

# Authenticated Multistep Nearest Neighbor Search

Stavros Papadopoulos, Lixing Wang, Yin Yang, Dimitris Papadias, and Panagiotis Karras

**Abstract**—Multistep processing is commonly used for nearest neighbor (NN) and similarity search in applications involving high-dimensional data and/or costly distance computations. Today, many such applications require a proof of result correctness. In this setting, clients issue NN queries to a server that maintains a database signed by a trusted authority. The server returns the NN set along with supplementary information that permits result verification using the data set signature. An adaptation of the multistep NN algorithm incurs prohibitive network overhead due to the transmission of *false hits*, i.e., records that are not in the NN set, but are nevertheless necessary for its verification. In order to alleviate this problem, we present a novel technique that reduces the size of each false hit. Moreover, we generalize our solution for a distributed setting, where the database is horizontally partitioned over several servers. Finally, we demonstrate the effectiveness of the proposed solutions with real data sets of various dimensionalities.

**Index Terms**—Query authentication, multistep nearest neighbors, similarity search.

## 1 INTRODUCTION

LET  $DB$  be a  $D$ -dimensional data set. Each record  $P \in DB$  can be thought of as a point in the space defined by the  $D$  attribute domains, and in the sequel we use the term record and point interchangeably. Given a point  $Q$ , a nearest neighbor (NN) query retrieves the record  $\{P \in DB: DST(Q, P) \leq DST(Q, P') \forall P' \in DB\}$ , where  $DST(Q, P)$  denotes the distance between  $Q$  and  $P$ . Likewise, a  $k$ NN query returns the  $k$  closest points to  $Q$ . NN and  $k$ NN queries are common in *similarity retrieval*. Specifically, since similarity between records is inversely proportional to their distance, a  $k$ NN query returns the  $k$  most similar records to  $Q$ . The *multistep* framework [11], [19] has been proposed for NN and similarity retrieval in domains that entail high-dimensional data (e.g., in time series, medical, image, biological and document databases), expensive distance functions (e.g., road network distance, dynamic time warping), or a combination of both factors.

In this paper, we focus on authenticated multistep NN search for applications that require a *proof of result correctness*. For instance, [3] argues that the most cost-effective way for medical facilities to maintain radiology images is to *outsource* all image management tasks to specialized commercial providers. Clients issue similarity queries to a provider. The latter returns the result set and additional verification information, based on which the client establishes that the result is indeed correct, i.e., it contains exactly the records of  $DB$  that satisfy the query conditions, and that these records

indeed originate from their legitimate data source (i.e., the corresponding medical facility). A similar situation occurs for *data replication*, i.e., when a data owner stores  $DB$  at several servers. Clients issue their queries to the closest (in terms of network latency) server, but they wish to be assured that the result is the same as if the queries were sent to the original source of  $DB$ . In other cases, correctness is guaranteed by a trusted third party. For instance, *notarization services* [20] have been proposed to safeguard against tampering in document databases (the motivating example being Enron). Authenticated query processing ensures the client that the received result complies with the validated  $DB$ .

Initially, we study the problem assuming that the entire  $DB$  resides at a single server. Our first contribution is AMN, an adaptation of a multistep algorithm that is optimal in terms of  $DST$  computations. AMN requires transmissions of *false hits*, i.e., records that are not in the result, but are nevertheless necessary for its verification. In addition to the network overhead, false hits impose a significant burden to the client, which has to verify them. The second contribution, C-AMN, alleviates this problem through an elaborate scheme that reduces the size of false hits. Finally, we consider a distributed setting, where the database is horizontally partitioned over several servers. Our third contribution, ID-AMN, incrementally retrieves data, gradually eliminating servers that cannot contribute results.

The rest of the paper is organized as follows: Section 2 surveys related work. Section 3 presents the indexing scheme and the query algorithms of AMN. Section 4 focuses on C-AMN and minimization of the false hits. Section 5 deals with distributed servers and ID-AMN. Section 6 contains an extensive experimental evaluation with real data sets and Section 7 concludes the paper.

## 2 BACKGROUND

Section 2.1 describes multistep query processing. Section 2.2 overviews similarity search for high-dimensional data. Section 2.3 surveys background on authenticated queries.

- S. Papadopoulos, L. Wang, Y. Yang, and D. Papadias are with the Department of Computer Science and Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. E-mail: {stavros, lxwang, yini, dimitris}@cse.ust.hk.
- P. Karras is with the School of Computing, National University of Singapore, Computing 1, #02-18, Computing Drive, Singapore 117417. E-mail: karras@comp.nus.edu.sg.

Manuscript received 18 Sept. 2009; revised 12 Dec. 2009; accepted 9 Feb. 2010; published online 30 Aug. 2010.

Recommended for acceptance by D. Srivastava.

For information on obtaining reprints of this article, please send e-mail to: tkde@computer.org, and reference IEEECS Log Number TKDE-2009-09-0665. Digital Object Identifier no. 10.1109/TKDE.2010.157.

**Algorithm MultistepNN( $Q, k$ )**

1. Retrieve the  $k$  NNs  $\{P_1, \dots, P_k\}$  of  $Q$  according to  $dst$
2.  $RS = \{P_1, \dots, P_k\}$ , sorted according to  $DST$
3.  $DST_{max} = DST(Q, P_k)$  // the current  $k^{\text{th}}$  NN  $DST$
4.  $P =$  next NN of  $Q$  according to  $dst$
5. While  $dst(Q, P) < DST_{max}$
6.     If  $DST(Q, P) < DST_{max}$
7.         Insert  $P$  into  $RS$  and remove previous  $k^{\text{th}}$  NN
8.         Update  $DST_{max}$  over  $RS$
9.      $P =$  next NN of  $Q$  according to  $dst$

Fig. 1. Optimal multistep  $k$ NN processing.

Section 2.4 focuses on the MR-Tree, which is used by the proposed techniques.

## 2.1 Multistep NN Framework

The *multistep NN* framework is motivated by applications that entail expensive distance computations. Specifically, let  $DST(Q, P)$  be the actual distance between a query  $Q$  and a data point  $P \in DB$ . The applicability of the multistep framework rests on the existence of a *filter* distance metric  $dst$ , which is cheap to evaluate and satisfies the *lower bounding property*, i.e., for every possible  $Q$  and  $P$ :  $dst(Q, P) \leq DST(Q, P)$ . Multistep NN search was introduced in [11]. Subsequently, Seidl and Kriegel [19] proposed the algorithm of Fig. 1, which is optimal in terms of  $DST$  computations. In order to provide a concrete context, our explanation focuses on road networks [18], where  $DST$  and  $dst$  refer to the network and euclidean distance, respectively. Compared to euclidean distance ( $dst$ ), network distance ( $DST$ ) computations are significantly more expensive because they entail shortest path algorithms in large graphs. Moreover, the euclidean  $k$ NNs can be efficiently retrieved using conventional NN search on a spatial index.

Assuming that  $DB$  is indexed by an  $R^*$ -Tree [1], the multistep  $k$ NN algorithm first retrieves the  $k$  euclidean NNs of  $Q$  using an incremental algorithm (e.g., [7]). These points are inserted into a result set  $RS$ , and their network ( $DST$ ) distances are computed. Let  $DST_{max}$  be the network distance<sup>1</sup> between  $Q$  and its current  $k^{\text{th}}$  NN  $P_k$ . The next euclidean NN  $P$  is then retrieved. As long as  $dst(Q, P) < DST_{max}$ , the algorithm computes  $DST(Q, P)$  and compares it against  $DST_{max}$ . If  $DST(Q, P) < DST_{max}$ ,  $P$  is inserted into  $RS$ , the previous  $P_k$  is expunged, and  $DST_{max}$  is updated. The loop of Lines 5-9 terminates when  $dst(Q, P) \geq DST_{max}$ ; because of the lower bounding property of the euclidean distance, any point lying further in the euclidean space cannot be closer than  $DST_{max}$  in the network.

Independently of the application domain, the algorithm of Fig. 1 performs the minimum number of  $DST$  computations. Specifically, in addition to  $RS$ , the  $DST$  distances are computed *only* for *false hits*, i.e., the set of points  $FH = \{P \in DB - RS : dst(Q, P) \leq DST(Q, P_k)\}$ , where  $P_k$  is the final  $k^{\text{th}}$  NN. The rest of the records are not accessed at all (if they reside in pruned nodes of the  $R^*$ -Tree), or they are eliminated using their  $dst$  to  $Q$ .

1. To avoid tedious details in our discussion, we assume that all data distances to the query point are different.

## 2.2 High-Dimensional Similarity Search Using Multistep NN

Several applications including image, medical, time series, and document databases involve high-dimensional data. Similarity retrieval in these applications based on low-dimensional indexes, such as the  $R^*$ -Tree [1], is very expensive due to the *dimensionality curse* [2]. Specifically, even for moderate dimensionality (i.e.,  $D = 20$ ) a sequential scan that computes  $DST(Q, P)$  for every  $P \in DB$  is usually cheaper than conventional NN algorithms using the index. Consequently, numerous specialized structures have been proposed for exact [8] and approximate [22]  $k$ NN search in high dimensions.

The GEMINI framework [6], [11] follows a different approach, combining *multistep search* with a *dimensionality reduction* technique that exhibits the lower bounding property. Specifically, each record  $P \in DB$  is mapped to a low-dimensional representation  $p$  in  $d$  dimensions ( $d \ll D$ ). The resulting  $d$ -dimensional data set  $db$  is indexed by an  $R^*$ -Tree, or any low-dimensional index. The query  $Q$  is also transformed to a  $d$ -dimensional point  $q$  and processed using a multistep method. For instance, in the algorithm of Fig. 1,  $DST$  (resp.  $dst$ ) computations involve high (low) dimensional points. The index prunes most nodes and records using the cheap, filter ( $dst$ ) distances,<sup>2</sup> whereas the expensive  $DST$  computations are necessary only for the points in result  $RS$  and false hit set  $FH$ .

GEMINI is the most common approach for performing similarity search over high-dimensional data, and especially time series. Numerous dimensionality reduction methods have been used extensively including *Discrete Fourier Transform* (DFT), *Singular Value Decomposition* (SVD), *Discrete Wavelet Transform* (DWT), *Piecewise Linear Approximation* (PLA), *Piecewise Aggregate Approximation* (PAA), *Adaptive Piecewise Constant Approximation* (APCA), and *Chebyshev Polynomials* (CP). Their effectiveness is measured by the number of records that they can prune using only the low-dimensional representations (i.e., it is inversely proportional to the cardinality of  $FH$ ). Ding et al. [5] experimentally compare various techniques, concluding that their effectiveness depends on the data characteristics.

## 2.3 Authenticated Query Processing

In authenticated query processing, a server maintains a data set  $DB$  signed by a trusted authority (e.g., the data owner, a notarization service). The signature  $sig$  is usually based on a *public-key cryptosystem* (e.g., RSA [16]). The server receives and processes queries from clients. Each query returns a result set  $RS \subseteq DB$  that satisfies certain predicates. Moreover, the client must be able to establish that  $RS$  is *correct*, i.e., that it contains all records of  $DB$  that satisfy the query conditions, and that these records have not been modified by the server or another entity. Since  $sig$  captures the entire  $DB$  (including records not in the query result), in addition to  $RS$ , the server returns a *verification object* ( $VO$ ). Given the  $VO$ , the client can verify  $RS$  based on  $sig$  and the signer's public key.

$VO$  generation at the server is usually performed using an *authenticated data structure* (ADS). The most influential ADS is

2. Note that in GEMINI  $DST$  and  $dst$  may be based on the same definition (e.g., they may both be euclidean distances). In this case,  $dst$  is cheaper because of the lower dimensionality.

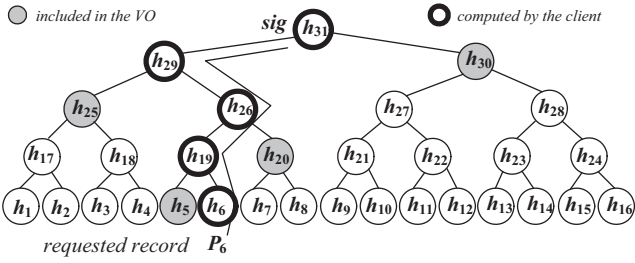


Fig. 2. MH-Tree example.

the *Merkle Hash Tree* (MH-Tree) [15], a main-memory binary tree, originally proposed for single record authentication. Each leaf in the MH-Tree stores the digest of a record, calculated using a one-way, collision-resistant hash function  $h(\cdot)$ , such as SHA-1 [16]. An inner node stores a digest computed on the concatenation of the digests in its children. The trusted authority signs the root digest. Fig. 2 illustrates an MH-Tree over 16 records. Assume that a client requests record  $P_6$ . When traversing the tree to locate  $P_6$ , the server produces a  $VO$  that contains the digests (shown in gray) of the siblings of the visited nodes:  $VO = [[h_{25}[[h_5 P_6] h_{20}] h_{30}]$ . Tokens “[” and “]” signify the scope of a node.  $VO$  and  $sig$  are transmitted to the client, which subsequently simulates a reverse tree traversal. Specifically, from  $h_5$  and  $P_6$  it computes  $h_{19}$ , then  $h_{26}$  (using  $h_{19}$  and  $h_{20}$ ),  $h_{29}$  (using  $h_{25}$  and  $h_{26}$ ), and finally  $h_{31}$  (using  $h_{29}$  and  $h_{30}$ ). Due to the collision resistance of the hash function, if  $P_6$  is modified, then the digest  $h_{31}$  reconstructed by the client will not match  $sig$ ; hence, the verification will fail.

The MH-Tree constitutes the basis of a large number of subsequent ADSs that support:

1. range queries on a single attribute [13],
2. multidimensional ranges [23],
3. continuous queries on data streams [24],
4. search in generalized directed acyclic graphs [14],
5. search over peer-to-peer networks [21],
6. XML queries [12], and
7. similarity-based document retrieval [17].

In the following, we focus on the MR-Tree [23], the state-of-the-art multidimensional ADS, which is utilized by our methods.

## 2.4 The MR-Tree

The MR-Tree [23] combines the concepts of the MH-Tree and the R\*-Tree [1]. A leaf node contains entries  $e_{lf}$  of the form  $(pg_P, P)$ , where  $P$  is an indexed point, and  $pg_P$  is a pointer to the page accommodating the record of  $P$ . An internal node  $N$  stores entries  $e_{in}$  of the form  $(pg_{N_c}, MBR_{N_c}, h_{N_c})$ , where  $pg_{N_c}$  points to  $e_{in}$ 's child node  $N_c$ . If  $N$  is at level 1 (the leaves being at level 0),  $MBR_{N_c}$  is the minimum bounding rectangle (MBR) of the points in  $N_c$  and  $h_{N_c}$  is a digest computed on the concatenation of the binary representation of the points in  $N_c$ . If  $N$  lies at higher levels,  $MBR_{N_c}$  is the MBR of all the MBR values in  $N_c$ , and  $h_{N_c}$  is the digest of the concatenation of all pairs  $(MBR_i, h_i)$  in  $N_c$ . Fig. 3 illustrates a two-dimensional MR-Tree assuming a maximum node capacity of three entries per node. A signature  $sig$  is produced on the root digest  $h_{root} = h(MBR_{N_2} | h_{N_2} | MBR_{N_3} | h_{N_3})$ .

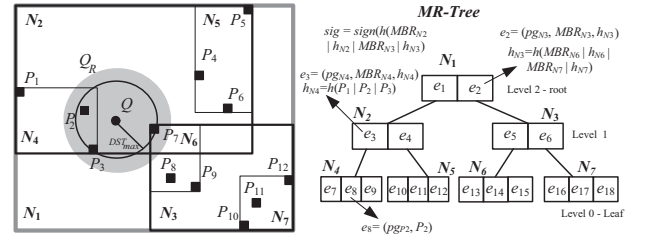


Fig. 3. MR-Tree example.

Upon receiving a range query  $Q_R$ , the server performs a depth-first traversal of the MR-Tree, using the algorithm of Fig. 4, to retrieve the set  $RS$  of points in  $Q_R$ . Furthermore, it generates a  $VO_R$  that contains: 1) all the points outside  $Q_R$  that reside in a leaf MBR overlapping  $Q_R$ , and 2) a pair  $(MBR_N, h_N)$ , for every node  $N$  pruned during query processing. In the example of Fig. 3, given the shaded range query  $Q_R$ , we have  $RS = \{P_2, P_3, P_7\}$ , and  $VO_R = [[P_1, result, result] (MBR_{N_5}, h_{N_5})] [[result, P_8, P_9] (MBR_{N_7}, h_{N_7})]$ . The token *result* signifies an object in  $RS$  according to the order of appearance in  $RS$ . For instance,  $[P_1, result, result]$  corresponds to node  $N_4$ ; the first occurrence of *result* refers to  $P_2$ , and the second one to  $P_3$ . In order to distinguish the type of each element in the  $VO$ ,  $MR\_Range$  includes a *header* prior to each token, digest, and point in the  $VO$ . This header consumes 3 bits, which suffice to represent eight different element types. For simplicity, we omit the headers in our presentation since the element type is implied by its name.

The server sends  $RS$ ,  $VO$ , and  $sig$  to the client, which first verifies that all points of  $RS$  indeed fall in  $Q_R$ . Then, it scans  $VO_R$  and reconstructs  $h_{root}$  bottom up (using a process similar to the MH-Tree verification). In our example, the client initially substitutes the first two result tokens with  $P_2$  and  $P_3$  from  $RS$ , and uses  $P_1, P_2$ , and  $P_3$  to compute digest  $h_{N_4} = h(P_1 | P_2 | P_3)$  and  $MBR_{N_4}$ . Then, it utilizes  $MBR_{N_4}$  and  $MBR_{N_5}$  to evaluate  $h_{N_2} = h(MBR_{N_4} | h_{N_4} | MBR_{N_5} | h_{N_5})$  and  $MBR_{N_2}$ . Likewise, it subsequently calculates  $h_{N_3}$ , and  $MBR_{N_3}$  from the partial  $VO$   $[[result, P_8, P_9] (MBR_{N_7}, h_{N_7})]$ . Eventually, it computes  $h_{root} = h(MBR_{N_2} | h_{N_2} | MBR_{N_3} | h_{N_3})$ . If  $h_{root}$  matches  $sig$ , the client is assured that the points in  $RS$  are sound. Furthermore, if all MBR values and points in  $VO$  lie outside  $Q_R$ , then  $RS$  is complete.

The MR-Tree can also process NN queries [23]. Assume, for instance, a query asking for the three NNs of point  $Q$  in

### Algorithm $MR\_Range(Q_R, N)$

1. Append [ to  $VO$
2. For each entry  $e$  in  $N$  // entries must be enumerated in original order
3. If  $N$  is leaf
4. If  $e$  falls in  $Q_R$
5. Insert  $e$  into  $RS$  and append a *result* token to  $VO$
6. Else //  $e$  falls out of  $Q_R$
7. Append  $e.P$  to  $VO$
8. Else //  $N$  is internal node
9. If  $e.MBR_{N_c}$  overlaps  $Q$ ,  $MR\_Range(Q, e.pg_{N_c})$
10. Else append  $e.MBR_{N_c}, e.h_{N_c}$  to  $VO$  // a pruned child node
11. Append ] to  $VO$

Fig. 4. Range query processing with the MR-Tree.

TABLE 1  
Summary of Symbols

Summary of symbols	
Symbol	Description
$DB$ ( $db$ )	Dataset in the original (reduced) space
$D$ ( $d$ )	Dimensionality (reduced dimensionality)
$P$ ( $p$ )	Original data point (reduced representation)
$Q$ ( $q$ )	Query (reduced representation)
$RS$ ( $FH$ )	Set of the actual $k$ NNs (false hits) of $Q$
$VO_R$ ( $VO_P$ )	VO that authenticates range $R$ (point $P$ )
$N$	MR-Tree node
$h_p$ / $h_p$ / $h_N$	Digest of $P$ / $p$ / $N$
$DST$ ( $dst$ )	Distance metric in the original (reduced) space

Fig. 3. The server: 1) retrieves the  $k$ NNs of  $Q$  (i.e.,  $P_2$ ,  $P_3$ , and  $P_7$ ) using any conventional algorithm, 2) computes the distance  $DST_{\max}$  of the  $k$ th NN (i.e.,  $DST_{\max} = DST(Q, P_7)$ ), and 3) finally executes  $MR\_Range$  treating  $(Q, DST_{\max})$  as the range. For this example, the VO is identical to that of  $Q_R$  in Fig. 3 since both the resulting points and accessed nodes are the same. The verification process of the client is also identical to the one performed for range queries. However, as an adaptation of the R\*-Tree, the MR-Tree also suffers from the dimensionality curse [2]. Therefore, the application of the afore-mentioned method on high-dimensional data has very limited pruning power. Specifically, for numerous dimensions, nearly all leaf nodes must be visited (leading to high-server cost); consequently, the majority of points are inserted in the VO (leading to high-communication overhead); finally, the client has to verify almost the entire data set.

### 3 AUTHENTICATED MULTISTEP NN

Our work adopts the GEMINI framework because 1) it has been proven effective in nonauthenticated similarity retrieval, especially for numerous (i.e.,  $D > 100$ ) dimensions, where even high-dimensional indexes fail;<sup>3</sup> 2) it can be extended to authenticated query processing based on a low-dimensional ADS, i.e., the MR-Tree, whereas, currently there are no authenticated high-dimensional structures; and 3) it is general, i.e., it can also be applied when the expensive distance computations are due to the nature of the distance definition (e.g., network distance), rather than the data dimensionality (in which case  $D = d$ ).

We assume a client-server architecture, where the server maintains data signed by a trusted authority. There are two versions of the signed data set: a  $D$ -dimensional  $DB$  and a  $d$ -dimensional  $db$  ( $d \ll D$ ), produced from  $DB$  using any dimensionality reduction technique that satisfies the lower bounding property. For instance,  $DB$  may be a set of high-dimensional time series and  $db$  their low-dimensional representations obtained by DFT. There is a single signature  $sig$ , generated by a public key cryptosystem (e.g., RSA), that captures both  $DB$  and  $db$ .  $DST$  ( $dst$ ) refers to the distance metric used in the  $D(d)$ -dimensional space. For ease of illustration, we use euclidean distance for both the  $DST$  and  $dst$  metrics. Nevertheless, the proposed techniques are

3. High-dimensional indexes for exact NN retrieval, such as the state-of-the-art  $i$ -distance [8], are designed for up to about 50 dimensions, whereas we perform experiments with  $D$  up to 1,024.

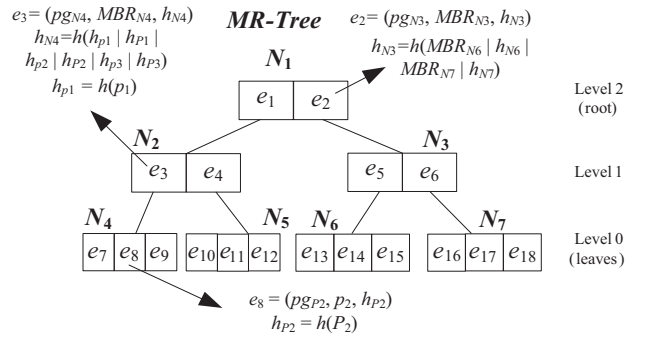


Fig. 5. Indexing scheme in AMN.

independent of these metrics, as well as of the underlying dimensionality reduction technique.

The proposed *Authenticated Multistep* NN (AMN) extends the multistep NN algorithm of [19] to our setting. As opposed to optimizing the processing cost at the server, the major objective of AMN (and any query authentication technique, in general) is to minimize 1) the network overhead due to the transmission of the VO, and 2) the verification cost at the client (which is assumed to have limited resources compared to the server). Section 3.1 describes the indexing scheme of AMN, while Section 3.2 presents the query processing and verification algorithms. Table 1 summarizes the most important symbols used throughout the paper.

#### 3.1 Indexing Scheme

The server indexes  $db$  using an MR-Tree. Since authentication information should capture both low- and high-dimensional representations, AMN necessitates the following modifications on the structure of the MR-Tree. Each leaf (level 0) entry  $e_{lf}$  has the form  $(pg_P, p, h_P)$ , where  $p \in db$  is the reduced representation of  $P \in DB$ , and  $h_P$  is the digest of the binary representation of  $P$ . Pointer  $pg_P$  points to the disk page(s) storing  $P$ . An intermediate MR-Tree node entry  $e_{in}$  has the form  $(pg_{N_c}, h_{N_c}, MBR_{N_c})$ , where  $pg_{N_c}$  is a pointer to a child node (let  $N_c$ ), and  $MBR_{N_c}$  is the MBR of the points in  $N_c$ . The value  $h_{N_c}$  depends on the level. For level 1,  $h_{N_c} = h(h_{p1}|h_{p1}|h_{p2}|h_{p2}| \dots |h_{pf}|h_{pf})$ , where  $h_{pi}$  ( $h_{pi}$ ) denotes the digest of  $p$  (resp.  $P$ ) in the  $i$ th entry in  $N_c$ . At higher levels,  $h_{N_c}$  is computed as in the traditional MR-Tree. Observe that the digests of *both* reduced and original points in the tree are incorporated into the root digest  $h_{root}$ . The trusted authority generates a signature  $sig$  by signing  $h_{root}$ . The server maintains  $DB$ , the MR-Tree and  $sig$ . Fig. 5 outlines the indexing scheme of AMN, assuming the data points and node structure of Fig. 3. Note that the proposed techniques are independent of the underlying index. For instance, an authenticated high-dimensional index (if such an ADS existed), would permit higher values of  $d$  (compared to the MR-Tree).

#### 3.2 Query Processing and Verification

Let  $Q$  be a  $D$ -dimensional query point received at the server. The goal of AMN is to return to the corresponding client the  $k$ NNs of  $Q$ , in a verifiable manner. The server starts processing  $Q$  by reducing it to a  $d$ -dimensional point  $q$ , using the same dimensionality reduction technique as for

**Algorithm AMN\_server(Q, k)**

```

// Q is the query and q is its reduced representation
1. Retrieve the k NNs {p1, ..., pk} of q in d-dimensional space
2. Retrieve the respective {P1, ..., Pk} in D-dimensional space
3. Set FH = ∅, RS = {P1, ..., Pk}, and compute DSTmax over RS
4. p = next NN of q in d-dimensional space
5. While (dstmin = dst(q, p)) < DSTmax
6.   Retrieve P // D-dimensional representation of p
7.   If DST(Q, P) > DSTmax, Insert P into FH
8.   Else
9.     Insert P into RS
10.    Remove Pk from RS and insert it into FH
11.    Update DSTmax over RS
12.    p = next NN of q in d-dimensional space
13. VOR = MR_Range(q, DSTmax, root)
14. Send RS, FH, VOR and sig to the client
    
```

 Fig. 6. Authenticated  $k$ NN processing at the server.

DB. Fig. 6 provides the pseudocode for AMN at the server for arbitrary values of  $k$ . Initially (Lines 1-4), the algorithm

1. extracts the  $k$  closest points  $\{p_1, \dots, p_k\}$  to  $q$  using an incremental NN algorithm (e.g., [7]) on the MR-Tree;
2. it retrieves their high-dimensional representations  $P_1, \dots, P_k$  from the disk, following the pointers from the leaf entries in the MR-Tree,
3. it initializes a false hit set  $FH = \emptyset$  and a result set  $RS = \{P_1, \dots, P_k\}$ , and computes  $DST_{max}$ , i.e., the  $DST$  of the current  $k$ th NN  $P_k$ ;
4. it obtains the next NN (i.e., the  $(k+1)$ th NN) point  $p$  of  $q$  from the tree.

The procedure then enters the loop of Lines 5-12, where it computes distance  $dst_{min} = dst(q, p)$ . Observe that  $dst_{min}$  is the minimum distance between  $q$  and any point  $p$ , whose high-dimensional representation has not yet been retrieved. If  $dst_{min} > DST_{max}$ , the algorithm terminates. Otherwise, it retrieves the true representation  $P$  of  $p$  from the disk. If  $DST(Q, P) > DST_{max}$ , then  $P$  is a false hit and appended to  $FH$ ; else,  $P$  is a candidate result and is inserted into  $RS$ . This causes the deletion of the current  $k$ th NN  $P_k$  from  $RS$ , and its insertion into  $FH$ . The algorithm updates  $DST_{max}$  over the new  $RS$ , and proceeds to retrieve the next NN of  $q$  in the tree. After the NN loop terminates, Line 13 performs the authenticated range query  $q_R = (q, DST_{max})$  using the most updated value of  $DST_{max}$ . This produces a  $VO_R$  that contains 1) a pair  $(h_N, MBR_N)$  for every pruned node  $N$ , 2) a pair  $(h_P, p)$  for every point  $p$  in a leaf node that intersects  $q_R$ , but whose  $P$  representation is not in  $RS \cup FH$ , 3) a *result* (*false\_hit*) token for every index point whose true point is in  $RS$  ( $FH$ ).

We illustrate AMN using Fig. 7 and assuming that  $k = 1$  and  $d = 2$ . Lines 1-3 (see Fig. 6) set  $FH = \emptyset$ ,  $RS = \{P_1\}$ , and  $DST_{max} = DST(Q, P_1)$ . Let  $SR$  (search region) be the area within distance  $(dst(q, p_1), DST_{max})$  from  $q$ , i.e., the shaded area in Fig. 7a. Only points in  $SR$  are candidate results. The server proceeds by retrieving the next NN  $p_2$  of  $q$  in  $db$ , and its high-dimensional representation  $P_2$ . Assuming that  $DST(Q, P_2) < DST(Q, P_1)$  ( $= DST_{max}$ ),  $P_2$  becomes the new NN of  $Q$  and it is inserted into  $RS$ . Moreover,  $P_1$  becomes a false hit, and is moved from  $RS$  to  $FH$ . The server then updates  $DST_{max}$  to  $DST(Q, P_2)$ , which leads to the shrinking of the  $SR$

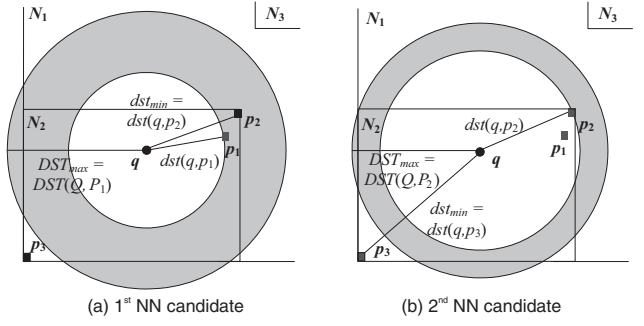


Fig. 7. Finding the 1NN. (a) 1st NN candidate. (b) 2nd NN candidate.

as shown in Fig. 7b. The next NN  $p_3$  of  $q$  falls out of  $SR$ , and NN retrieval terminates with  $RS = \{P_2\}$  and  $FH = \{P_1\}$ . Next, the server performs an authenticated range query  $q_R = (q, DST_{max})$ , where  $DST_{max} = DST(Q, P_2)$ . The result of  $q_R$  contains the low-dimensional representations of  $p_1$  and  $p_2$ . Considering that the MR-Tree in Fig. 7 consists of root node  $N_1$ , and its two children  $N_2$  and  $N_3$ , the  $VO$  of  $q_R$  is  $VO_R = [[result, false\_hit, (p_3, h_{P_3})](h_{N_3}, MBR_{N_3})]$ , i.e., it is computed as in the traditional MR-Tree, with two differences: 1)  $p_3$  is inserted along with its associated digest  $h_{P_3}$  (since this is necessary for computing  $h_{root}$ ), and 2) two tokens are used as placeholders, one corresponding to the reduced representation of an actual result (*result*), and one of a false hit (*false\_hit*). Note that it is redundant to provide  $p_1$  and  $p_2$  because the client can derive them from  $P_1$  and  $P_2$  included in  $RS \cup FH$ . Signature  $sig$ ,  $RS$ ,  $FH$ , and  $VO_R$  are transmitted to the client.

Having  $RS$ ,  $FH$ , and  $VO_R$ , the client can reconstruct the signed digest  $h_{root}$  using the verification algorithm of the MR-Tree with two alterations: 1) it derives the reduced point  $p$  and digest  $h_P$  of every point  $P$  in  $RS$  ( $FH$ ), and substitutes its corresponding *result* (*false\_hit*) token in  $VO_R$  with  $(p, h_P)$ ; 2) it computes the level-1 digests as in Fig. 5. Fig. 8 provides the pseudocode for the verification algorithm. After reconstructing and matching  $h_{root}$  against  $sig$ , the procedure computes  $DST_{max}$  over  $RS$ . It then establishes that the points in  $FH$  are indeed false hits. Finally, it confirms that all points and MBRs in  $VO_R$  (other than the *result*/*false\_hit* tokens) lie further than  $DST_{max}$  from  $q$ . In the running example, the client first computes  $p_1$  and  $p_2$  from  $P_1$  and  $P_2$ , and  $MBR_{N_2}$  using  $p_1, p_2$ , and  $p_3$ . Then, it generates  $h_{N_2} = h(h_{p_1}|h_{p_2}|h_{p_3}|h_{P_3})$ , and subsequently  $h_{root} = (h_{N_2}|MBR_{N_2}|h_{N_3}|MBR_{N_3})$ . If  $h_{root}$  matches  $sig$ , the client evaluates  $DST_{max} = DST(Q, P_2)$  and ascertains that  $DST(Q, P_1) > DST_{max}$ . Eventually, it establishes that the distance of  $p_3$  from  $q$ , as well as the minimum distance between  $MBR_{N_3}$  and  $q$ , is indeed larger than  $DST_{max}$ .

**Algorithm AMN\_client(R, VO<sub>R</sub>)**

```

1. MR_verify(RS, FH, VOR)
2. Compute DSTmax over RS
3. For each point Pi in FH, Verify DST(Q, Pi) > DSTmax
4. For each point pi (MBRi) in VOR,
5.   Verify dst(q, pi) > DSTmax (mindist(q, MBRi) > DSTmax)
    
```

 Fig. 8.  $k$ NN verification at the client.

Full Representations																
$i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$Q$ :	5	6	10	3	7	40	2	2	5	20	25	10	5	3	3	7
$P_1$ :	25	6	10	4	7	20	2	5	5	20	25	10	5	3	13	7
$P_2$ :	24	15	11	3	22	40	1	0	4	18	20	20	6	5	4	7

Reduced Representations		
$q$ :	9.5625	-2.1908
$p_1$ :	10.4375	-0.1698
$p_2$ :	12.5	-0.0279

Distances			
$DST(Q, P_1)$	= 30.1662	$dst(q, p_1)$	= 2.2023
$DST(Q, P_2)$	= 28.443 ( $DST_{max}$ )	$dst(q, p_2)$	= 3.6479

Fig. 9. Representations of  $Q, P_1, P_2, q, p_1, p_2$  (see Fig. 7).

**Proof of Correctness.** We distinguish two cases: 1) the server returns to the client a set  $RS'$ , which is derived from the correct  $RS$  after substituting a point  $P$  with another  $P'$  that does not exist in  $DB$  (i.e.,  $P'$  is bogus). Since the signature does not incorporate authentication information about  $P'$ , the reconstructed  $h_{root}$  does not match  $sig$  due to the collision-resistance of the hash function, and thus, the client is alarmed. 2) The server returns to the client a set  $RS'$ , which is derived from  $RS$  after substituting a point  $P$  with another  $P'$  that belongs to  $DB$  (i.e.,  $P'$  is legitimate). Let  $DST_{max}$  ( $DST'_{max}$ ) denote the distance between  $Q$  and its  $k$ th NN in  $RS$  ( $RS'$ ). Since  $P'$  is not a true result,  $DST'_{max} > DST_{max}$ . The server also generates a  $VO'_R$ , which verifies  $RS'$ , i.e., it authenticates range  $q'_R = (q, DST'_{max})$ . Point  $p$  lies in range  $q'_R$  because  $dst(q, p) \leq DST(Q, P) \leq DST_{max} < DST'_{max}$ . Given that  $P$  is not in  $RS'$ , it must be included in  $FH$  (otherwise, the verification of  $VO'_R$  will fail). Thus, Line 3 of algorithm  $AMN\_client$  detects the violation and the client is alarmed.  $\square$

Since AMN follows the optimal framework of Fig. 1, it is also optimal in terms of  $DST$  distance computations. A more important question in our setting regards its performance in terms of the communication overhead and the verification cost at the client. Recall that, along with the result, the client needs a representation for each  $P \in FH$  in order to verify that indeed  $P$  is a false hit. For typical data series applications,  $D$  can be up to 1-3 orders of magnitude larger than  $d$ . Therefore,  $FH$  emerges as the most important performance factor in authenticated NN search, especially for very high-dimensional data. However, as we show next,  $FH$  does not have to include the complete representations of false hits.

#### 4 COMMUNICATION-EFFICIENT AMN

Depending on the dimensionality reduction technique, the values  $D, d$ , and  $k$ , and the data set characteristics, there may be numerous false hits in  $FH$ , each containing hundreds or thousands (i.e.,  $D$ ) values. Next, we propose communication-efficient AMN (C-AMN), which decreases the size of the false hits, significantly reducing the transmission and verification cost without compromising the security of AMN. Section 4.1, explains the main concepts of C-AMN, whereas Section 4.2 presents the concrete algorithm for false hit reduction.

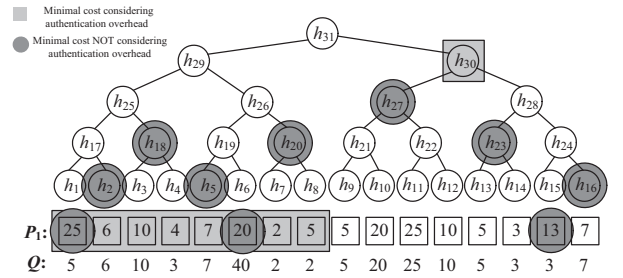


Fig. 10. MH-Tree over  $P_1$ .

#### 4.1 General Framework

We illustrate the main idea of C-AMN through Fig. 9, where 1)  $Q, P_1, P_2$  are 16-dimensional time series (i.e.,  $D = 16$ ), 2)  $q, p_1, p_2$  are their two-dimensional representations obtained by taking the first two coefficients of the DFT decomposition (i.e.,  $d = 2$ ), and 3)  $DST$  and  $dst$  correspond to euclidean distances. The coefficients and distances are computed using the real values of  $Q, P_1$ , and  $P_2$ . Since  $DST(Q, P_2) < DST(Q, P_1)$ ,  $P_2$  is the 1NN of  $Q$ . Furthermore,  $dst(q, p_2) > dst(q, p_1)$ , which signifies that  $P_1$  is a false hit, thus all its 16 values are included in  $FH$  by AMN.

Let  $P[i] (1 \leq i \leq D)$  be the  $i$ th value of  $P$  (e.g.,  $P_1[2] = 6$  see Fig. 9), and  $SP$  be an ordered subset of  $P$  values, e.g.,  $SP_1 = (P_1[1], P_1[6], P_1[15]) = (25, 20, 13)$ .  $SQ_1$  contains the values of  $Q$  in the same positions as  $SP_1$ , e.g.,  $SQ_1 = (Q[1], Q[6], Q[15]) = (5, 40, 3)$ . Then,  $DST(SQ_1, SP_1) = 28.4605 > DST(Q, P_2) (= 28.443)$ . Thus, instead of the complete  $P_1$ , it suffices for the server to include into  $FH$  only  $SP_1$ . The client computes  $SQ_1$  from  $Q$ , establishes that  $DST(Q, P_2) < DST(SQ_1, SP_1) \leq DST(Q, P_1)$ , and confirms  $P_1$  as a false hit. Assuming that each value has size  $S_v = 8$  bytes,  $SP_1$  consumes 24 bytes, as opposed to 128 bytes for  $P_1$ . By sending  $SP_1$  to the client, the server reduces the communication cost significantly (this is the largest possible reduction for the current example). However, it must also prove that  $SP_1$  is not falsified.

We next demonstrate a solution to this problem, assuming, for simplicity, that  $D$  is a power of 2. The server maintains an MH-Tree over  $P_1$  as shown in Fig. 10. Recall that in AMN, the digest  $h_{P_1}$  included in the MR-Tree leaf accommodating  $p_1$  is computed on the concatenation of the binary representation of the values of  $P_1$ , i.e.,  $h_{P_1} = h(P_1[1]||P_1[2]|\dots|P_1[16])$ . Instead, in C-AMN,  $h_{P_1}$  is the digest  $h_{31}$  in the root of the MH-Tree, which summarizes authentication information about the entire  $P_1$ . The rest of the MR-Tree structure remains the same. Returning to our running example, after producing  $SP_1$ , the server constructs a  $VO$  (denoted as  $VO_{SP_1}$ ) that can verify  $SP_1$ .  $VO_{SP_1}$  contains the necessary components for reconstructing  $h_{P_1}$ . These components are highlighted in dark gray in Fig. 10:  $VO_{SP_1} = [ [ [ [25 \ h_2] \ h_{18}] [h_5 \ 20] \ h_{20}] [h_{27} [h_{23} [13 \ h_{16}]]]]]$ . Note that the positions<sup>4</sup> of  $SP_1$  values can be computed based on the structure of  $VO_{SP_1}$ . For example, prior to 20 in  $VO_{SP_1}$ , there is one value (25), two level-0 digests ( $h_2, h_5$ ) and a level-1 digest ( $h_{18}$ ). This implies that 20 is the 6th value in  $P_1$ .

4. The positions of the  $P_1$  values used in  $SP_1$  are necessary so that the client can compute  $SQ_1$  from  $Q$ .

Given  $VO_{SP_1}$ , the client produces  $h_{p_1}$  by performing successive hash operations on the contents of corresponding “[and]” tokens as explained in Section 2.3. Finally, recall that the digest of  $p_1$ ,  $h_{p_1}$ , is needed for computing the level-1 digest of  $N_2$ ,  $h_{N_2} = h(h_{p_1}|h_{p_2}|h_{p_3}|h_{p_4})$ . In AMN, the client could compute  $h_{p_1}$  by first reducing  $P_1$  (fully included in  $FH$ ) to  $p_1$ , and then, calculating  $h_{p_1} = h(p_1)$ . In C-AMN, however, the client does not have the entire  $P_1$ , and it cannot generate  $p_1$  from  $SP_1$ . Therefore, the server must send  $h_{p_1}$  to the client as well. The additional authentication information increases the communication overhead. Continuing the example, and assuming that a digest has size  $S_h = 20$  bytes (a typical value),  $VO_{SP_1}$  and  $h_{p_1}$  consume a total of 184 bytes, which is larger than  $P_1$  (128 bytes). This is due to the fact that the values of  $SP_1$  are dispersed in the MH-Tree, causing the insertion of numerous digests in  $VO_{SP_1}$ . Consider now that we modify  $SP_1$  to include values  $P_1[1]-P_1[8]$ . In this case,  $DST(SQ_1, SP_1) = 28.46 > DST_{max}$ , and  $VO_{SP_1} = [[[[[25\ 6][10\ 4]][[7\ 20][2\ 5]]] h_{30}]$  contains a single digest,  $h_{30}$  (light gray see Fig. 10). The total cost is now 106 bytes, which is actually the lowest possible in this example. Note that for simplicity, we omit the headers of the  $VO$  elements (see Section 2.4) in the size computations because their space consumption is negligible.

Summarizing, C-AMN aims at replacing each false hit  $P$  with a verifiable representation  $SP$  that consumes less space. C-AMN necessitates some modifications over the AMN indexing scheme: 1) for every  $P \in DB$ , the server maintains a MH-Tree, and 2) each digest  $h_P$  at the leaf level of the MR-Tree is set to the root digest of the MH-Tree of  $P$ . Query processing at the server proceeds as in Fig. 6, but after computing  $RS$ ,  $FH$ , and  $VO_R$ , the server calls a function *ReduceFH* to replace every false hit  $P$  with a pair  $(VO_{SP}, h_p)$ , where  $VO_{SP}$  contains the subset  $SP$  of  $P$  that proves that  $P$  is a false hit, along with verification information, and  $h_p$  is the digest of  $P$ 's indexed point  $p$ .

Verification at the client is similar to *AMN\_client* (see Fig. 8), with the following alterations: 1) *MR\_verify* computes, for every pair  $(VO_{SP}, h_p) \in FH$ , the digest  $h_P$  of the corresponding false hit  $P$ , simply simulating the initial calculation of  $h_{root}$  in the MH-Tree of  $P$ . 2) For every  $VO_{SP}$  in  $FH$ , Line 3 extracts  $SP$  from  $VO_{SP}$ , computes the respective  $SQ$  of  $Q$ , and verifies that  $DST(SQ, SP) > DST_{max}$ . The proof of correctness of C-AMN is identical to that in AMN, given that there is no way for the server to falsify  $SP$  (otherwise, the reconstructed  $h_{root}$  would not match *sig* due to the collision-resistance property of the hash function).

## 4.2 False Hit Reduction Algorithm

Ideally, for each false hit  $P$ , *ReduceFH* should derive the subset  $SP$  with the minimum length. Intuitively, this task is at least as difficult as the *Knapsack Problem*; we need to select a subset of items ( $SP$  of  $P$  values), each assigned a cost (communication overhead) and a weight (distance  $DST(SQ, SP)$ ), such that the sum of costs is minimized and the sum of weights exceeds  $DST_{max}$ . An additional complication is that when we select one item, the cost of the rest changes (i.e., unlike knapsack, where the cost is fixed).

**Theorem 1.** *Determining the optimal subset  $SP$  of  $P$ , which leads to the minimal  $VO_{SP}$  under the constraint that  $DST(SQ, SP) > DST_{max}$ , is NP-hard.*

**Proof.** This can be proven by a straightforward polynomial reduction from the NP-Hard *Precedence Constraint Knapsack Problem* [9].  $\square$

We next propose a *greedy* version of *ReduceFH*. Let  $dist_i = (P[i] - Q[i])^2$  be the contribution of  $P[i]$  to  $DST(SQ, SP)$ , and  $comm_i$  the additional cost if  $P[i]$  is inserted in the current  $VO_{SP}$ . To assist the iterative decisions of our greedy algorithm, we assign to each unselected value  $P[i]$  a *benefit*  $B[i] = dist_i / comm_i$  that captures the potential gain if  $P[i]$  is included in  $SP$ . For example, in Fig. 10, suppose that  $SP_1 = (P_1[6])$ ; hence  $VO_{SP_1} = [[h_{25}[[h_5\ 20]h_{20}] h_{30}]$ . If we insert  $P_1[1]$  in  $SP_1$ , the new  $VO_{SP_1}$  verifying  $SP_1 = (P_1[1], P_1[6])$  becomes  $[[[[[20\ h_2] h_{18}][[h_5\ 20]h_{20}] h_{30}]]$ . The increase of  $VO_{SP_1}$  due to the insertion of  $P_1[1]$  is  $comm_1 = S_h + S_v = 28$  ( $[[20\ h_2]h_{18}]$  substitutes  $h_{25}$  in  $VO_{SP_1}$ ). At each step, the unselected  $P[i]$  with the *highest* benefit  $B[i]$  is greedily inserted in  $SP$ . Intuitively, we prefer values with large *dist* (to satisfy  $DST(SQ, SP) > DST_{max}$  with as few values as possible), and small *comm* (to keep  $VO_{SP}$  as small as possible). After inserting  $P[i]$  in  $SP$ , we set  $B[i] = 0$  (so that it cannot be selected again). We also update (if necessary) the benefits of the yet unselected  $P$  values, since *comm* depends on the  $VO_{SP}$  verifying the *current*  $SP$ . The algorithm terminates when the condition  $DST(SQ, SP) > DST_{max}$  is satisfied.

The most expensive operation is the update of the benefits at each step. A naive algorithm would take  $O(D^2)$  time in overall; upon each selection, it would scan *all* unselected  $P$  values and update their benefits as necessary. We next present an implementation with complexity  $O(D \cdot \log D)$ . We demonstrate the process on  $P_1$  of our running example using Fig. 11. The algorithm initially sets  $SP_1 = \emptyset$ , and  $VO_{SP_1} = \emptyset$ . Moreover, it assigns  $comm_i = 4 \cdot S_h + S_v = 84$  for all  $P_1[i]$ , since inserting any  $P_1[i]$  in  $VO_{SP_1}$  would cause its increase by four digests and one value. Subsequently, it calculates the benefit vector  $B$  of  $P_1$ , and constructs an auxiliary binary tree on  $B$ , where 1) each leaf node  $n_i$  stores  $B[i]$ , and 2) inner nodes are created bottom-up, storing the *maximum* value in their subtree. In the first step, *ReduceFH* performs a root to leaf traversal of the tree, visiting always the node with the maximum value. In Fig. 11a, the leaves with the maximum benefits are  $n_1$  and  $n_6$  because they have the largest distances to  $Q_1[1]$  and  $Q_1[6]$ , respectively. Suppose that the algorithm (randomly) visits  $n_6$ ; it inserts  $P_1[6] = 20$  into  $SP_1$ , and updates the value of  $n_6$  to 0 (so that  $n_6$  will not be selected in the future).  $VO_{SP_1}$  becomes  $[[h_{25}[[h_5\ 20]h_{20}] h_{30}]$ , and  $DST(SQ, SP_1) = 20 < DST_{max}$ . Therefore, the algorithm continues.

The next action regards the updating of the benefits in the rest of the tree. Observe that, if after the inclusion of  $P_1[6]$  in  $SP_1$ , we also insert  $P_1[5]$ , digest  $h_5$  in  $VO_{SP_1}$  will be replaced by a value. Thus, the size of  $VO_{SP_1}$  will *decrease* because  $S_h$  (20 bytes) is larger than  $S_v$  (8 bytes). In general, depending on the relative size of  $S_h$  and  $S_v$ , there is a *cutoff level* ( $cl$ ), such that, if a value  $P[i]$  is inserted in  $SP$ , all the benefits in the same subtree as  $n_i$  rooted at  $cl$  will become negative because of the reduction of the communication cost. To include the corresponding values in  $SP$ , we set their respective benefits to  $\infty$ . In our example  $cl = 2$ , and the

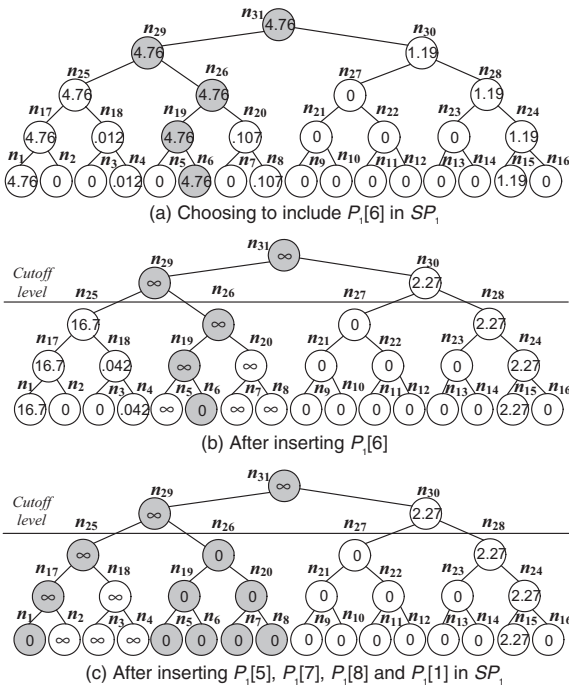


Fig. 11. Using the auxiliary tree of *ReduceFH*. (a) Choosing to include  $P_1[6]$  in  $SP_1$ . (b) After inserting  $P_1[6]$ . (c) After inserting  $P_1[5]$ ,  $P_1[7]$ ,  $P_1[8]$  and  $P_1[1]$  in  $SP_1$ .

selection of  $P_1[6]$  will assign  $\infty$  to the benefits of nodes  $n_5$ ,  $n_7, n_8, n_{19}, n_{20}, n_{26}, n_{29}, n_{31}$ . The benefits of the remaining nodes are updated as shown in Fig. 11b.

In the three next steps, *ReduceFH* follows the nodes with  $\infty$  benefit and inserts  $P_1[5]$ ,  $P_1[7]$ , and  $P_1[8]$  into  $SP_1$ , updating their corresponding benefits (as well as the values of their ancestors) to 0. Observe that these insertions do not influence the benefits of the nodes in the subtrees rooted at  $n_{25}$  and  $n_{30}$ . The general rule is that  $comm_i$  changes, only if the insertion of a  $P[j]$  into  $SP$  causes the substitution (in  $VO_{SP}$ ) of a digest that was taken into account in  $comm_i$ . Note that each such digest corresponds to a subtree of the auxiliary tree in *ReduceFH*, since the latter has a one-to-one correspondence with the MH-Tree of  $P$ . For example, in Fig. 11b (i.e., after  $P_1[6]$  has been inserted in  $SP_1$ ),  $comm_9$  considers digests  $h_{10}$ ,  $h_{22}$ , and  $h_{28}$ , corresponding to nodes  $n_{10}$ ,  $n_{22}$ , and  $n_{28}$ , respectively. After the insertion of  $P_1[5]$ ,  $comm_9$  still involves the above digests, and thus, does not change. *ReduceFH* marks every visited node of the tree during the selection of the next  $P_1$  value.

The marked nodes in Fig. 11 are highlighted in gray color. A marked node signifies that there is at least one leaf  $n_i$  in its subtree such that  $P_1[i]$  is in  $SP_1$ . In each new traversal, if a visited node  $n$  is not marked, then the benefits in the subtree of  $n$ 's sibling  $n_s$  must be updated. For instance, in Fig. 11b,  $comm_9$  takes into account  $h_{28}$  corresponding to  $n_{28}$ , which is not marked. If  $n_{28}$  is visited, the benefit of  $n_9$  (i.e., of  $P_1[9]$ ), and of all leaves in the subtree rooted at  $n_{27}$ , will change. Returning to our example, at this point,  $SP_1 = (P_1[6], P_1[5], P_1[7], P_1[8]) = (20, 7, 2, 5)$ , and  $DST(SQ, SP_1) = 20.224 < DST_{max}$ . In the next step, the procedure inserts  $P_1[1] = 25$  into  $SP_1$  (it has the largest benefit), and updates the benefits of  $n_2, n_3, n_4, n_{18}, n_{25}, n_{29}, n_{31}$  to  $\infty$ , as shown in Fig. 11c. Subsequently,  $P_1[2], P_1[3], P_1[4]$  are inserted because their

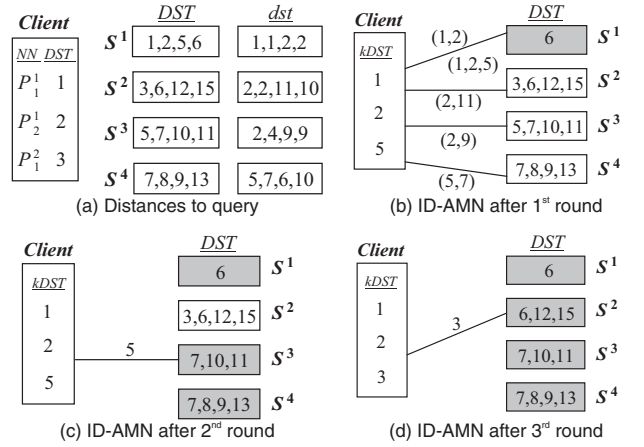


Fig. 12. Algorithm *ReduceFH*.

benefit is  $\infty$ . The resulting  $SP_1 = (P_1[6], P_1[5], P_1[7], P_1[8], P_1[1], P_1[2], P_1[3], P_1[4]) = (20, 7, 2, 5, 25, 6, 10, 4)$  satisfies the constraint  $DST(SQ, SP_1) = 28.461 > DST_{max}$ . Its corresponding  $VO$  is equal to  $VO_{SP_1} = [[[[[25\ 6][10\ 4]][[7\ 20][2\ 5]]] h_{30}]$ . Observe that  $SP_1$  and  $VO_{SP_1}$  are the same as the ones we constructed for Fig. 9, and are optimal for this example.

Fig. 12 provides the generalized pseudocode of *ReduceFH*. Function *ConstructTree* in Line 2 computes the initial benefit vector  $B$  of  $P$  and builds a binary tree over it. The loop in Lines 3-5 first calls function *FindNextValue*, which retrieves the next value  $P_v$  with the maximum benefit. It also updates the benefits of the affected nodes. As soon as the condition in Line 3 is satisfied, Lines 6-8 include the values with benefit  $\infty$  into  $SP$  as well. *ReduceFH* eventually returns  $SP$ .

**Complexity analysis.** The construction of the tree takes  $O(D)$  time. Furthermore, the loops in Lines 3-5 and 6-8 involve  $O(D)$  steps. *FindNextValue* involves two operations: 1) finding the value with the maximum benefit, which takes  $O(\log D)$  time, and 2) updating the benefits of the affected nodes. Instead of calculating the latter cost in each step of *FindNextValue*, we will compute it collectively in the entire execution of *ReduceFH*. We mentioned that the benefit of a node  $n_i$  is updated, only when the subtree corresponding to a digest involved in  $comm_i$  is marked for the first time. Since the maximum number of digests entailed in  $comm_i$  is  $\log D$ , the total time required for updating all values is  $O(D \cdot \log D)$ , which is also the complexity of *ReduceFH* for each false hit.

## 5 AMN IN DISTRIBUTED SERVERS

In this setting, we assume that the database is horizontally partitioned and distributed over  $m$  ( $>1$ ) servers. Specifically, each server  $S^i$  stores a subset  $DB^i$  such that:  $DB^1 \cup \dots \cup DB^m = DB$  and  $DB^i \cap DB^j = \emptyset, \forall i, j \leq m$ . In addition,  $S^i$  maintains an MR-Tree on the corresponding reduced data set  $db^i$ , which is signed by a signature  $sig^i$ . A query result comprises the  $k$ NNs over all servers. Minimization of transmissions (of the high-dimensional data) is particularly important for this setting, especially for large values of  $m$ . Section 5.1 presents SD-AMN (short for *simple distributed AMN*), used as a benchmark in our experimental evaluation. Section 5.2 proposes ID-AMN



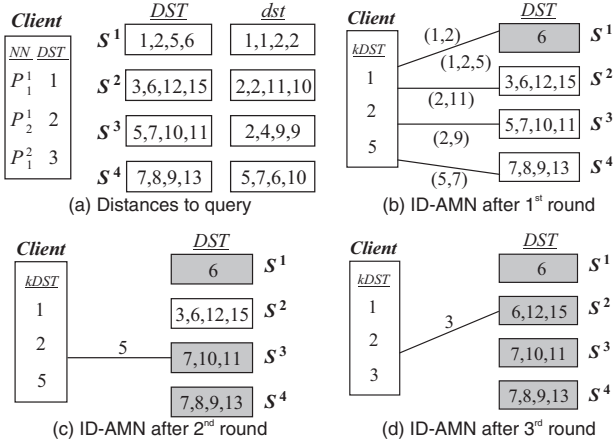


Fig. 13. Distributed authenticated  $k$ NN processing. (a) Distances to query. (b) ID-AMN after 1<sup>st</sup> round. (c) ID-AMN after 2<sup>nd</sup> round. (d) ID-AMN after 3<sup>rd</sup> round.

(short for *incremental distributed AMN*), a more elaborate method, which quickly eliminates servers that cannot contribute results.

## 5.1 Simple Distributed AMN

In SD-AMN, a client sends its  $k$ NN query  $Q$  to all servers. Each server  $S^i$  retrieves the partial result  $RS^i$  on the local  $DB^i$  using the conventional multistep algorithm of Fig. 1, and generates a vector  $kDST^i$  with the *distance values* of the  $k$ NN set  $RS^i$  in  $S^i$ . The client collects the vectors from the servers and determines the global  $k$ th nearest distance  $DST_{max}$  over all  $m \cdot k$  collected distances. Then, it transmits a range  $q_R = (q, DST_{max})$ . Each server  $S^i$  executes  $q_R$  using its MR-Tree and returns  $VO_R^i$ ,  $RS^i$ , and  $FH^i$ .  $VO_R^i$  has the same meaning as in centralized processing, i.e., it is the  $VO$  of  $q_R$ .  $RS^i$  (resp.  $FH^i$ ) is a set of results (resp. false hits), i.e., points of  $db^i$  that fall in  $q_R$  and whose high-dimensional representations have distance from  $Q$  smaller (resp. larger) than  $DST_{max}$ . The size of  $FH$  can be reduced through C-AMN.

We demonstrate SD-AMN using the example of Fig. 13a, assuming a 3NN query  $Q$ . There are four servers, each containing four points. For ease of illustration, we sort these points on their distance to  $Q$  and display their  $DST$ , e.g.,  $P_1^2$  is the first point in  $S^2$  and  $DST(Q, P_1^2) = 3$ . The diagram also includes the distances in the reduced space, e.g.,  $dst(q, p_1^2) = 2$ . Given  $kDST^1 = (1, 2, 5)$ ,  $kDST^2 = (3, 6, 12)$ ,  $kDST^3 = (5, 7, 10)$ , and  $kDST^4 = (7, 8, 9)$ , the client computes  $DST_{max} = 3$  (the first two NNs are in  $S^1$ , and the third one in  $S^3$ ), and transmits the range  $q_R = (q, 3)$ .  $S^1$  returns  $VO_R^1$ ,  $RS^1 = \{P_1^1, P_2^1\}$ , and  $FH^1 = \{P_3^1, P_4^1\}$ .  $P_3^1$  and  $P_4^1$  are necessary, in order for the client to establish that they are indeed false hits. At the other extreme,  $S^4$  returns  $VO_R^4$ ,  $RS^4 = FH^4 = \emptyset$ , as it does not contain results or false hits (for all points in  $S^4$ , their  $dst$  from  $q$  exceeds 3). If each  $VO_R^i$  is verified successfully, and for every point  $P$  in each  $FH^i$  it holds  $DST(Q, P) = DST_{max}$ , then the client is assured that  $RS$  is correct.

**Proof of Correctness.** In SD-AMN, the client computes a  $DST_{max}$  (e.g., 3 see Fig. 13a) such that the range  $(Q, DST_{max})$  contains exactly  $k$  high-dimensional points in

$DB^1 \cup \dots \cup DB^m$ . At the last step of SD-AMN all servers perform a verifiable range  $q_R = (q, DST_{max})$ , during which they cannot cheat. Thus, they can only misreport the distance vectors  $kDST^i$ , leading the client to compute a  $DST'_{max}$  that is different from the real  $DST_{max}$ . We distinguish two cases with respect to the relationship between  $DST'_{max}$  and the  $DST_{max}$ . If  $DST'_{max} > DST_{max}$ , the client will obtain all results (possibly, in addition to more points) through the verifiable range  $(q, DST'_{max})$ , and will detect the discrepancy between  $DST'_{max}$  and  $DST_{max}$ . If  $DST'_{max} < DST_{max}$ , the client receives fewer than  $k$  objects in  $RS^1 \cup \dots \cup RS^m$  and is alarmed. Therefore, it can always establish result correctness.  $\square$

As an example, suppose that in Fig. 13a  $S^2$  misreports  $DST(Q, P_1^2)$  as 4 (instead of 3). Consequently, the client computes the distance to its 3<sup>rd</sup> NN as  $DST'_{max} = 4 > DST_{max}$ , and receives a  $VO$  for range  $(q, 4)$  from every server.  $S^2$  will then have to return  $P_1^2$  as a result, and the difference between the reported and actual  $DST(Q, P_1^2)$  will be revealed. For the second case, consider that server  $S^2$  misreports  $DST(Q, P_2^2)$  as 2, leading the client to compute  $DST'_{max} = 2 < DST_{max}$ . Upon receiving the  $VO$ s for range  $(q, 2)$  from all servers, it discovers that there are only two points ( $P_1^1, P_2^1$ ) in  $RS^1 \cup \dots \cup RS^4$ . Note that servers can misreport distances of points in  $DB$ -RS-FH without being caught, provided that the false  $DST$  reported are larger than  $DST_{max}$ . In the example of Fig. 13a, there are no low-dimensional objects of  $S^4$  within  $(q, 3)$ . Therefore,  $S^4$  can send to the client any vector that contains distances larger than 3 because the verification of  $(q, 3)$  does not involve the transmission of any point ( $RS^4 = FH^4 = \emptyset$ ). Clearly, this type of false values does not affect the result.

## 5.2 Incremental Distributed AMN

SD-AMN is optimal in terms of high-dimensional point transmissions because the client receives  $D$ -dimensional representations only for points in  $q_R$ . All these points (results and false hits) are necessary to establish correctness anyway. However, it must transmit  $Q$  to all servers. Moreover, each server  $S^i$  has to compute  $RS^i$  although none of the points of  $RS^i$  may participate in the global result (e.g.,  $S^4$  see Fig. 13). ID-AMN avoids these problems by gradually eliminating servers that cannot contribute results. Specifically, ID-AMN incrementally retrieves distance values from servers to compute the final  $DST_{max}$ , postponing local NN computations at the servers until they are required. We present the pseudocode of ID-AMN (at the client) in Fig. 14, and explain its functionality by continuing the example of Fig. 13 ( $k = 3$ ).

Initially, the client marks each server  $S^i$  as a candidate, and transmits  $q$  to  $S^i$ , which responds with two values: the  $dst^i$  ( $kdst^i$ ) of its 1<sup>st</sup> ( $k$ th) NN in the low-dimensional space. For instance, in Fig. 13, the client receives  $(1, 2)$ ,  $(2, 11)$ ,  $(2, 9)$ ,  $(5, 7)$  from  $S^1, S^2, S^3, S^4$ , respectively. Intuitively, a low  $kdst^i$  implies a promising server that may contain  $k$  good results.  $DST^i$  and  $dst^i$  are used for server pruning and selection, to be discussed shortly. Let  $S^j$  be the server (e.g.,  $S^1$ ) with the minimum  $kdst^j$ . The client directs  $Q$  to  $S^j$  and obtains a vector  $kDST$  with the distance values of the  $k$ NN set  $RS^j$  in  $S^j$ .  $DST_{max}$  is set to the  $k$ th distance in  $kDST$  and  $S^j$  ceases to be a candidate. Continuing the example,

**Algorithm ID-AMN\_client (Q, k)**


---

```

1. For each server  $S^i$ 
2.   Set  $Candidate[i]=1$ ;
3.    $(dst^i, kdst^i) = get\_smallest\_dist(q, S^i)$ 
4.    $DST^i = dst^i$ 
5. Let  $S^j$  be the server with the minimum  $kdst^i$ 
6. Set vector  $kDST = get\_k\_smallest\_DSTs(Q, S^i)$ 
7. Set  $DST_{max} =$  maximum value in  $kDST$ ; Set  $Candidate[j]=0$ 
8. While there are candidate servers
9.   For each server  $S^i$ 
10.    If  $DST^i \geq DST_{max}$ , Set  $Candidate[i]=0$ 
11.   Select candidate server  $S^i$  with minimum  $DST^i$ 
12.   Set  $DST_{new} = get\_next\_smallest\_DST(Q, S^i)$  from server  $S^i$ 
13.   If  $DST_{new} \geq DST_{max}$ , Set  $Candidate[i]=0$ 
14.   Else //  $DST_{new} < DST_{max}$ 
15.     Insert  $DST_{new}$  into  $kDST$ 
16.     Set  $DST_{max} =$  maximum value in  $kDST$ ;
17.      $DST^i = DST_{new}$ 
18. For each server  $S^i$ 
19.    $(VO_R^i, RS^i, FH^i) = MR\_Range(q, DST_{max}, root^i)$ 
20.    $Verify(VO_R^i)$  and incorporate  $RS^i$  into  $RS$ 

```

---

Fig. 14. Incremental distributed AMN (client).

$kDST = (1, 2, 5)$ ,  $DST_{max} = 5$ . Fig. 13b illustrates the server-to-client transmissions during these steps.

The while loop (Lines 8-17) starts by eliminating each server such that  $DST^i \geq DST_{max}$  (initially  $DST^i = dst^i$ ). For instance,  $DST^4 = 7 \geq DST_{max} = 5$ , and the client discards  $S^4$  without sending  $Q$ . Since the subsequent verification of  $S^4$  does not require  $Q$  either, there is no transmission of high-dimensional data (query, or points) between the client and  $S^4$ . Line 11 selects the candidate server  $S^i$  with the minimum  $DST^i$ , and asks for the distance  $DST_{new}$  of the next NN in  $S^i$ . If  $DST_{new} \geq DST_{max}$ ,  $S^i$  is purged. Assuming that the selected server is  $S^3$  ( $DST^3 = DST^2 = 2$ ), then  $DST(Q, P_1^3) = 5 \geq DST_{max} = 5$ , causing the elimination of  $S^3$  without changing  $kDST$ . Fig. 13c shows the pruned servers  $S^1, S^3, S^4$  in gray. The next iteration of the loop selects the last candidate  $S^2$ , and retrieves  $DST_{new} = 3$ . Since  $3 < DST_{max}$ ,  $DST_{new}$  is inserted into  $kDST$ , and  $DST_{max}$  changes to 3. The loop terminates because all servers have been eliminated (see Fig. 13d). Lines 18-20 simply verify the range  $q_R = (q, DST_{max})$  in each server. All the result points ( $RS^i$ ), as well as false hits ( $FH^i$ ) are transmitted during this step. The client generates the final result  $RS$  locally from the union of all  $RS^i$ . C-AMN can be applied to reduce the size of false hits. Note that Line 12 may call  $get\_next\_smallest\_DST$  multiple times on the same server  $S^i$ . In this case, the client needs to transmit the full query  $Q$  only the first time; for subsequent requests, it suffices to send the query ID.

**Proof of Correctness.** The client obtains all results and false hits at the end through the verifiable range (Lines 18-20). As shown in the proof SD-AMN, any  $DST$  misreporting that leads to the computation of a  $DST_{max}^i \neq DST_{max}$  can be detected by the client. Let us now consider that some server  $S^i$  sends false  $dst^i$  and  $kdst^i$ . The value of  $kdst^i$  is only used as an estimator for the selection of the initial server (Line 5), and it only affects the efficiency (but not the correctness) of the algorithm. For instance, if  $S^3$  reports  $kdst^3 = 1$  (instead of 9), it will become the initial

server, increasing the communication overhead ( $S^4$  cannot be immediately eliminated), without however altering the result. Moreover, as discussed in Section 5.1, any false distance smaller than  $DST_{max}$  will be caught by the verification. Similarly,  $dst^i$  is used as a lower bound for  $DST^i$ . If  $S^i$  sends a value of  $dst^i$  lower than the actual one, it can only be selected earlier than necessary during the while loop without affecting correctness. On the other hand, if the transmitted  $dst^i$  exceeds the actual one, 1)  $S^i$  is selected later during the loop, or 2) eliminated altogether if the reported  $dst^i$  exceeds  $DST_{max}$ . Case 1) only affects the efficiency, whereas case 2) is detected during the verification because  $S^i$  has to send objects within the range  $q_R = (q, DST_{max})$ .  $\square$

Similar to SD-AMN, ID-AMN is optimal in terms of data point transmissions. Now let us consider query transmissions. Ideally, an optimal algorithm would send the complete query  $Q$  only to servers that contribute nearest neighbors ( $S^1, S^2$  see Fig 13) because the distances of these NNs are necessary for obtaining the final value of  $DST_{max}$ . Additionally, ID-AMN sends  $Q$  to some *false candidates* (e.g.,  $S^3$ ) that cannot be eliminated. Specifically, let  $DST_1^i$  be the distance of the 1st NN in  $S^i$ .  $S^i$  is a false candidate, if  $dst^i < DST_{max} < DST_1^i$ . Regarding the CPU cost, in ID-AMN some eliminated servers (e.g.,  $S^4$ ) do not perform any high-dimensional distance computations. False candidates return a single  $DST_1^i$ , thus they need to retrieve the first local NN (which may involve multiple  $DST$  computations, if there are false hits). The rest of the servers must retrieve either  $k^i$  or  $(k^i + 1)$  NNs, where  $k^i (\leq k)$  is the contribution of  $S^i$  to the final  $k$ NN set. The distance of the additional (+1) NN is sometimes required to eliminate  $S^i$ . In comparison, SD-AMN transmits  $Q$  to all servers, and each server performs the necessary computations to obtain the  $k$  local NNs.

## 6 EXPERIMENTAL EVALUATION

We use four real data sets that capture different combinations of dimensionality  $D$ , cardinality  $N$ , and application domain:

1. *Corel* ( $D = 64, N = 68,040$ ),
2. *Chlorine* ( $D = 128, N = 4,310$ ),
3. *Stock* ( $D = 512, N = 10,000$ ),
4. *Mallat* ( $D = 1,024, N = 2,400$ ).

*Corel*<sup>5</sup> can be downloaded from [archive.ics.uci.edu/ml/](http://archive.ics.uci.edu/ml/), while the rest are available at: [www.cs.ucr.edu/~eamonn/time\\_series\\_data/](http://www.cs.ucr.edu/~eamonn/time_series_data/). We decrease the dimensionality of each data set using Chebyshev polynomials [4]. The value of  $d$  is a parameter with range [2-16] and default value 8. Each reduced data set is indexed by an MR-Tree using a page size of 4 KB. Every digest is created by SHA-1 [16]. We assume that both  $DST$  and  $dst$  are based on the euclidean distance. Section 6.1, compares AMN and C-AMN considering a single server. Section 6.2, evaluates SD-AMN and ID-AMN assuming multiple servers.

<sup>5</sup> *Corel* has four attribute sets. We use the first two sets (Color Histogram and Color Histogram Layout, of 32 attributes each) to derive the 64 D data set used in the experiments.

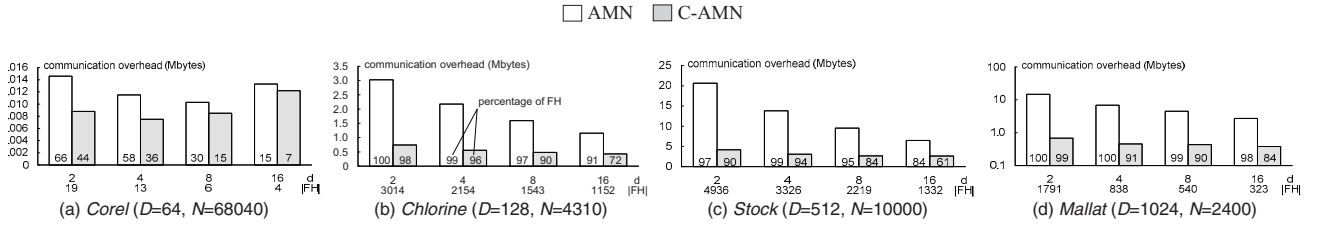


Fig. 15. Communication overhead versus  $d$  ( $k = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

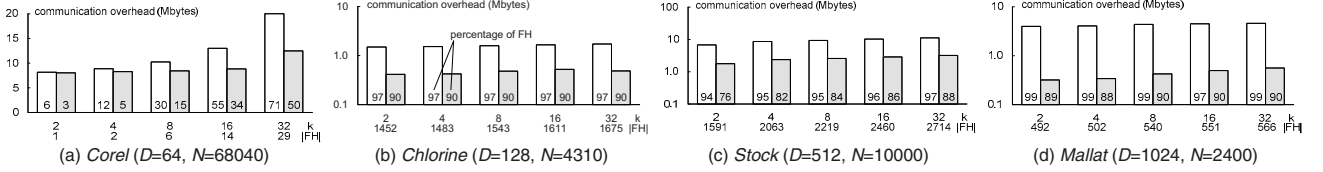


Fig. 16. Communication overhead versus  $k$  ( $d = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

## 6.1 Single Server

The measures of interest are the communication overhead, and the CPU cost at the server and the client. We assess the communication overhead based on the verification information sent to the client. The transmission of the query and the result is omitted because it is necessary in any method. The CPU cost is measured in terms of the *elementary distance computations*. Specifically,  $D$  elementary computations are required to derive the euclidean distance of two  $D$ -dimensional points. We exclude the I/O cost at the server because it is identical for both AMN and C-AMN (and similar to that of the conventional multistep algorithm) since in any case, we have to retrieve the low-dimensional NNs using the MR-Tree. For each experiment, we select a random data point as the query, and report the average results over 10 queries.

Fig. 15 fixes the number  $k$  of NNs to 8, and investigates the effect of  $d$  on the communication overhead of the verification information. Specifically, the overhead is measured in *Mbytes*, assuming that each value consumes  $S_v = 8$  bytes (a double precision number) and each digest is  $S_h = 20$  bytes (typical size for SHA-1). We indicate the number  $|FH|$  of false hits below the  $x$ -axis. As  $d$  increases,  $|FH|$  drops because the reduced representation captures more information about the corresponding point. In all cases, C-AMN leads to a significant decline of the overhead. The savings grow with  $D$ , and exceed an order of magnitude for *Mallat*, because long series provide more optimizations opportunities. On the other hand, the gains decrease as  $d$  grows due to the smaller  $FH$ . In order to demonstrate the effect of the false hits, we include inside each column of the diagrams, the contribution of  $FH$  as a percentage of the total overhead. For high  $D$  and low  $d$ ,  $FH$  constitutes the dominant factor, especially for AMN (e.g., at least 98 percent in *Mallat*), corroborating the importance of C-AMN.

The absolute overhead is lower (in both AMN and C-AMN) for high values of  $d$  due to the decrease of  $|FH|$ . The exception is *Corel*, where the communication cost actually grows when  $d$  exceeds a threshold (8 for AMN, 4 for C-AMN). This is explained as follows: A typical record (i.e., image) in *Corel* has very low values ( $<0.005$ ) on most ( $>60$ )

dimensions, and relatively large values ( $>0.1$ ) on the rest. Furthermore, the large values of different records usually concentrate on different dimensions. Dimensionality reduction using Chebyshev polynomials [4] captures effectively those important dimensions even for low  $d$ . Consequently, there is a small number of false hits (for  $d = 2, |FH| \approx 0.28\%$  of  $N$ , whereas in the other data sets  $|FH|$  is 50-75 percent of  $N$ ). As  $d$  grows,  $|FH|$  does not drop significantly. On the other hand, the verification information transmitted to the client contains more boundary records and node MBRs, increasing the *VO* size.

Fig. 16 illustrates the communication overhead as a function of the number of neighbors  $k$  to be retrieved ( $d = 8$ ). Note that the minimum value of  $k$  is 2 because the query is also a data point, i.e., its first NN is itself. The number of false hits increases with  $k$ , leading to higher transmission cost. However, compared to Fig. 15, the difference is rather small because  $k$  has a lower impact on  $|FH|$  than  $d$ . For the same reason, the absolute performance of both methods is rather insensitive to  $k$ . The benefits of C-AMN are again evident, especially if  $D \geq 128$ .

Fig. 17 investigates the elementary distance computations at the server as a function of  $d$ . C-AMN is always more expensive than AMN due to the additional cost of *ReduceFH*. This cost drops with increasing  $d$  because there are fewer false hits to be reduced. The numbers inside each column denote the CPU time (in milliseconds) using a Pentium Intel Core 2 Duo 2.33 GHz processor. For *Corel*, the CPU time is too low to be accurately measured. However, for *Chlorine*, *Stock*, and *Mallat* it may reach several seconds due to the large values of  $D$  and  $|FH|$ .

Fig. 18 shows the server computations versus the number of required neighbors. The cost increases slightly with  $k$ , but similar to Fig. 16, the effect is not as pronounced as that of  $d$ . Note that the diagrams do not include the I/O cost, which is identical to both methods. I/O operations normally dominate the processing overhead (since large records must be retrieved from the disk) and the performance difference of the two methods in terms of the overall cost diminishes. Moreover, the difference of C-AMN and AMN would decrease further, if *DST* were based on a more

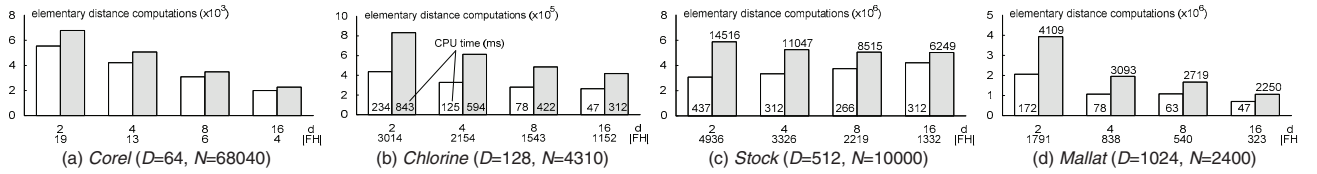


Fig. 17. Server computations versus  $d$  ( $k = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

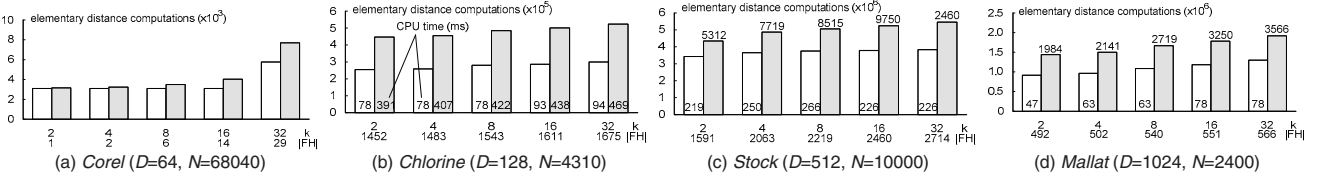


Fig. 18. Server computations versus  $k$  ( $d = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

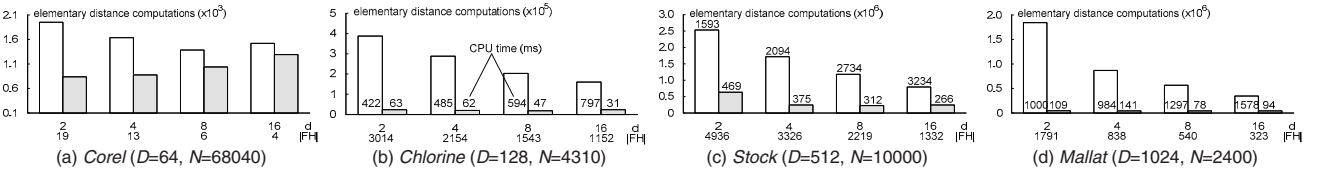


Fig. 19. Client computations versus  $d$  ( $k = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

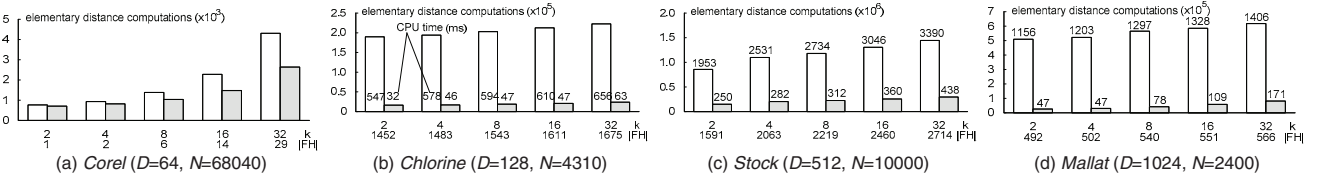


Fig. 20. Client computations versus  $k$  ( $d = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

expensive distance function than  $dst$  (e.g., DTW versus euclidean distance as in [10]) and applied the optimization of Section 4.3. This is because *ReduceFH* entails only cheap  $dst$  computations, which would be dominated by the more expensive  $DST$  calculations, common in both methods.

Fig. 19 illustrates the number of elementary distance computations at the client as a function of  $d$ . C-AMN leads to significant gains, sometimes dropping the processing cost by more than an order of magnitude. Since this cost is proportional to the amount of data received by the client, the diagrams are correlated to those in Fig. 15; accordingly, the benefits of C-AMN are more significant for settings that involve large values of  $D$  and  $|FH|$ . Fig. 20 investigates the effect of  $k$  on the client. Similar to Figs. 16 and 18, the CPU cost increases with  $k$ , but the impact of  $k$  is rather small.

Summarizing, compared to AMN, C-AMN imposes additional CPU cost for the server, in order to reduce the communication overhead and the verification effort at the client. This is a desirable tradeoff in client-server architectures because 1) data transmissions constitute the main bottleneck in most applications, especially those involving wireless networks, and 2) clients are assumed to have limited resources, whereas servers are powerful. Finally,

note that the transmission overhead can also be reduced by conventional compression techniques. We do not include experiments with this approach since it benefits both AMN and C-AMN. Moreover, it increases the computational burden of the client, which has to decompress the data before their verification.

## 6.2 Distributed Servers

Next, we compare SD-AMN and ID-AMN considering that the database is horizontally partitioned over  $m$  servers. Recall that the methods first collect distance information, based on which they determine the range that contains the result. The NNs and the false hits are obtained during the verification of this range, which is identical in SD-AMN and ID-AMN. Thus, when measuring the communication cost, we focus on their differences, which regard the transmission of query points and the distance information. The CPU overhead is based again on elementary distance computations. Finally, due to the identical verification process, the client cost is similar, and the corresponding experiments are omitted.

Fig. 21 shows the communication cost as a function of the number  $m$  of servers. Since we do not count the

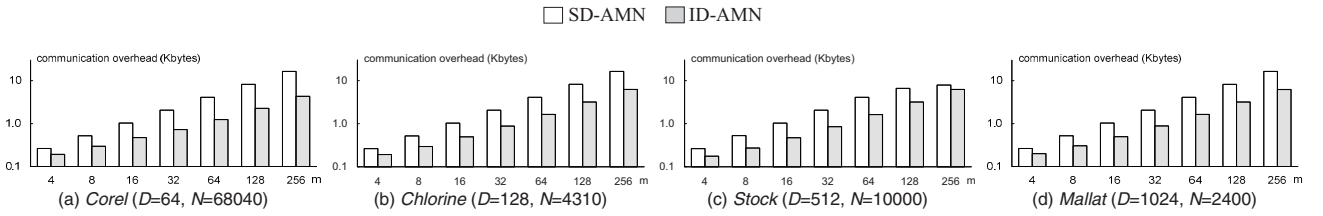


Fig. 21. Communication overhead versus  $m$  ( $d = 8, k = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

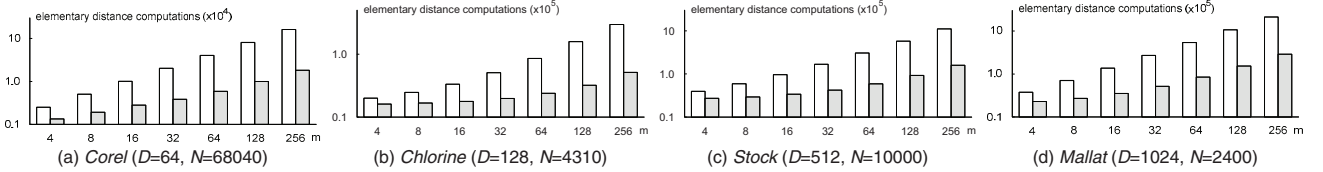


Fig. 22. Server computations versus  $m$  ( $d = 8, k = 8$ ). (a) *Corel* ( $D = 64, N = 68,040$ ). (b) *Chlorine* ( $D = 128, N = 4,310$ ). (c) *Stock* ( $D = 512, N = 10,000$ ). (d) *Mallat* ( $D = 1,024, N = 2,400$ ).

common data transmissions, the dominant factor is the number of high-dimensional query ( $Q$ ) transmissions. SD-AMN sends  $Q$  to all servers yielding an overhead of  $D \cdot m$  values. On the other hand, ID-AMN transmits  $Q$  only to candidate servers. In the best case, all results may be found in a single server, and the rest are eliminated using the  $dst$  bound; in the worst case,  $Q$  must be sent to all servers, if they all constitute candidates. In general, the number of eliminated servers increases with their total number, leading to the savings of ID-AMN.

Fig. 22 compares the two methods on elementary distance computations at the server versus  $m$ . The retrieval of a  $k$ NN set involves a number of computations linear to  $(k + |FH|) \cdot (d + D)$  because the distances of all results and false hits must be evaluated in both low and high-dimensional spaces. In SD-AMN, each of the  $m$  servers must retrieve the  $k$ NNs; thus, the total cost increases linearly with both  $m$  and  $k$ . In ID-AMN a server has to perform a number of computations that is proportional to its contribution  $k^i (\leq k)$  in the result set. The value of  $m$  affects the number of computations only indirectly, by increasing the false candidates. In general, ID-AMN clearly outperforms SD-AMN in all settings.

## 7 CONCLUSIONS

The importance of authenticated query processing increases with the amount of information available at sources that are untrustworthy, unreliable, or simply unfamiliar. This is the first work addressing authenticated similarity retrieval from such sources using the multistep  $k$ NN framework. We show that a direct integration of optimal NN search with an authenticated data structure incurs excessive communication overhead. Thus, we develop C-AMN, a technique that addresses the communication specific aspects of NN, and minimizes the transmission overhead and verification effort of the clients. Furthermore, we propose ID-AMN, which retrieves distance information from distributed servers, eliminating those that cannot contribute results.

## ACKNOWLEDGMENTS

This work was supported by grant HKUST 618108 from Hong Kong RGC.

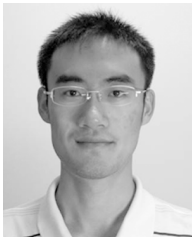
## REFERENCES

- [1] N. Beckmann, H.-P. Kriegel, R. Schneider, and B. Seeger, "The R\*-Tree: An Efficient and Robust Access Method for Points and Rectangles," *Proc. ACM SIGMOD*, 1990.
- [2] K. Beyer, J. Goldstein, R. Ramakrishnan, and U. Shaft, "When is "Nearest Neighbor" Meaningful?," *Proc. Int'l Conf. Database Theory (ICDT '99)*, 1999.
- [3] R. Bryan, "The Digital Revolution: The Millennial Change in Medical Imaging," *Radiology*, vol. 229, pp. 299-304, Nov. 2003.
- [4] Y. Cai and R. Ng, "Indexing Spatio-Temporal Trajectories with Chebyshev Polynomials," *Proc. ACM SIGMOD*, 2004.
- [5] H. Ding, G. Trajcevski, P. Scheuermann, X. Wang, and E. Keogh, "Querying and Mining of Time Series Data: Experimental Comparison of Representations and Distance Measures," *Proc. Int'l Conf. Very Large Data Base Endowment (VLDB '08)*, vol. 1, pp. 1542-1552, 2008.
- [6] C. Faloutsos, M. Ranganathan, and Y. Manolopoulos, "Fast Subsequence Matching in Time-Series Databases," *Proc. ACM SIGMOD*, 1994.
- [7] G. Hjaltason and H. Samet, "Distance Browsing in Spatial Databases," *Trans. Database Systems (TODS '99)*, vol. 24, no. 2, pp. 265-318, 1999.
- [8] H. Jagadish, B. Ooi, K. Tan, C. Yu, and R. Zhang, "I-Distance: An Adaptive B+-Tree Based Indexing Method Nearest Neighbor Search," *Trans. Database Systems (TODS '05)*, vol. 30, no. 2, pp. 364-397, 2005.
- [9] H. Kellerer, U. Pferschy, and D. Pisinger, *Knapsack Problems*. Springer, 2004.
- [10] E.J. Keogh, C.A. Ratanamahatana, "Exact Indexing of Dynamic Time Warping," *Knowledge and Information Systems*, vol. 7, no. 3, pp. 358-386, 2005.
- [11] F. Korn, N. Sidiropoulos, C. Faloutsos, E. Siegel, and Z. Protopapas, "Fast Nearest Neighbor Search in Medical Image Databases," *Proc. Int'l Conf. Very Large Data Base Endowment (VLDB '96)*, 1996.
- [12] A. Kundu and E. Bertino, "Structural Signatures for Tree Data Structures," *Proc. Int'l Conf. Very Large Data Base Endowment (VLDB '08)*, 2008.
- [13] F. Li, M. Hadjieleftheriou, G. Kollios, and L. Reyzin, "Dynamic Authenticated Index Structures for Outsourced Databases," *Proc. ACM SIGMOD*, 2006.
- [14] C. Martel, G. Nuckolls, P. Devanbu, M. Gertz, A. Kwong, and S. Stubblebine, "A General Model for Authenticated Data Structures," *Algorithmica*, vol. 39, no. 1, pp. 21-41, 2004.
- [15] R. Merkle, "A Certified Digital Signature," *CRYPTO*, 1989.

- [16] A. Menezes, P. van Oorschot, and S. Vanstone, *Handbook of Applied Cryptography*. CRC Press, 1996.
- [17] H. Pang and K. Mouratidis, "Authenticating the Query Results of Text Search Engines," *Proc. Int'l Conf. Very Large Data Base Endowment (VLDB '08)*, 2008.
- [18] D. Papadias, J. Zhang, N. Mamoulis, and Y. Tao, "Query Processing in Spatial Network Databases," *Proc. Int'l Conf. Very Large Data Base Endowment (VLDB '03)*, 2003.
- [19] T. Seidl and H.-P. Kriegel, "Optimal Multi-Step  $k$ -Nearest Neighbor Search," *Proc. ACM SIGMOD*, 1998.
- [20] R.T. Snodgrass, S.S. Yao, and C. Collberg, "Tamper Detection in Audit Logs," *Proc. Int'l Conf. Very Large Data Base Endowment (VLDB '04)*, 2004.
- [21] R. Tamassia and N. Triandopoulos, "Efficient Content Authentication over Distributed Hash Tables," *Proc. Int'l Conf. Applied Cryptography and Network Security (ACNS '07)*, 2007.
- [22] Y. Tao, K. Yi, C. Sheng, and P. Kalnis, "Quality and Efficiency in High Dimensional Nearest Neighbor Search," *Proc. ACM SIGMOD*, 2009.
- [23] Y. Yang, S. Papadopoulos, D. Papadias, and G. Kollios, "Spatial Outsourcing for Location-Based Services," *Proc. Int'l Conf. Data Eng. (ICDE '08)*, 2008.
- [24] K. Yi, F. Li, M. Hadjieleftheriou, G. Kollios, and D. Srivastava, "Randomized Synopses for Query Assurance on Data Streams," *Proc. Int'l Conf. Data Eng. (ICDE '08)*, 2008.



**Stavros Papadopoulos** received the BSc degree in computer science from the Aristotle University of Thessaloniki, Greece. He is currently working toward the PhD degree at the Department of Computer Science and Engineering, Hong Kong University of Science and Technology. His research interests include authenticated query processing, private information retrieval, spatio-temporal databases, and cloud computing.



**Lixing Wang** received the BE degree in software engineering from Fudan University, China and the BS degree from the University College Dublin, Ireland, both in computer science. He is currently working toward the PhD degree at the Department of Computer Science and Engineering, Hong Kong University of Science and Technology. His research interests include spatio-temporal databases and query processing in wireless sensor networks.



keyword search, and spatio-temporal databases.

**Yin Yang** received the BE degree in computer science and engineering from Shanghai Jiao Tong University and the PhD degree in computer science from the Hong Kong University of Science and Technology. He is currently a visiting scholar at the Department of Computer Science and Engineering, Hong Kong University of Science and Technology. His research interests include database outsourcing, query processing in data stream management systems,



university (Canada), and University of Patras (Greece). He serves or has served in the editorial boards of the *VLDB Journal*, *IEEE Transactions on Knowledge and Data Engineering*, and *Information Systems*.

**Dimitris Papadias** is currently a professor of Computer Science and Engineering, Hong Kong University of Science and Technology. In 1997, he worked and studied at the German National Research Center for Information Technology (GMD), the National Center for Geographic Information and Analysis (NCGIA, Maine), the University of California at San Diego, the Technical University of Vienna, the National Technical University of Athens, Queen's University (Canada), and University of Patras (Greece). He serves or has served in the editorial boards of the *VLDB Journal*, *IEEE Transactions on Knowledge and Data Engineering*, and *Information Systems*.



interests include data mining, algorithms, spatial data management, anonymization, indexing, and similarity search. His work has been published in major database and data mining conferences and journals.

**Panagiotis Karras** received the PhD degree in computer science from the University of Hong Kong and the MEng degree in electrical and computer engineering from the National Technical University of Athens. He is currently a Lee Kuan Yew postdoctoral fellow at the National University of Singapore. In 2008, he received the Hong Kong Young Scientist Award. He has also held positions at the University of Zurich and the Technical University of Denmark. His research

► For more information on this or any other computing topic, please visit our Digital Library at [www.computer.org/publications/dlib](http://www.computer.org/publications/dlib).