

and the same strategic behavior of the players in both games. This observation applies to the proof of Theorem 7.1 as well.

Finally, we believe that the liquid welfare is an appropriate efficiency benchmark for auctions with budget-constrained players. The recent paper by Azar et al. [1] studies the LPoA of simultaneous first-price auctions; obtaining similar results for other auction formats (e.g., see the recent survey of Roughgarden et al. [19]) is certainly important. Needless to say, we do not expect that the liquid welfare is unique as a measure of efficiency in settings with budgets. Defining alternative efficiency benchmarks and studying the price of anarchy with respect to them would shed extra light to the strengths and weaknesses of auction mechanisms.

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