Passively Mobile Communicating Machines that Use Restricted Space

Andreas Pavlogiannis[2]

Joint work with: I. Chatzigiannakis[1], O. Michail[1]
S. Nikolaou[1], P. G. Spirakis[1]

[1]Research Academic Computer Technology Institute (RACTI)
Patras, Greece

[2]UC Davis, Computer Science Davis, CA

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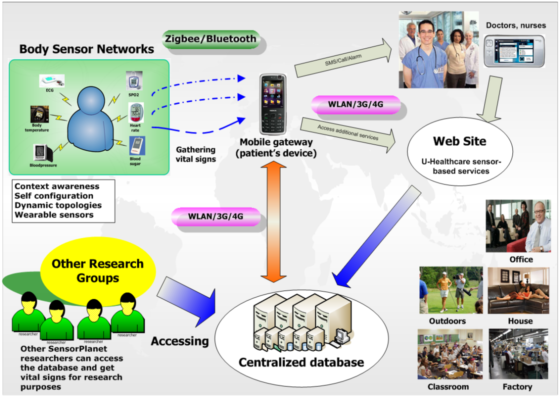
The Motivation



- Wireless Sensor Networks have received great attention recently due to their wide range of applications.

The Background Work

- Theoritical models for WSNs have become significantly important in order to understand their capabilities and limitations.
- Population Protocols [Angluin, Aspnes, Diamadi, Fischer, and Peralta, PODC '04] is a model for WSNs where:
  - Tiny Entities: *finite-state* machine + *sensor* (**agent**).
  - Communication: **Pairwise - Interaction Based**.
  - **Passively mobile** agents: incapable to choose participants in interactions.
    - Models: unstable environment, like water flow or wind, or the natural mobility of their carriers.
  - **Anonymity:** No enough space for unique identifiers.

The Population Protocols Model and Characteristics

- Agents sense their environment and receive an input symbol.
  - Computation on predicates defined by this input.
- Agents interact in pairs according to a **communication graph** $G = (V, E)$ where:
  - V: A **population** of $|V| = n$ agents of constant memory (independent of n).
  - E: The permissible interactions between the agents.
- Interaction pattern: **adversary**
- Adversarial choices: **fairness condition**
- **fairness condition**: forbids population partition (the adversary cannot avoid a possible interaction forever)

Computational Power

- Due to the minimalistic nature of the model the class of computable predicates is fairly small.
- In [Angluin et al. 2004, 2006] it was proven that it is exactly the class of **semilinear predicates**.
- Formulas such as $N_a \geq 10$ or $N_a < N_b$.
  - *Threshold, Majority...*
- This class does not include multiplication, exponentiation and other important operations.

Relaxing the PP constraints

- **Tiny** (constant) space $\rightarrow$ **Restricted** space
  - Allowing for logarithmic memory is reasonable.
  - 10^9 agents only need $\propto$ 30 bits!
- Preserve passive mobility - no control over the interactions.
  - But still, fair.
- **Passively Mobile Communicating Machines**
- Study space complexity of various problems.
  - Interest remains on problems that use *restricted space*.

Agent

- **Sensor**: Receive the input $x \in X$.
- **Working Tape**: Internal computation.
- **Output Tape**: Agent's output.
- **Outgoing Message Tape**: Send messages to other agents.
- **Incoming Message Tape**: Receive messages from other agents.
- **Working Flag**: When set, the agent is busy doing internal computation and *cannot interact*.

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The Passively Mobile Machines Model (*PM*)

Definition

PM protocol: 6-tuple $(X, \Gamma, Q, \delta, \gamma, q_0)$

- **X**: *input alphabet*, $\sqcup \notin X$,
- **Γ**: *tape alphabet*, $\sqcup \in \Gamma$ and $X \subset \Gamma$,
- **Q**: set of *states*,
- $\delta : \mathbf{Q} \times \mathbf{\Gamma^4} \to \mathbf{Q} \times \mathbf{\Gamma^4} \times \{\mathbf{L}, \mathbf{R}, \mathbf{S}\}^4 \times \{\mathbf{0}, \mathbf{1}\}$, the *internal transition function*,
  - Internal computation, Message processing...
- $\gamma : \mathbf{Q} \times \mathbf{Q} \to \mathbf{Q} \times \mathbf{Q}$, the *external transition function*,
  - Upon interaction, transition to a state that starts reading the incoming message.
- $\mathbf{q_0} \in \mathbf{Q}$, the *initial state*.

Computation in PM

- **Agent Configuration** $B \in \mathcal{B}$: A tuple specifying the agent "state".
- **Population Configuration** $C \in \mathcal{C}$: A tuple capturing the population state. Configuration yieldability $C \to C'$: C' occurs from C in one step.
- Initially, every agent is assigned an *input sybmol*.
- The adversary chooses:
  - An agent to execute one internal step (application of δ).
  - A pair of agents to interact (message exchange and application of γ)
    - initiator - responder distinction.
- But **fairly**.
  - If $C \to C'$ and C appears infinite times, C' also appears infinite times.

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Computation in PM (Continued)

- **Execution**: a sequence of population configurations $(C_1, C_2, \dots)$ such that $C_i \rightarrow C_{i+1}$.
- **Computation**: an infinite fair execution.
- PM protocols **stabilize**: $\exists i : \forall v \in V,\ \forall j \geq i$, agent v does not change his output tape in C_j, and all agents agree on output.
- Stable computation of predicates $p : X^{|V|} \rightarrow \{0, 1\}$.
  - Symmetric predicates: $p(a) = 1 \iff p(\tilde{a}) = 1$, $\tilde{a}$: permutation of a.
- **Space Complexity Classes**:
  - **PMSPACE(f(n))**: Predicates computable by a PM protocol using $O(f(n))$ space.
  - **SSPACE(f(n))**, **SNSPACE(f(n))**: Symmetric subsets of predicates in $SPACE(f(n))$, $NSPACE(f(n))$.
  - **SEM**: Class of Semilinear predicates.

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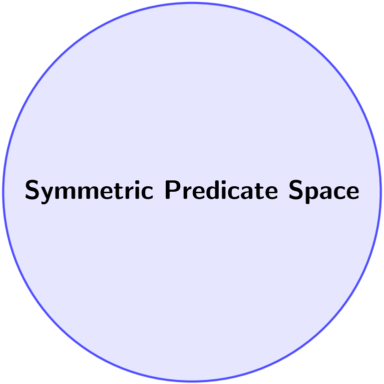
Dividing the predicate space

- Study of the impact of passive mobility in computational capabilities of distributed systems.



Goal: Divide predicate space according to predicate space complexity.

Assigning Unique Ids

Theorem

Any PM protocol $\mathcal{A}$ can assume the existance of unique ids, at the cost of $O(\log n)$ space.

Proof: A protocol $\mathcal{I}$ for UID assignment.

- All agents start with $uid = 0$.
- **Effective** interactions only between agents with the same uid.
  - Initiator increments uid.
- **$\mathcal{I}$ does not terminate. Every time a uid is incremented, the agent broadcasts a message for $\mathcal{A}$ to reinitiate computation.**
- Agents ignore such messages with uid smaller than the last one (ignore *late* messages).
  - After $uid = n - 1$, reinitiations stop, and $\mathcal{A}$ finally is executed correctly.

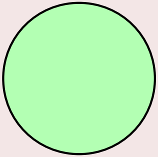
□

Assigning Unique Ids (Continued)

Theorem

Any PM protocol $\mathcal{A}$ can assume the existance of unique ids and knowledge of the population size, at the cost of $O(\log n)$ space.

Proof.



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$\square$

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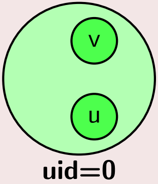

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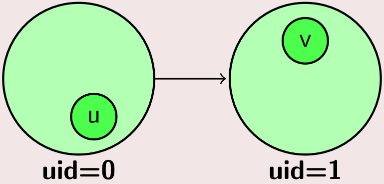

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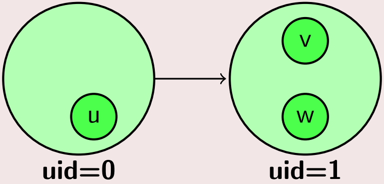

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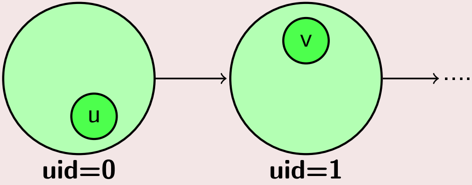

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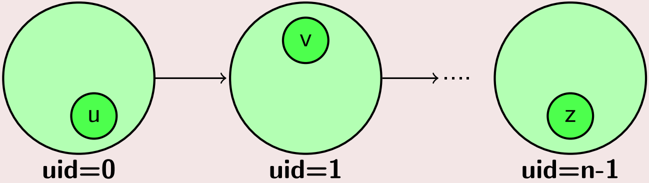

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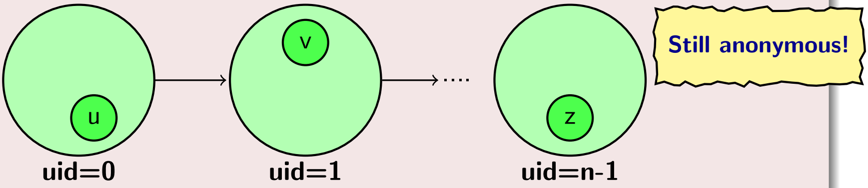

Simulating a Deterministic Turing Machine

Theorem

For any $f(n) = \Omega(\log n)$, $\mathbf{SSPACE(nf(n))} \subseteq \mathbf{PMSPACE(f(n))}$.

Proof.

Input string $w \in SSPACE(\Omega(n \log n))$ decided by a TM D, $|w| = n$.

- Each agent receives a symbol of w.
- Use $\mathcal{I}$ to align all agents.
- Use this alignment as a tape in a **modular** fashion.
  - The **local tape** of each agent provides $O(\log n)$ cells.
- Each time, one **active** agent carries the simulation.
- State transition rules of D embedded in the PM protocol.
- Head move $\rightarrow$ pass **control** + **current state** to neighbor.
- **Simulation accepts a permutation of** w. ✓

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- Initial configuration C: all agents set output to **reject**.
- Use simulation of D.
- **Non deterministic** choice out of k possible.
- **Exploit fairness of the adversary!**
  - Pause simulation and wait for interaction.
  - Pick choice based on *uid* of the other participant.
- Simulating branch N **rejects:** Reset population to C.
- N **accepts:** A *good* simulating branch starting from C exists.
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Exact Characterization

Theorem

$\mathbf{PMSPACE(f(n))} \subseteq \mathbf{SNSPACE(nf(n))}$.

Proof Idea.

- A NTM N simulates the behaviour of the population.
- Encode n agent configurations of size $f(n)$.
- Non deterministic guesses of adversarial strategies.

$\square$

- For any $f(n) = \Omega(\log n)$, $\mathbf{PMSPACE(f(n))} = \mathbf{SNSPACE(nf(n))}$.

A Space Hierarchy

Theorem

For $h(n) \in \Omega(\log n)$ and recursive $l(n)$, separated by a nondeterministically fully space constructible function $g(n)$, with $h(n) \in \Omega(g(n))$ but $l(n) \notin \Omega(g(n))$, $\exists$ language in $\mathbf{PMSPACE}(h(n)) - \mathbf{PMSPACE}(l(n))$.

Proof.

- A unary seperation language has been shown to exist for **NSPACE**.
  - *V. Geffert. Space hierarchy theorem revised.*
- Unary languages are symmetric: $\mathbf{NSPACE} = \mathbf{SNSPACE}$.
- But when $h(n) \in \Omega(\log n) \rightarrow$ $SNSPACE(nh(n)) = PMSPACE(h(n))$.

$\square$

A Computational Threshold

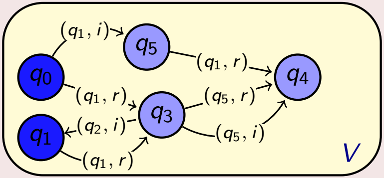
Theorem

Threshold. $\mathbf{PMSPACE}(\mathbf{o}(\log\log \mathbf{n})) = \mathbf{SEM}$.

Proof Idea

Agent Configuration Graph: Describes the effects of interactions of protocol A, but ignores the *deterministic* internal computation.

- Fixed for specific A, V.



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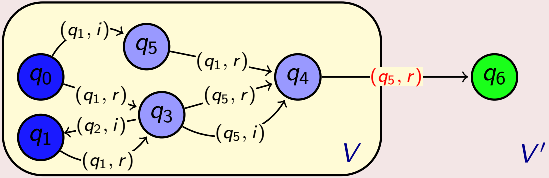
- Fixed for specific A, V.
- Moving to V', $|V'| > |V|$ adds new configurations k.
  - Accessible through interacting configurations (a, b) existing in V.
  - Since k does not exist in V, a and b cannot exist concurently in V.



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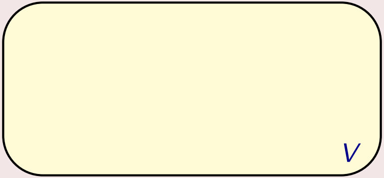

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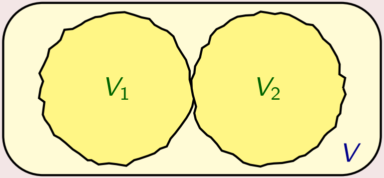

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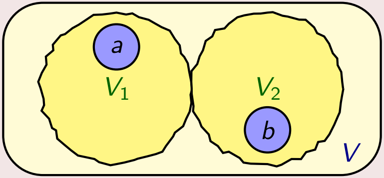

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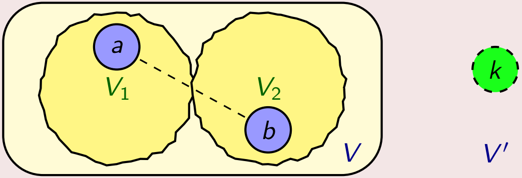
- Interaction creates $k \to k$ not new in V'!



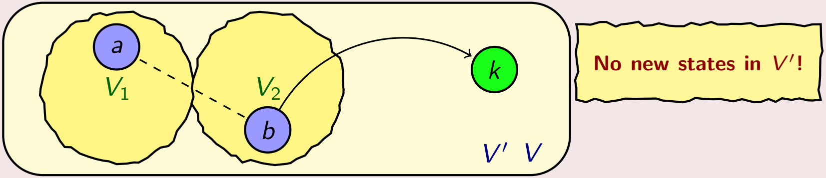
A Computational Threshold (Continued)

Theorem

Threshold. $\mathbf{PMSPACE}(\mathbf{o}(\log\log\mathbf{n})) = \mathbf{SEM}$.

Proof Idea

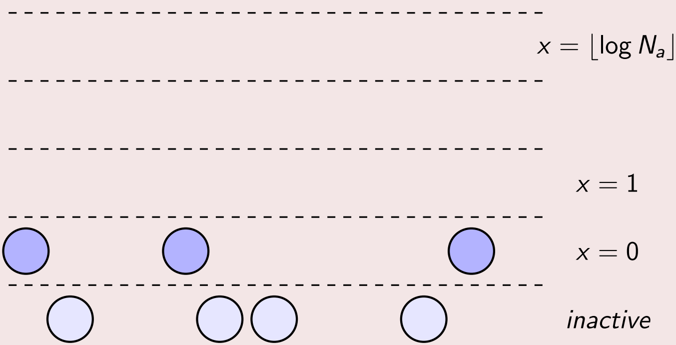
- **Important Lemma:** When $f(n) = o(\log\log n)$, $\exists V$ such that any configuration can occur in a subpopulation of size $\frac{|V|}{2}$.
- Partition V in V_1, V_2.
- V_1 creates a, V_2 creates b.
- Interaction creates $k \to k$ not new in V'!



Theorem

Predicate $p: \log N_a = t$*, for some* t *is in* $PMSPACE(\log\log n)$.

- *Whenever* $x_v = x_v + 1$ *for some* v*, there are at least* 2^{x_v+1} a*'s.*
- $x_v \neq 0$ **for only one** $v \iff 2^{x_v+1}$.
- **Max**$(x_v) = \log N_a \leq \log n \implies O(\log\log n)$ **space**.



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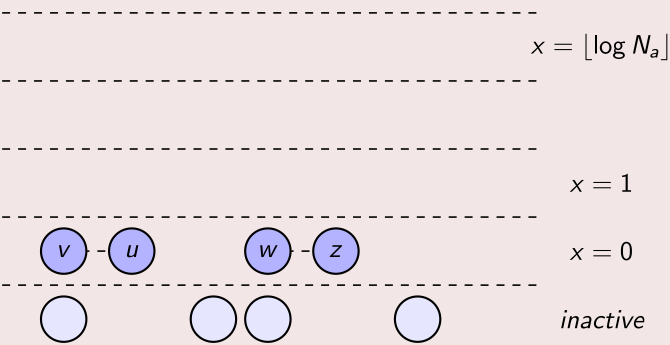

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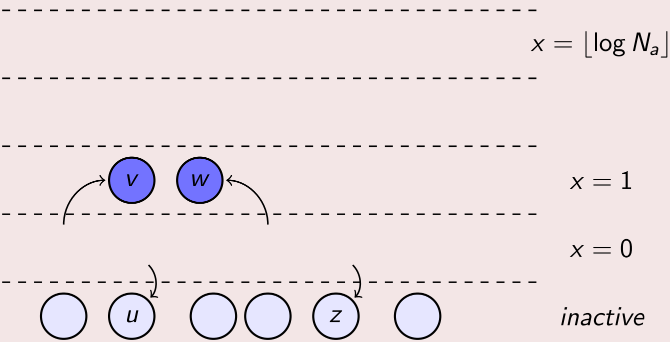

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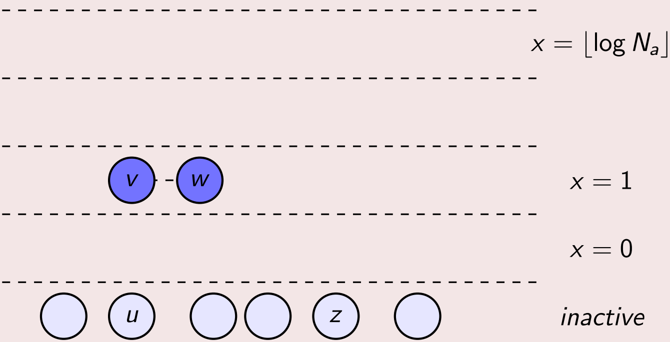

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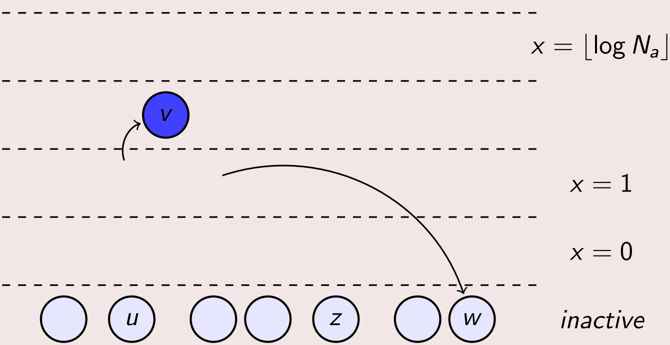

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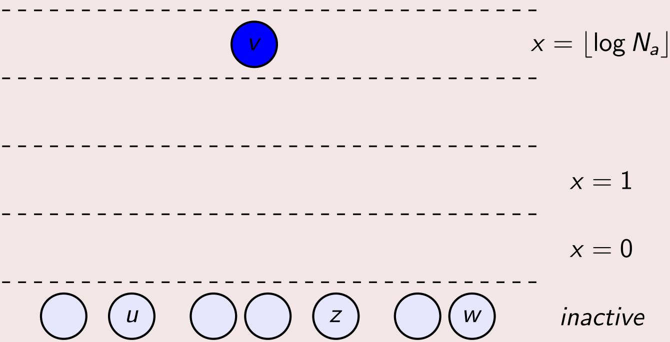
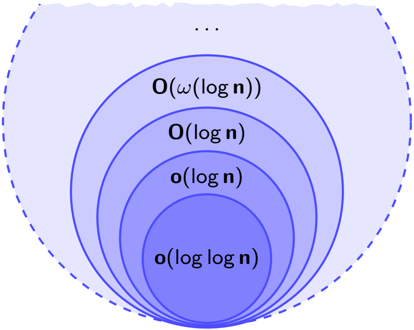
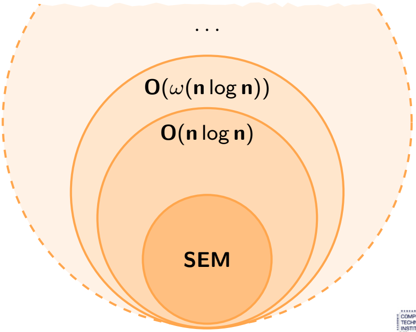


- **PMSPACE**



- **SNSPACE**



Conclusions - Further Research

- **Our contribution:**
  - We have presented a new model to study passive mobility in interaction-based, distributed, anonymous systems.
  - We have given a space hierarchy for functions $\Omega(\log n)$.
  - We have proved an interesting threshold in $o(\log\log n)$.
    - Tight.
- **Further research:**
  - Computational characterization between $\log\log n$ and $\log n$.
  - Fault tolerance.
  - Probabilistic assumptions & time complexity.
  - Adversarial perspective.

Thank You!

SEVENTH FRAMEWORK PROGRAMME