Mathematics: Algorithms and Proofs

8-12 January 2007

Aim

The goal of the workshop was to bring together people from the communities of formal proofs, constructive mathematics and computer algebra (in a wide sense). It is a continuation of previous meetings in Schloss Dagstuhl (2003 and 2005), Luminy (2004) and Castro Urdiales (2006). The previous meetings have been quite successful in bringing these communities closer together.

One objective of the workshop is to bridge the gap between conceptual (abstract) and computational (constructive) mathematics, by providing a computational understanding of abstract mathematics. It is becoming clear that many parts of abstract mathematics can be made constructive and even computational and that abstract mathematical techniques contain an underlying constructive content. We are not only interested in algorithms however, but also in formal proofs of the correctness of these algorithms.

Computer algebra provides a variety of interesting basic algorithms, from exact linear algebra to various aspects of elimination and real root counting, which are the foundations for much more sophisticated results like nullstellensatz, quantifier elimination etc... It is remarkable that in constructive and computer algebra, progress in sophisticated algorithms often implies progress on these basic algorithms.

Moreover the scope of computer algebra is now widened by the consideration of seminumerical algorithms. When such algorithms are correctly controlled, they actually deal with real and complex numbers in the constructive meaning of these objects. So computer algebra fills many objectives of computational analysis.

Providing formal proofs of correctness to the computer algebra community is very useful, specially for algorithms which are basic and used everywhere. On the other hand, a collection of mathematically non trivial examples is very useful for the formal proof community, which also needs powerful automatic methods from computer algebra.

For more information about the MAP-community we refer to `http://www.disi.unige.it/map` where one can also find the MAP manifesto. MAP is an acronym for 'Mathematics: Algorithms and Proofs'.

Participants

There were 59 participants from many different countries.

Program

Tutorials

- Arjeh Cohen (Eindhoven University) - *Three aspects of exact computation*
- Thierry Coquand (Chalmers, Sweden) - *A logical approach to abstract algebra*
- Per Martin-Löf (Stockholm, Sweden) - *Topos theory and type theory*

Invited talks

- Michael Beeson (San José State, USA) - *Algorithms and Proofs in Geometry*
- Dirk van Dalen (Utrecht University) - *The lonely revolutionary. Brouwer's first program.*
- Hendrik Lenstra (Universiteit Leiden) - *Ordering fields*

Contributed talks

Thorsten Altenkirch (Nottingham, UK) - *Quotient Types in Observational Type Theory*

Jesus Aransay (La Rioja, Spain) - *A mechanised proof of the Basic Perturbation Lemma*
Eyvind Briseid (Darmstadt, Germany) - *Using proof mining to find computable rates of convergence for the Picard iteration sequence for non-nonexpansive functions.*
Jan Draisma (Technische Universiteit Eindhoven) - *A scenic tour in tropical geometry*
Gilles Dowek (l'École polytechnique, France) - *The linear-algebraic lambda-calculus* (joint work with Pablo Arrighi)
Harold Edwards (New York, USA) - *Addition on Elliptic Curves*
Peter Hancock (Edinburgh, UK) - *Stream Processing, a la Brouwer* (Joint work with Neil Ghani and Dirk Pattinson)
Assia Mahboubi (INRIA/MSR, France) - *A formal correctness proof for the subresultant algorithm*
Sean McLaughlin (CMU, USA) - A Methodology for Implementing Reusable Decision Procedures
Henri Lombardi (France Comté, France) - *Constructive real algebra*
Russell O'Connor (Radboud University) - *Implementing Analysis*
Paulo Oliva (Queen Mary, University of London, UK) - *Modified realizability of classical linear logic*
Peter Paule (Research Institute for Symbolic Computation (RISC), J. Kepler University Linz, Austria) - *A Computer Proof of a Conjecture of Moll*
Ana Romero Ibañez (Rioja, Spain) - *Constructive Spectral Sequences*
Peter Schuster (Munich, Germany) - *Labelled Trees of Finite Depth* (joint work with Hervé Perdry)
Helmut Schwichtenberg (Munich, Germany) - *Goedels Dialectica Interpretation*
Bas Spitters (Radboud University) - *Located and overt locales* (jww Thierry Coquand)
Roland Zumkeller (l'École polytechnique, France) - *Towards a formal proof of the Kepler conjecture*

Outcome

The workshop was highly successful. The presentations were of excellent quality as can be judged from the slides which may be found on the workshop website. The friendly atmosphere in Lorentz center stimulated interaction between the participants. The coffee room and offices were used for discussions from early to very late in the day. We received many very positive reactions, both on the scientific quality and on the stimulating well-organized center.

Acknowledgement

Scientific Committee

Thierry Coquand (Chalmers, Sweden)
Henri Lombardi (France Comté, France)
Peter Paule (University of Linz, Austria)
Bas Spitters (Radboud Univerisity Nijmegen, the Netherlands)

Organizing Committee

Herman Geuvers (Radboud Univerisity Nijmegen)
Bas Spitters (chair, Radboud Univerisity Nijmegen)
Freek Wiedijk (Radboud Univerisity Nijmegen)