True*} e {x.  $\phi(x)$ }

*in Iris* for a suitable  $\phi$ , then  $Correct_{\phi}(e)$ 

### Program Logic



**Examples:**  $\{n \ge 0\}$  fact  $n\{x, x = n!\}$   $\{\text{True}\} \text{ letrec } f x = f x \text{ in } f 4\{x, \text{ False}\}$ 

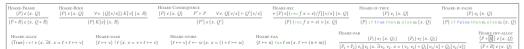
### Theorem (Adequacy)

*If we prove* 

 $\vdash \{True\} e \{x. \phi(x)\}$ 

*in Iris* for a suitable  $\phi$ , then  $Correct_{\phi}(e)$ 

#### Proof rules for reasoning about programs:



### Expressivity: Higher-Order Logic

Specifying abstract data types:<sup>1</sup>

 $\exists isStack : Val \rightarrow list(Val \rightarrow Prop) \rightarrow Prop.$   $\{True\} mk\_stack() \{s.isStack(s, [])\} \land$   $\forall s. \forall \Phi. \forall \Phis. \{isStack(s, \Phi s) * \Phi(x)\} push(x, s) \{v.v = () \land isStack(s, \Phi :: \Phi s)\} \land$  $\forall s. \forall \Phi. \forall \Phis. \{isStack(s, \Phi :: \Phi s)\} pop(s) \{v.\Phi(v) * isStack(s, \Phi s)\}$ 

#### Note the higher-order quantification of a predicate that takes a list of predicates

<sup>&</sup>lt;sup>1</sup>Taken verbatim from Iris lecture notes.

### Expressivity: Separation Logic

Separating conjunction:



P \* Q holds if P and Q hold for *disjoint* resources

Example: exclusive ownership of a memory location (points-to proposition)

 $\ell \mapsto \nu \ast \ell' \mapsto \nu' \vdash \ell \neq \ell'$ 

HOARE-ALLOC  

$$\{\text{True}\} \text{ ref } v \{x. \exists \ell. x = \ell * \ell \mapsto v\}$$
  
 $\{\ell \mapsto v\} ! \ell \{x. x = v * \ell \mapsto v\}$ 

HOARE-STORE  $\{\ell \mapsto \nu\} \ell \leftarrow w \{x. \ x = () * \ell \mapsto w\}$ 

### Expressivity: Separation Logic

#### In separation logic a Hoare triple specifies *footprint* of the program.

Hence the *frame* rule:

HOARE-FRAME  $\frac{\{P\} e\{x. Q\}}{\{P * R\} e\{x. Q * R\}}$ 

#### Important for modular verification:

Verify modules working on separate parts of memory in isolation and combine proofs

#### What if two modules share memory?

We use invariants (and resources) to specify sharing protocols

### Expressivity: User-Defined Resources

Users can introduce resources as partial commutative monoids (PCM's)

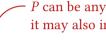
logical equivalence user-defined operation - *update* modality  $[\overline{a}]^{\gamma} * [\overline{b}]^{\gamma} + [\overline{a} + \overline{b}]^{\gamma}$ 

 $\Rightarrow$  *P* holds if *P* holds after updating resources

Idea: for verifying stateful programs we need a stateful logic

### **Expressivity: Step-Indexing and Invariants**

Iris invariants are *impredicative*:



*P* can be any proposition; it may also including invariants

Step-indexing is necessary for impredicative invariants to avoid self-referential paradoxes

These features are necessary for defining logical relations models for programming languages with advanced features

Goal: we want to prove type safety (well-typed programs do not crash)

#### Using logical relations:

We prove by induction on typing derivation:

 $e:\tau \Longrightarrow LR_{\tau}(e)$ 

where

$$LR_{\tau}(e) \Rightarrow Safe(e)$$

However, we cannot take  $LR_{\tau}(e)$  to be Safe(*e*):

 $\operatorname{Safe}(e_1) \wedge \operatorname{Safe}(e_2) \not\Rightarrow \operatorname{Safe}(e_1 - e_2)$ 

Counter example: Safe(true) and Safe(3) but  $\neg$ Safe(true - 3)

We should take  $LR_{\tau}$  to be:

 $LR_{\tau}(e) \triangleq \operatorname{Correct}_{\llbracket \tau \rrbracket}(e)$ 

where  $\llbracket \tau \rrbracket(v)$  means that *v* is a value of type  $\tau$ .

Ideally, we should define this by induction on types:

$$\begin{bmatrix} int \end{bmatrix} (v) \triangleq v \in \mathbb{Z}$$

$$\begin{bmatrix} (\tau_1 \times \tau_2) \end{bmatrix} (v) \triangleq \exists v 1, v_2.v = (v_1, v_2) \land \llbracket \tau_1 \rrbracket (v_1) \land \llbracket \tau_2 \rrbracket (v_2)$$

$$\vdots$$

$$\llbracket \mu X. \tau \rrbracket (v) \triangleq \exists w. v = \texttt{fold} (w) \land \llbracket \tau \rrbracket_{X \mapsto \llbracket \mu X. \tau \rrbracket} (w)$$

$$\llbracket \mathsf{ref}(\tau) \rrbracket (v) \triangleq \exists \ell. v = \ell \land \underline{\ell} \text{ always stores a value of } \tau$$
how do we express this?

#### We use Iris and define

 $LR_{\tau}(e) \triangleq \{\text{True}\} e \{x. [[\tau]](v)\}$ 

We define  $\llbracket \tau \rrbracket(v)$  inductively as follows:







#### This approach to type safety is called semantic type safety

It has been used for reasoning about correctness of the Rust type system.

See Derek Dreyer's POPL'18 keynote for more details.

### Example: Shared Memory Concurrency

Consider the following concurrent program where threads share memory:



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Consider the following concurrent program where threads share memory:

#### {True}

```
let c = ref 0 in
(faa c 1 \| faa c 2);
! c
{x. x \ge 0}
```

### Example: Shared Memory Concurrency

Consider the following concurrent program where threads share memory:  $\{\mathsf{True}\}$ 

let c = ref 0 in  $\{c \mapsto 0\}$  $\{\exists n. n \ge 0 * c \mapsto n\}$  $\begin{pmatrix} \{ \exists n. \ n \ge 0 * c \mapsto n \} \\ \{ \exists n. \ n \ge 0 * c \mapsto n \} \\ faa c 1 \\ \{ x. \ \exists n. \ n \ge 0 * c \mapsto n \} \\ \{ x. \ \exists n. \ n \ge 0 * c \mapsto n \} \\ \{ x. \ \exists n. \ n \ge 0 * c \mapsto n \} \end{pmatrix} | \begin{cases} \exists n. \ n \ge 0 * c \mapsto n \\ faa \ell 2 \\ \{ x. \ \exists n. \ n \ge 0 * c \mapsto n \} \end{cases} ;$  $\{\exists n. n \ge 0 * c \mapsto n\}$ !c

 $\{x. \ x \ge 0\}$ 

Can we also prove the following stronger specs for our code?

#### {True}

let c = ref 0 in
 (faa c 1 || faa c 2);
 ! c
{x. x = 3}

Can we also prove the following stronger specs for our code?

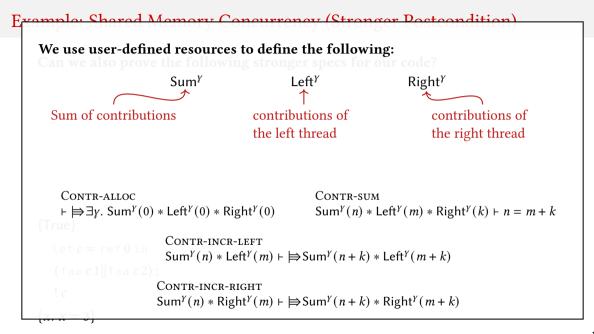
With which invariant should we proceed?

$$\exists n. \ n \ge 0 * c \mapsto n \qquad c \mapsto 3$$

Neither works. We need to be able to refer to the value outside the invariant!

{True}

let c = ref 0 in
 (faa c 1 || faa c 2);
 ! c
{x. x = 3}



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#### {True}

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 (faa c 1 || faa c 2);
 ! c
{x. x = 3}

### Can we also prove the following stronger specs for our code?

{True}

 $\{x, x = 3\}$ 

let c = ref 0 in  $\{c \mapsto 0\}$  $\{c \mapsto 0 * \operatorname{Sum}^{\gamma}(0) * \operatorname{Left}^{\gamma}(0) * \operatorname{Right}^{\gamma}(0)\}$  $\{\exists n. \ c \mapsto n * \operatorname{Sum}^{\gamma}(n)\} * \operatorname{Left}^{\gamma}(0) * \operatorname{Right}^{\gamma}(0)\}$  $\begin{pmatrix} \left\{ \exists n. \ c \mapsto n * \operatorname{Sum}^{\gamma}(n) \right\} * \operatorname{Left}^{\gamma}(0) \\ \text{faa } c 1 \\ \left\{ x. \ \operatorname{Left}^{\gamma}(1) \right\} \end{cases} \\ \left\{ \exists n. \ c \mapsto n * \operatorname{Sum}^{\gamma}(n) \\ \text{faa } c 2 \\ \left\{ x. \ \operatorname{Right}^{\gamma}(2) \right\} \end{cases};$  $\{\exists n. c \mapsto n * \operatorname{Sum}^{\gamma}(n)\} * \operatorname{Left}^{\gamma}(1) * \operatorname{Right}^{\gamma}(2)\}$ !c

### Proofs and Iris Proof Mode

- ► I simplified the proofs that I just presented
- However, Iris features a Proof Mode (IPM)
- IPM makes program verification in Coq very close to what I presented
- To the right: screenshot of the proofs we just saw in Iris in Coq



### Online resources

I hope this talk has made you interested in learning more about Iris, separation logic, program verification, *etc*.

#### See http://iris-project.org

- Iris Tutorial material
- Iris related publications
- PhD theses that include Iris works
- Other manuscripts

#### See https://cs.au.dk/~timany/talks/plmw21/

- These slides
- A list of Iris related talks at POPL'21

#### Iris Project Iris is a Higher-Order Concurrent Separation Logic Framework implemented and verified in the Cog proof assistant Coo Formalization Technical Documentation (v3.3) Mailing List Learning Iris Events Publications PhD dissertations Other material Iris is a framework that can be used for reasoning about safety of concurrent programs, as the logic in logical relations, to reason about type-systems, data-abstraction etc. In case of questions, please contact us on the Iris Club list or in our chat room Learning Iris Some useful resources designed to learn Iris and its Cog implementation The Iris lecture notes provide a tutorial style introduction to Iris, including a number of exercises (but most of it not in Coo) The Iris Tutorial at POPI '18 contains a number of exercises to practice the Iris tactics in Con- The Iris Tutorial at POPL'20 shows how to use Iris to build logical relations for establishing type safety. A selection of naners that are suited to get started with Iris: The Iris From The Ground Up paper contains an extensive description of the rules and the model of the Iris logic. The Iris Proofmode paper (Section 3) contains a brief tutorial to the Iris tactics in Con. The Iris Proof Mode (IPM) / MoSel, and the Heapl and documentation provide a reference of the Iris factics in Cool Events 18 January 2021: Tutorial on Iris at POPL. Virtual · 20 January 2020; Tutorial on Proving Semantic Type Soundness in Iris at POPL. New Orleans, USA 28 October -1 November 2019: The Einst Iris Workshop, Aarbur, Denmark 8 January 2018: Tutorial on Iris at POPL. Los Angeles, USA Publications Below, we give an overview of the research that uses Iris one way or another. [1] Reasoning about Monotonicity in Separation Logic Amin Timany, Lars Rickedal .pdf Cog development [2] Machine-Checked Semantic Session Typing Jonas Kastberg Hinrichsen, Daniël Louwrink, Robbert Krebbers, Jesper Bengtson Recipient of CPP 2021 Distinguished Paper Award Integrint1 adf Con development [3] Efficient and Provable Local Capability Revocation using Uninitialized Capabilities

## Iris related talks at POPL'21 and co-events (Time in CET)

Toward Complete Stack Safety for Capability Machines Aïna Linn Georges, Armaël Guéneau, Alix Trieu, Lars Birkedal	@PriSC, on 17 <sup>th</sup> at 20:12
Contextual Refinement of the Micheal-Scott Queue (Proof Pearl) Amin Timany, Lars Birkedal	@CPP, on 18 <sup>th</sup> at 17:00
Reasoning About Monotonicity in Separation Logic Amin Timany, Lars Birkedal	@CPP, on 18 <sup>th</sup> at 17:15
[T4] Iris – A Modular Foundation for Higher-Order Concurrent Separation Logic Tej Chajed, Ralf Jung, Joseph Tassarotti	@Tutorial, on 18 <sup>th</sup> at 18:00
Machine-Checked Semantic Session Typing Jonas Kastberg Hinrichsen, Daniël Louwrink, Robbert Krebbers, Jesper Bengtson	@CPP, on 18 <sup>th</sup> at 18:30
Fully Abstract from Static to Gradual Koen Jacobs, Amin Timany, Dominique Devriese	@POPL, on 20 <sup>th</sup> at 16:00
Efficient and Provable Local Capability Revocation using Uninitialized Capabilities Aïna Linn Georges, Armaël Guéneau, Thomas Van Strydonck, Amin Timany, Alix Trieu, Sander Huyghebaert, Dominique Devriese, Lars Birkedal	@POPL, on 21 <sup>st</sup> at 16:00
Mechanized Logical Relations for Termination-Insensitive Noninterference Simon Oddershede Gregersen, Johan Bay, Amin Timany, Lars Birkedal	@POPL, on 21 <sup>st</sup> at 16:10
A Separation Logic for Effect Handlers Paulo Emílio de Vilhena, François Pottier	@POPL, on 22 <sup>nd</sup> at 16:15
Distributed Causal Memory: Modular Specification and Verification in Higher-Order Distributed Separation Logic Léon Gondelman, Simon Oddershede Gregersen, Abel Nieto, Amin Timany, Lars Birkedal	@POPL, on 22 <sup>nd</sup> at 16:15 20