CIS 2018

Efficient MPC

Optimizations for Garbled Circuits

SODA
Scalable Oblivious Data Analytics

erc
European Research Council
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DANMARKS FRIE FORSKNINGSFOND
INDEPENDENT RESEARCH FUND DENMARK

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Part 3: Garbled Circuits

- **GC: Definitions and Applications**
- Garbling gate-by-gate: Basic and optimizations
- Active security 101: simple-cut-and choose, dual-execution

Garbled Circuit

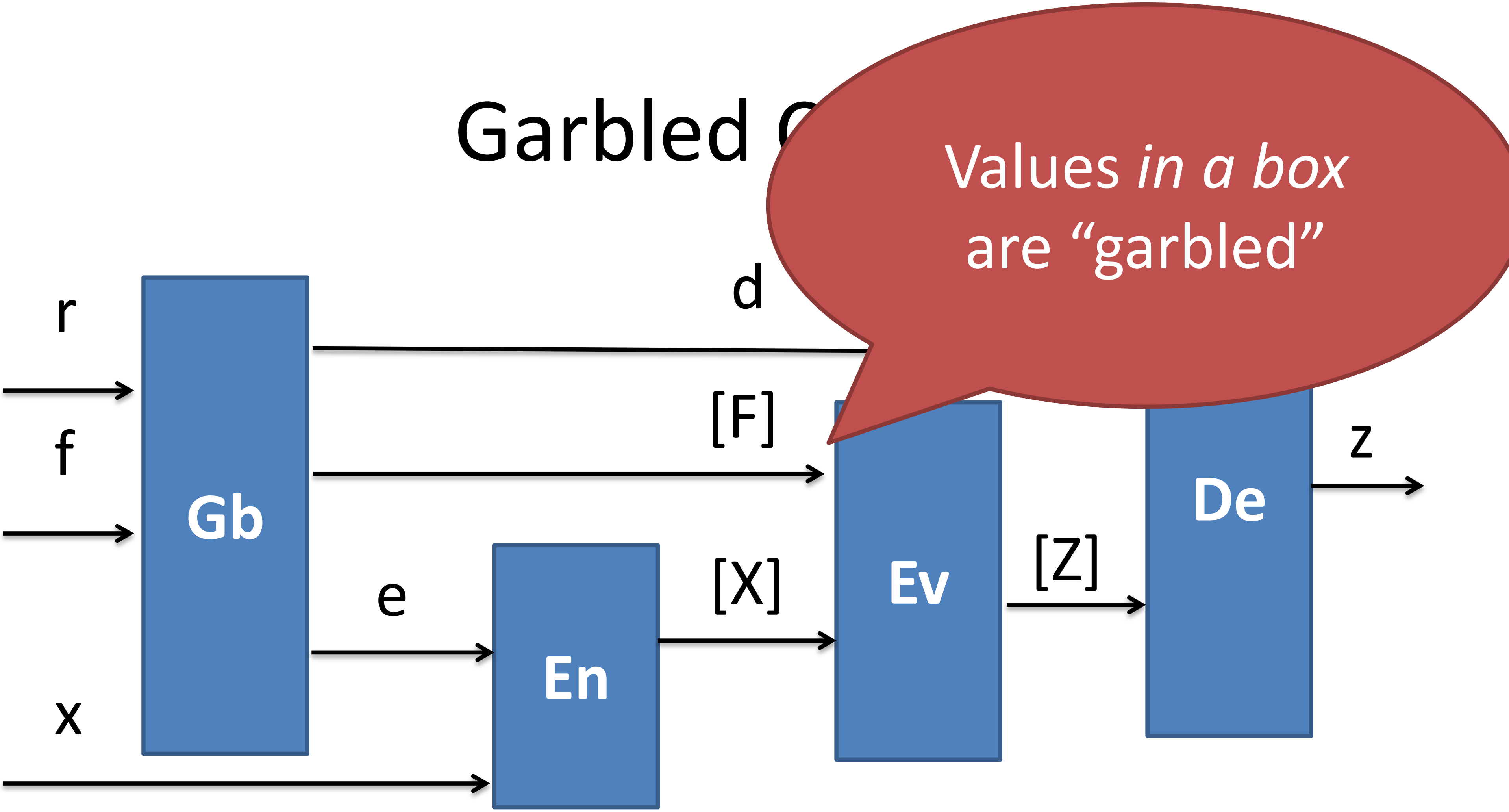
Cryptographic primitive that allows to evaluate

encrypted functions

on

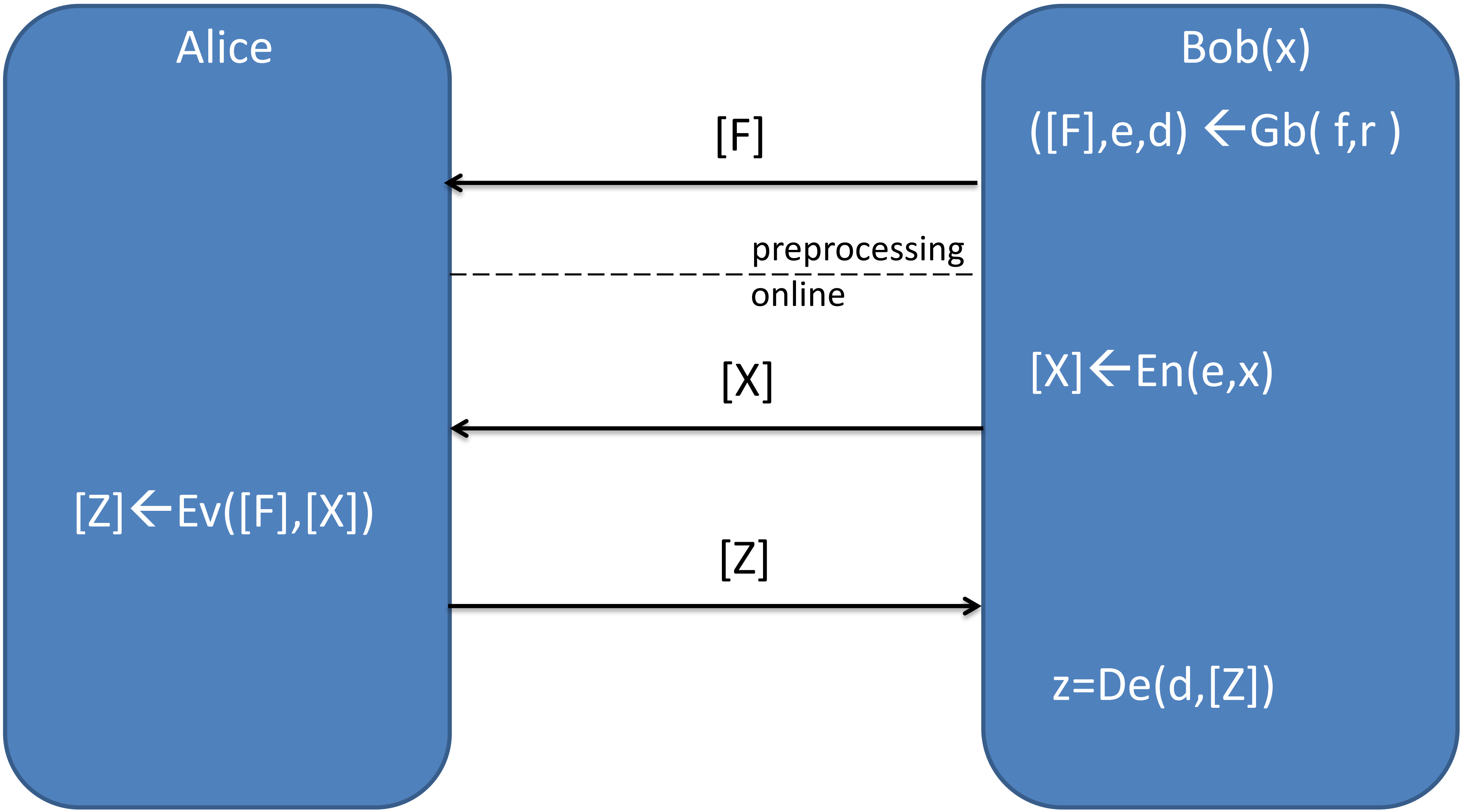
encrypted inputs

Garbled C



Correct if z=f(x)

Application 1: One-time Delegation via GC



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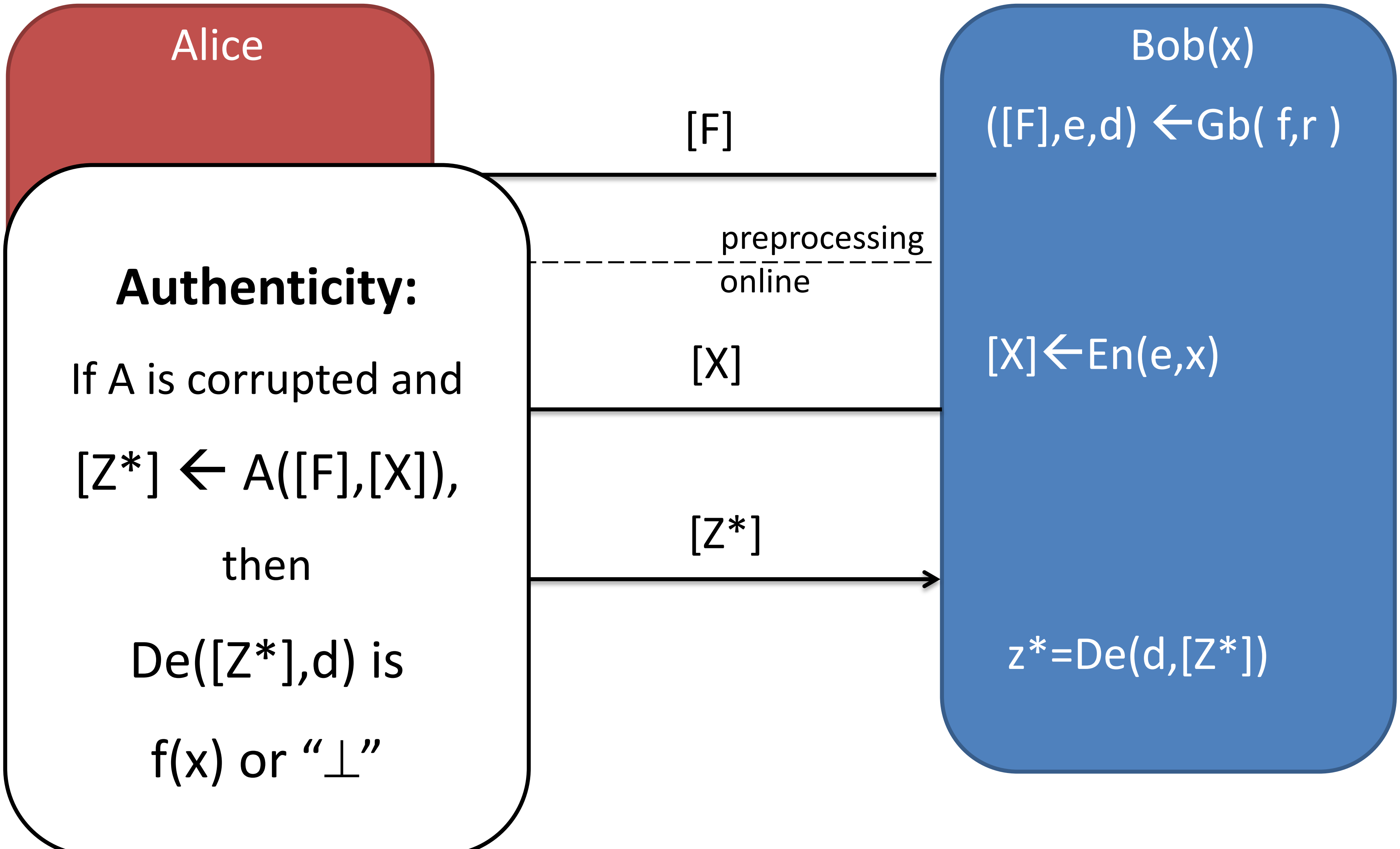
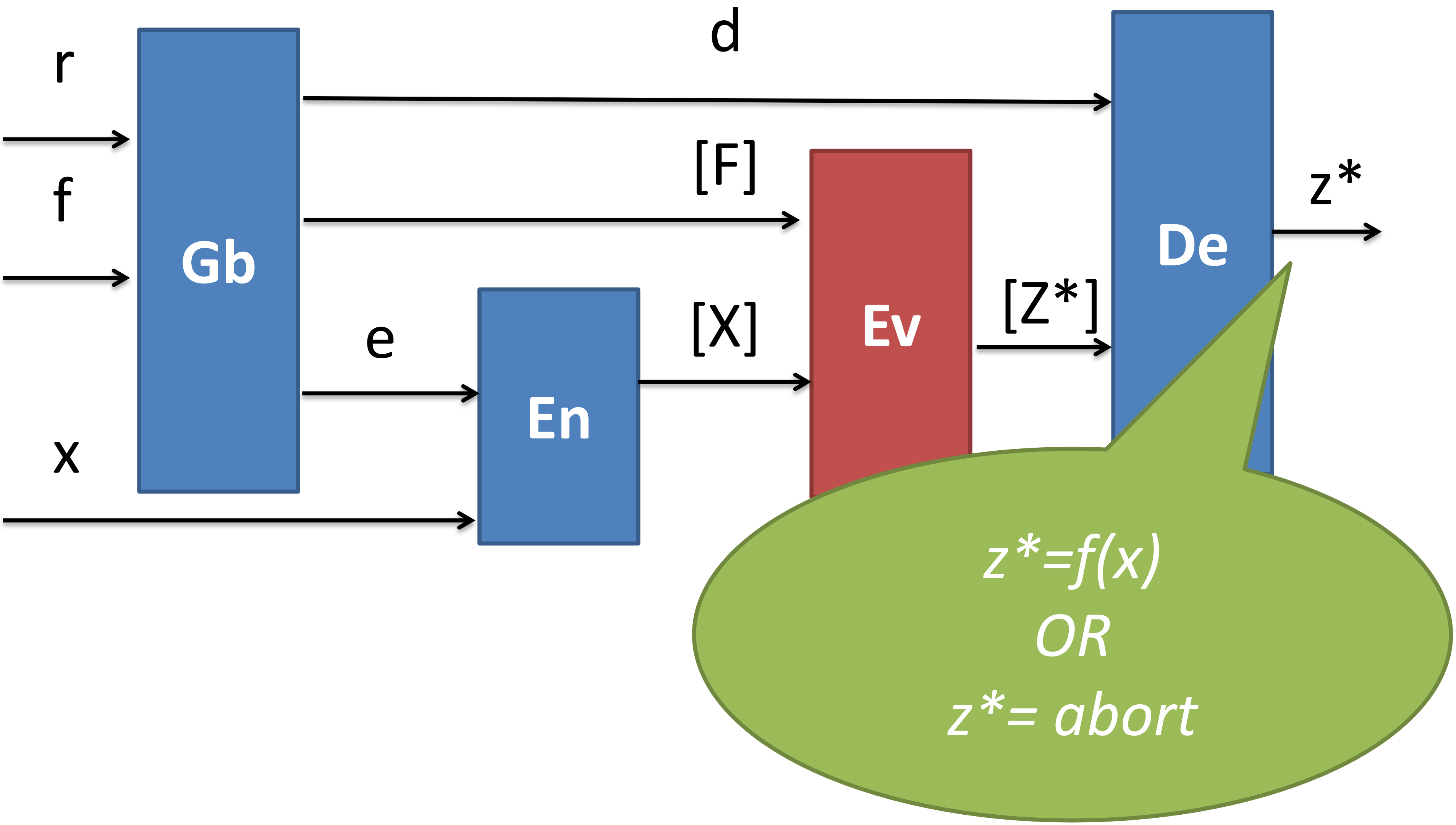
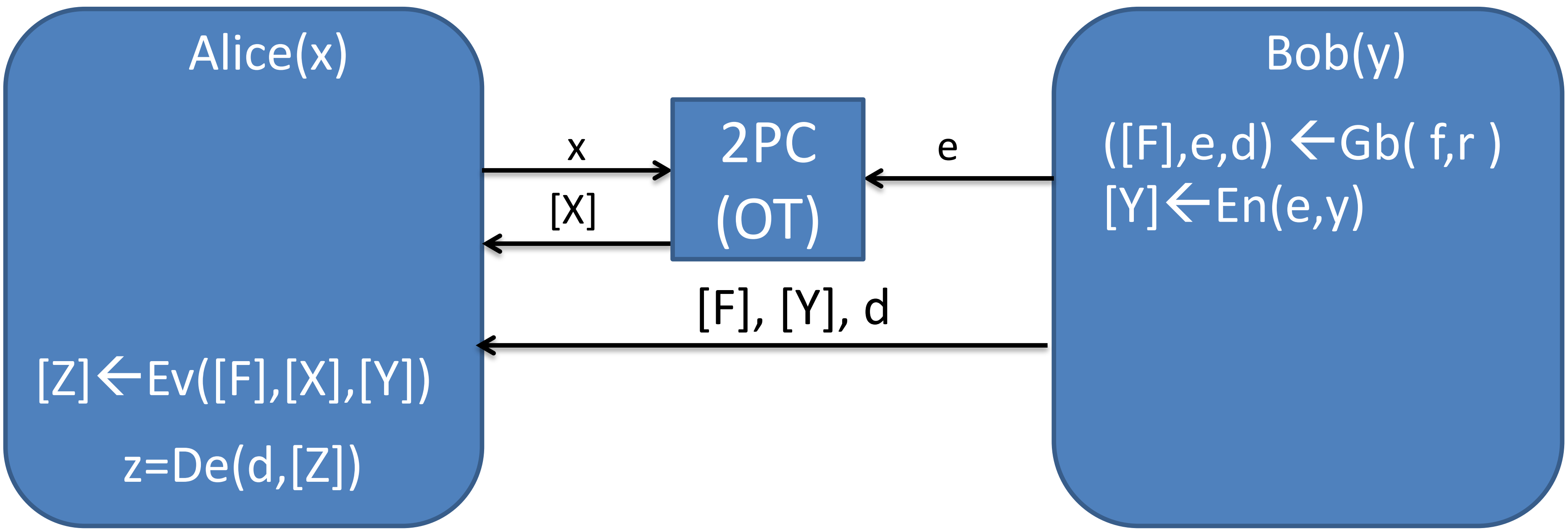


Garbled Circuits: Authenticity



Application 2:
Passive Constant Round 2PC (Yao)



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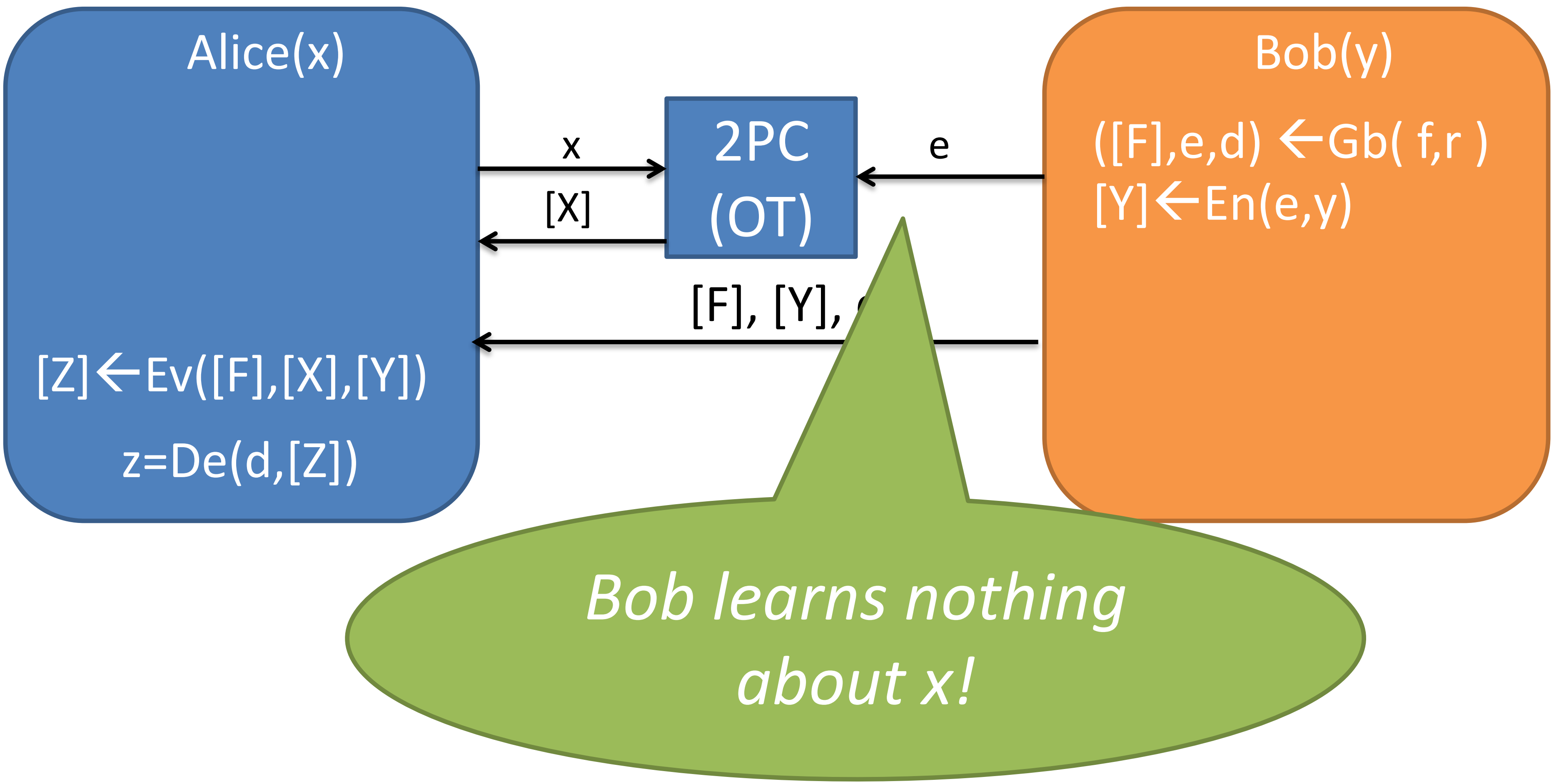

Application 2:
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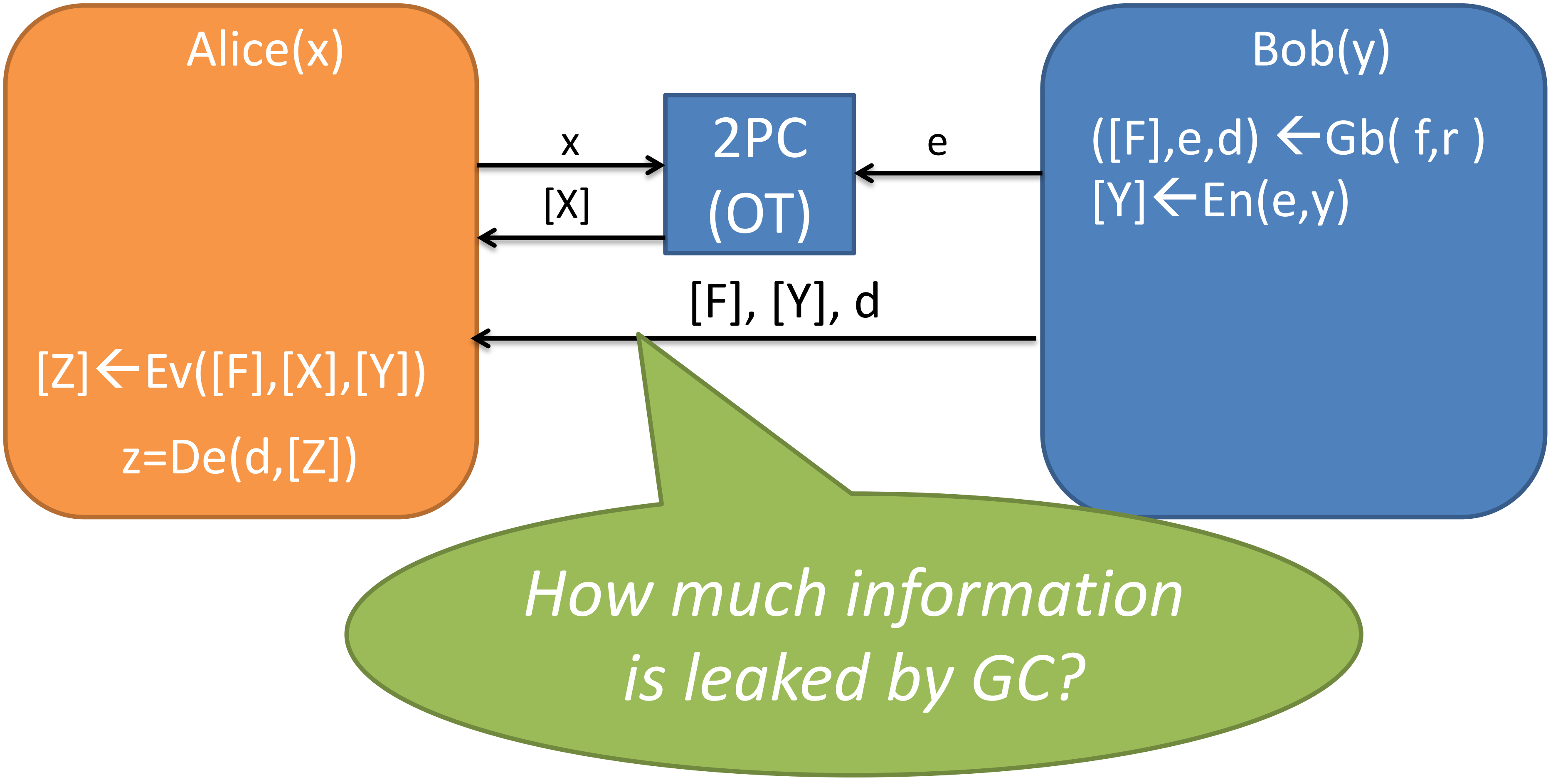
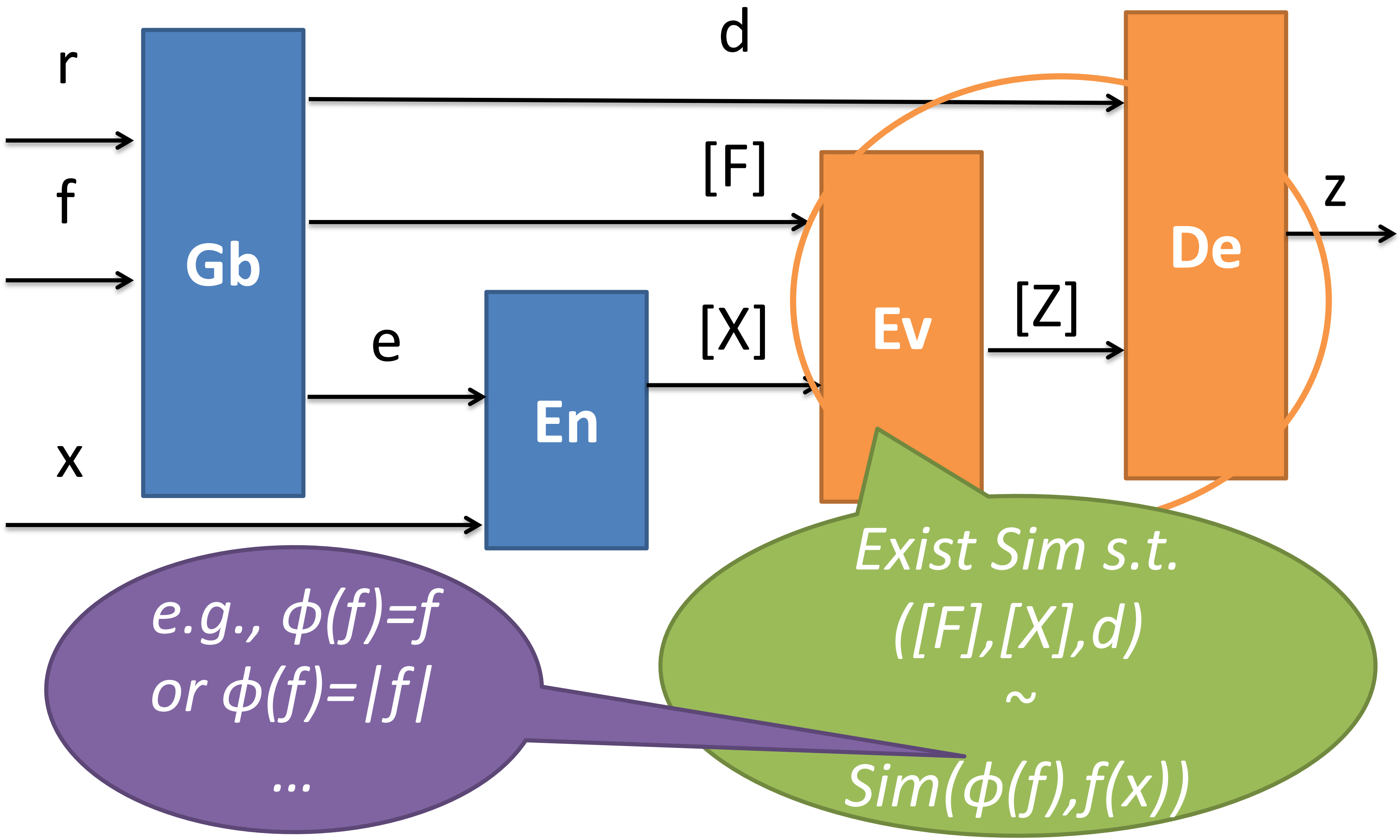


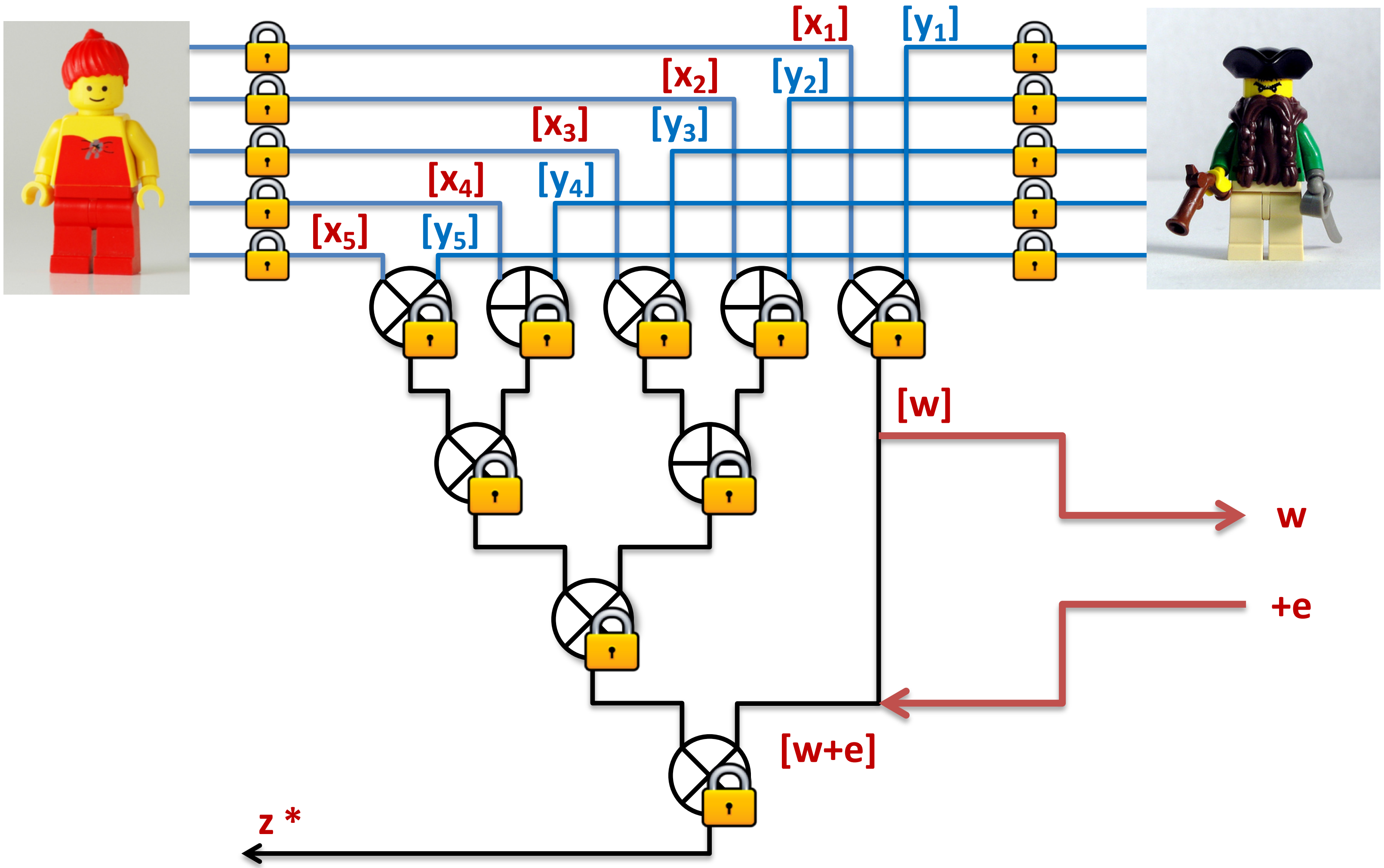
Garbled Circuits: Privacy



Part 3: Garbled Circuits

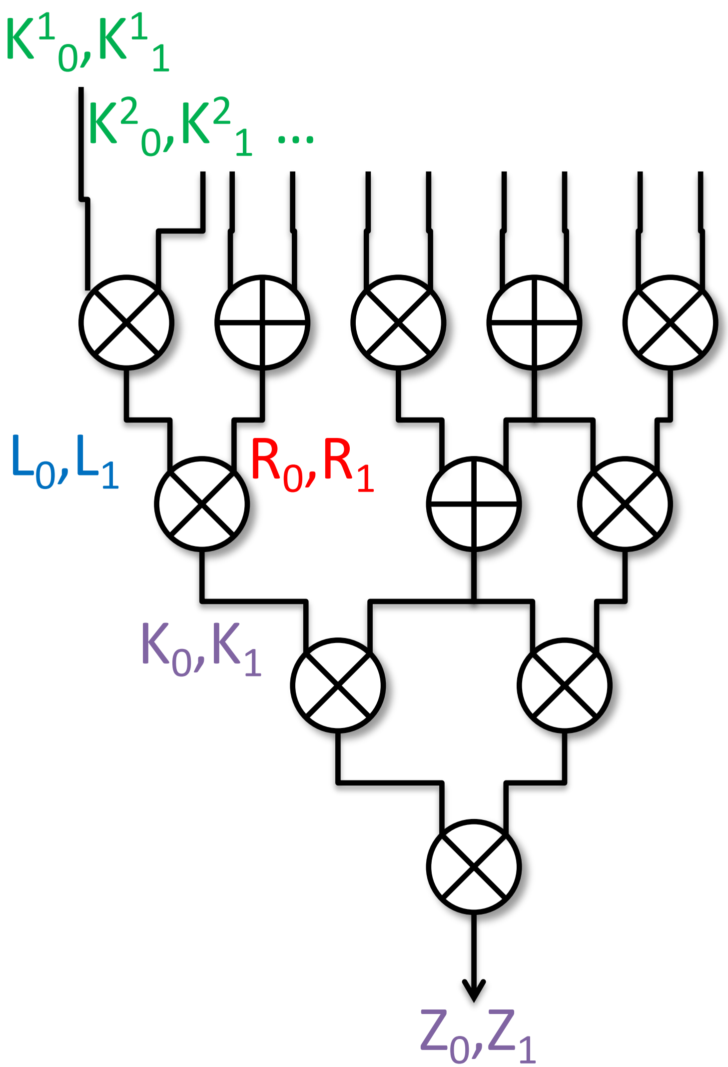
- Definitions and Applications
- **Garbling gate-by-gate: Basic and optimizations**
- Active security 101: simple-cut-and choose, dual-execution

Garbling: Gate-by-gate



PROJECTIVE SCHEMES:
CIRCUIT BASED GARBLING/EVALUATIONS

Garbling a Circuit : ([F],e,d)← Gb(f)



- Choose 2 random keys K^i_0,K^i_1 for each wire in the circuit
  - *Input, internal and, output wires*
- For each gate g compute
  - gg ← Gb(g,L_0,L_1,R_0,R_1,K_0,K_1)
- Output
  - e=(K^i_0,K^i_1) for all input wires
  - d=(Z_0,Z_1)
  - [F]=(gg^i) for all gates i

Encoding and Decoding

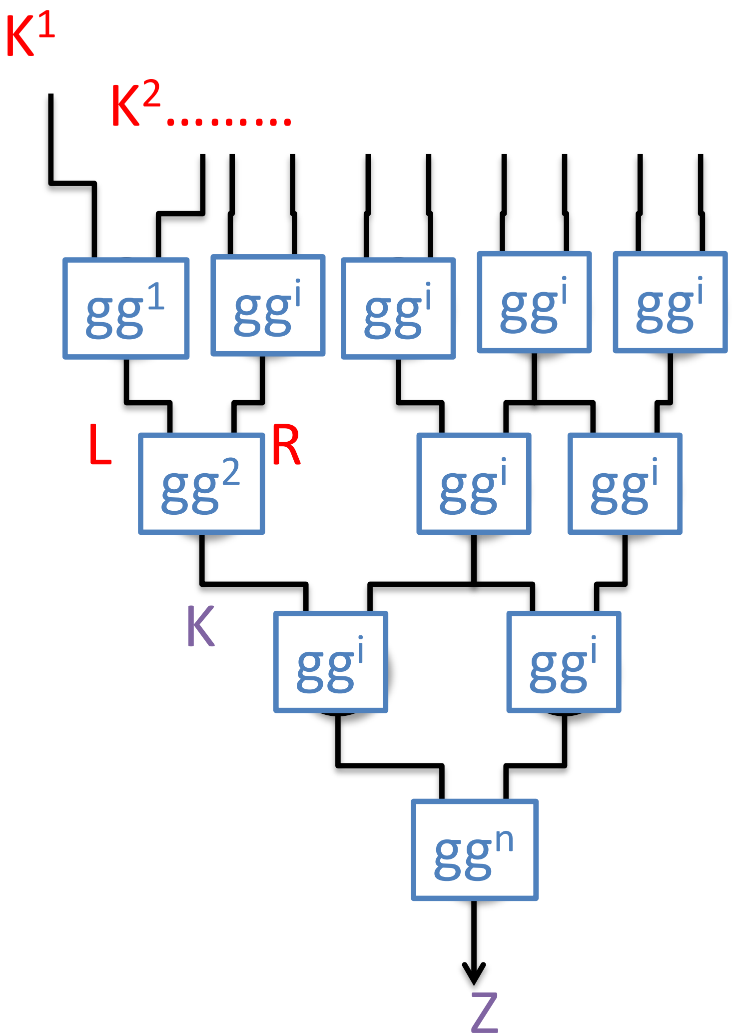
$[X] = En(e, x)$

- $e = \{ K^i_0, K^i_1 \}$
- $x = \{ x_1, \ldots, x_n \}$
- $[X] = \{ K^1_{x1}, \ldots, K^n_{xn} \}$

$z = De(d, [Z])$

- $d = \{ Z_0, Z_1 \}$
- $[Z] = \{ K \}$
- $z =$
  - 0 if $K = Z_0$,
  - 1 if $K = Z_1$,
  - "abort" else

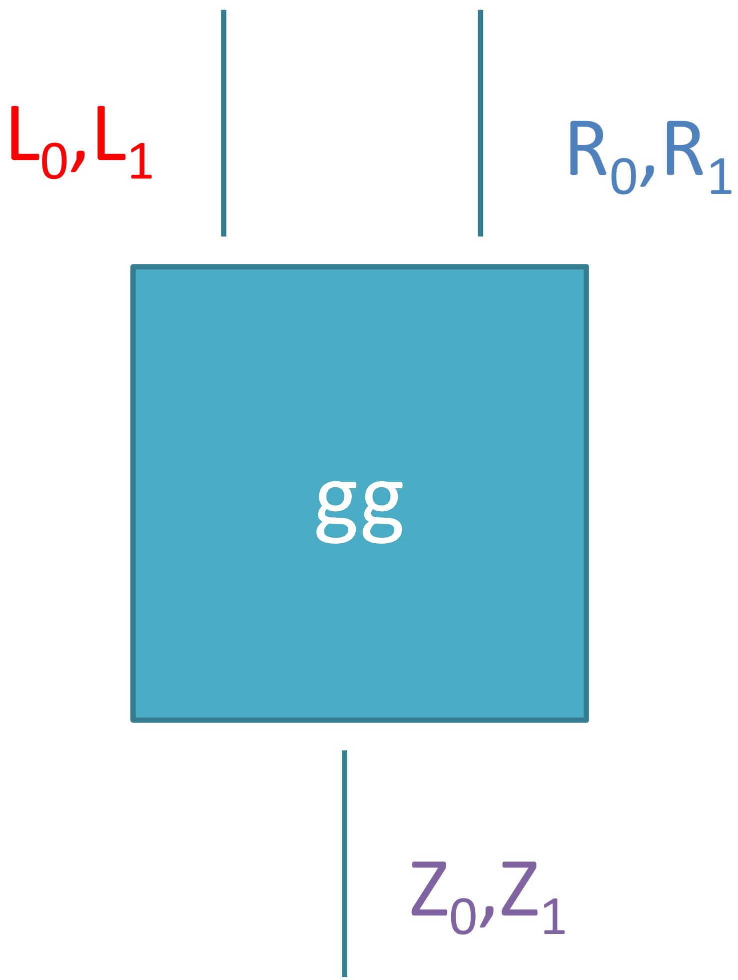
Evaluating a GC : [Z] ← Ev([F],[X])



- Parse [X]={K^1,…,K^n}
- Parse [F]={gg^i}
- For each gate i compute
  - K ← Ev(gg^i,L,R)
- Output
  - Z

INDIVIDUAL GATES GARBLING/EVALUATION

Notation



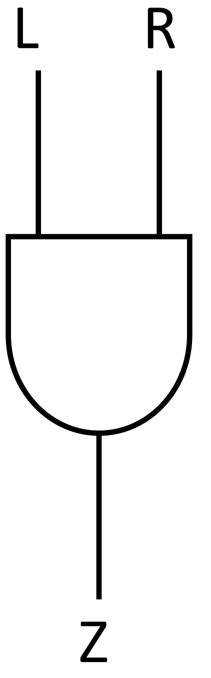
- A garbled gate is a gadget that given two inputs keys gives you the right output key *(and nothing else)*

- gg ← Gb($g, L_0, L_1, R_0, R_1, Z_0, Z_1$)
- $Z_{g(a,b)}$ ← Ev(gg, L_a, R_b)

- //and not $Z_{1-g(a,b)}$

Yao Gate Garbling (1)

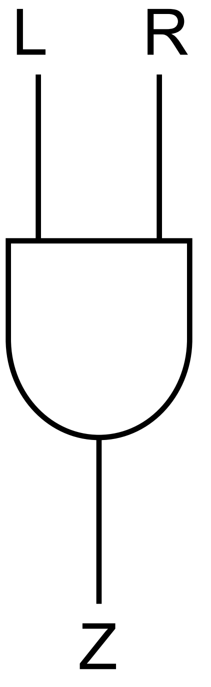
L	R	Z
0	0	0
0	1	0
1	0	0
1	1	1



- AND gate

Yao Gate Garbling (2)

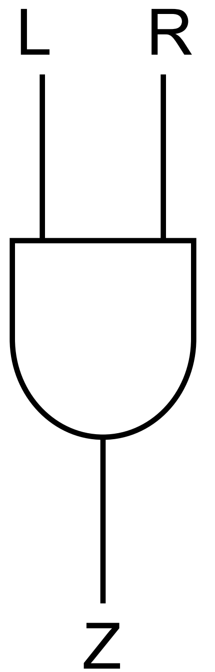
L	R	Z
L_0	R_0	Z_0
L_0	R_1	Z_0
L_1	R_0	Z_0
L_1	R_1	Z_1



- Choose labels (e.g., 128 bits strings) for every value on every wire

Yao Gate Garbling (3)

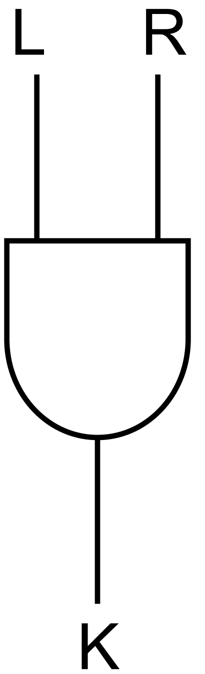
C
$C_1 = G(L_0,R_0) \oplus Z_0$
$C_2 = G(L_0,R_1) \oplus Z_0$
$C_3 = G(L_1,R_0) \oplus Z_0$
$C_4 = G(L_1,R_1) \oplus Z_1$



- Encrypt the output key with the input keys
  - G is some “key derivation function” so that the encryption is secure

Yao Gate Garbling (4)

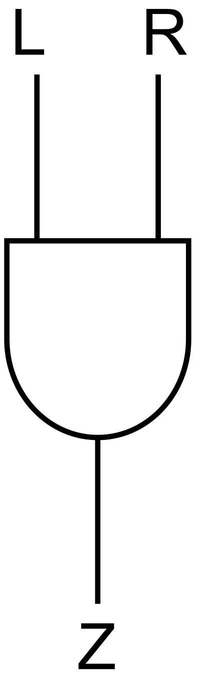
C
$C_1 = G(L_0,R_0) \oplus (Z_0,0^k)$
$C_2 = G(L_0,R_1) \oplus (Z_0,0^k)$
$C_3 = G(L_1,R_0) \oplus (Z_0,0^k)$
$C_4 = G(L_1,R_1) \oplus (Z_1,0^k)$



- Add redundancy (later used to check if decryption is successful)

Yao Gate Garbling (5)

C
$C_1 = G(L_0,R_0) \oplus (Z_0,0^k)$
$C_2 = G(L_0,R_1) \oplus (Z_0,0^k)$
$C_3 = G(L_1,R_0) \oplus (Z_0,0^k)$
$C_4 = G(L_1,R_1) \oplus (Z_1,0^k)$



$C'_1,C'_2,C'_3,C'_4 = \text{perm}(C_1,C_2,C_3,C_4)$

- Permute the order of the ciphertexts (to hide information about inputs/outputs)

Yao Gate Evaluation (1)

Eval(gg, L_a, R_b) //not a,b

- For i=1..4
  - $(K,t)=C'_i \oplus G(L_a,R_b)$
  - If $t=0^k$ output K

gg (permuted)
$C_1 = G(L_0,R_0) \oplus (K_1,0^k)$
$C_2 = G(L_0,R_1) \oplus (K_1,0^k)$
$C_3 = G(L_1,R_0) \oplus (K_1,0^k)$
$C_4 = G(L_1,R_1) \oplus (K_0,0^k)$

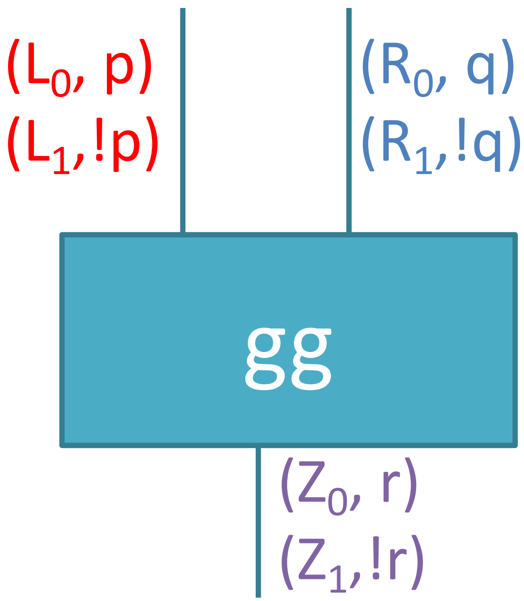
- Output is correct:
  - $t=0^k$ only for right row
- Evaluator learns nothing else:
  - Encryption + permutation

Efficiency

	|gg|	G/Gb	G/Eval	Assumption on G
Classic	8k	4	4	Standard

Point-and-permute

- Problem: Evaluator needs to try to decrypt all 4 rows
- Solution: add permutation bits to keys

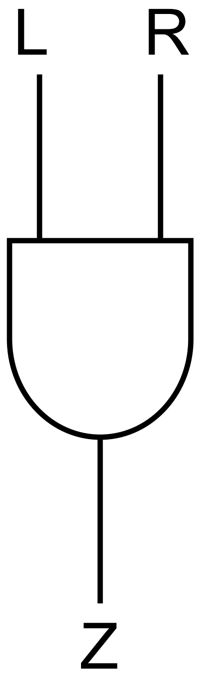


$gg \leftarrow Gb(g, L_0, L_1, p, R_0, R_1, q, Z_0, Z_1, r)$

$(Z_{g(a,b)}, r \oplus ab) \leftarrow Ev(gg, L_a, a \oplus p, R_b, b \oplus q)$

Point-and-permute Garbling (4)

C
$C_1 = G(L_0,R_0) \oplus (Z_0, r \oplus 0)$
$C_2 = G(L_0,R_1) \oplus (Z_0, r \oplus 0)$
$C_3 = G(L_1,R_0) \oplus (Z_0, r \oplus 0)$
$C_4 = G(L_1,R_1) \oplus (Z_1, r \oplus 1)$



- Remove redundancy
- Add random permutation bit

Point-and-permute Garbling (5)

C
$C'_0 = G(L_p, R_q) \oplus (Z_{p \cdot q},\ r \oplus p \cdot q)$
$C'_1 = G(L_p, R_{!q}) \oplus (Z_{p \cdot !q},\ r \oplus p \cdot !q)$
$C'_2 = G(L_{!p}, R_q) \oplus (Z_{!p \cdot q},\ r \oplus !p \cdot q)$
$C'_3 = G(L_{!p}, R_{!q}) \oplus (Z_{p \cdot !q},\ r \oplus !p \cdot !q)$

- Permute rows using p,q

Point-and-permute Evaluation

Eval(gg, L, u, R, v) //not a,b

- (Z,r)=$C'_{2u+v} \oplus G(L,R)$

C
$C'_0 = G(L_p,R_q) \oplus (Z_{p\cdot q},\ r \oplus p\cdot q)$
$C'_1 = G(L_p,R_{!q}) \oplus (Z_{p\cdot !q},\ r \oplus p\cdot !q)$
$C'_2 = G(L_{!p},R_q) \oplus (Z!_{p\cdot q},\ r \oplus !p\cdot q)$
$C'_3 = G(L_{!p},R_{!q}) \oplus (Z_{!p\cdot !q},\ r \oplus !p\cdot !q)$

- Output is correct:
  - (Check permutation)
- Privacy:
  - $u=p\oplus a$, $v=q\oplus b$
  - p,q are “one time pads” for a,b

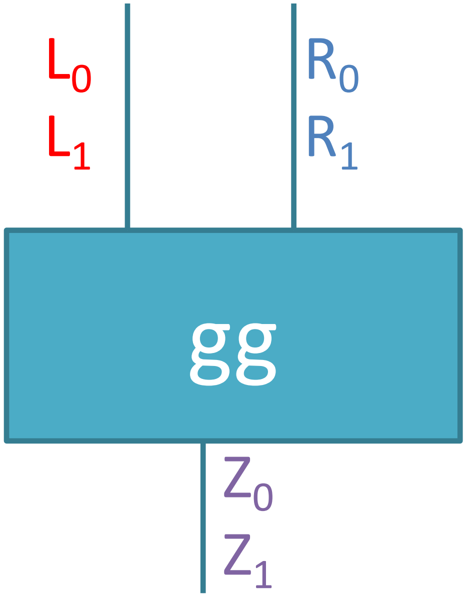
Efficiency

	|gg|	G/Gb	G/Eval	Assumption on G
Classic	8k	4	4	Standard
P&P	4k	4	1	Standard

GARBLING OPTIMIZATIONS:
SIMPLE GARBLED ROW REDUCTION

Changing the syntax

- Problem: each gg is 4 ciphertexts
- Solution: define output key pseudorandomly as functions of input keys, reduce comm. complexity

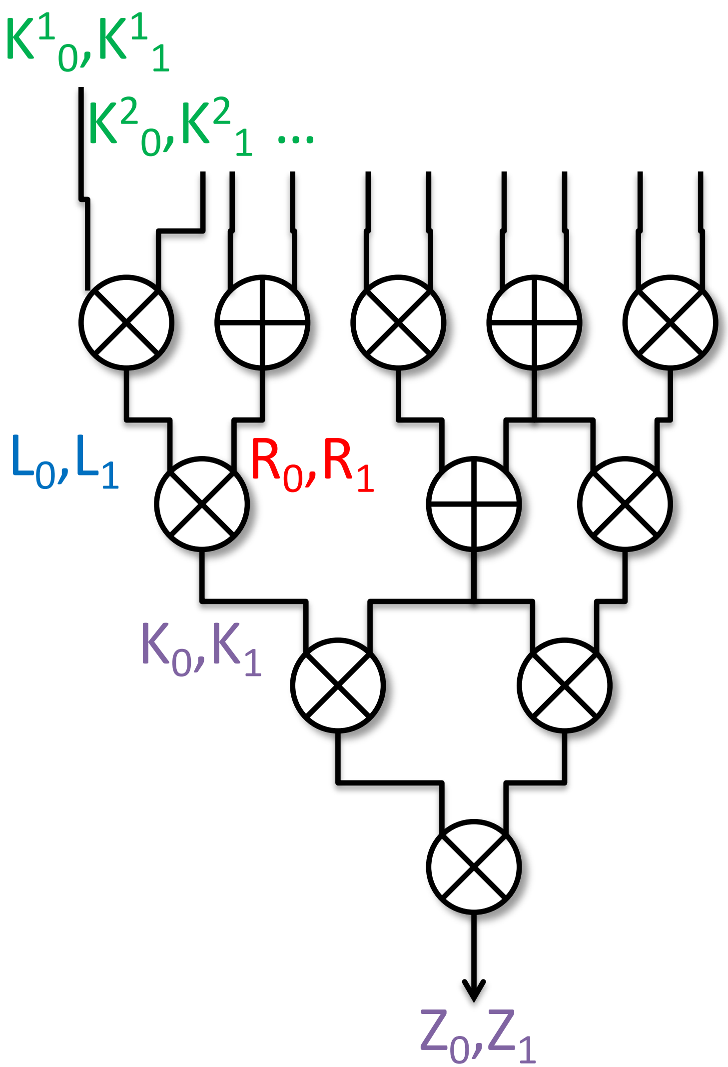


$(gg, Z_0, Z_1) \leftarrow Gb(g, L_0, L_1, R_0, R_1)$

$(Z_{g(a,b)}) \leftarrow Ev(gg, L_a, R_b)$

Note, now garbling cannot be done in parallel anymore!

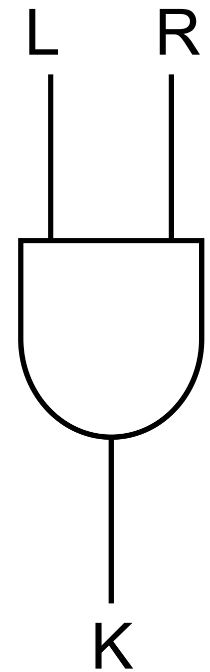
Garbling a Circuit : ([F],e,d)← Gb(f)



- Choose 2 random keys K^i_0,K^i_1 for each wire in the circuit
  - *Input wire only!*
- For each gate g compute
  - $(gg,K_0,K_1) \leftarrow Gb(g,L_0,L_1,R_0,R_1)$
- Output
  - $e=(K^i_0,K^i_1)$ for all input wires
  - $d=(Z_0,Z_1)$
  - $[F]=(gg^i)$ for all gates i

Yao Gate Garbling (3)

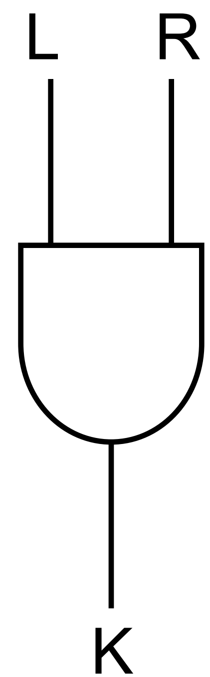
C
$C_1 = G(L_0,R_0) \oplus Z_0$
$C_2 = G(L_0,R_1) \oplus Z_0$
$C_3 = G(L_1,R_0) \oplus Z_0$
$C_4 = G(L_1,R_1) \oplus Z_1$



- Encrypt the output key with the input keys

Garbled Row Reduction Garbling

C
$Z_0 = G(L_0,R_0)$ $(C_1=0^k)$
$C_2 = G(L_0,R_1) \oplus Z_0$
$C_3 = G(L_1,R_0) \oplus Z_0$
$C_4 = G(L_1,R_1) \oplus Z_1$



- Define output keys as function of input keys
  - (compatible with p&p)
  - Can reduce 2 rows, but 1 is compatible with Free-XOR (coming up!)

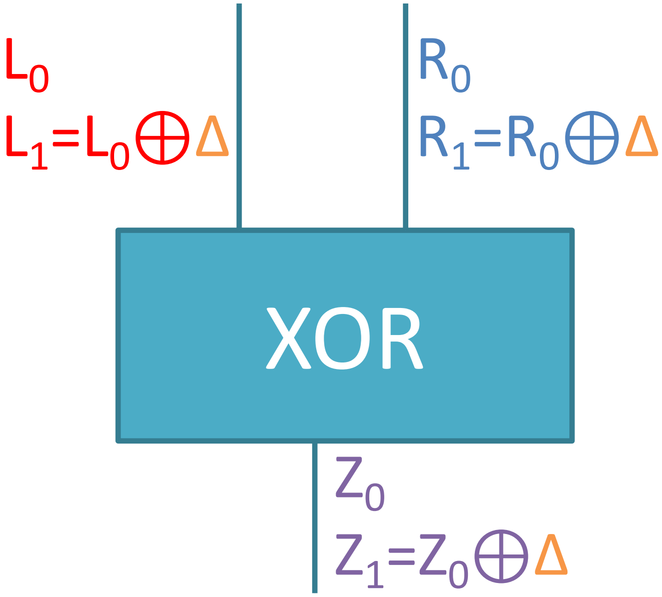
Efficiency

	|gg|	G/Gb	G/Eval	Assumption on G
Classic	8k	4	4	Standard
P&P	4k	4	1	Standard
+GRR	3k/2k	4	1	Standard

GARBLING OPTIMIZATIONS:
FREE XOR

Free-XOR

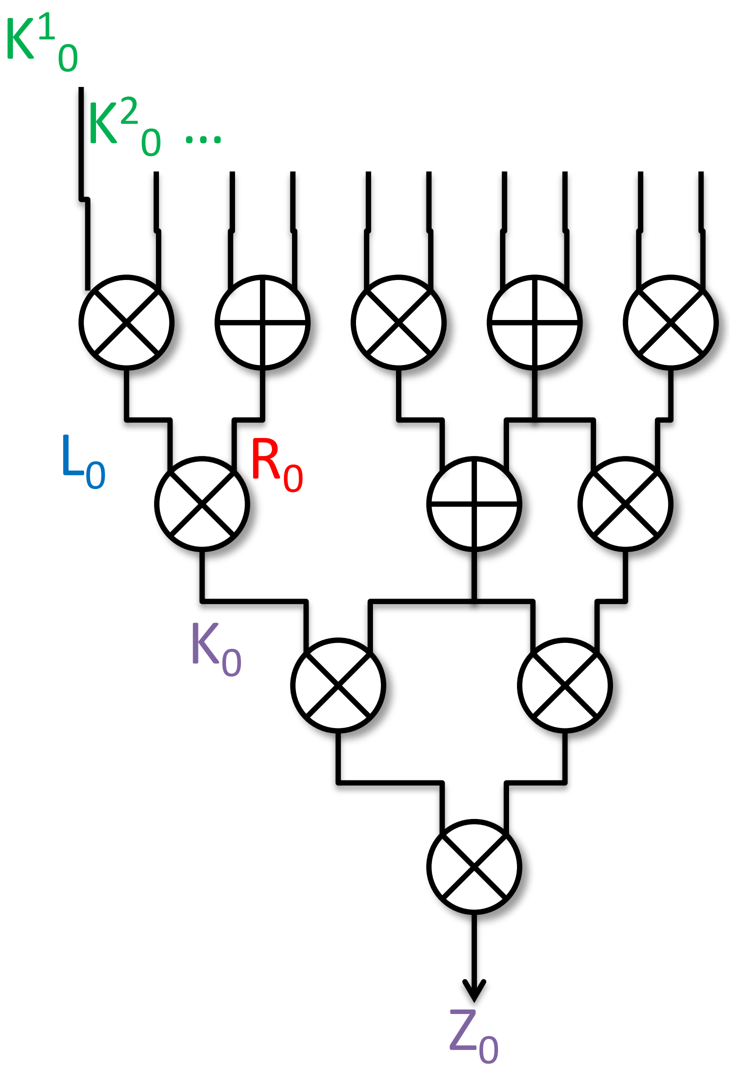
- Problem: with secret sharing linear gates are for free. What about GC?
- Solution: introduce correlation between keys, make XOR computation "free"



$(gg, Z_0) \leftarrow Gb(g, L_0, R_0, \Delta)$

$(Z_{g(a,b)}) \leftarrow Ev(gg, L_a, R_b)$

Changing syntax, again!



- Choose 1 random key K^i_0 for each input wire in the circuit
  - *And global difference* Δ
- For each gate g compute
  - $(gg, K_0) \leftarrow Gb(g, L_0, R_0, \Delta)$
- Output
  - $e=(K^i_0, K^i_1)$ for all input wires
  - $d=(Z_0, Z_1)$
  - $[F]=(gg^i)$ for all gates i

What about AND Gates?

- Like before, but requires “circular security assumption”

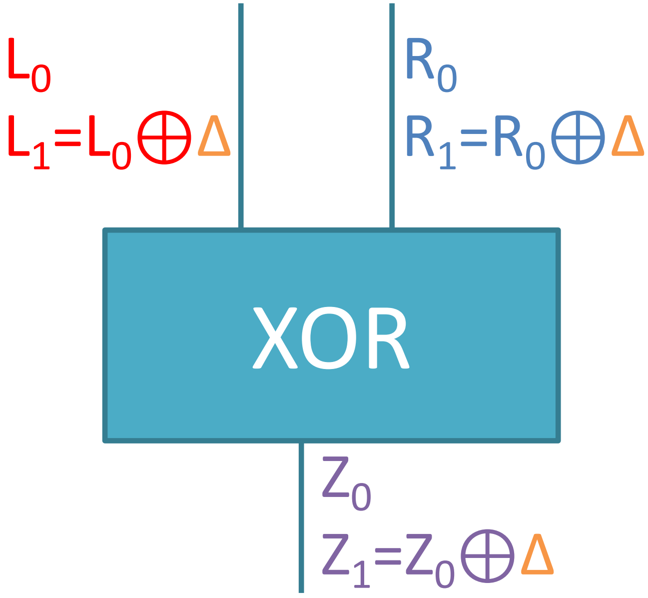
- Evaluator sees

$$L_0,\ R_0,\ Z_0, \text{ and}$$

$$G(L_0 \oplus \Delta,\ R_0 \oplus \Delta) \oplus Z_0 \oplus \Delta$$

- And should not be able to compute Δ !
- Effectively an encryption of Δ under Δ !

Garbling/Evaluating XOR Gates



$(gg, Z_0) \leftarrow Gb(g, L_0, R_0, \Delta)$

$Z_{a \oplus b} \leftarrow Ev(gg, L_a, R_b)$

$Gb(XOR, L_0, R_0, \Delta)$

- Output $Z_0 = L_0 \oplus R_0$
- (gg is empty)

$Ev(XOR, L_a, R_b, \Delta)$

- Output $Z_{a \oplus b} = L_a \oplus R_b$

$$L_a \oplus R_b = L_0 \oplus a\Delta \oplus R_0 \oplus b\Delta = Z_0 \oplus (a \oplus b)\Delta = Z_{a \oplus b}$$

Efficiency

	AND			XOR			
	|gg|	G/Gb	G/Eval	|gg|	G/Gb	G/Eval	Assumption on G
Classic	8k	4	4	8k	4	4	Standard
P&P	4k	4	1	4k	4	1	Standard
+GRR	3k/2k	4	1	3k/2k	4	1	Standard
+Free-XOR	3k	4	1	0	0	0	Circular

Privacy Free Garbing

- In some application (example, the delegation of computation from the first slide) we don't care about hiding the input/output of the circuit to the evaluator.

- Can we construct more efficient garbling if we don't care about *privacy*, but only *authenticity*?

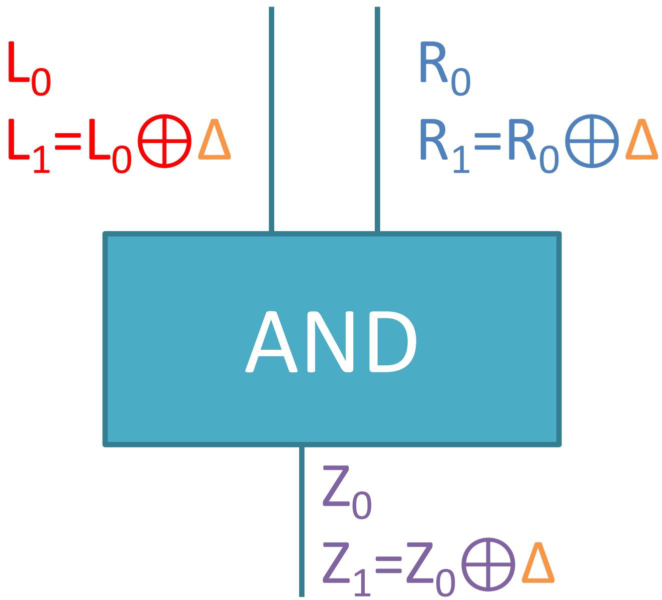
Privacy Free with Free XOR

- For XOR-gates it is hard to do better than *free*-XOR

- What about AND gates?

- Let c=AND(a,b) then
  - If a = 0 → c = 0
  - If a =1 → c = b

Privacy-Free AND gates



$(gg, Z_0) \leftarrow Gb(g, L_0, R_0, \Delta)$

$Z_{ab} \leftarrow Ev(gg, L_a, a, R_b, b)$

$Gb(AND, L_0, R_0, \Delta)$

- $Z_0 = G(L_0)$ // GRR
- $C = G(L_1) \oplus Z_0 \oplus R_0$

$Ev(gg, L_a, a, R_b, b, \Delta)$

- If $a=0$: output
  $Z_0 = G(L_0)$
- If $a=1$: output
  $Z_b = C \oplus G(L_1) \oplus R_b$

$$Z_b = C \oplus G(L_1) \oplus R_b = Z_0 \oplus R_0 \oplus (R_0 \oplus b\Delta) = Z_b$$

Efficiency

	AND			XOR			
	|gg|	G/Gb	G/Eval	|gg|	G/Gb	G/Eval	Assumption on G
Classic	8k	4	4	8k	4	4	Standard
P&P	4k	4	1	4k	4	1	Standard
+GRR	3k/2k	4	1	3k/2	4	1	Standard
+Free-XOR	3k	4	1	0	0	0	Circular
Privacy-Free*	k	4	1	0	0	0	Circular

Privacy-Free Garbling: Extension?

- Can the same trick help us also in garbling for 2PC?
- Can we let the evaluator learn the bit of some internal wires?
  - No!
- Can we let the evalutor learn a one-time pad encryption of some internal wire?
  - Sure, why not!

Half-Gate (Two Halves Make a Whole)

- Note that we can write:

$$a \cdot b = (a \cdot r) \oplus (a \cdot (r \oplus b))$$

r known to garbler → How to garble efficiently?

$r \oplus b$ known to evaluator → can use PF garbling

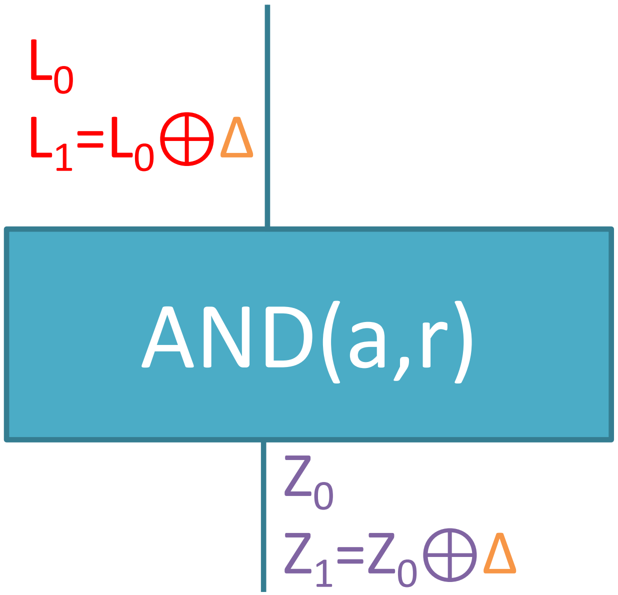
- 1 AND → 2 ANDs. How is this better?
  - The garbled can choose a random r at garbling time
  - Make sure that the evaluator learns $(r \oplus b)$
  - How? Use the permutation bits from point&permute!

How to garble with hidden constant

- The garbled knows r
  and wants to garble $c = a \cdot r$
  – If $r = 0 \rightarrow c=0$
  – If $r = 1 \rightarrow c=a$

- How to garble an unary gate, which is either the 0 gate or the identity gate depending on r?

Garbling AND with hidden-constant



$(gg, Z_0) \leftarrow Gb(g, L_0, r, \Delta)$

$Z_{ar} \leftarrow Ev(gg, L_a)$

$Gb(AND, L_0, p, \Delta)$

- $Z_p = G(L_p)$ //GRR $C_0 = 0^k$
- $C_1 = G(L_{!p}) \oplus Z_p \oplus r\Delta$

$Ev(gg, L_a, a \oplus p, \Delta)$

- Output
  $Z_{ar} = C_{a \oplus p} \oplus G(L_a)$

Let p=0 for simplicity

$$C_a \oplus G(L_a) = Z_0 \oplus a(r\Delta) = Z_{ar}$$

Efficiency

	AND			XOR			
	|gg|	G/Gb	G/Eval	|gg|	G/Gb	G/Eval	Assumption on G
Classic	8k	4	4	8k	4	4	Standard
P&P	4k	4	1	4k	4	1	Standard
+GRR	3k/2k	4	1	3k/2k	4	1	Standard
+Free-XOR	3k	4	1	0	0	0	Circular
Privacy-Free*	k	4	1	0	0	0	Circular
Half-Gate	2k	4	2	0	0	0	Circular
GLP	2k	4	1	1	4	1	Standard

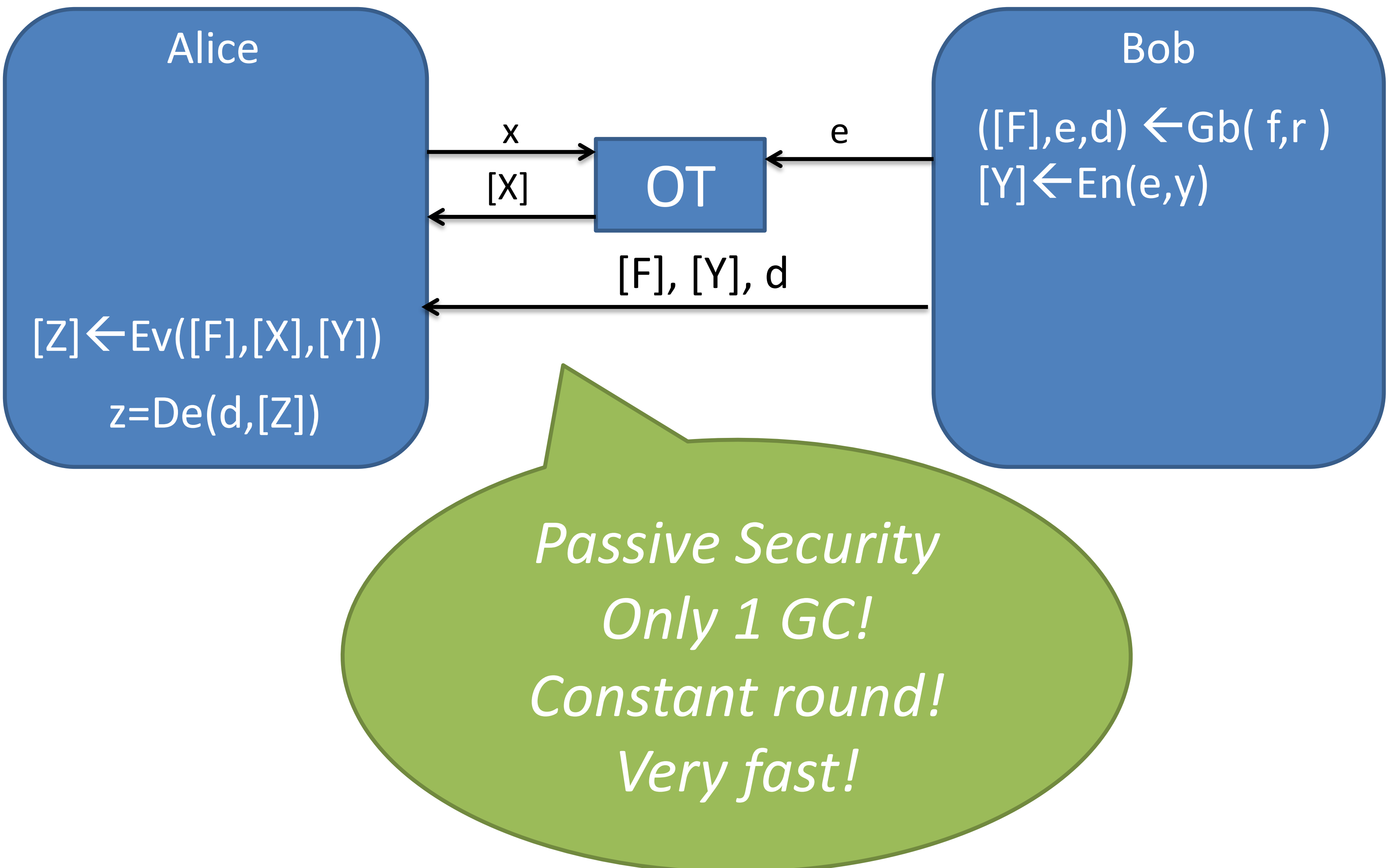
- GLP : Fast Garbling of Circuits Under Standard Assumptions (Gueron, Lindell, Pinkas)
- (The measure of G in this table is somehow arbitrary, in practice the size of the input to G makes a difference in runtime)

Part 3: Garbled Circuits

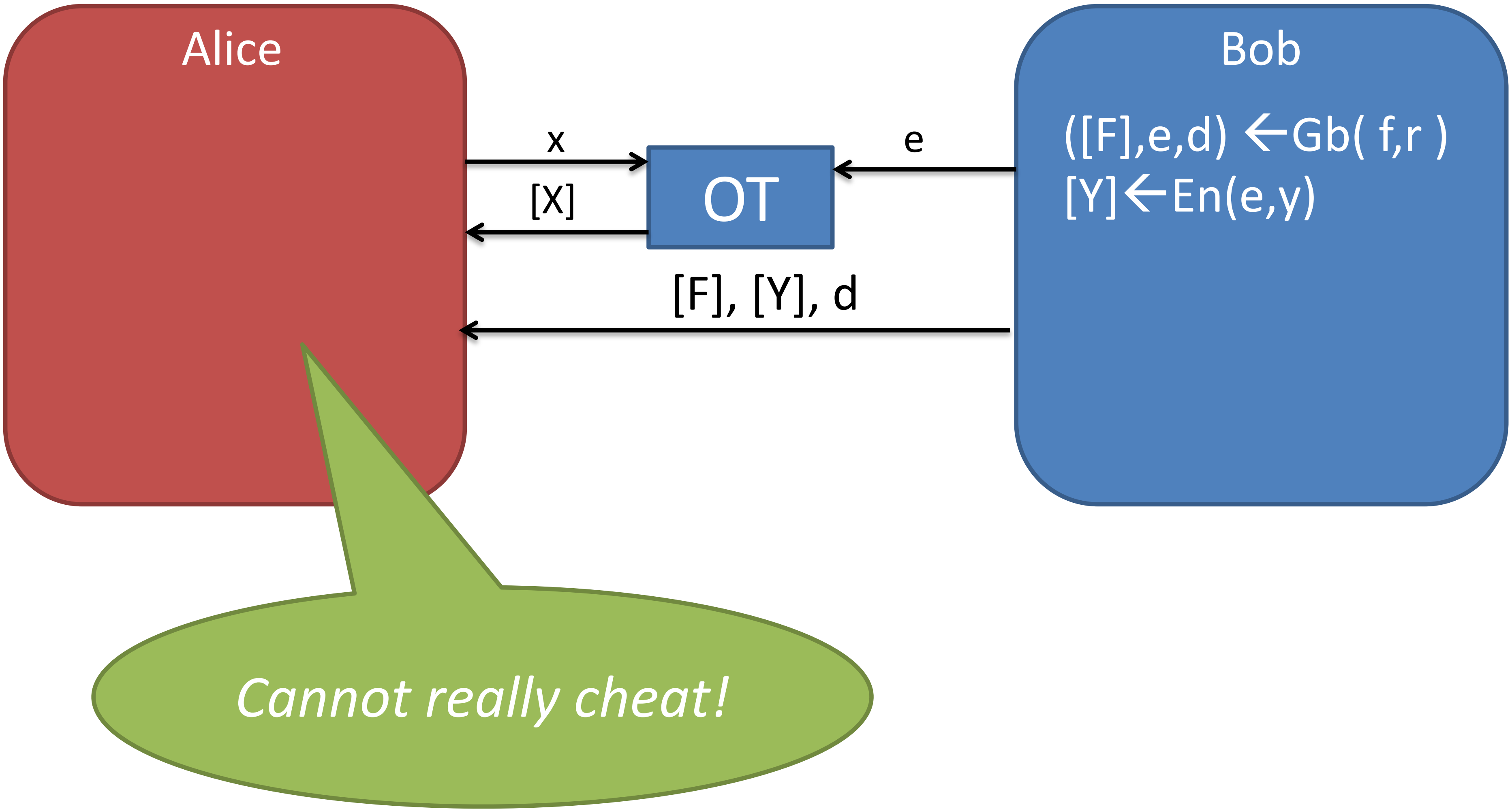
- Definitions and Applications
- Garbling gate-by-gate: Basic and optimizations
- **Active security 101: simple-cut-and choose, dual-execution**

ACTIVE ATTACKS VS YAO

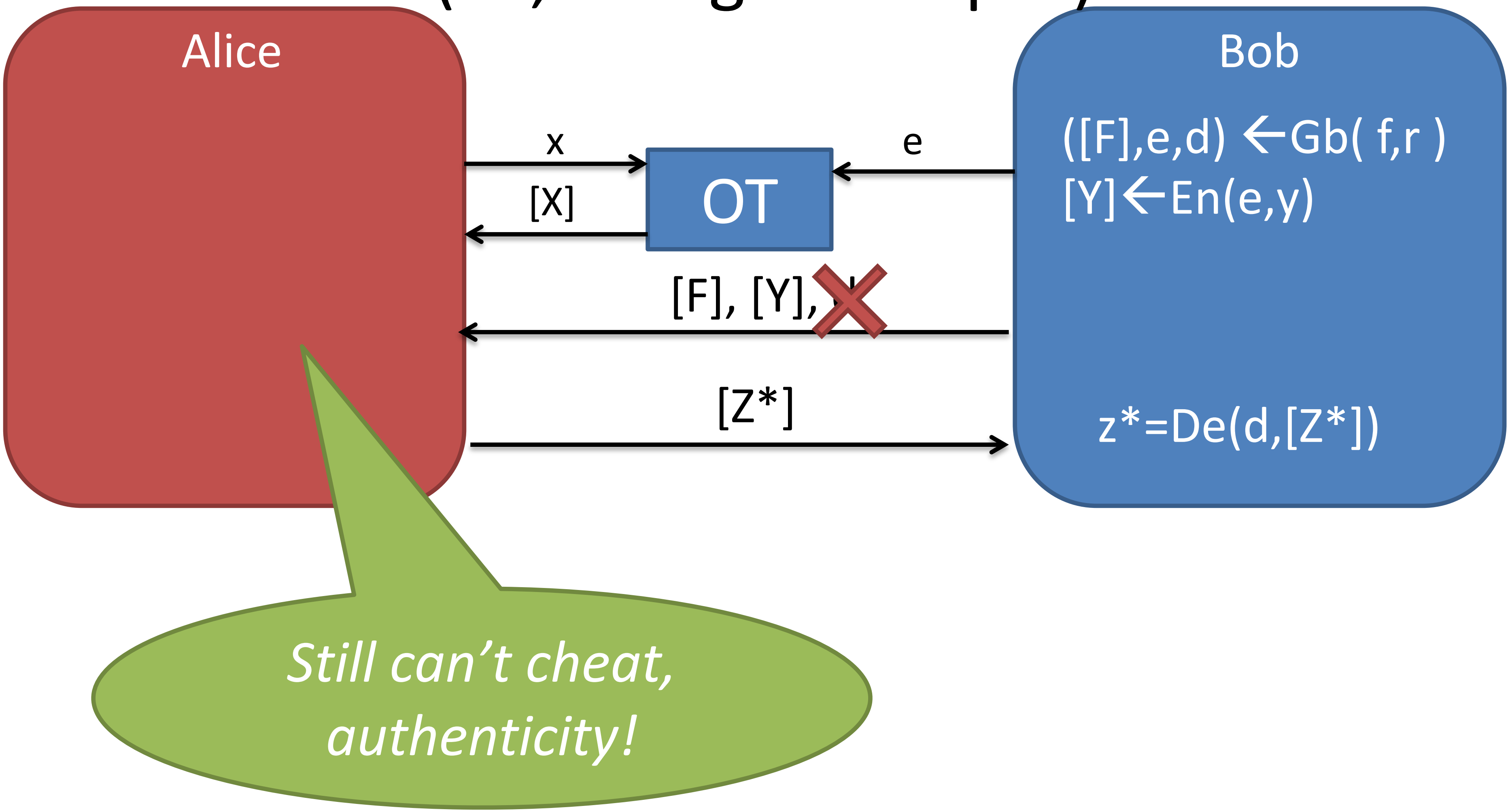
Yao´s protocol



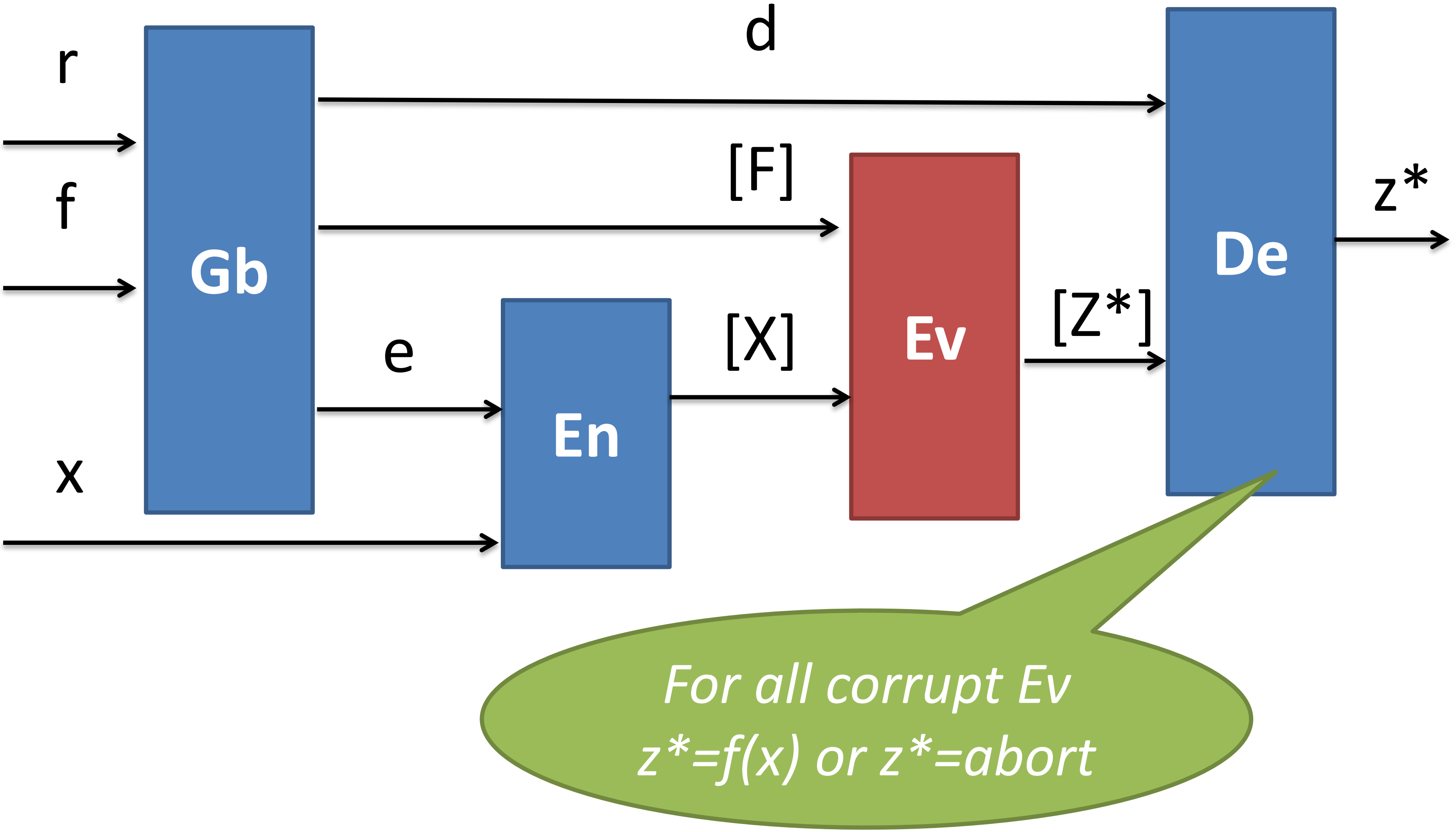
Active security of Yao



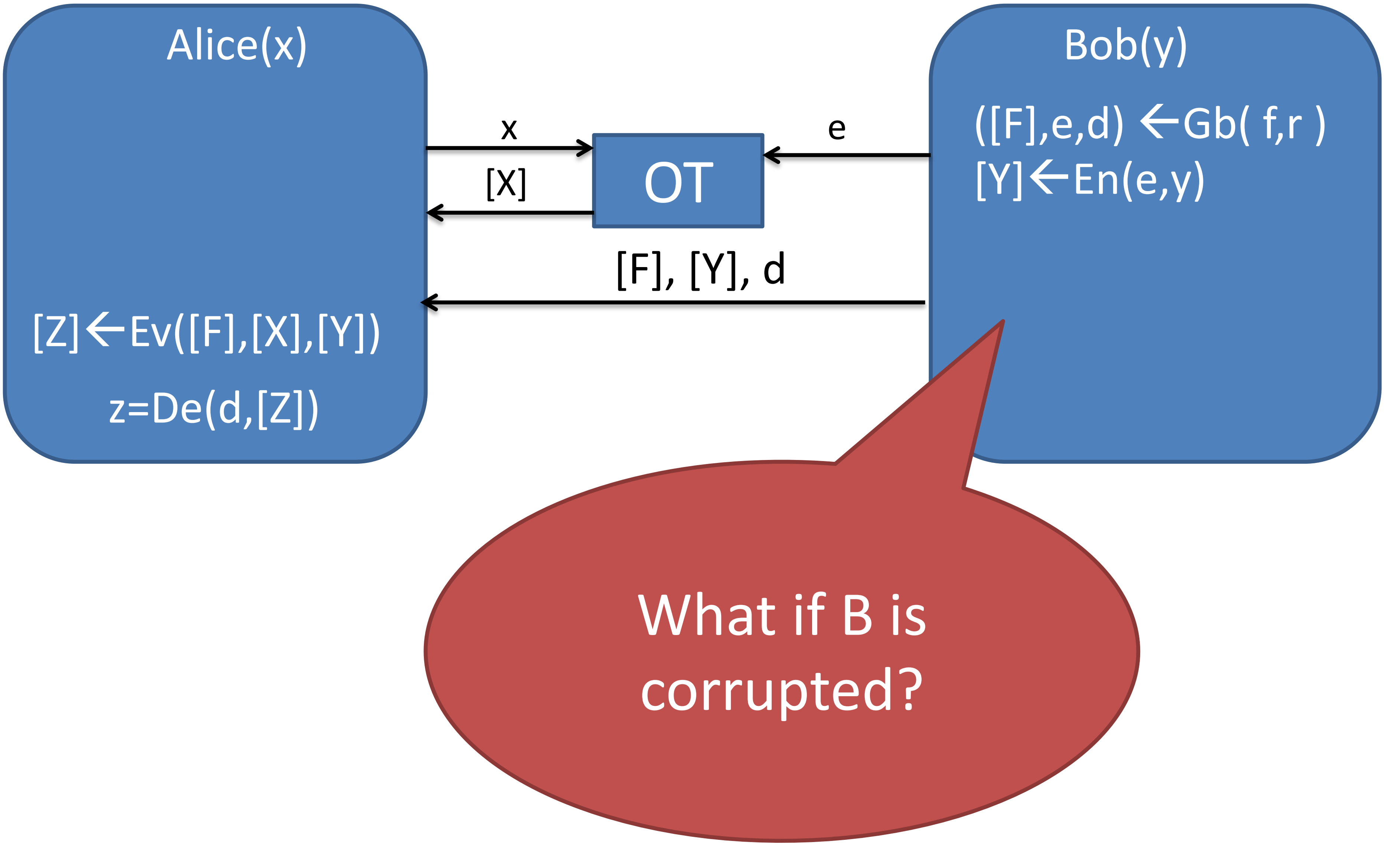
Active security of Yao (v2, Bob gets output)



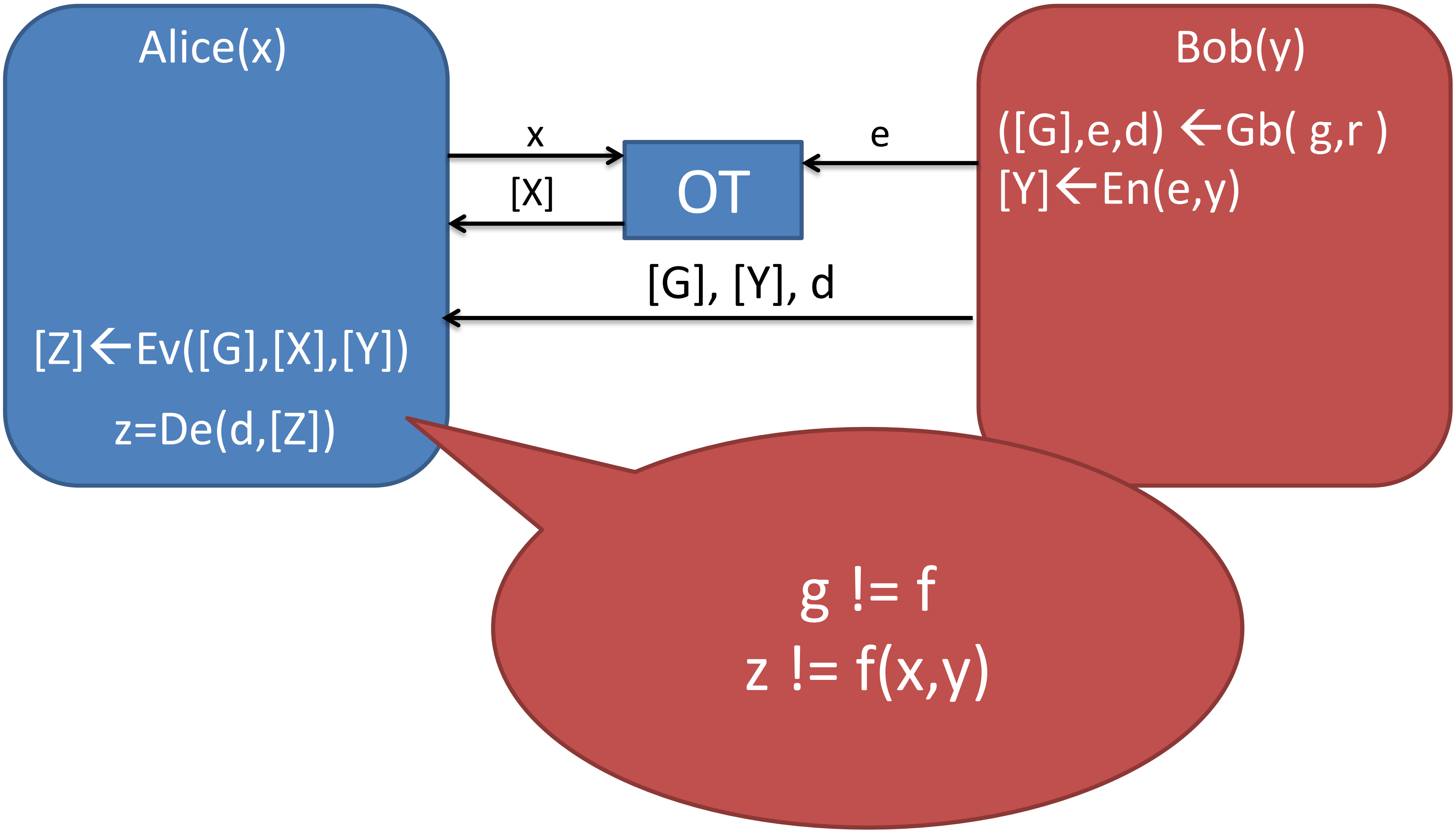
Garbled Circuits: Authenticy



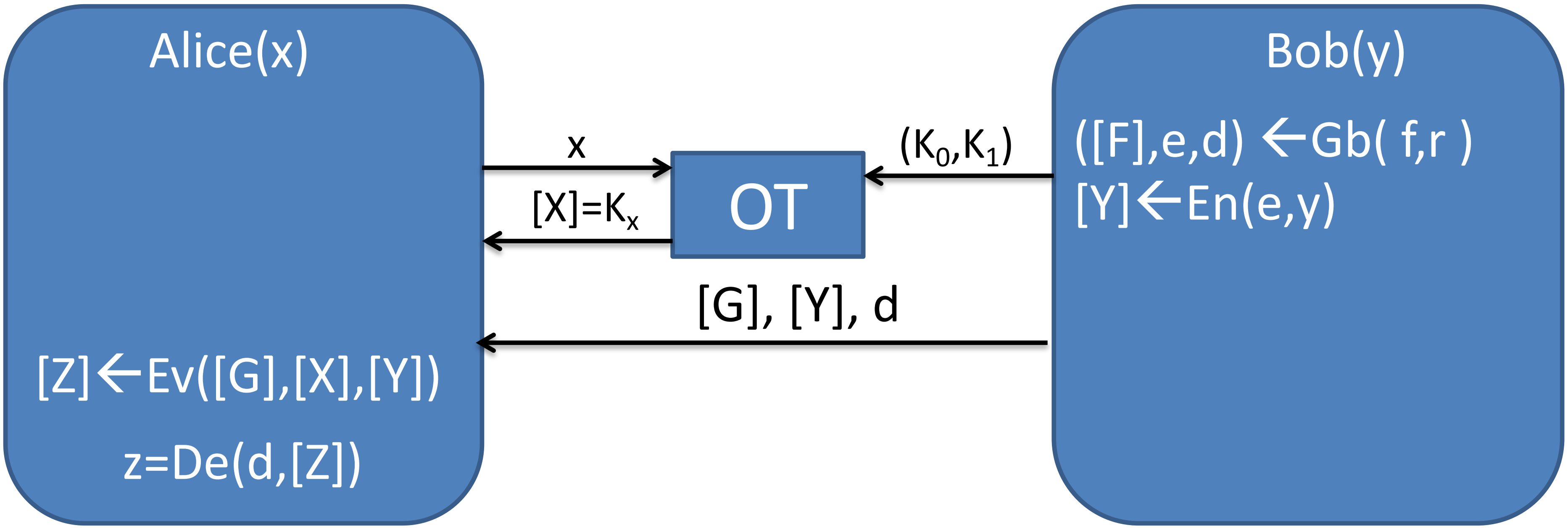
Active security of Yao



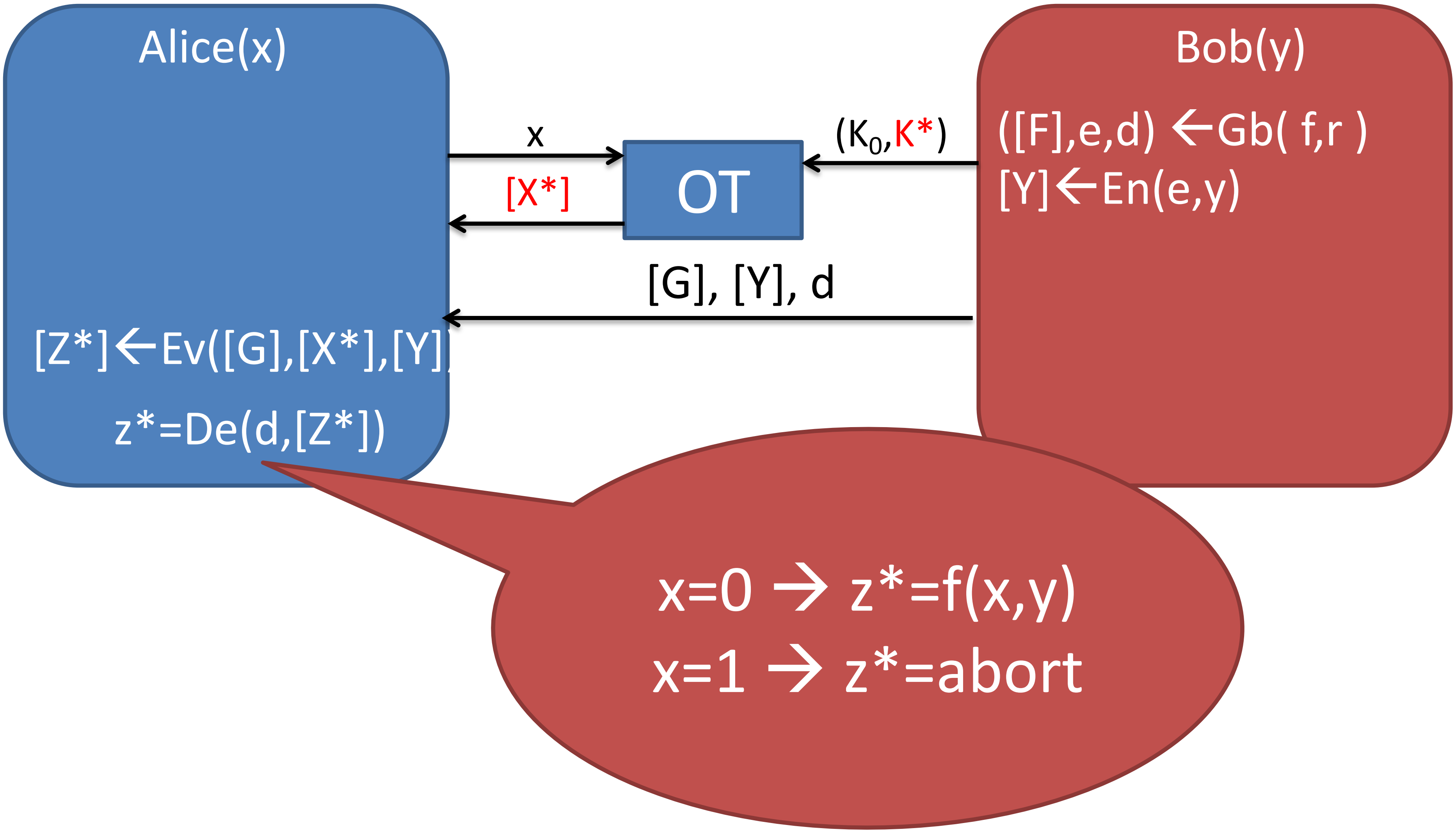
Insecurity 1 (wrong f)



Insecurity 2 (selective failure)

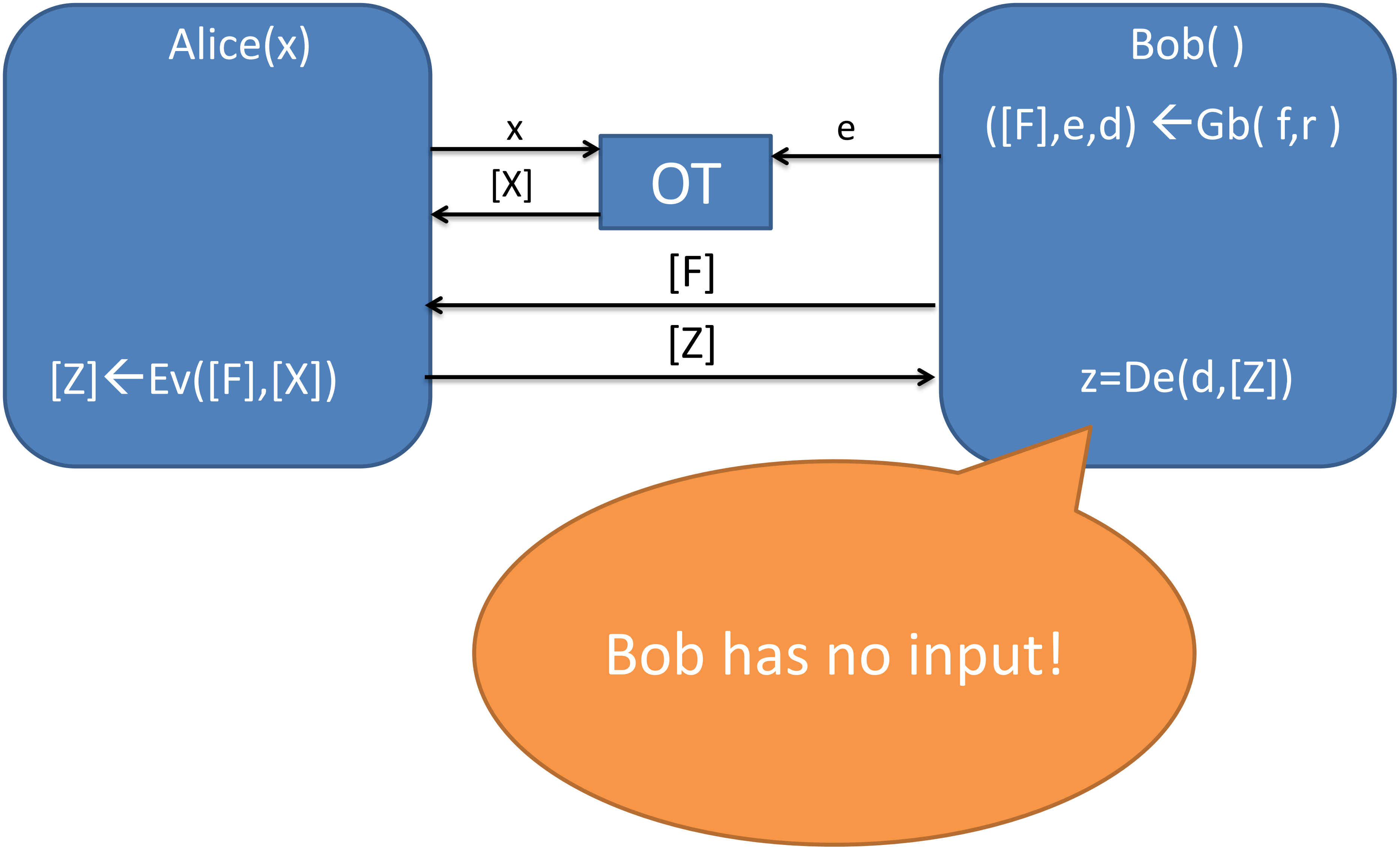


Insecurity 2 (selective failure)

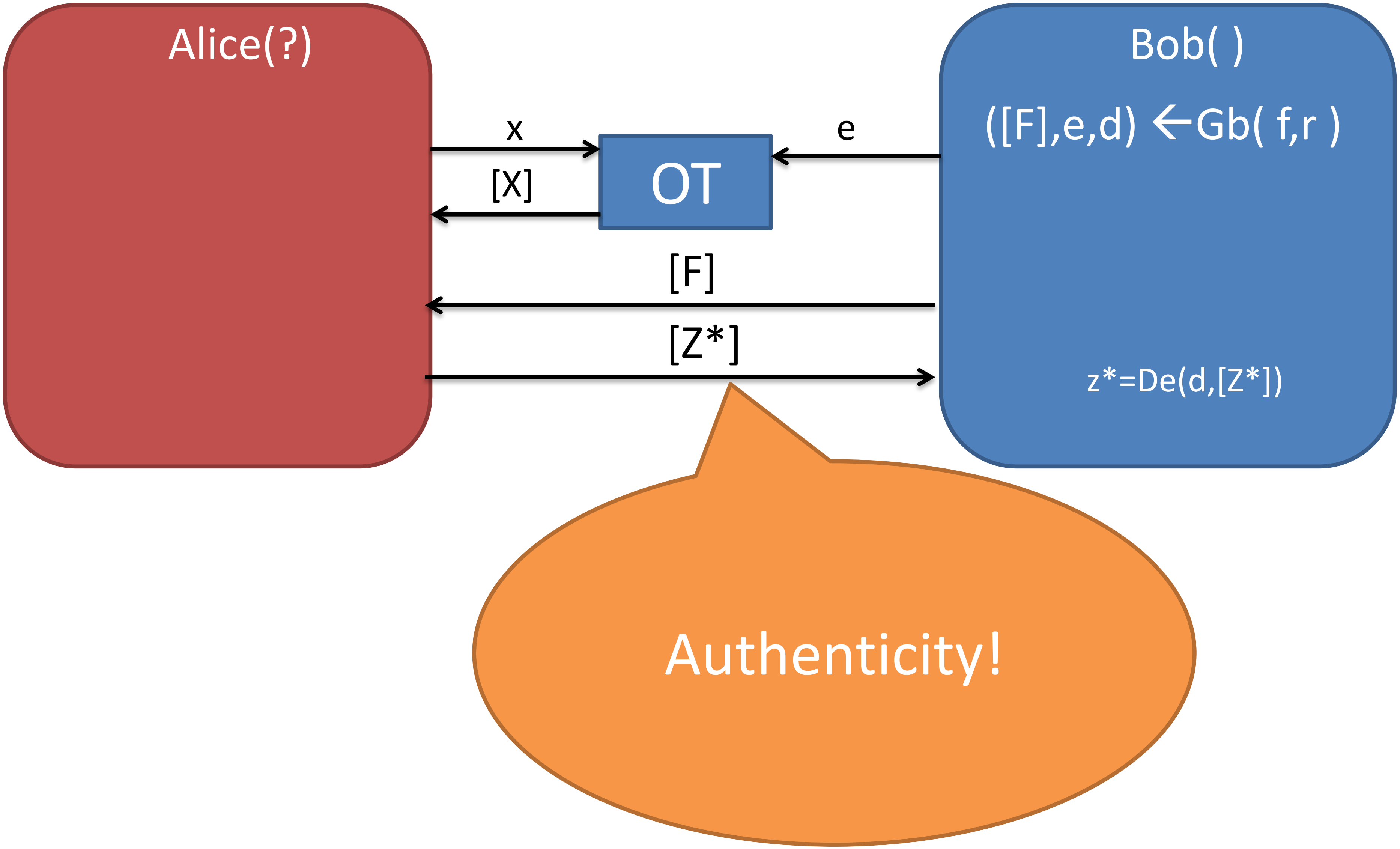


SIMPLE TRICKS FOR ACTIVE SECURITY

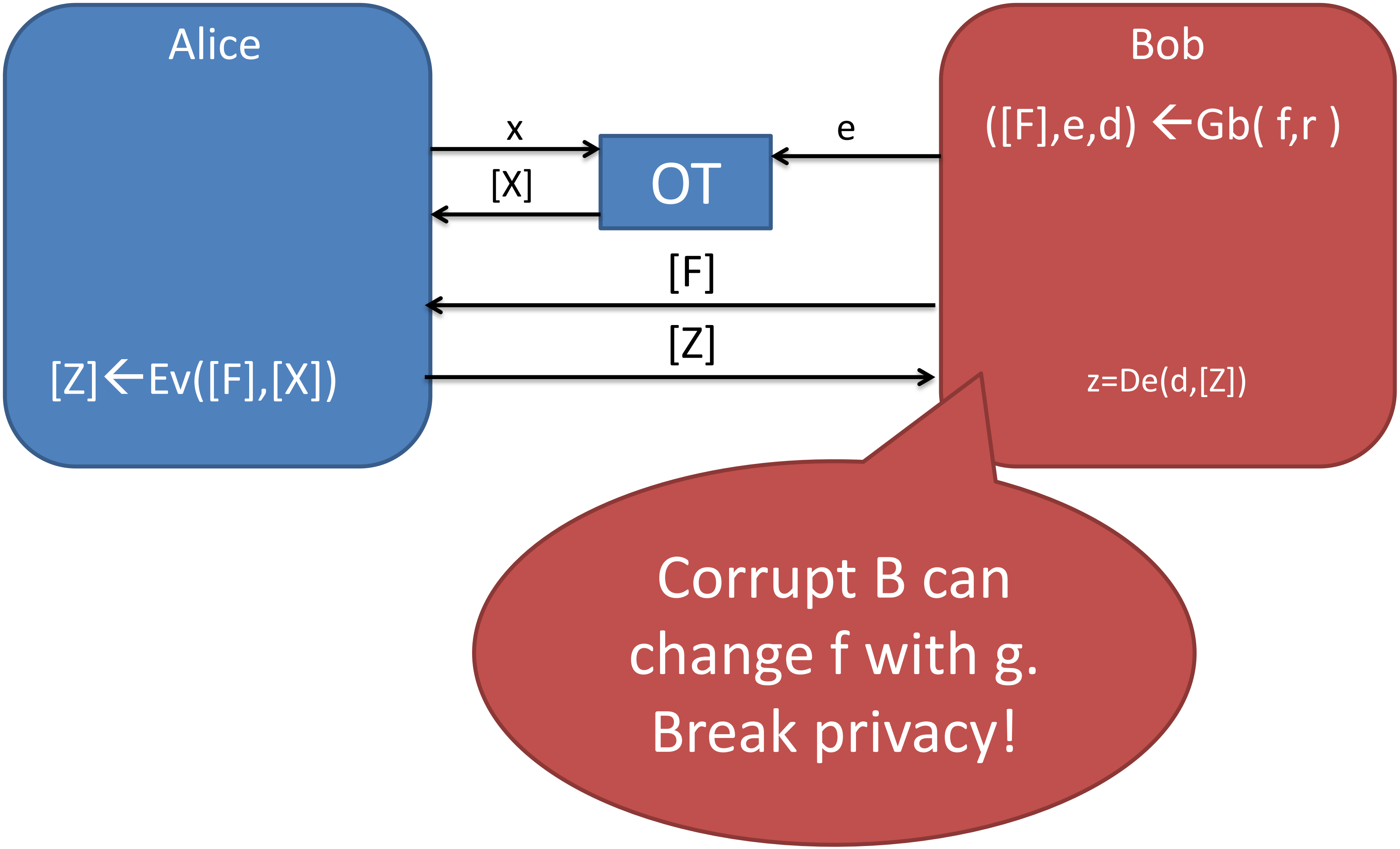
ZKGC (Alice proves f(x)=z)



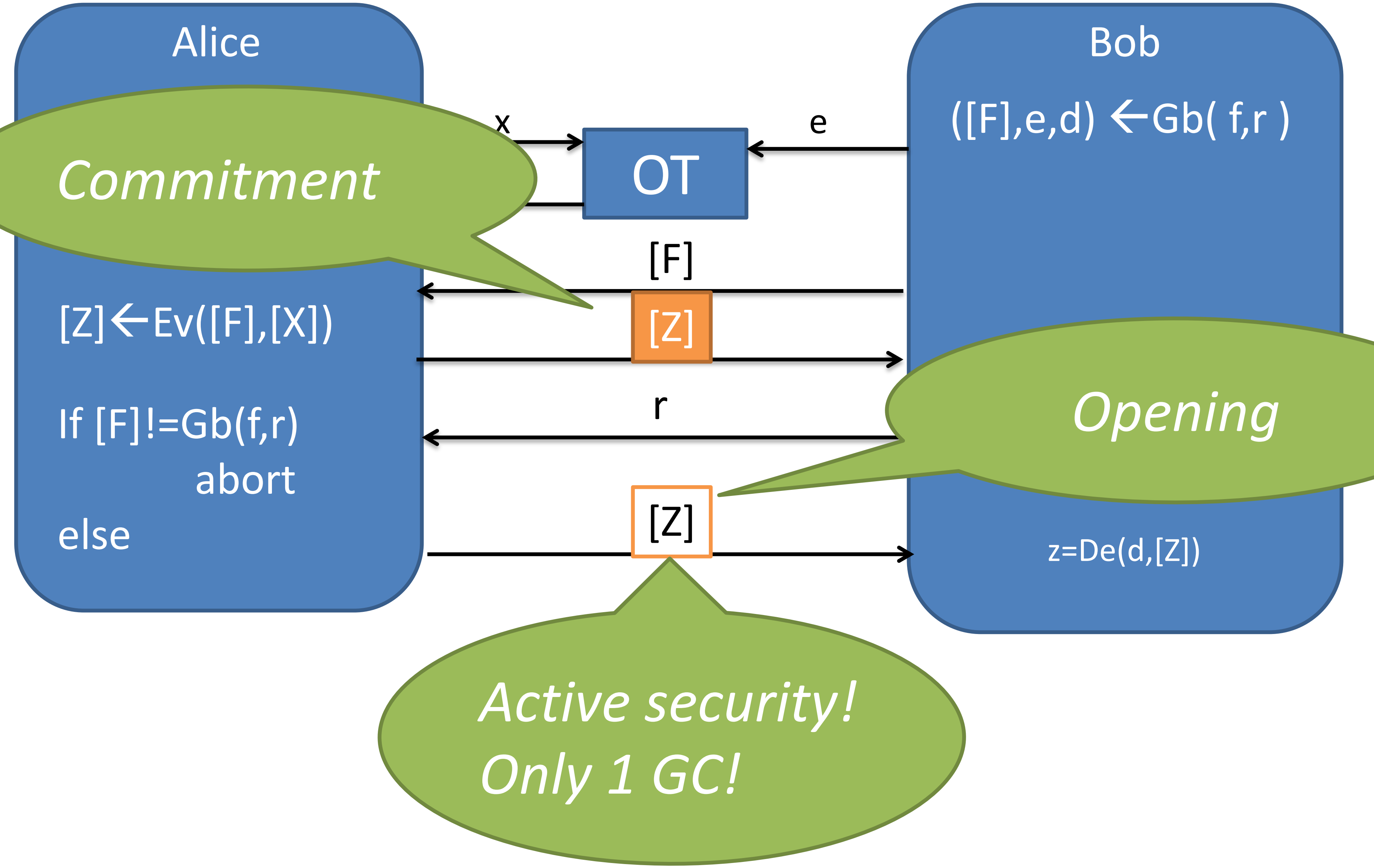
ZKGC (Alice proves f(x)=z)



ZKGC (Alice proves f(x)=z)

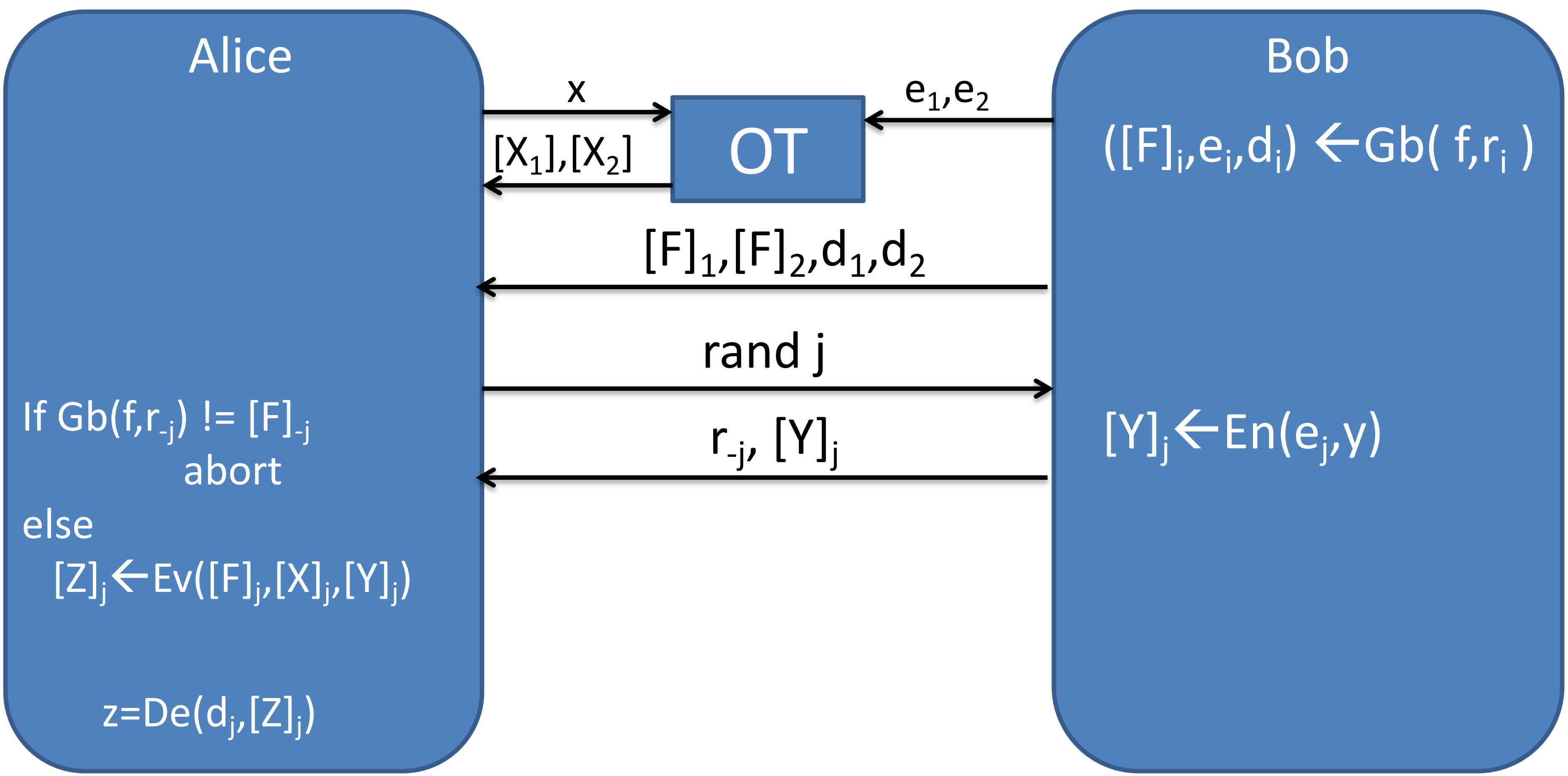


ZKGC (Alice proves f(x)=z)

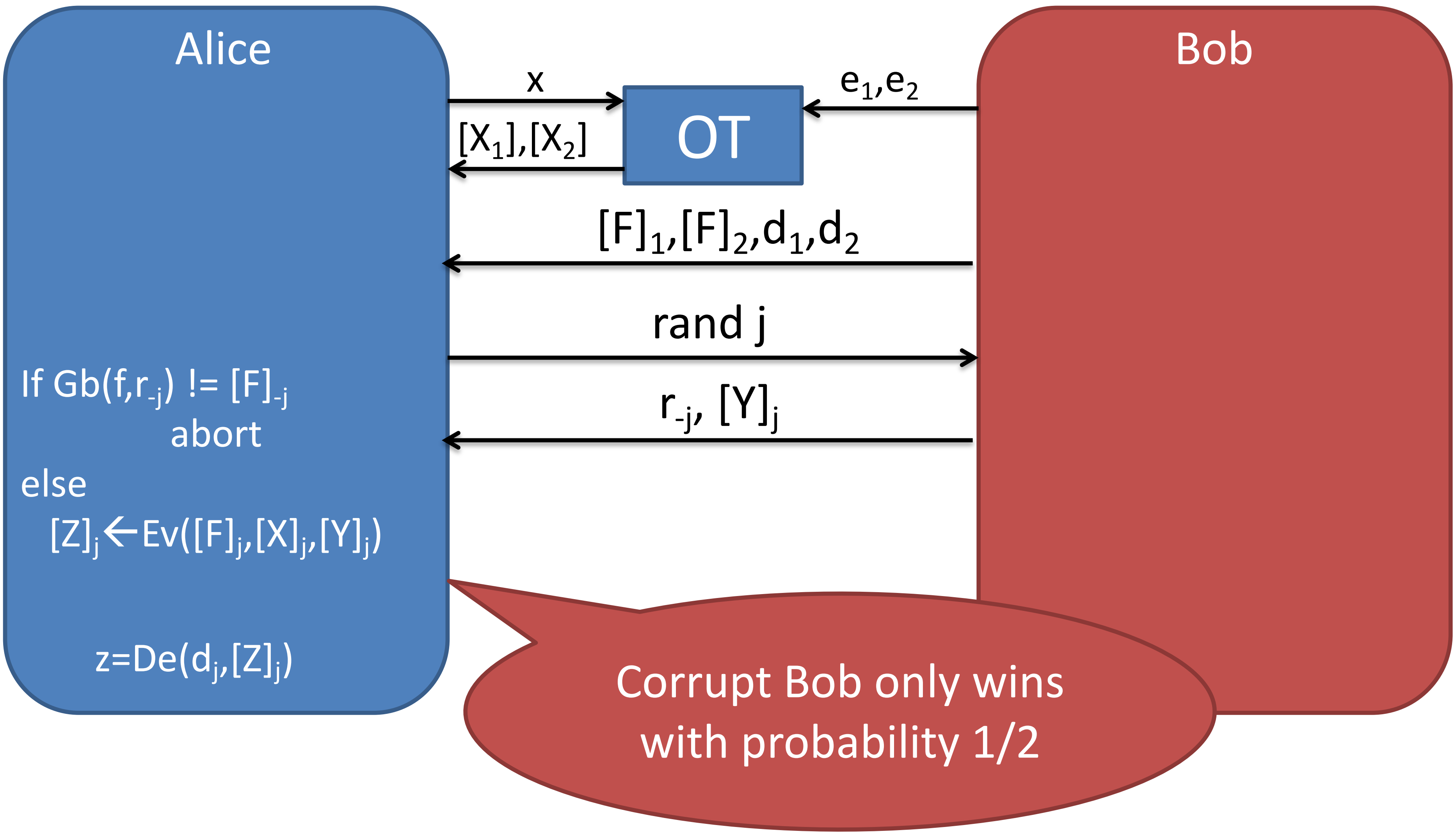


Cut-And-Choose

2PC, simple cut-and-choose



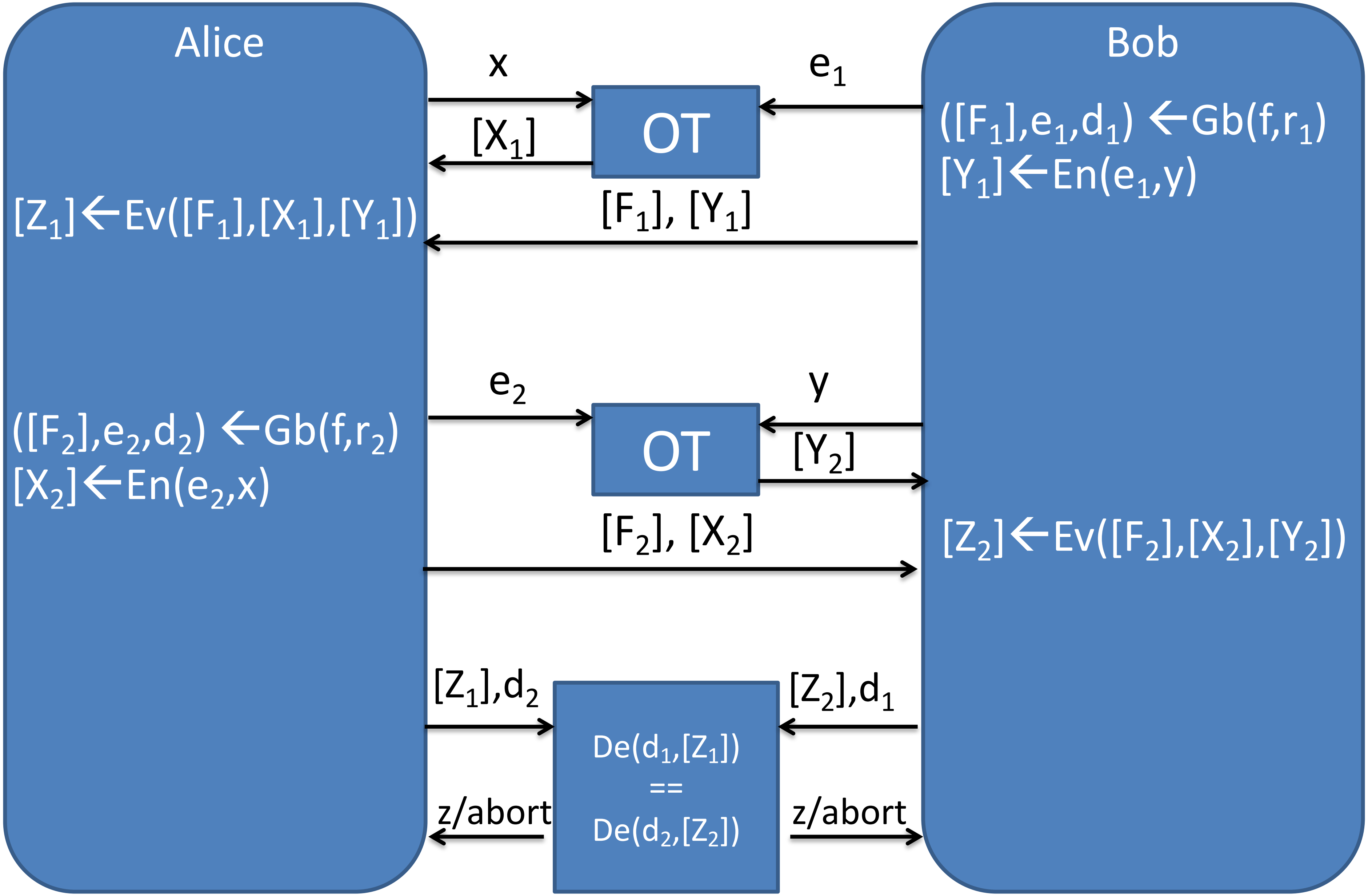
2PC, simple cut-and-choose

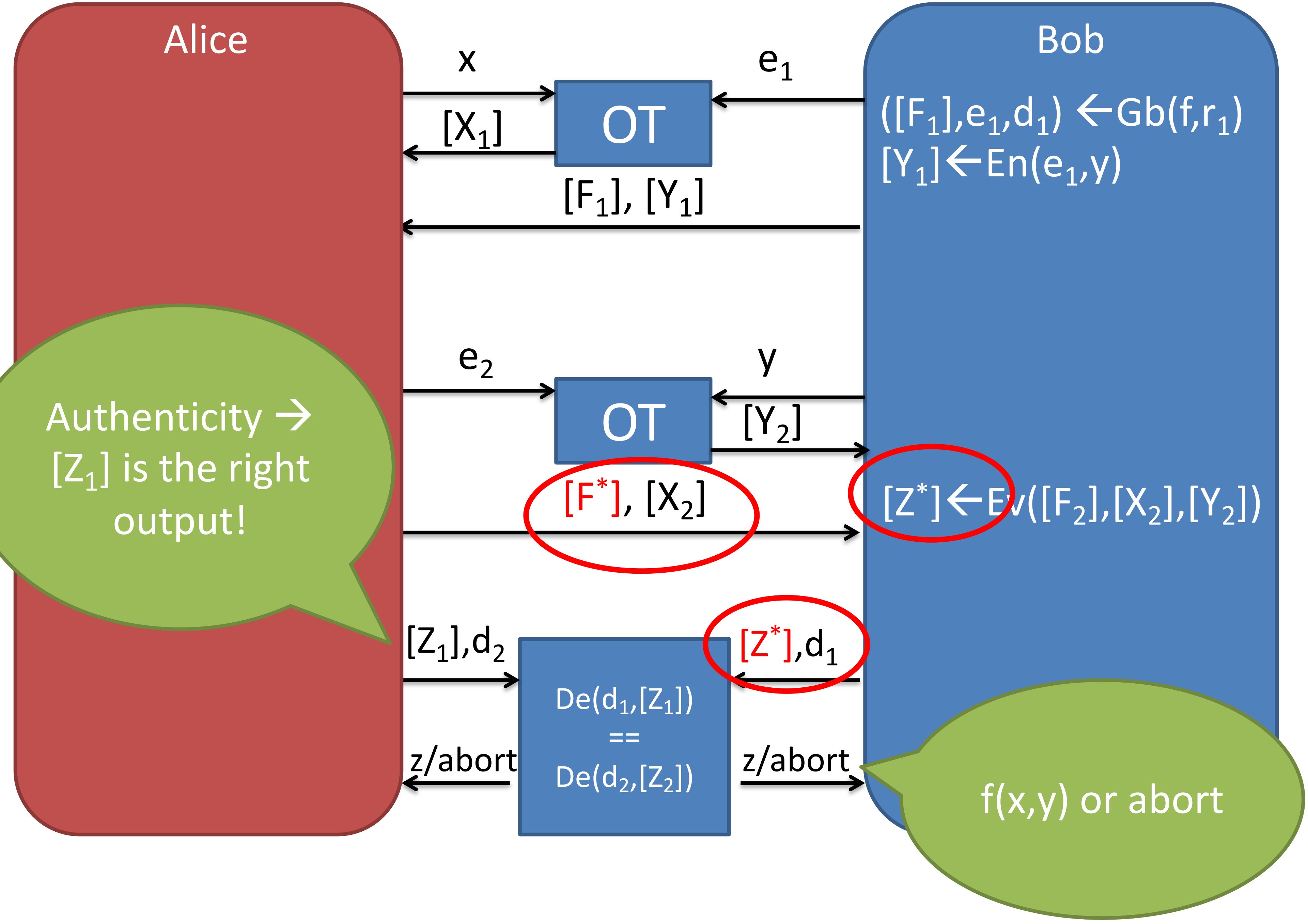


2PC, cut-and-choose

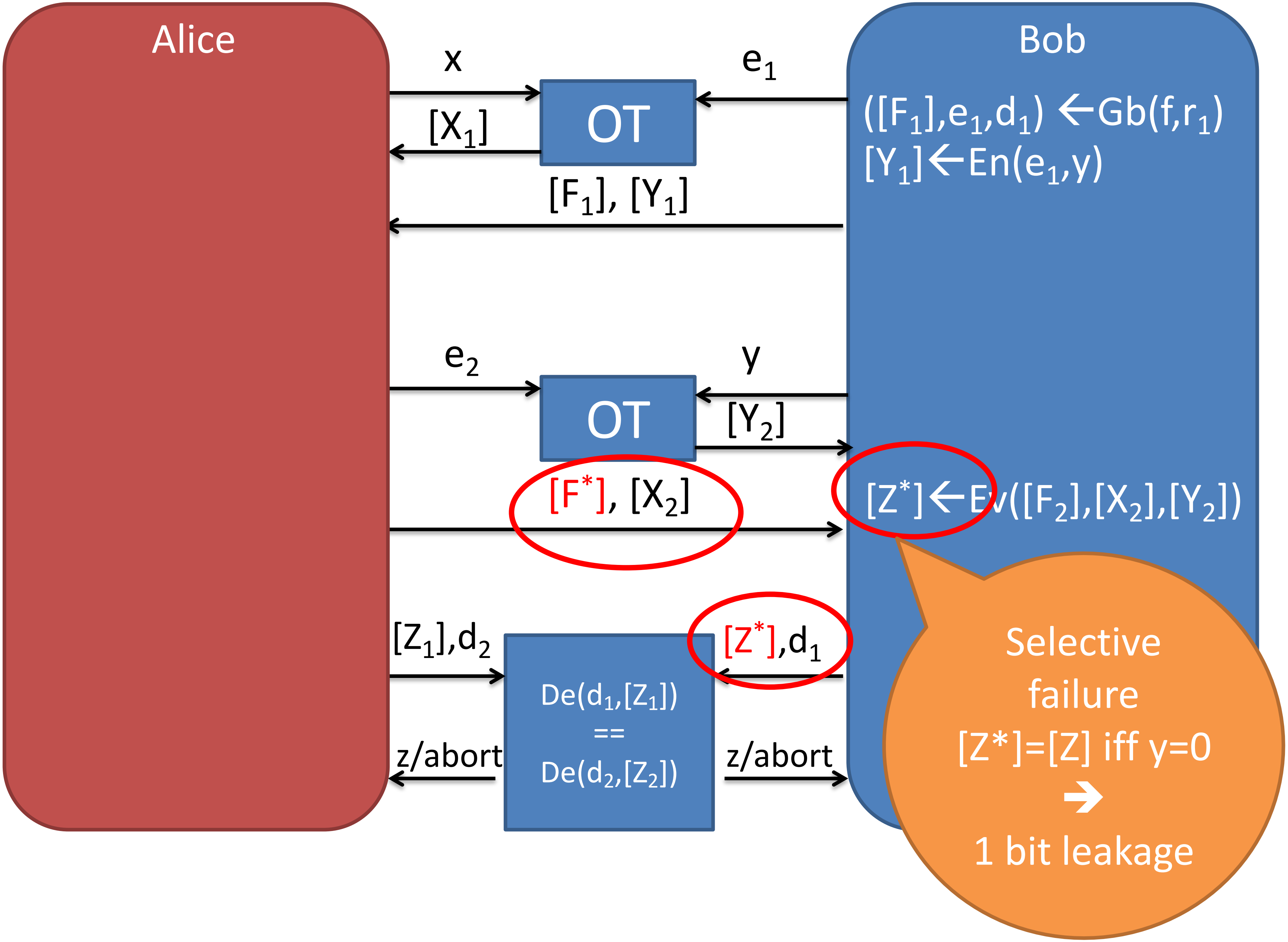
- Simple cut-and-choose
  - Garble k, check k-1, evaluate 1.
  - Security 1-1/k
- Advanced cut-and-choose (see references)
  - Garble 2k, check k, evaluate k
  - Output majority result
  - Security with $2^{-O(k)}$
  - (Need mechanisms to ensure the same input is used!)

Dual Execution


Alice
Bob
x
e1
OT
[X1]
([F1],e1,d1) ←Gb(f,r1)
[Y1]←En(e1,y)
[F1], [Y1]
[Z1]←Ev([F1],[X1],[Y1])
e2
y
OT
[Y2]
([F2],e2,d2) ←Gb(f,r2)
[X2]←En(e2,x)
[F2], [X2]
[Z2]←Ev([F2],[X2],[Y2])
[Z1],d2
[Z2],d1
De(d1,[Z1])
==
De(d2,[Z2])
z/abort
z/abort

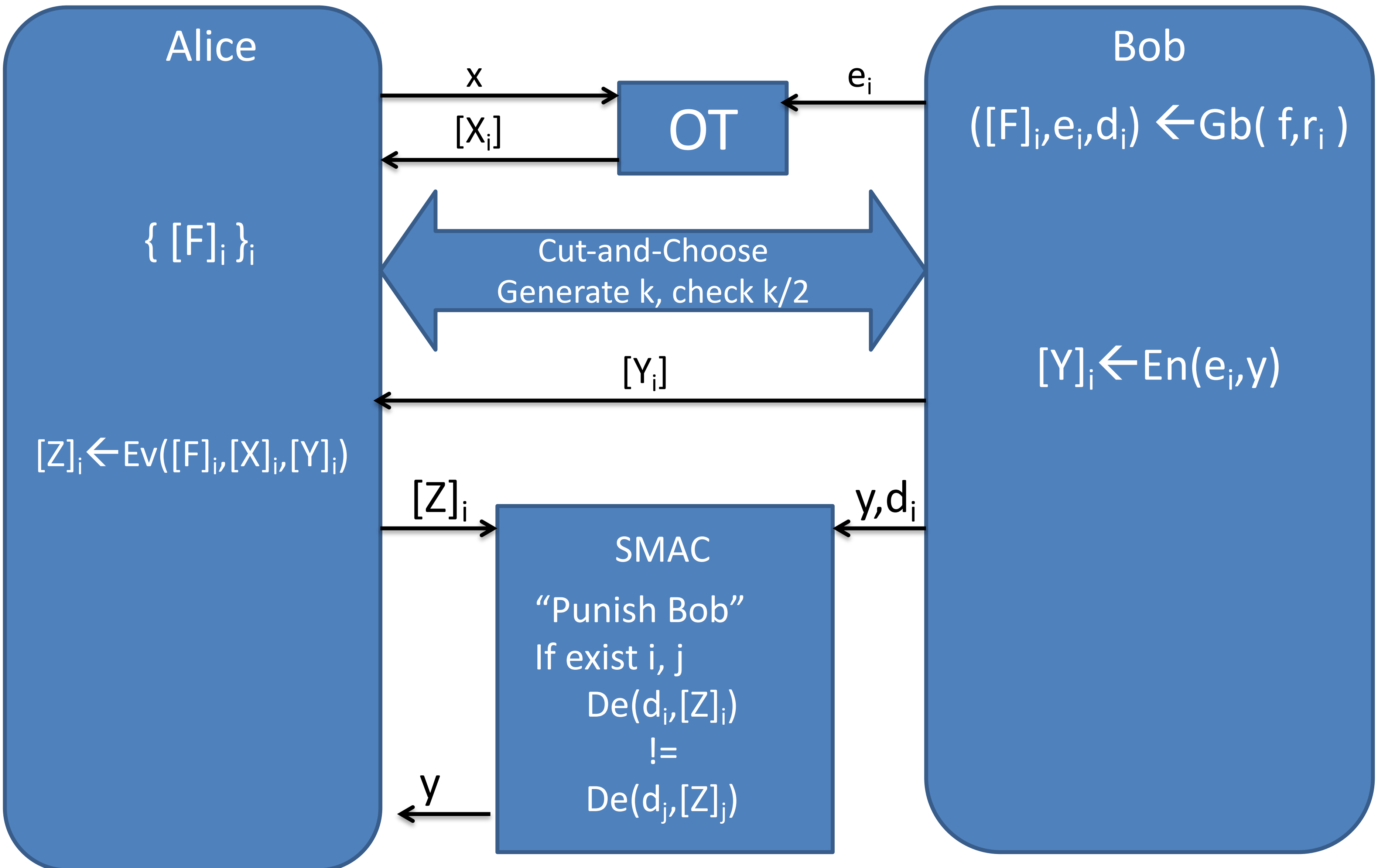


Alice
Bob
x
e_1
OT
$[X_1]$
$([F_1],e_1,d_1) \leftarrow Gb(f,r_1)$
$[Y_1] \leftarrow En(e_1,y)$
$[F_1], [Y_1]$
e_2
y
OT
$[Y_2]$
Authenticity → $[Z_1]$ is the right output!
$[F^*], [X_2]$
$[Z^*] \leftarrow Ev([F_2],[X_2],[Y_2])$
$[Z_1],d_2$
$[Z^*],d_1$
$De(d_1,[Z_1]) == De(d_2,[Z_2])$
z/abort
z/abort
f(x,y) or abort

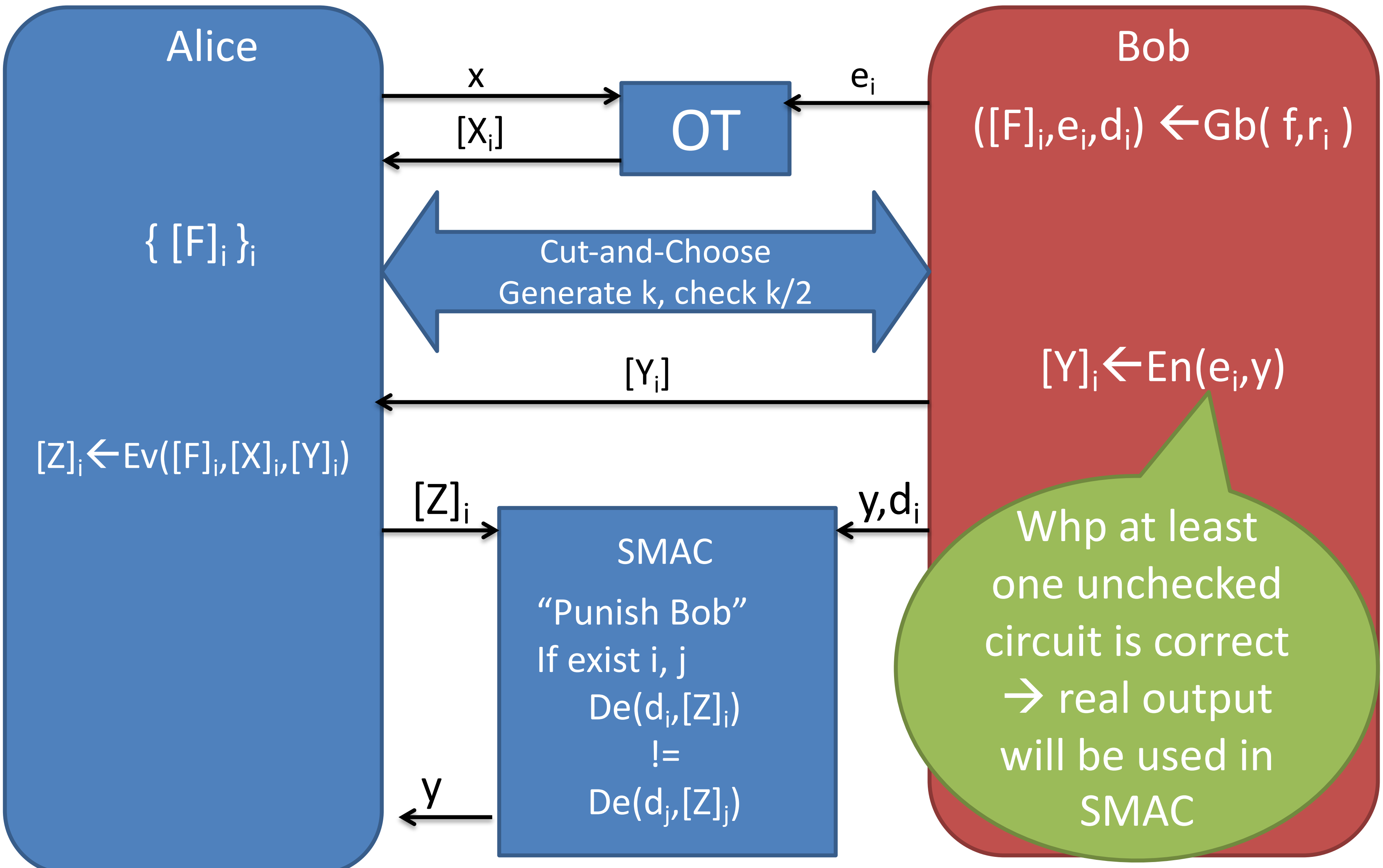

Alice
Bob
x
e_1
OT
$[X_1]$
$([F_1],e_1,d_1) \leftarrow Gb(f,r_1)$
$[Y_1] \leftarrow En(e_1,y)$
$[F_1], [Y_1]$
e_2
y
OT
$[Y_2]$
$[F^*], [X_2]$
$[Z^*] \leftarrow Ev([F_2],[X_2],[Y_2])$
$[Z_1],d_2$
$[Z^*],d_1$
$De(d_1,[Z_1])$
==
$De(d_2,[Z_2])$
z/abort
z/abort
Selective
failure
$[Z^*]=[Z]$ iff y=0
→
1 bit leakage

Forge And Lose

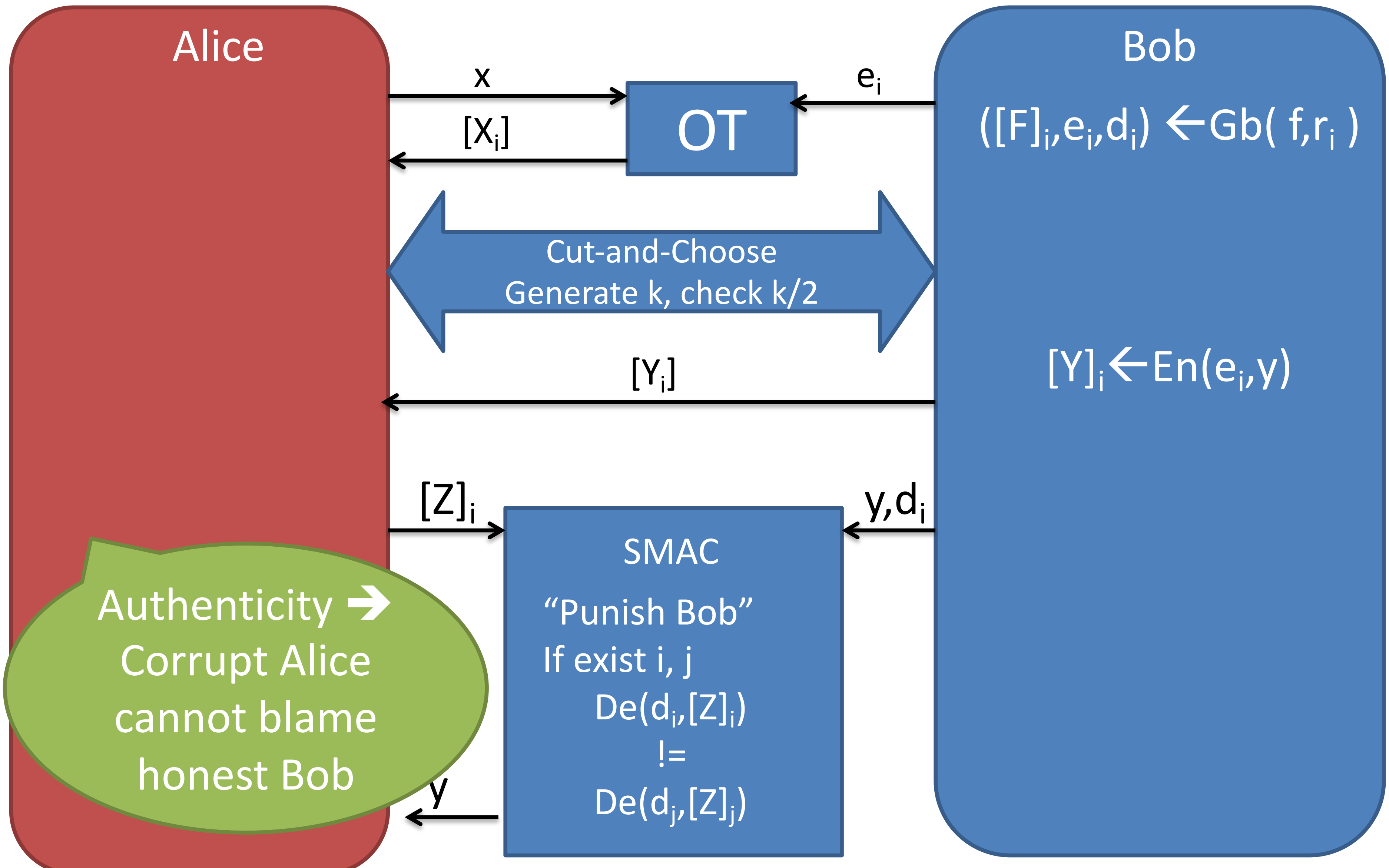
2PC, forge-and-lose (idea)



2PC, forge-and-lose (idea)



2PC, forge-and-lose (idea)



Summary

- Garbling Schemes
  - Definitions and Applications
- Efficient Garbling Techniques
  - Point&Permute, Garbled Row Reduction, Free-XOR, Half-Gate, Privacy-Free, …
- How to use Garbled Circuits with active corruptions

Primary References

- Cryptographic Computing, lecture notes, http://orlandi.dk/crycom (with theory and programming exercises)
- A Brief History of Practical Garbled Circuit Optimizations (Rosulek)
- Fast Cut-and-Choose-Based Protocols for Malicious and Covert Adversaries (Lindell)
- Two Halves Make a Whole - Reducing Data Transfer in Garbled Circuits Using Half Gates (Zahur et al.)
- Privacy-Free Garbled Circuits with Applications to Efficient Zero-Knowledge (Frederiksen et al.)
- Zero-knowledge using garbled circuits: how to prove non-algebraic statements efficiently (Jawurek et al.)
- Improved Garbled Circuit: Free XOR Gates and Applications (Kolesnikov et al.)
- Foundations of Garbled Circuits (Bellare et al.)

Other References

- Fast Garbling of Circuits Under Standard Assumptions (Gueron et al.)
- Garbling Gadgets for Boolean and Arithmetic Circuits (Ball et al.)
- FleXOR: Flexible Garbling for XOR Gates That Beats Free-XOR (Kolesnikov et al.)
- MiniLEGO: Efficient Secure Two-Party Computation From General Assumptions (Frederiksen et al.)
- Secure Two-Party Computation with Reusable Bit-Commitments, via a Cut-and-Choose with Forge-and-Lose Technique (Brandao)
- Secure Two-Party Computation via Cut-and-Choose Oblivious Transfer (Lindell, Pinkas)
- An Efficient Protocol for Secure Two-Party Computation in the Presence of Malicious Adversaries (Lindell, Pinkas)
- Efficiency Tradeoffs for Malicious Two-Party Computation (Mohassel et al.)