Python Crash Course

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Program		
Monday	November 9, 13:00-16:00:	I: Python language introduction
Wednesday	November 11, 14:00-16:00:	II: Some hands on, adjusted to how it goes
Monday	November 23, 13:00-16:00:	III: More Python, Numpy, Matplotlib, Pandas, Numba, pypy, mypy, …

About me

- Gerth Stølting Brodal, professor (PhD Aarhus University 1997, cv)
- Research *is not* AI, ML, Python, Programming languages, …
- Research *is* Algorithms and Data Structures
  - Basic research on how to make algorithms and data structures efficient
  - Data structures
  - External memory algorithms
  - Computational geometry
- Teaching
  - Algorithms and Data Structures (1st year Computer Science BSc, since 2002)
  - **Introduction to Programming with Scientific Applications (Python)**
    (1st year Data Science BSc, 2nd year Mathematics BSc, since 2018)

Google Colaboratory (Colab)

- https://colab.research.google.com/
- Online version of Jupyter documents hosted by Google in the cloud
- Documents stored in your Google Drive
- Allows Python to be executed in the cloud using a web browser

Introduction to Programming with Scientific Applications 2020

Basic programming
Advanced / specific Python
Libraries & applications

1. Introduction to Python	**10. Functions as objects**	**19. Linear programming**
2. Python basics / if	**11. Object oriented programming**	**20. Generators, iterators, with**
3. Basic operations	**12. Class hierarchies**	**21. Modules and packages**
4. Lists / while / for	**13. Exceptions and files**	**22. Working with text**
5. Tuples / comprehensions	**14. Doc, testing, debugging**	**23. Relational data**
6. Dictionaries and sets	**15. Decorators**	**24. Clustering**
7. Functions	**16. Dynamic programming**	**25. Graphical user interfaces (GUI)**
8. Recursion	**17. Visualization and optimization**	**26. Java vs Python**
9. Recursion and Iteration	**18. Multi-dimensional data**	**27. Final lecture**

www.cs.au.dk/~gerth/ipsa20/

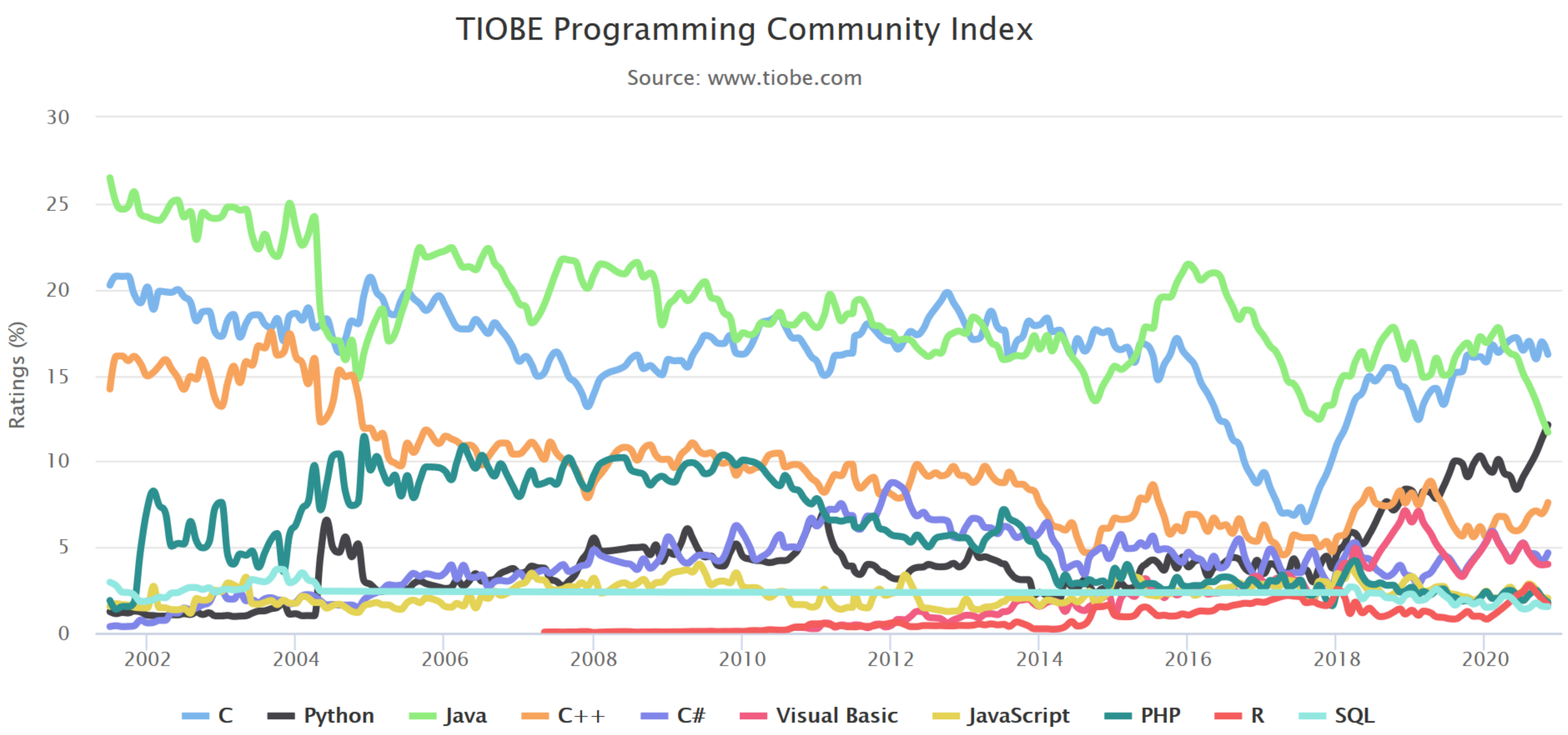
Why Python ?

TIOBE Index November 2020

Nov 2020	Nov 2019	Change	Programming Language	Ratings	Change
1	2	^	C	16.21%	+0.17%
2	3	^	Python	12.12%	+2.27%
3	1	v	Java	11.68%	-4.57%
4	4		C++	7.60%	+1.99%
5	5		C#	4.67%	+0.36%
6	6		Visual Basic	4.01%	-0.22%
7	7		JavaScript	2.03%	+0.10%
8	8		PHP	1.79%	+0.07%
9	16	^^	R	1.64%	+0.66%
10	9	v	SQL	1.54%	-0.15%

The TIOBE Programming Community index is an indicator of the *popularity of programming languages*. The index is updated once a month. The ratings are based on the number of skilled engineers world-wide, courses and third party vendors. Popular search engines such as Google, Bing, Yahoo!, Wikipedia, Amazon, YouTube and Baidu are used to calculate the ratings. It is important to note that the TIOBE index is not about the *best* programming language or the language in which *most lines of code* have been written. www.tiobe.com

Popularity of programming languages



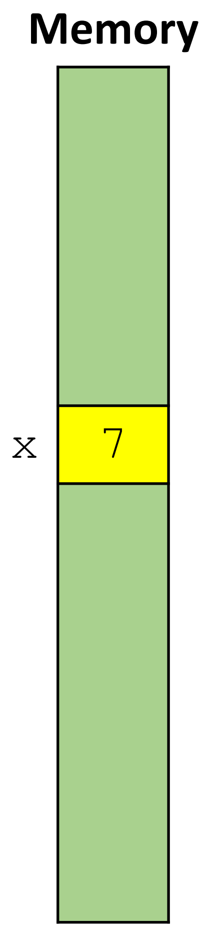
[Most Popular Programming Languages 1965 – 2019 (YouTube)]

Two Python programs

A Python program

Python shell

```
> x = 7
> print(x * x)
| 49
```



- `7` is an *integer literal* – in Python denoted an "`int`"
- `x` is the name of a *variable* that can hold some value
- `=` is assigning a value to a variable
- `*` denotes multiplication
- `print` is the name of a built-in *function*,
  here we call `print` to print the result of 7*7
- A program consists of a sequence of *statements*, executed sequentially

Question – What is the result of this program?

Python shell

```
> x = 3
> y = 5
> x = 2
> print(x * y)
```

x assigned new value

a) 10

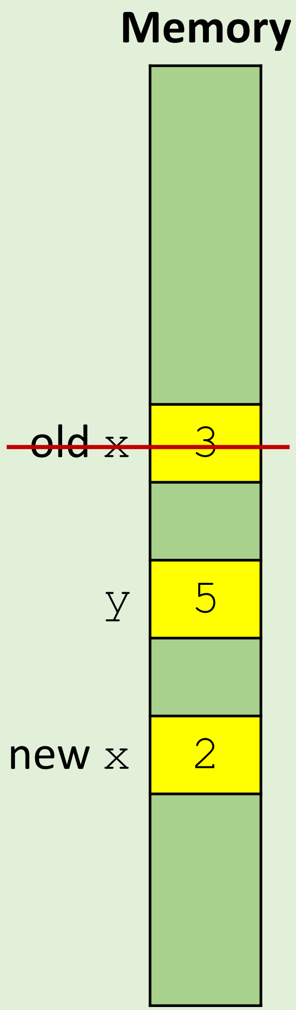
b) 15

c) 25

d) [15, 10]

e) Error

f) Don't know



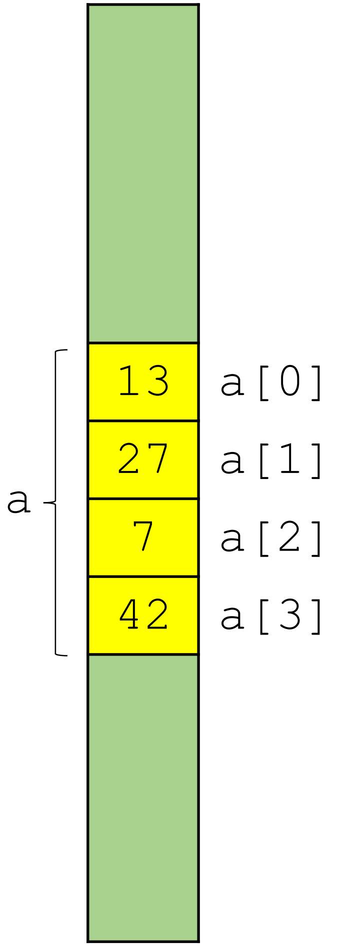
Another Python program using lists

Python shell

```
> a = [13, 27, 7, 42]
> print(a)
| [13, 27, 7, 42]
> print(a[2])
| 7
```

- `[13, 27, 7, 42]` is a *list* containing four integers
- `a[2]` refers to the entry in the list with *index* 2
  (the first element has index 0, i.e. `a[2]` is the 3rd element of the list)
- Note that `print` also can print a list

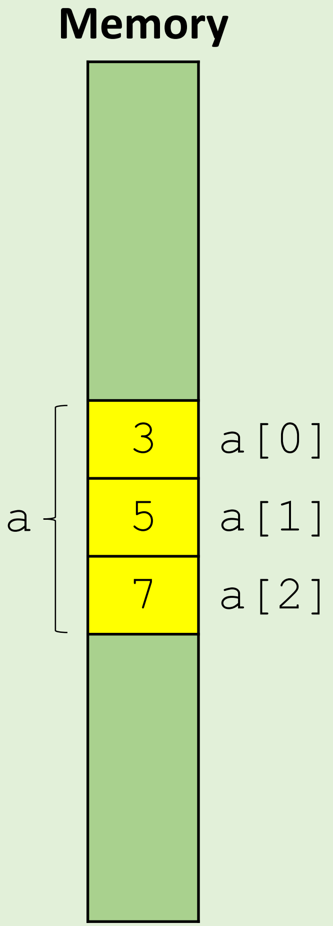
Memory



Question – What is the result of this program?

Python shell

```
> a = [3, 5, 7]
> print(a[1] + a[2])
```



a) 8

b) 10

c) 12

d) 15

e) Don't know

$$1 + 2 + \cdots + n$$

add.py

```python
import sys

n = int(sys.argv[1])
sum = 0
for i in range(1, n + 1):
    sum += i
print("Sum = %d" % sum)
```

add.c

```c
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char *argv[]) {
  int n = atoi(argv[1]);
  int sum = 0;
  for (int i=1; i<=n; i++)
    sum += i;
  printf("Sum = %d\n", sum);
}
```

add.cpp

```cpp
#include <iostream>
#include <cstdlib>
using namespace std;

int main(int argc, char *argv[]) {
  int n = atoi(argv[1]);
  int sum = 0;
  for (int i=1; i<=n; i++)
    sum += i;
  cout << "Sum = " << sum << endl;
}
```

add.java

```java
class Add{
    public static void main(String args[]){
        int n = Integer.parseInt(args[0]);
        int sum = 0;
        for (int i=1; i<=n; i++)
            sum += i;
        System.out.println("Sum = " + sum);
    }
}
```

Timing results

					Python		
n	**C (gcc 9.2)**	**C++, int (g++ 9.2)**	**C++, long (g++ 9.2)**	**Java (12.0)**	**Python (3.8.1)**	**PyPy (7.3.0)**	**Numba, int64**
10^7	0.001 sec*	0.001 sec*	0.003 sec	0.006 sec*	1.5 sec	0.27 sec	0.002 sec
10^9	0.10 sec**	0.10 sec**	0.30 sec	0.40 sec**	145 sec	27 sec	0.2 sec

Wrong output (overflow)

* -2004260032 instead of 50000005000000

** -243309312 instead of 500000000500000000

- since C, C++, and Java only uses 32 bits to represent integers (and 64 bits for ”long” integers)

Have fun debugging!

```
Bit                 6666666666555555555544444444443333333333222222222211111111110000000000
position            9876543210987654321098765432109876543210987654321098765432109876543210

bin(10**9)                                                  111011100110101100101000000000

bin(50000005000000)                         1011010111100110001000100010010110101101000000
bin(-2004260032 + 2**32)                                  10001000100010010110101101000000

bin(500000000500000000)        11011110000010110110101100111110001011111110110010100000000
bin(-243309312 + 2**32)                                   11110001011111110110010100000000
```

[Try Google: civilization gandhi overflow]

Timing results

n	C (gcc 9.2)	C++, int (g++ 9.2)	C++, long (g++ 9.2)	Java (12.0)	Python (3.8.1)	PyPy (7.3.0)	Numba, int64
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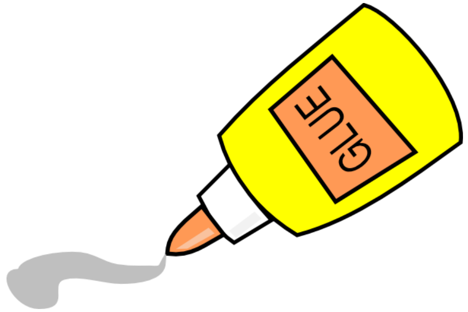
- Relative speed

C ≈ C++ > Java >> Python

- C, C++, Java need to care about integer overflows – select integer representation carefully with sufficient number of bits (8, 16, 32, 64, 128)
- Python natively works with arbitrary long integers (as memory on your machine allows). Also possible in Java using the class java.math.BigInteger
- Python programs can (sometimes) run faster using PyPy
- Number crunching in **Python** should be delegated to **specialized modules (e.g. Numpy, CPLEX, Numba)** – often written in C or C++

Why Python ?

- Short concise code
- Index out of range exceptions
- Elegant for-each loop
- Garbage collection is done automatically
- **Exact integer arithmetic (no overflows)**
- **Can delegate number crunching to C, C++, ...**



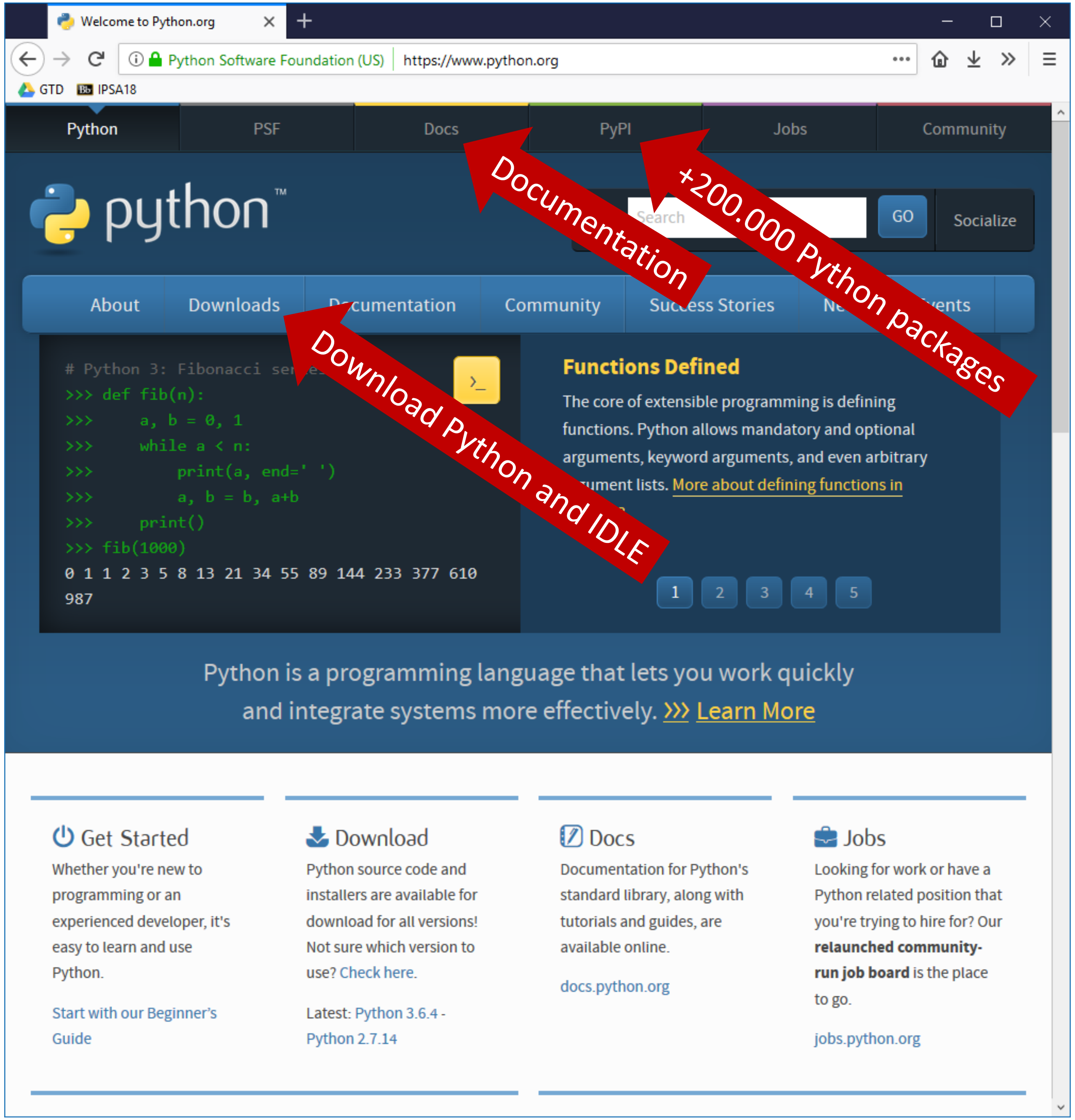
History of Python development

- Python created by Guido van Rossum in 1989, first release 0.9.0 1991
- **Python 2 → Python 3** (clean up of Python 2 language)
  - Python 2 – version 2.0 released 2000, final version 2.7 released mid-2010
  - Python 3 – released 2008, current release 3.8.1
- Python 3 is *not* backward compatible, libraries incompatible

Python 2	Python 3
print 42	print(42)
int = C long (32 bits)	int = arbitrary number of digits (= named "long" in Python 2)
7/3 → 2 returns "int"	7/3 → 2.333... returns "float"
range() returns list (memory intensive)	range() returns iterator (memory efficient; xrange in Python 2)

[100th episode of Talk Python To Me: **Python past, present, and future with Guido van Rossum**]

Python.org


Welcome to Python.org
Python Software Foundation (US)
https://www.python.org
GTD
IPSA18
Python
PSF
Docs
PyPI
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python™
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GO
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# Python 3: Fibonacci series
>>> def fib(n):
>>>   a, b = 0, 1
>>>   while a < n:
>>>   print(a, end=' ')
>>>   a, b = b, a+b
>>>   print()
>>> fib(1000)
0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610
987
Functions Defined
The core of extensible programming is defining functions. Python allows mandatory and optional arguments, keyword arguments, and even arbitrary
More about defining functions in
1 2 3 4 5
Python is a programming language that lets you work quickly
and integrate systems more effectively. >>> Learn More
Get Started
Whether you're new to programming or an experienced developer, it's easy to learn and use Python.
Start with our Beginner's Guide
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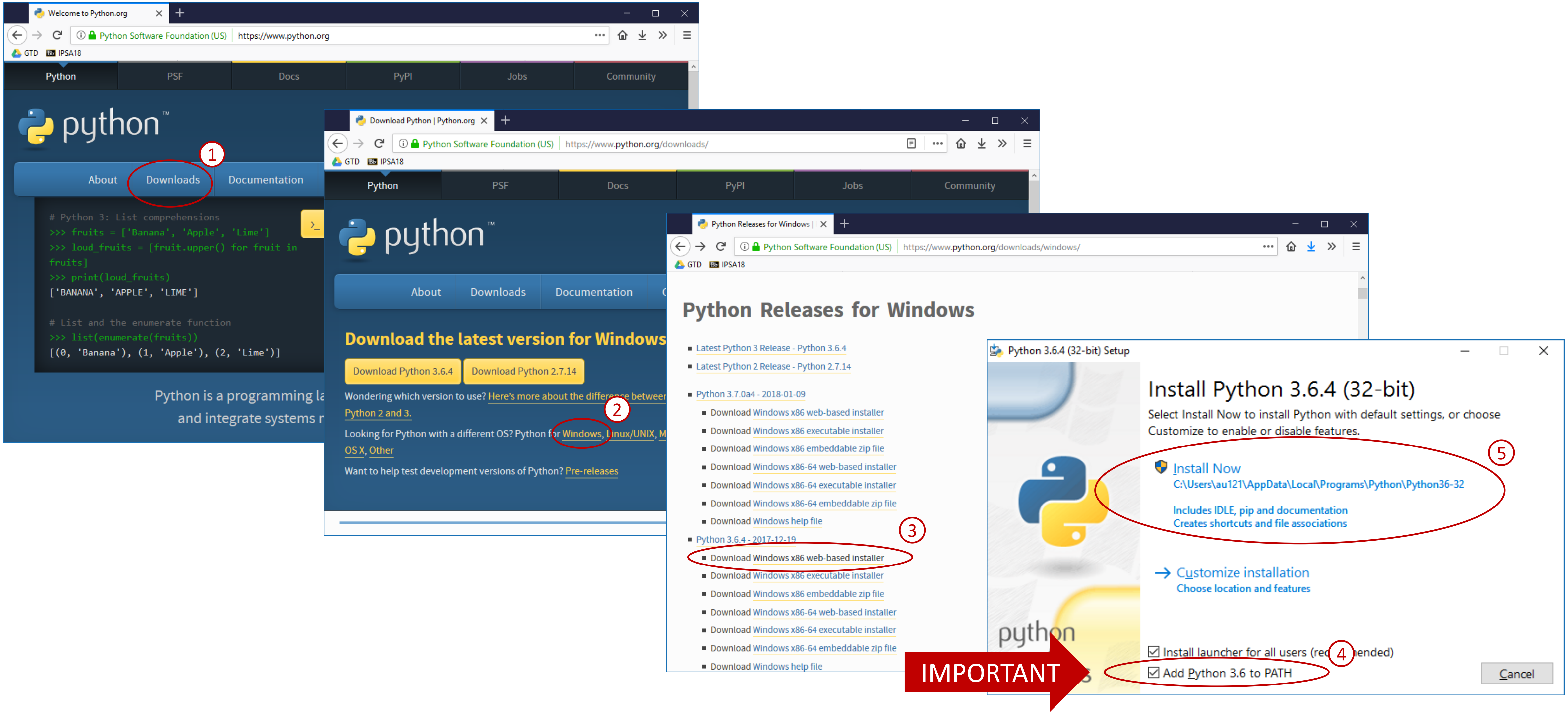
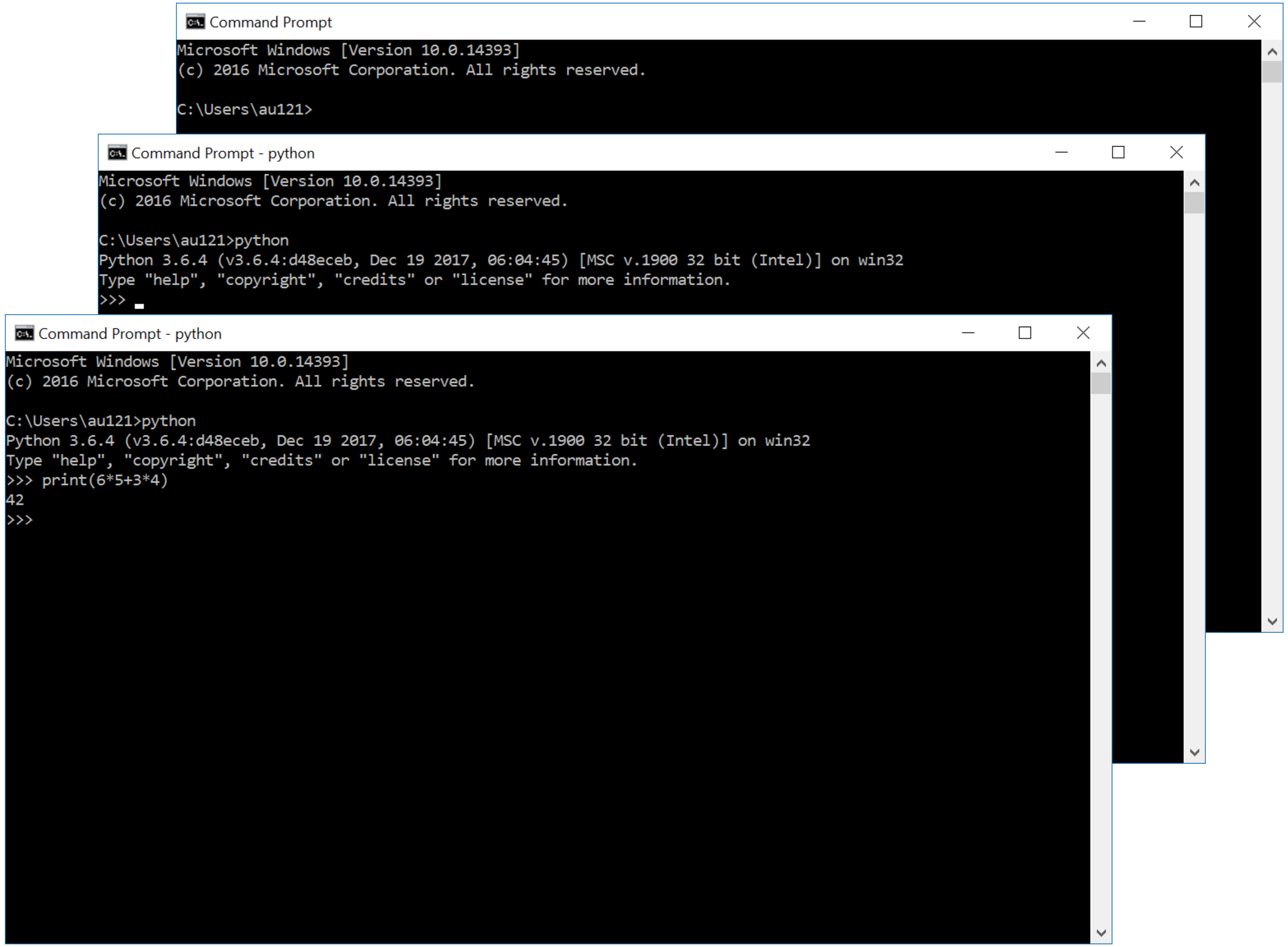
Python source code and installers are available for download for all versions!
Not sure which version to use? Check here.
Latest: Python 3.6.4 - Python 2.7.14
Docs
Documentation for Python's standard library, along with tutorials and guides, are available online.
docs.python.org
Jobs
Looking for work or have a Python related position that you're trying to hire for? Our relaunched community-run job board is the place to go.
jobs.python.org

Installing Python



Running the Python Interpreter

- Open Command Prompt (Windows-key + cmd)
- Type “python” + return
- Start executing Python statements



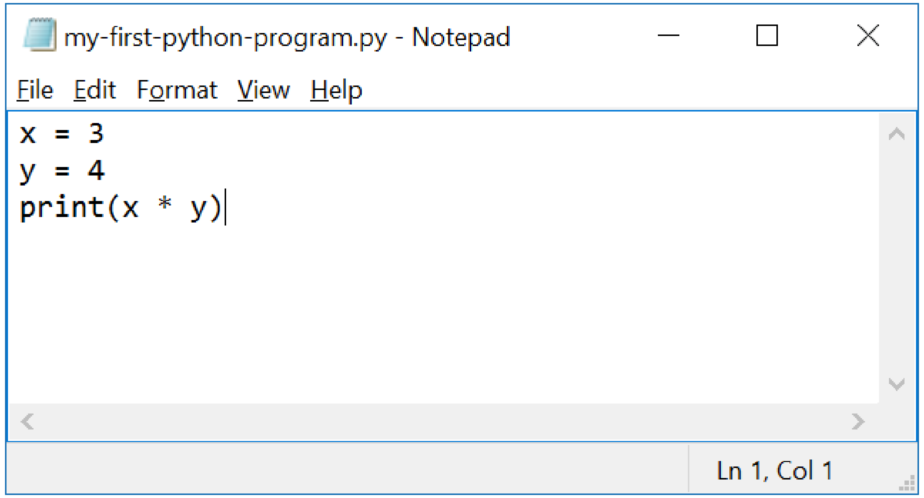
- To exit shell:
  Ctrl-Z + return *or*
  exit() + return

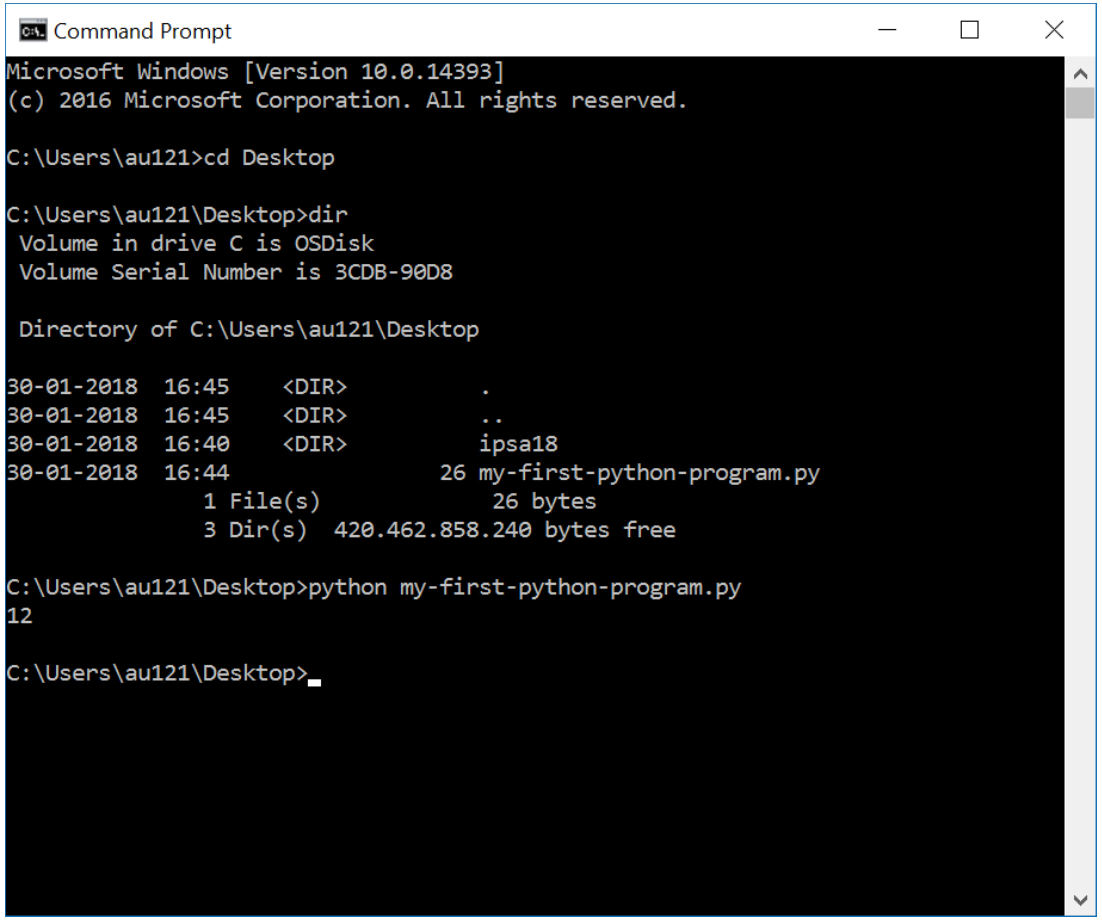
Some other usefull packages

- Try installing some more Python packages:

`pip install numpy`	linear algebra support (N-dimensional arrays)
`pip install scipy`	numerical integration and optimization
`pip install matplotlib`	2D plotting library
`pip install pylint`	Python source code analyzer enforcing a coding standard

Creating a Python program the very basic way

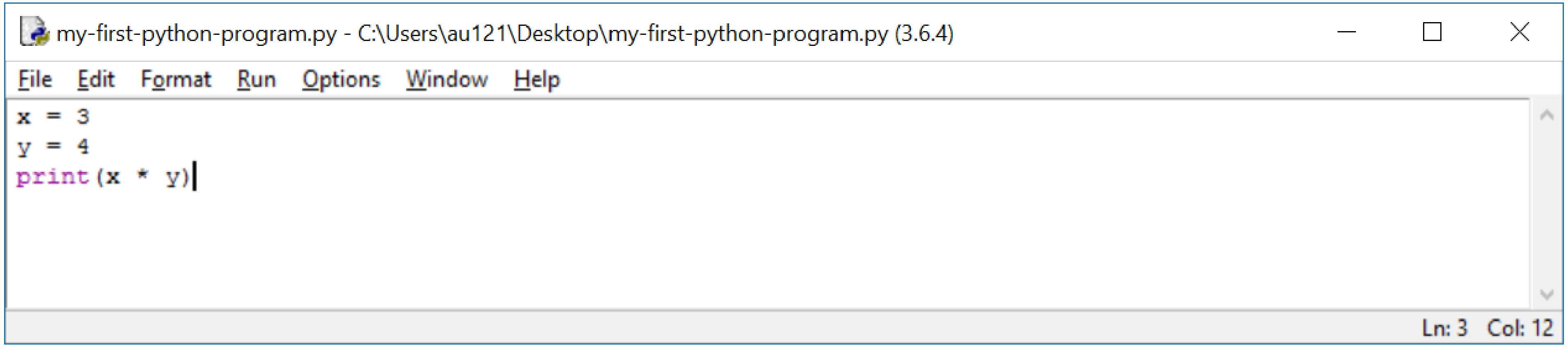


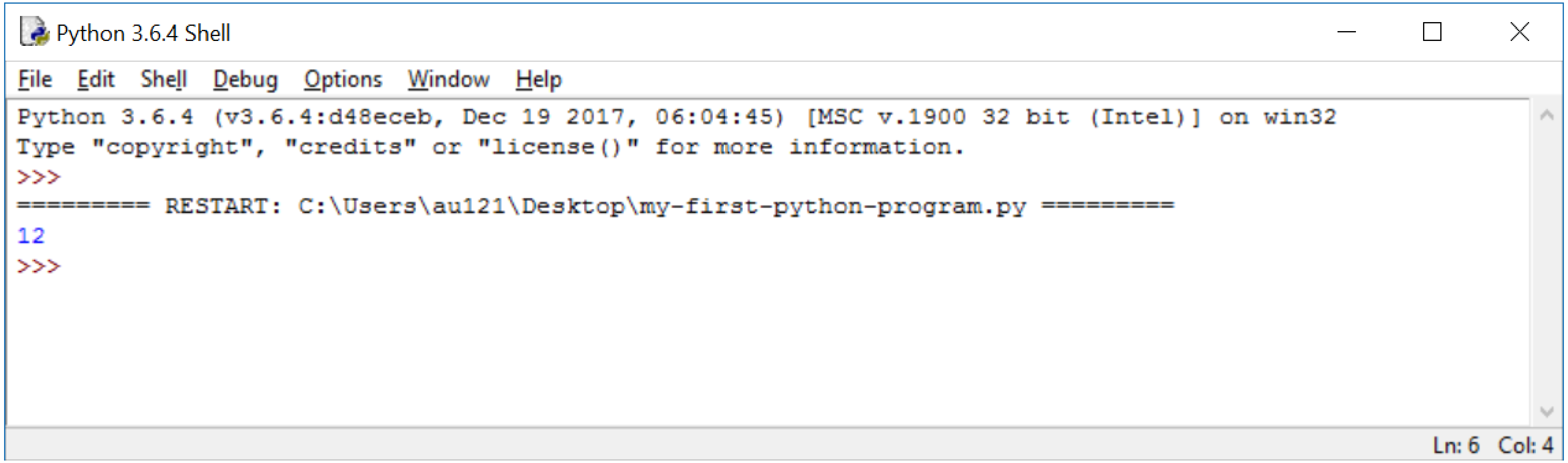


- Open Notepad
  - write a simple Python program
  - save it
- Open a command prompt
  - go to folder (using cd)
  - run the program using

```
python <program name>.py
```

... or open IDLE and run program with F5





The Python Ecosystem

- **Interpreters/compiler**
  - CPython – reference C implementation from python.org
  - PyPy – written in RPython (a subset of Python) – faster than Cpython
  - Jython – written in Java and compiles to Java bytecode, runs on the JVM
  - IronPython – written in C#, compiles to Microsoft's Common Language Runtime (CLR) bytecode
  - Cython – project translating Python-ish code to C
- **Shells (IPython, IDLE)**
- **Libraries/modules/packages**
  - pypi.python.org/pypi (PyPI - the Python Package Index, +200.000 packages)
- **IDEs (Integrated development environment)**
  - IDLE comes with Python (docs.python.org/3/library/idle.html)
  - Anaconda w. Spyder, IPython (www.anaconda.com/download)
  - Canopy (enthought.com/product/canopy)
  - Python tools for Visual Studio (github.com/Microsoft/PTVS)
  - PyCharm (www.jetbrains.com/pycharm/)
  - Emacs (Python mode and ElPy mode)
  - Notepad++
- **Python Style guide (PEP8)**
  - pylint, pep8, flake8

Python basics

- Comments
- ";"
- Variable names
- int, float, str
- type conversion
- assignment (=)
- print(), help(), type()

Python comments

A ‘#’ indicates the beginning of a comment. From ‘#’ until of end of line is ignored by Python.

```
x = 42  # and here goes the comment
```

Comments useful to describe what a piece of code is supposed to do, what kind of input is expected, what is the output, side effects...

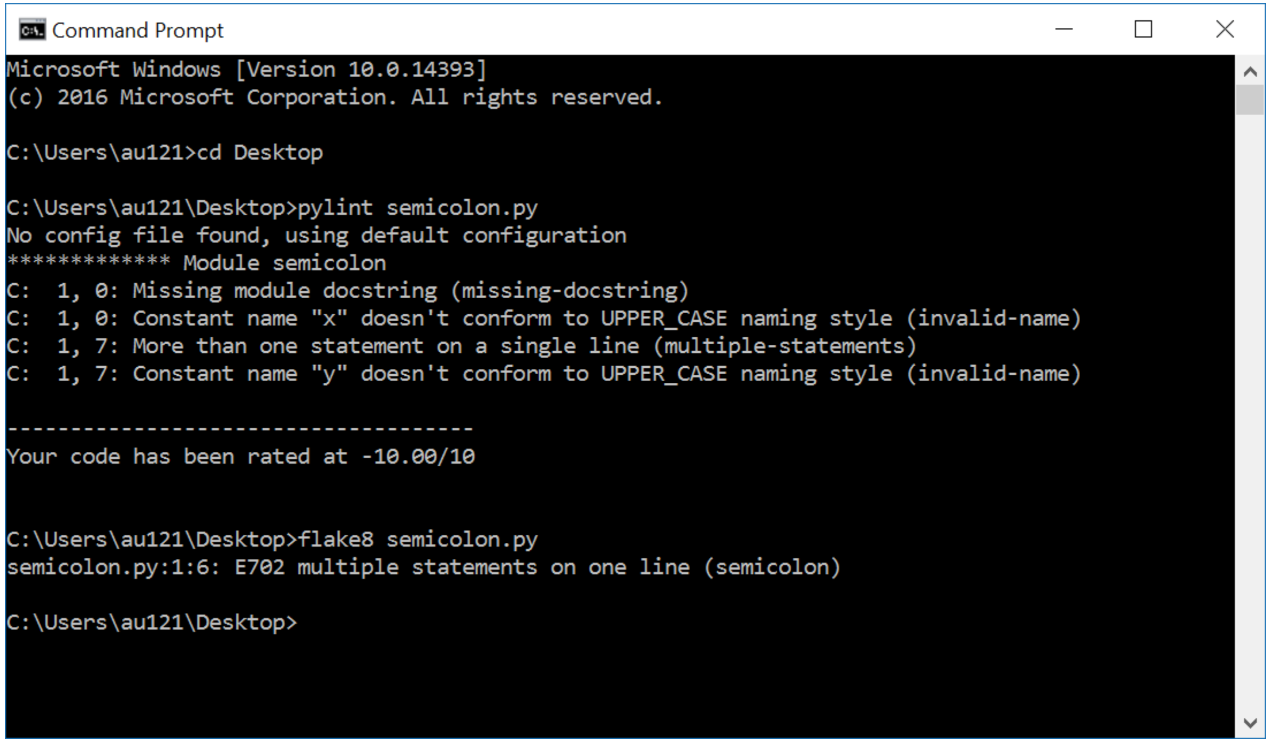
The “;” in Python

- Normally statements follow in consecutive lines with identical indentation

```
x = 1
y = 1
```

- but Python also allows multiple statements on one line, separated by “;”

```
x = 1; y = 1
```



neither **pylint** or **flake8** like “;”

- General Python PEP 8 guideline: **avoid using “;”**
- Other languages like C, C++ and Java require “;” to end/separate statements

Variable names

- Variable name = sequence of **letters** 'a'-'z', 'A'-'Z', **digits** '0'-'9', and **underscore** '_'

```
v, volume, height_of_box, WidthOfBox, x0, _v12_34B, _
```

(snake_case) (CamelCase)

  – a name cannot start with a digit
  – names are case sensitive (`AB`, `Ab`, `aB` and `ab` are different variables)

- Variable names are references to objects in memory
- **Use meaningful variables names**
- **Python 3 reserved keywords:** `and`, `as`, `assert`, `break`, `class`, `continue`, `def`, `del`, `elif`, `else`, `except`, `False`, `finally`, `for`, `from`, `global`, `if`, `import`, `in`, `is`, `lambda`, `nonlocal`, `None`, `not`, `or`, `pass`, `raise`, `return`, `True`, `try`, `while`, `with`, `yield`

Question – Not a valid Pyton variable name?

a) `print`

b) `for` ← Python reserved keyword

c) `_100`

d) `x`

e) `_`

f) `python_for_ever`

g) Don’t know

Python shell

```
> print = 7
> print(42)
| Traceback (most recent call last):
|   File "<stdin>", line 1, in <module>
| TypeError: 'int' object is not callable
```

`print` is a valid variable name, with default value a builtin *function* to print output to a shell – assigning a new value to `print` is very likely a bad idea (like many others `sum`, `int`, `str`, ...)

Integer literals

- .... `-4, -3, -2, -1, 0, 1, 2, 3, 4` ....
- Python integers can have an arbitrary number of digits
  (only limited by machine memory)
- Can be preceded by a plus `(+)` or minus (–)
- For readability underscores (_) can be added between digits,

`2_147_483_647`

(for more, see PEP 515 - Underscores in Numeric Literals)

Question – What statement will not fail?

a) `x = _42`


b) `_10 = -1_1`

c) `x = 1__0`

d) `x = +1_0_`

e) Don't know

Float literals

- Decimal numbers are represented using **float** – contain "." or "e"
- Examples
  - `3.1415`
  - `-.00134`
  - `124e3 = 124·10`^{3}
  - `-2.345e2 = -234.5`
  - `12.3e-4 = 0.00123`
- Floats are often only approximations, e.g. `0.1` is *not* 1/10
- Extreme values (CPython 3.8.1)
  - max = 1.7976931348623157e+308
  - min = 2.2250738585072014e-308
- NB: Use module `fractions` for exact fractions/rational numbers

Python shell

```
> 0.1 + 0.2 + 0.3
| 0.6000000000000001
> (0.1 + 0.2) + 0.3
| 0.6000000000000001
> 0.1 + (0.2 + 0.3)
| 0.6
> type(0.1)
| <class 'float'>
> 1e200 * 1e300
| inf
> 0.1+(0.2+0.3) == (0.1+0.2)+0.3
| False
> x = 0.1 + 0.2
> y = 0.3
> x == y
| False
> print(f'{x:.30f}')   # 30 decimals
| 0.300000000000000044408920985006
> print(f'{y:.30f}')   # 30 decimals
| 0.299999999999999988897769753748
> import sys
> sys.float_info.min
| 2.2250738585072014e-308
> sys.float_info.max
| 1.7976931348623157e+308
```

Associativity rule does not apply to floats

Question – What addition order is ”best”?

a) `1e10 + 1e-10 + -5e-12 + -1e10`

b) `1e10 + -1e10 + 1e-10 + -5e-12`

c) `1e-10 + 1e10 + -1e10 + -5e-12`

d) `-5e-12 + -1e10 + 1e10 + 1e-10`

e) Any order is equally good

f) Don’t know

```
 1e10   =  10000000000
-1e10   = -10000000000
 1e-10  =             0.0000000001
-5e-12  =            -0.000000000005
```

Python shell

```
> 1e10 + 1e-10 + -5e-12 + -1e10
| 0.0
> 1e10 + -1e10 + 1e-10 + -5e-12
| 9.500000000000001e-11
> 1e-10 + 1e10 + -1e10 + -5e-12
|  -5e-12
> -5e-12 + -1e10 + 1e10 + 1e-10
|  1e-10
```

a) - d) give four different outputs

Approximating
π = 3.14159265359...

$$\frac{\pi^2}{6} = \sum_{k=1}^{+\infty} \frac{1}{k^2} = 1.6449340668\ldots$$

Riemann zeta function ζ(2)

pi_approximation_riemann.py

```
apx = 0.0
k = 0.0
while True:
    k = k + 1.0
    apx = apx + 1.0 / (k * k)
    print(k, apx)
```

Output

```
...
94906261.0 1.6449340578345741
94906262.0 1.6449340578345744
94906263.0 1.6449340578345746
94906264.0 1.6449340578345748
94906265.0 1.644934057834575
94906266.0 1.644934057834575
94906267.0 1.644934057834575
94906268.0 1.644934057834575
94906269.0 1.644934057834575
94906270.0 1.644934057834575
...
```

⚠ This is not a course in numeric computations – but now you are warned....

Python float ≡ IEEE-754 double precision*

- A binary number is a number in base 2 with digits/bits from {0,1}

$$10110_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 16 + 4 + 2 = 22_{10}$$

- IEEE-754 64-bit double

sign s 1 bit	exponent e 11 bits	coefficient c 52 bits

$s = 0$ c

+1 0011101101011100111010101010101010101010101011101100 1001010101100.01000111100101011110011101010101111101011001101O1

e – 1023 bits

2^{-6}

Float value	Case
$(-1)^s \cdot (1 + c \cdot 2^{-52}) \cdot 2^{e-1023}$	$0 < e < 2047$
$(-1)^s \cdot c \cdot 2^{-1074}$	$e = 0, c \neq 0$
+0 and -0	$e = 0, c = 0$
+∞ and -∞	$e = 2047, c = 0$
NaN ("not a number")	$s = 0, e = 2047, c \neq 0$

Python shell

```
> 1e200 * 1e200
| inf
> -1e200 * 1e200
| -inf
> 1e-200 * 1e-200
| 0.0
> -1e-200 * 1e-200
| -0.0
> 1e200 * 1e200 * 0.0
| nan
```

(*most often, but there is no guarantee given in the Python language specification that floats are represented using IEEE-754)

String literals (type `str`)

- Sequence of characters enclosed by single (') or double (") quotes

```
"a 'quoted' word"    "Hello World"    'abc'
'a "quoted" word'    '_"_\'_"_'
```

- Escape characters

`\n`	newline
`\t`	tab
`\\`	backslash
`\'`	single quote
`\"`	double quote

- A backslash (\ ) a the end of line, will continue line/string on next line
- Use triple single or double quotes (''' or """) for enclosing strings spanning more lines

  (in particular for Python Dosctrings, see [PEP 257](PEP 257))

string-test.py

```
print("abc")
print('de\'f')
print("'ghi'")
print("'jk\nl'\"")
print("mn\
o")
print("p\\q\tr")
```

Output

```
$ python string-test.py
abc
de'f
'ghi'
'jk
l'"
mno
p\q     r
```

Question – What does the following print ?

```
print("\\\"\\n\n'")
```

a) `\\\"\\n\n'`

b) `\"\nn'`

c)
```
\"\n
'
```

d) `"nn'`

e)
```
\"


'
```

f) Don't know

Long string literals

- Long string literals often need to be split over multiple lines
- In Python two (or more) string literals following each other will be treated as a single string literal (they can use different quotes)
- Putting parenthesis around multiple literals allows line breaks
- Advantages:
  - avoids the backslash at the end of line
  - can use indentation to increase readability
  - allows comments between literals

long-string-literals.py

```python
s1 = 'abc' "def"  # two string literals
print(s1)
s2 = '"' "'" '"'  # avoid escaping quotes
print(s2)
s3 = 'this is a really, really, really, \
really, really, long string'
print(s3)
s4 = ('this is a really, really, '
      'really, really, really, '
      'long string')
print(s4)
very_very_long_variable_name = (
    'this is a really, really, '  # line 1
    'really, really, really, '    # line 2
    "long string"                 # line 3
)
print(very_very_long_variable_name)
```

Python shell

```
| abcdef
| "'"
| this is a really, really, really, really,
  really, long string
| this is a really, really, really, really,
  really, long string
| this is a really, really, really, realy,
  really, long string
```

Raw string literals

- By prefixing a string literal with an `r`, the string literal will be considered a raw string and backslashes become literal characters
- Useful in cases where you actually need backslashes in your strings, e.g. when working with Python's regular expression module `re`

Python shell

```
> print('\let\epsilon\varepsilon')      # \v = vertical tab
| \let\epsilon
  arepsilon
> print('\\let\\epsilon\\varepsilon')   # many backslashs
| \let\epsilon\varepsilon
> print(r'\let\epsilon\varepsilon')     # more readable
| \let\epsilon\varepsilon
```

print(...)

- `print` can print zero, one, or more values
- default behavior
  - print a space between values
  - print a line break after printing all values
- default behavior can be changed by **keyword arguments** “`sep`“ and “`end`“

Python shell

```
> print()
|
> print(7)
| 7
> print(2, 'Hello')
| 2 Hello
> print(3, 'a', 4)
| 3 a 4
> print(3, 'a', 4, sep=':')
| 3:a:4
> print(5); print(6)
| 5
| 6
> print(5, end=', '); print(6)
| 5, 6
```

print(...) and help(...)

Python shell

```
> help(print)
| Help on built-in function print in module builtins:
|
| print(...)
|     print(value, ..., sep=' ', end='\n', file=sys.stdout, flush=False)
|
|     Prints the values to a stream, or to sys.stdout by default.
|     Optional keyword arguments:
|     file:  a file-like object (stream); defaults to the current sys.stdout.
|     sep:   string inserted between values, default a space.
|     end:   string appended after the last value, default a newline.
|     flush: whether to forcibly flush the stream.
```

Assignments

- *variable = expression*

```
x = 42
```

- Multiple assignments – right hand side evaluated before assignment

```
x, y, z = 2, 5, 7
```

- Useful for swapping

```
x, y = y, x
```

- Assigning multiple variables same value in left-to-right

```
x = y = z = 7
```

Warning

```
i = 1
i = v[i] = 3  # v[3] is assigned value 3
```

In languages like C and C++ instead v[1] is assigned 3

Python is dynamically typed, type(...)

- The current type of a value can be inspected using the **type()** function (that returns a type object)
- In Python the values contained in a variable over time can be of different type
- In languages like C, C++ and Java variables are declared with a given type, e.g.

```
int x = 42;
```

and the different values stored in this variable must remain of this type

Python shell

```
> x = 1
> type(x)
| <class 'int'>
> x = 'Hello'
> type(x)
| <class 'str'>
> type(42)
| <class 'int'>
> type(type(42))
| <class 'type'>
```

x new type

Type conversion

- Convert a value to another type:

  new-type(*value*)

- Sometimes done automatically:

`1.0+7=1.0+float(7)=8.0`

Python shell

string concatenation

```
> float(42)
| 42.0
> int(7.8)
| 7
> x = 7
> print("x = " + x)
| Traceback (most recent call last):
|   File "<stdin>", line 1, in <module>
| TypeError: must be str, not int
> print("x = " + str(x))
| x = 7
> print("x = " + str(float(x)))
| x = 7.0
> int("7.3")
| Traceback (most recent call last):
|   File "<stdin>", line 1, in <module>
| ValueError: invalid literal for int() with base 10: '7.3'
> int(float("7.3"))
| 7
```

Questions – `str(float(int(float("7.5"))))` ?

a) 7
b) 7.0
c) 7.5
d) "7"
e) "7.0"
f) "7.5"
g) Don’t know

Control structures

- input()
- if-elif-else
- while-break-continue

input

- The builtin function `input(`*message*`)` prints *message*, and waits for the user provides a line of input and presses return. The line of input is returned as a `str`
- If you e.g. expect input to be an `int`, then remember to convert the input using `int()`

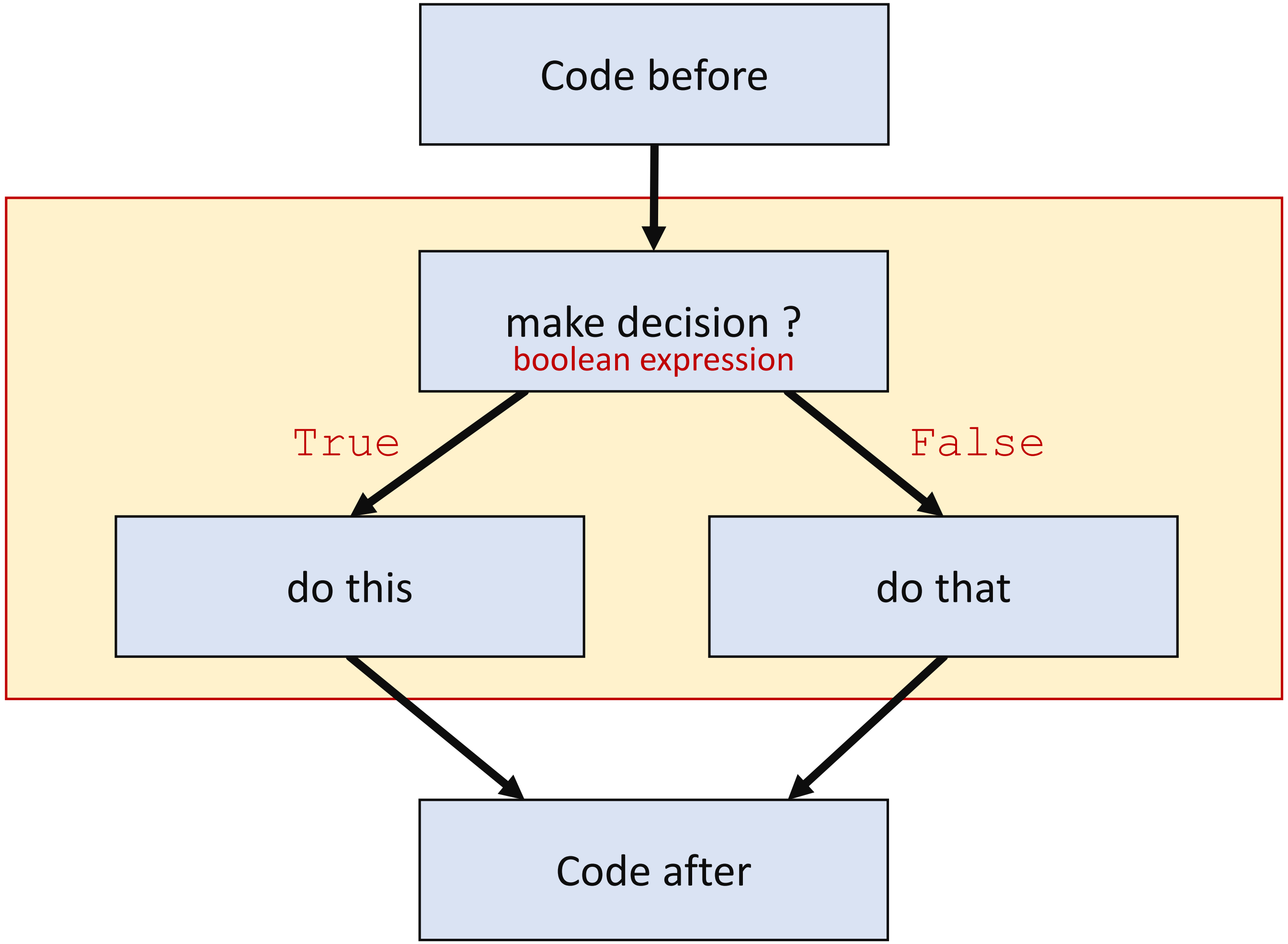
name-age.py

```
name = input('Name: ')
age = int(input('Age: '))
print(name, 'is', age, 'years old')
```

Python shell

```
> Name: Donald Duck
> Age: 84
| Donald Duck is 84 years old
```

Branching – do either this or that ?



Basic if-else

`if` *boolean expression*:

identical indentation

code
code
code

`else:`

identical indentation

code
code
code

if-else.py

```
if x % 2 == 0:
    print('even')
else:
    print('odd')
```

Identical indentation for a sequence of lines = the same spaces/tabs should precede code

pass

- `pass` is a Python statement doing nothing. Can be used where a statement is required but you want to skip (e.g. code will be writen later)

- Example (bad example, since `else` could just be omitted):

if-else.py

```
if x % 2 == 0:
    print('even')
else:
    pass
```

if-elif-else

`if` *condition*:

code

`elif` *condition*: # zero or more “elfi“ ≡ “else if”

code

`else:` # optional

code

```
if (condition) {
  code
} else if (condition) {
  code
} else {
  code
}
```

Java, C, C++ syntax

Other languages using indentation for blocking: ABC (1976), occam (1983), Miranda (1985)

if.py

```python
if x == 0:
    print('zero')
```

if-else.py

```python
if x % 2 == 0:
    print('even')
else:
    print('odd')
```

elif.py

```python
if x < 0:
    print('negative')
elif x == 0:
    print('zero')
elif x == 1:
    print('one')
else:
    print('>= 2')
```

Questions – What value is printed?

```
x = 1
if x == 2:
    x = x + 1
else:
    x = x + 1
    x = x + 1
x = x + 1
print(x)
```

a) 1

b) 2

c) 3

d) 4

e) 5

f) Don't know

Nested if-statements

nested-if.py

```
if x < 0:
    print('negative')
elif x % 2 == 0:
    if x == 0:
        print('zero')
    elif x == 2:
        print('even prime number')
    else:
        print('even composite number')
else:
    if x == 1:
        print('one')
    else:
        print('some odd number')
```

if-else *expressions*

- A very common computation is

    `if` *test*:
        `x =` *true-expression*
    `else:`
        `x =` *false-expression*

- In Python there is a shorthand for this:

think of this as the "common case" and the "exceptional case"

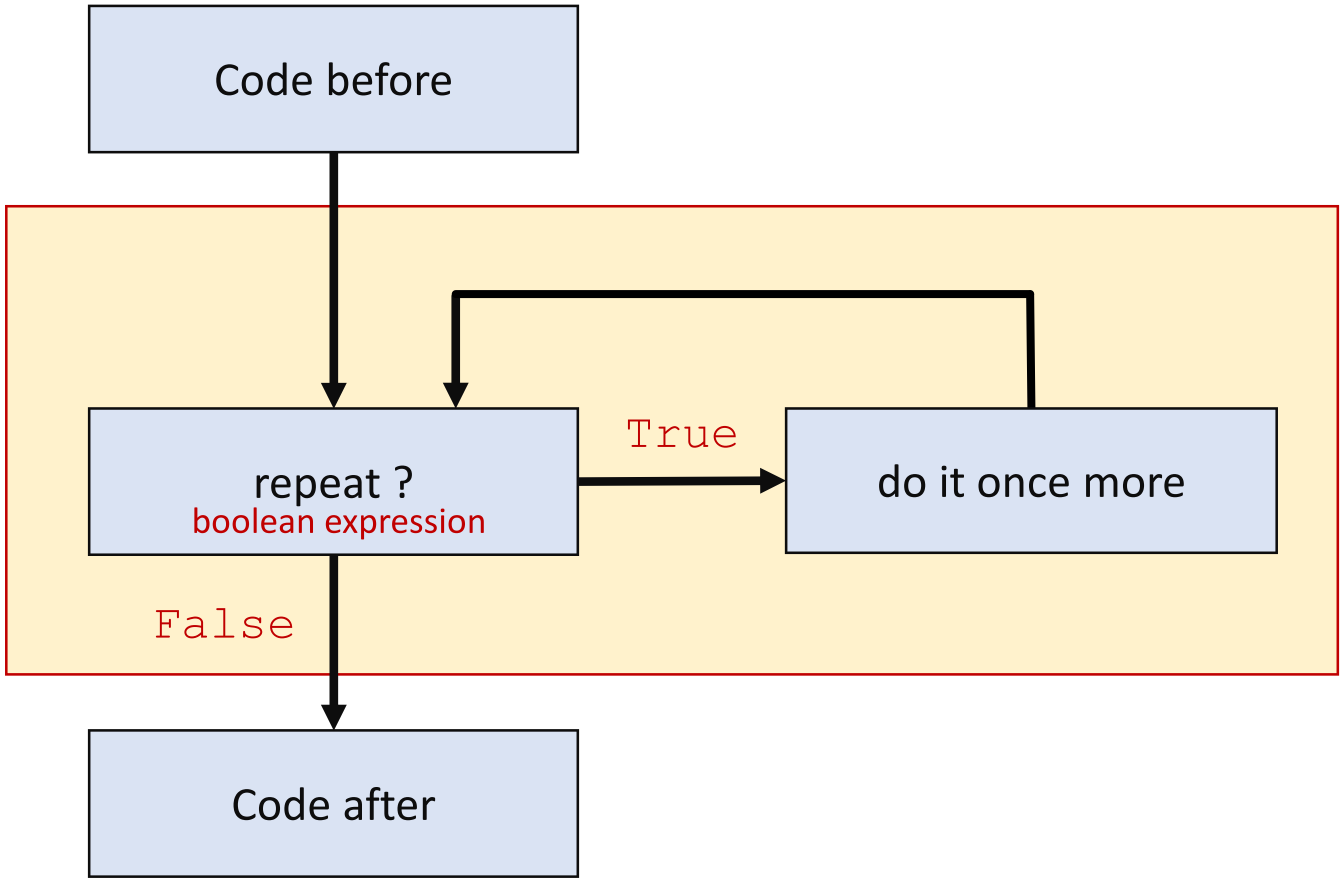
`x =` *true-expression* `if` *test* `else` *false-expression*

(see [What's New in Python 2.5 - **PEP 308: Conditional Expressions**])

- In C, C++ and Java the equivalent notation is (note the different order)

`x =` *test* `?` *true-expression* `:` *false-expression*

Repeat until done



while-statement

```
while condition:
    code
    ...
    break  #  jump to code after while loop
    ...
    continue #  jump to condition at the
    ...      #  beginning of while loop
```

The function `randint(a, b)` from module `random` returns a random integer from {a, a + 1,..., b – 1, b}

random-pair.py

```python
from random import randint
while True:
    x = randint(1, 10)
    y = randint(1, 10)
    if abs(x - y) >= 2:
        break
    print('too close', x, y)
print(x, y)
```

Python shell

```
| too close 4 4
| too close 10 9
| 8 5
```

count.py

```python
x = 1
while x <= 5:
    print(x, end=' ')
    x = x + 1
print('and', x)
```

Python shell

```
| 1 2 3 4 5 and 6
```

```
while (condition) {
  code
}
```

Java, C, C++ syntax

Computing $\lfloor\sqrt{x}\rfloor$ using binary search

An exercise asks to simplify the code

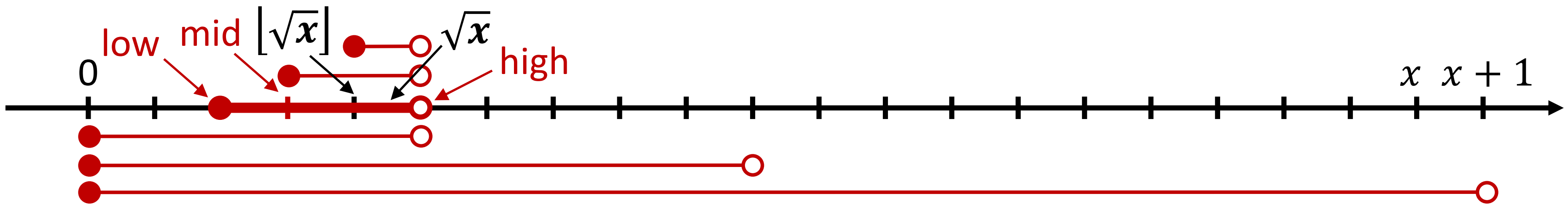
int-sqrt.py

```
x = 20
low = 0
high = x + 1
while True:   # low <= sqrt(x) < high
    if low + 1 == high:
        break
    mid = (high + low) // 2
    if mid * mid <= x:
        low = mid
        continue
    high = mid
print(low)   # low = floor(sqrt(x))
```

Integer division

$$\left\lfloor\frac{\texttt{high+low}}{2}\right\rfloor$$

$$\texttt{mid} \le \sqrt{x}$$
$$\Updownarrow$$
$$\texttt{mid}^2 \le x$$

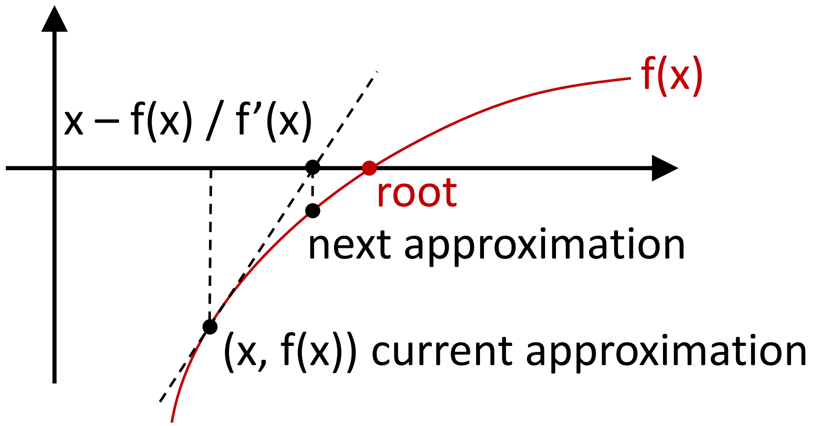


Division using the Newton-Raphson method

- **Goal:** Compute 1 / n only using +, -, and *
- $x = 1 / n \iff f(x) = n - 1 / x = 0$
- Problem reduces to finding root of f
- Newton-Raphson:

$$x := x - f(x)/f'(x) = x - (n-1/x)/(1/x^2) = (2 - n \cdot x) \cdot x$$

since $f'(x) = 1 / x^2$ for $f(x) = n - 1 / x$



division.py

```python
n = 0.75  # n in [0.5, 1.0]
x = 1.0
last = 0.0
while last < x:
    print(x)
    last = x
    x = (2 - n * x) * x
print('Apx of 1.0 /', n, '=', x)
print('Python 1.0 /', n, '=', 1.0 / n)
```

Python shell

```
| 1.0
| 1.25
| 1.328125
| 1.3331298828125
| 1.333333330228925
| 1.3333333333333333
| Apx of 1.0 / 0.75 = 1.3333333333333333
| Python 1.0 / 0.75 = 1.3333333333333333
```

en.wikipedia.org/wiki/Newton's_method

Operations

- None, bool
- basic operations
- strings
- += and friends

NoneType

- The type None has only one value: `None`
- Used when context requires a value, but none is really available
- **Example**: All functions must return a value. The function `print` has the *side-effect* of printing something to the standard output, but returns `None`
- **Example**: Initialize a variable with no value, e.g. list entries `mylist = [None, None, None]`

Python shell

```
> x = print(42)
| 42
> print(x)
| None
```

Named after mathematician George Boole (1815-1864)

Type bool

- The type `bool` only has two values: `True` and `False`

- Logic truth tables:

x **or** *y*	True	False
True	True	True
False	True	False

x **and** *y*	True	False
True	True	False
False	False	False

x	**not** *x*
True	False
False	True

Scalar vs Non-scalar Types

- **Scalar types** (atomic/indivisible): `int`, `float`, `bool`, `None`
- **Non-scalar**: Examples `strings` and `lists`

```
"string"[3] = "i"
[2, 5, 6, 7][2] = 6
```

Questions – What is `[7,3,5][[1,2,3][1]]` ?

a) 1

b) 2

c) 3

 d) 5

e) 7

f) Don’t know

Operations on int and float

Result is float if and only if at least one argument is float, except ** with negative exponent always gives a float

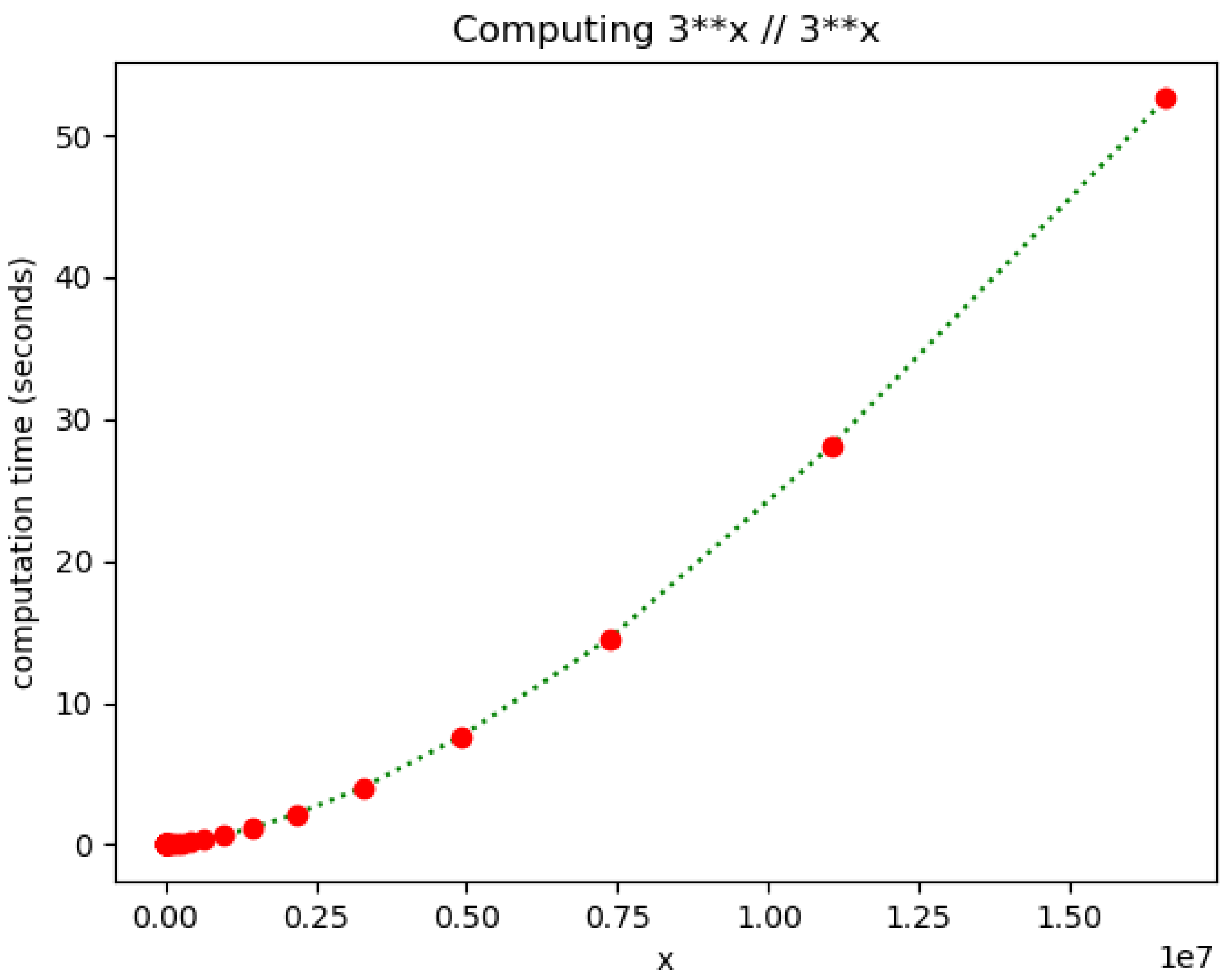
- `+`, `-`, `*` addition multiplication, e.g. `3.0*2` = `6.0`
- `**` and `pow(x, y)` power, e.g. `2**3=pow(2,3)=8`, `2**-2=0.25`
- `//` integer division = ⌊x / y⌋
  e.g. `15.0//4= 3.0`. Note: `-8//3=-3`
- `/` division returns float, `6/3=2.0`
- `abs(x)` absolute value
- `%` integer division remainder (modulo)
  `11%3 =2`
  `4.7%0.6=0.5000000000000003`

Python shell

```
> 0.4 // 0.1
| 4.0
> 0.4 / 0.1
| 4.0
> 0.3 // 0.1
| 2.0
> 0.3 / 0.1
| 2.9999999999999996
> 10**1000 / 2
| OverflowError: integer division
  result too large for a float
```

Running time for 3**x // 3**x

Working with larger integers takes slightly more than linear time in the number of digits



integer-division-timing.py

```
from time import time
import matplotlib.pyplot as plt

bits, compute_time = [], []

for i in range(42):
    x = 3**i // 2**i
    start = time()
    result = 3**x // 3**x     # the computation we time
    end = time()
    t = end - start
    print("i =", i, "x =", x, "Result =", result, "time(sec) =", t)
    bits.append(x)
    compute_time.append(t)

plt.title('Computing 3**x // 3**x')
plt.xlabel('x')
plt.ylabel('computation time (seconds)')
plt.plot(bits, compute_time, "g:")
plt.plot(bits, compute_time, "ro")
plt.show()
```

module math

Many standard mathematical functions are available in the Python module "`math`", e.g.

sqrt, sin, cos, tan, asin, acos, atan, log(natural), log10, exp, ceil, floor, ...

- To use all the functions from the math module use `import math`
  Functions are now available as e.g. `math.sqrt(10)` and `math.ceil(7.2)`
- To import selected functions you instead write `from math import sqrt, ceil`
- The library also contains some constants, e.g.
  `math.pi` = 3.141592... and `math.e` = 2.718281...
- Note: `x ** 0.5` significantly faster than `sqrt(x)`

Python shell

```
> (0.1 + 0.2) * 10
| 3.0000000000000004
> math.ceil((0.1 + 0.2) * 10)
| 4
```

docs.python.org/3/library/math.html

Python shell

```
> import math
> math.sqrt(8)
| 2.8284271247461903
> from math import pi, sqrt
> pi
| 3.141592653589793
> sqrt(5)
| 2.23606797749979
> from math import sqrt as kvadratrod
> kvadratrod(3)
| 1.7320508075688772

> import timeit
> timeit.timeit("1e10**0.5")
| 0.021124736888936863
> timeit.timeit("sqrt(1e10)", "from math import sqrt")
| 0.136631405286578‌9
> timeit.timeit("math.sqrt(1e10)", "import math")
| 0.19466084163458‌2
```

docs.python.org/3.6/library/timeit.html

Rounding up integer fractions

- Python: $\lceil x/y \rceil = -(-x//y)$

-(-13/3)		
Python	Java	C
-(-13//3) = 5	-(-13/3) = 4 ⚠	-(-13/3) = 4 ⚠

⚠ The intermediate result `x/y` in

`math.ceil(x/y)`

is a float with limited precision

- Alternative computation:

$\lceil x/y \rceil = (x+(y-1))//y$

Python shell

```
> from math import ceil
> from timeit import timeit
> 13 / 3
| 4.333333333333333
> 13 // 3
| 4
> -13 // 3
| -5
> -(-13 // 3)
| 5
> ceil(13 / 3)
| 5
> -(-22222222222222222223 // 2)
| 11111111111111111112
> ceil(22222222222222222223 / 2)
| 11111111111111110656
> timeit('ceil(13 / 3)', 'from math import ceil')
| 0.27746671276099973
> timeit('-(-13 // 3)')  # negation trick is fast
| 0.05231945830200857
```

floats : Overflow, inf, -inf, nan

- There exists special float values
  `inf`, `-inf`, `nan`
  representing “+infinity”, “-infinity” and “not a number”
- Can be created using e.g.
  `float('inf')`
  or imported from the `math` module
- Some overflow operations generate an `OverflowError`, other return `inf` and allow calculations to continue !
- Read the [IEEE 754 standard]() if you want to know more details...

Python shell

```
> 1e250 ** 2
| OverflowError:
  (34, 'Result too large')
> 1e250 * 1e250
| inf
> -1e250 * 1e250
| -inf
> import math
> math.inf
| inf
> type(math.inf)
| <class 'float'>
> math.inf / math.inf
| nan
> type(math.nan)
| <class 'float'>
> math.nan == math.nan
| false
> float('inf') - float('inf')
| nan
```

Operations on bool

- The operations `and`, `or`, and `not` behave as expected when the arguments are `False`/`True`.
- The three operators also accept other types, where the following values are considered *false*:

  `False, None, 0, 0.0, "", [],…`

  (see The Python Standard Library > 4.1. True Value Testing for more *false* values)
- *Short-circuit evaluation*: The rightmost argument of `and` and `or` is only evaluated if the result cannot be determined from the leftmost argument alone. The result is either the leftmost or rightmost argument (see truth tables), i.e. the result is not necessarily `False`/`True`.

  `True or 7/0` is completely valid since 7/0 will never be evaluated (which otherwise would throw a ZeroDivisionError exception)

`x`	`x or y`
false	`y`
otherwise	`x`

`x`	`x and y`
false	`x`
otherwise	`y`

`x`	`not x`
false	`True`
otherwise	`False`

Questions – What is `"abc" and 42` ?

a) `False`
b) `True`
c) `"abc"`
d) `42`
e) TypeError
f) Don’t know

Comparison operators (e.g. int, float, str)

`==`	test if two objects are equal, returns bool not to be confused with the assignment operator (=)
`!=`	not equal
`>`	
`>=`	
`<`	
`<=`	

Python shell

```
> 3 == 7
| False
> 3 == 3.0
| True
> "-1" != -1
| True
> "abc" == "ab" + "c"
| True
> 2 <= 5
| True
> -5 > 5
| False
> 1 == 1.0
| True
> 1 == 1.0000000000000001
| True
> 1 == 1.000000000000001
| False
```

Chained comparisons

- A recurring condition is often

```
x < y and y < z
```

- If `y` is a more complex expression, we would like to avoid computing `y` twice, i.e. we often would write

```
tmp = complex expression
x < tmp and tmp < z
```

- In Python this can be written as a chained comparisons (which is shorthand for the above)

```
x < y < z
```

- Note: Chained comparisons do not exist in C, C++, Java, ...

Questions – What is `1 < 0 < 6/0` ?

a) `True`
b) `False`
c) `0`
d) `1`
e) `6`
f) ZeroDivisionError
g) Don't know

Binary numbers and operations

- Binary number = integer written in base 2: $101010_2 = 42_{10}$

- Python constant prefix 0b: `0b101010` → `42`
- `bin(x)` converts integer to string: `bin(49)` → `"0b110001"`
- `int(x,2)` converts binary string value to integer: `int("0b110001",2)` → `49`
- Bitwise operations

|	Bitwise OR
&	Bitwise AND
~	Bitwise NOT (`~ x` equals to `-x - 1`)
^	Bitwise XOR

- Example: `bin(0b1010 | 0b1100)` → `"0b1110"`
- Hexadecimal = base 16, Python prefix 0x: `0x30` → `48`, `0xA0` → `160`, `0xFF` → `255`
- `<<` and `>>` integer bit shifting left and right, e.g. `12 >> 2` → `3`, and `1 << 4` → `16`

Operations on strings

- `len(`*str*`)` returns length of *str*
- *str*`[`*index*`]` returns *index*+1'th symbol in *str*
- str_1 + str_2 returns concatenation of two strings
- *int* * *str* concatenates *str* with itself *int* times
- Formatting: % operator or .format() function
  old Python 2 way — since Python 3.0
  or formatted string literals (f-strings) with prefix
  since Python 3.6
  letter `f` and Python expressions in `{}`

(see pyformat.info for an introduction)

From "What's New In Python 3.0", 2009: *A new system for built-in string formatting operations replaces the % string formatting operator. (However, the % operator is still supported; it will be deprecated in Python 3.1 and removed from the language at some later time.) Read PEP 3101 for the full scoop.*

Python shell

```
> len("abcde")
| 5
> "abcde"[2]
| 'c'
> x = 2; y = 3
> "x = %s, y = %s" % (x, y)
| 'x = 2, y = 3'
> "x = {}, y = {}".format(x,y)
| 'x = 2, y = 3'
> f'x + y = {x + y}'
| 'x + y = 5'
> f'{x + y = }'   # >= Python 3.8
| 'x + y = 5'
> f'{x} / {y} = {x / y:.3}'
| '2 / 3 = 0.667'
> "abc" + "def"
| 'abcdef'
> 3 * "x--"
| 'x--x--x--'
> 0 * "abc"
| ''
```

% formatting (inherited from C's sprint() function) was supposed to be on the way out - but is still going strong in Python 3.8

… more string functions

- *str*`[`−*index*`]` returns the symbol i positions from the right, the rightmost *str*`[-1]`
- *str*`[`*from*`:`*to*`]` substring starting at index *from* and ending at index *to*-1
- *str*`[`*from*`:-`*to*`]` substring starting at *form* and last at index `len(`*str*`)` − *to* `-1`
- *str*`[`*from*`:`*to*`:`*step*`]` only take every *step*’th symbol in *str*[*from*:*to*]
  - *from* or/and *to* can be omitted and defaults to the beginning/end of string
- `chr(`*x*`)` returns a string of length 1 containing the *x*’th Unicode character
- `ord(`*str*`)` for a string of length 1, returns the Unicode number of the symbol
- *str*`.lower()` returns string in lower case
- *str*`.split()` split string into list of words, e.g.

```
"we love python".split() = ['we', 'love', 'python']
```

docs.python.org/3/library/string.html

Questions – What is `s[2:42:3]`?

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

`s = 'abwwdexy   lwtopavghevt xypxxyattx hxwoadnxxx'`

a) `'wwdexy___lwtopavghevt_xypxxyattx_hxwoadn'`

b) `'we_love_python'`

c) `'we_love_java'`

d) Don’t know

Strings are immutable

- Strings are non-scalar, i.e. for `s = "abcdef"`, `s[3]` will return `"d"`
- Strings are immutable and cannot be changed once created. I.e. the following natural update is not possible (but is e.g. allowed in C)

```
s[3] = "x"
```

- To replace the `"d"` with `"x"` in `s`, instead do the following update

```
s = s[:3] + "x" + s[4:]
```

Operators
Precedence rules & Associativity

Example: * has higher precedence than +

2 + 3 * 4 ≡ 2 + (3 * 4) → 14 and (2 + 3) * 4 → 20

All operators in same group are evaluated left-to-right

2 + 3 - 4 - 5 ≡ ((2 + 3) - 4) - 5 → -4

except for **, that is evaluated right-to-left

2**2**3 ≡ 2**(2**3) → 256

Rule: Use parenthesis whenever in doubt of precedence!

docs.python.org/3/reference/expressions.html

Precedence (low to high)		
`or`		
`and`		
`not x`		
`in`	`not in`	
`is`	`is not`	
`==`	`<`	`<=`
`!=`	`>`	`>=`
`|`		
`^`		
`&`		
`<<`	`>>`	
`+`	`-`	
`*`	`@`	
`/`	`//`	`%`
`+x`	`-x`	`~x`
`**`		

Long expressions

- Long expressions can be broken over several lines by putting parenthesis around it
- The PEP8 guidelines recommend to limit **all** lines to a maximum of 79 characters

Python shell

```
> (1
    + 2 +
        3)
| 6
```

https://www.python.org/dev/peps/pep-0008/#maximum-line-length

+= and friends

- Recurring statement is

```
x = x + value
```

- In Python (and many other languages) this can be written as

```
x += value
```

- This also applies to other operators like

```
+=   -=   *=   /=   //=   **=
|=   &=   ^=   <<=   >>=
```

Python shell

```
> x = 5
> x *= 3
> x
| 15
> a = 'abc'
> a *= 3
> a
| 'abcabcabc'
```

:= assignment expressions (the "Walrus Operator")

New in Python 3.8

- Syntax

  `name := expression`

- Evaluates to the value of `expression`, with the side effect of assigning result to `name`
- Useful for naming intermediate results/repeating subexpressions for later reusage
- See PEP 572 for further details and restrictions of usage

Python shell

```
> (x := 2 * 3) + 2 * x
| 18
> print(1 + (x := 2 * 3), 2 + x)
| 7 8
> x := 7
| SyntaxError
> (x := 7)  # valid, but not recommended
> while line := input():
      print(line.upper())
> abc
| ABC
```

- In some languages, e.g. Java, C and C++, "`=`" also plays the role of "`:=`", implying "`if (x=y)`" and "`if (x==y)`" mean quite different things (common typo)

Lists

- List syntax
- List operations
- copy.deepcopy
- range
- while-else
- for
- for-break-continue-else

List operations

- List syntax `[`*value*$_1$`,` *value*$_2$`, ...,` *value*$_k$`]`
- List indexing `L[`*index*`]`, `L[`−*index*`]`
- List slices `L[`*from*`:`*to*`]`, `L[`*from*`:`*to*`:`*step*`]` or `L[slice(`*from*`,`*to*`,`*step*`)]`
- Creating a copy of a list `L[:]` or `L.copy()`
- List concatenation (creates new list) `X + Y`
- List repetition (repeated concatenation with itself) `42 * L`
- Length of list `len(L)`
- Check if element is in list `e in L`
- Index of first occurrence of element in list `L.index(e)`
- Number of occurrences of element in list `L.count(e)`
- Check if element is not in list `e not in L`
- `sum(L)` `min(L)` `max(L)`

docs.python.org/3/library/stdtypes.html#sequence-types-list-tuple-range

List modifiers (lists are mutable)

- Extend list with elements (`X` is modified) `X.extend(Y)`
- Append an element to a list (`L` is modified) `L.append(42)`
- Replace sublist by another list (length can differ) `X[i:j] = Y`
- Delete elements from list `del L[i:j:k]`
- Remove & return element at position `L.pop(i)`
- Remove first occurrence of element `L.remove(e)`
- Reverse list `L.reverse()`
- `L *= 42`
- `L.insert(i, x)` same as `L[i:i] = [x]`

Python shell

```
> x = [1, 2, 3, 4, 5]
> x[2:4] = [10, 11, 12]
> x
| [1, 2, 10, 11, 12, 5]
> x = [1, 2, 11, 5, 8]
> x[1:4:2] = ['a', 'b']
| [1, 'a', 11, 'b', 8]
```

docs.python.org/3/library/stdtypes.html#sequence-types-list-tuple-range

Questions – What is `x` ?

```
x = [1,2,3,4,5,6,7,8,9,10]
x[2:8:3] = ['a', 'b']
```

a) `[1,2,'a','b',5,6,7,8,9,10]`
b) `[1,'a',3,4,5,6,7,'b',9,10]`
c) `[1,2,3,4,5,6,7,'a','b']`
d) `[1,2,'a',4,5,'b',7,8,9,10]`
e) ValueError
f) Don’t know

Questions – What is `y` ?

```
y = [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19]
y = y[3:15:3][1:4:2]
```

a) `[3,6,9,12,15]`

b) `[7,13]`

c) `[1,9]`

d) `[4,7,10,13,2,4]`

e) TypeError

f) Don’t know

Nested lists (multi-dimensional lists)

- Lists can contain lists as elements, that can contain lists as elements, that ...
- Can e.g. be used to store multi-dimensional data (list lengths can be non-uniform)

Note: For dealing with matrices the NumPy module is a better choice

multidimensional-lists.py

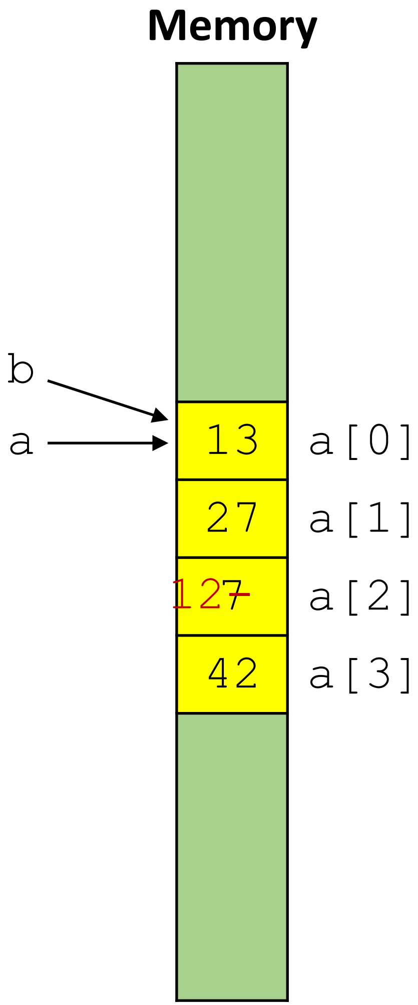
```
list1d = [1, 3, 5, 2]
list2d = [[1, 2, 3, 4],
          [5, 6, 7, 9],
          [0, 8, 2, 3]]
list3d = [[[5,6], [4,2], [1,7], [2,4]],
          [[1,2], [6,3], [2,5], [7,5]],
          [[3,8], [1,5], [4,3], [2,4]]]

print(list1d[2])
print(list2d[1][2])
print(list3d[2][0][1])
```

Python shell

```
| 5
| 7
| 8
```

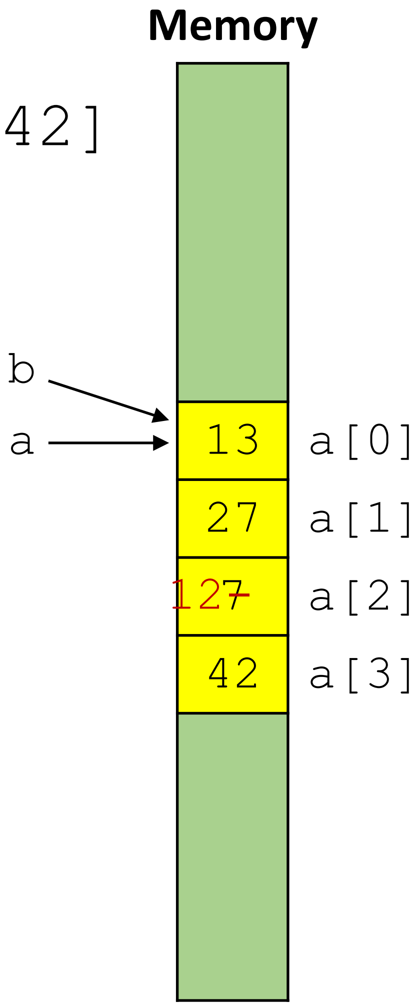
aliasing

```
a = [13, 27, 7, 42]
b = a
a[2] = 12
```

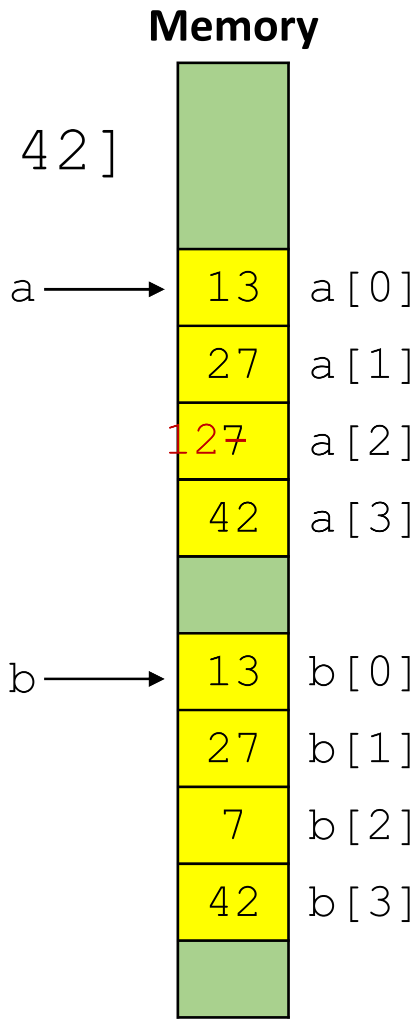


`y = x` vs `y = x[:]`

```
a = [13, 27, 7, 42]
b = a
a[2] = 12
```

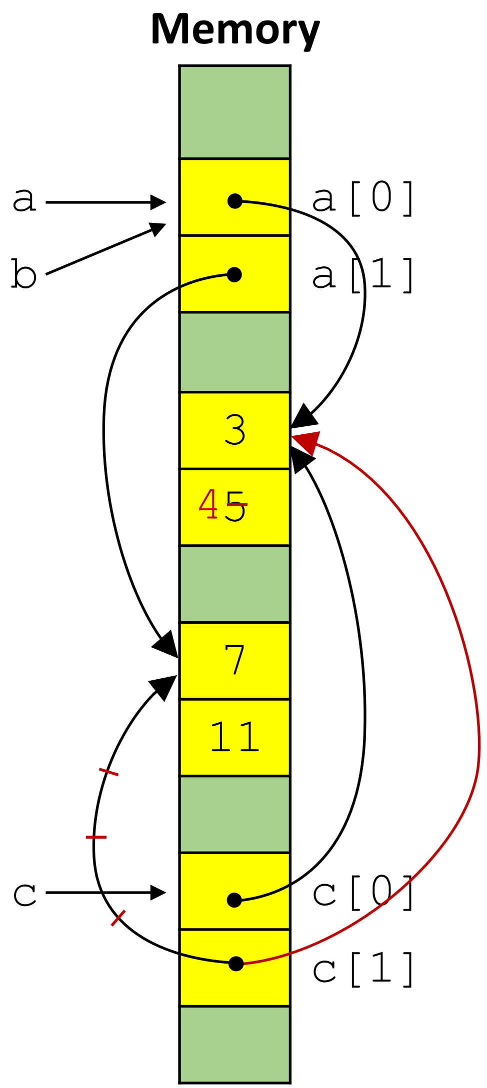



```
a = [13, 27, 7, 42]
b = a[:]
a[2] = 12
```



`x[:]` vs nested structures

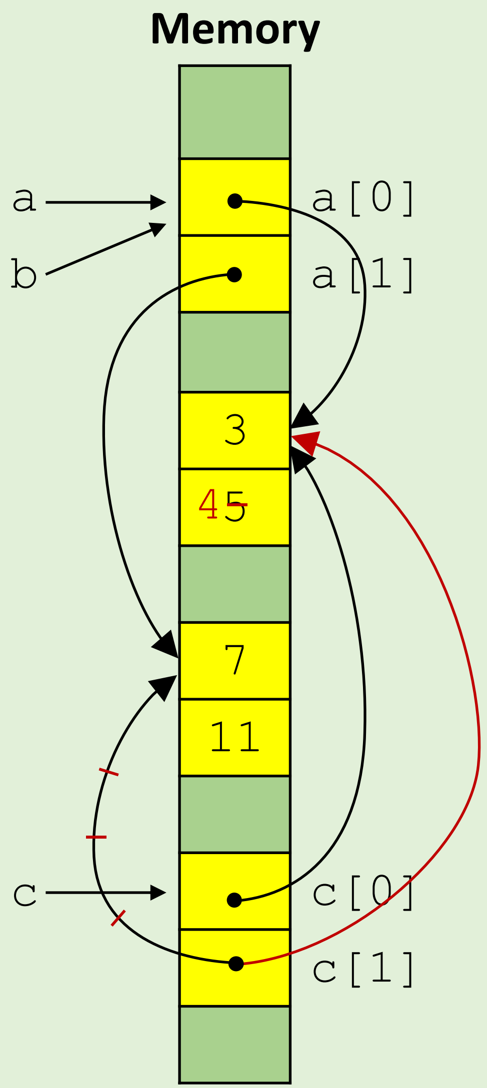
```
a = [[3,5],[7,11]]
b = a
c = a[:]
a[0][1] = 4
c[1] = b[0]
```



Question – what is `c` ?

a) `[[3,5],[7,11]]`
b) `[[3,5],[3,5]]`
c) `[[3,4],[3,5]]`
d) `[[3,4],[3,4]]`
e) Don't know

```
a = [[3,5],[7,11]]
b = a
c = a[:]
a[0][1] = 4
c[1] = b[0]
```

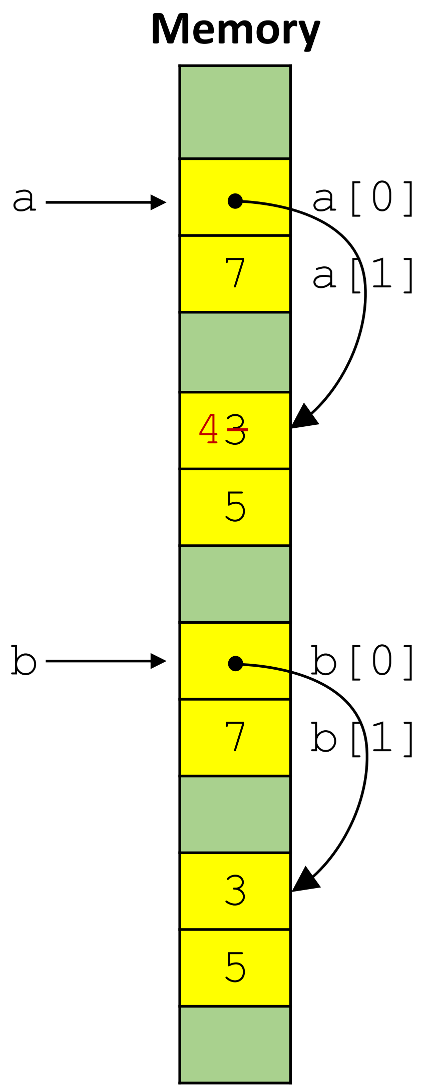


copy.deepcopy

- To make a copy of all parts of a composite value use the function deepcopy from module copy

Python shell

```
> from copy import deepcopy
> a = [[3, 5], 7]
> b = deepcopy(a)
> a[0][0] = 4
> a
| [[4,5],7]
> b
| [[3,5],7]
```



Initializing a 2-dimensional list

Python shell

```
> x = [1] * 3
> x
| [1, 1, 1]
> y = [[1] * 3] * 4
> y
| [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]]
> y[0][0] = 0
> y
| [[0, 1, 1], [0, 1, 1], [0, 1, 1], [0, 1, 1]]
```

Python shell

```
> y = []
> for _ in range(4): y.append([1] * 3)
> y[0][0] = 0
> y
| [[0, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]]
```

range(*from*, *to*, *step*)

In Python 2, `range` generates the explicit list, i.e. always use memory proportional to the length; `xrange` in Python 2 corresponds to `range` in Python 3; Python 3 is more memory friendly

- `range(`*from*`,` *to*`,` *else*`)` generates the sequence of numbers starting with *from*, with increments of *step*, and smaller/greater than *to* if *step* positive/negative

`range(5)`	`: 0, 1, 2, 3, 4`	(default *from* = 0, *step* = 1)
`range(3, 8)`	`: 3, 4, 5, 6, 7`	(default *step* = 1)
`range(-2, 7, 3)`	`: -2, 1, 4`	(*from* and *to* can be any integer)
`range(2, -5, -2)`	`: 2, 0, -2, -4`	(decreasing sequence if *step* negative)

- Ranges are immutable, can be indexed like a list, sliced, and compared (i.e. generate the same numbers)
- `list(range(...))` generates the explicit list of numbers

Python shell

```
> range(1, 10000000, 3)[2]
| 7
> range(1, 10000000, 3)[100:120:4]
| range(301, 361, 12)
> range(1, 10000000, 3)[100:120:4][2:3]
| range(325, 337, 12)
> list(range(5, 14, 3))
| [5, 8, 11]
```

Question – What is `range(3,20,4)[2:4][1]` ?

a) 3
b) 7
c) 11
d) 15
e) 19
f) Don't know

for - loop

- For every element in a sequence execute a block of code:

  `for` *var* `in` *sequence*:

  block

- Sequences can e.g. be lists, strings, ranges
- `break` and `continue` can be used like in a while-loop to break out of the for-loop or continue with the next element in the sequence

Python shell

```
> for x in [1, "abc", [2, 3], 5.0]:
>       print(x)
| 1
| abc
| [2, 3]
| 5.0
> for x in "abc":
>       print(x)
| a
| b
| c
> for x in range(5, 15, 3):
>       print(x)
| 5
| 8
| 11
| 14
```

Question – What is printed ?

Python shell

```
> for i in range(1, 4):
>     for j in range(i, 4):
>         print(i, j, sep=':', end=' ')
```

a) 1:1 1:2 1:3 2:1 2:2 2:3 3:1 3:2 3:3

b) 1:1 1:2 1:3 2:2 2:3 3:3

c) 1:1 2:1 3:1 1:2 2:2 3:2 1:3 2:3 3:3

d) 1:1 2:1 3:1 2:2 3:2 3:3

e) Don’t know

Question – `break`, what is printed ?

Python shell

```
> for i in range(1, 4):
>     for j in range(1, 4):
>         print(i, j, sep=':', end=' ')
>         if j >= i:
>             break
```

a) *nothing*

b) `1:1`

c) `1:1 2:1 2:2 3:1 3:2 3:3`

d) `1:1 2:2 3:3`

e) Don’t know

In nested `for`- and `while`-loops, `break` only breaks the innermost loop

Palindromic substrings

- Find all palindromic substrings of length ≥ 2, i.e. substrings spelled identically forward and backwards:

  abracadrabratrallalla

  i j i j

- **Algorithm:** Test all possible substrings (brute force/exhaustive search)
- **Note**: the slice `t[::-1]` is `t` reversed

palindrom.py

```python
s = "abracadrabratrallalla"

for i in range(len(s)):
    for j in range(i + 2, len(s) + 1):
        t = s[i:j]
        if t == t[::-1]:
            print(t)
```

Python shell

```
| aca
| alla
| allalla
| ll
| llall
| lal
| alla
| ll
```

Sieve of Eratosthenes

- Find all prime numbers ≤ n

- Algorithm:

2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...

eratosthenes.py

```python
n = 100
prime = [True] * (n + 1)

for i in range(2, n):
    for j in range(2 * i, n + 1, i):
        prime[j] = False

for i in range(2, n + 1):
    if prime[i]:
        print(i, end=' ')
```

Python shell

```
| 2 3 5 7 11 13 17 19 23 29 31 37 41
  43 47 53 59 61 67 71 73 79 83 89
  97
```

en.wikipedia.org/wiki/Sieve_of_Eratosthenes

while-else and for-else loops

- Both for- and while-loops can have an optional “else”:

```
for var in sequence:
        block
else:
        block

while condition:
        block
else:
        block
```

- The “else” block is only executed if no `break` is performed in the loop
- The “else” construction for loops is specific to Python, and does not exist in e.g. C, C++ and Java

Linear search

linear-search-while.py

```
L = [7, 3, 6, 4, 12, 'a', 8, 13]
x = 4

i = 0
while i < len(L):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        break
    i = i + 1

if i >= len(L):
    print(x, "not in", L)
```

linear-search-while-else.py

```
i = 0
while i < len(L):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        break
    i = i + 1
else:
    print(x, "not in", L)
```

linear-search-for.py

```
found = False
for i in range(len(L)):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        found = True
        break

if not found:
    print(x, "not in", L)
```

linear-search-for-else.py

```
for i in range(len(L)):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        break
else:
    print(x, "not in", L)
```

linear-search-builtin.py

```
if x in L:
    print(x, "at position", L.index(x), "in", L)
else:
    print(x, "not in", L)
```

Some performance considerations

String concatenation

- To concatenate two (or few) strings use

  str_1 + str_2
  var += *str*

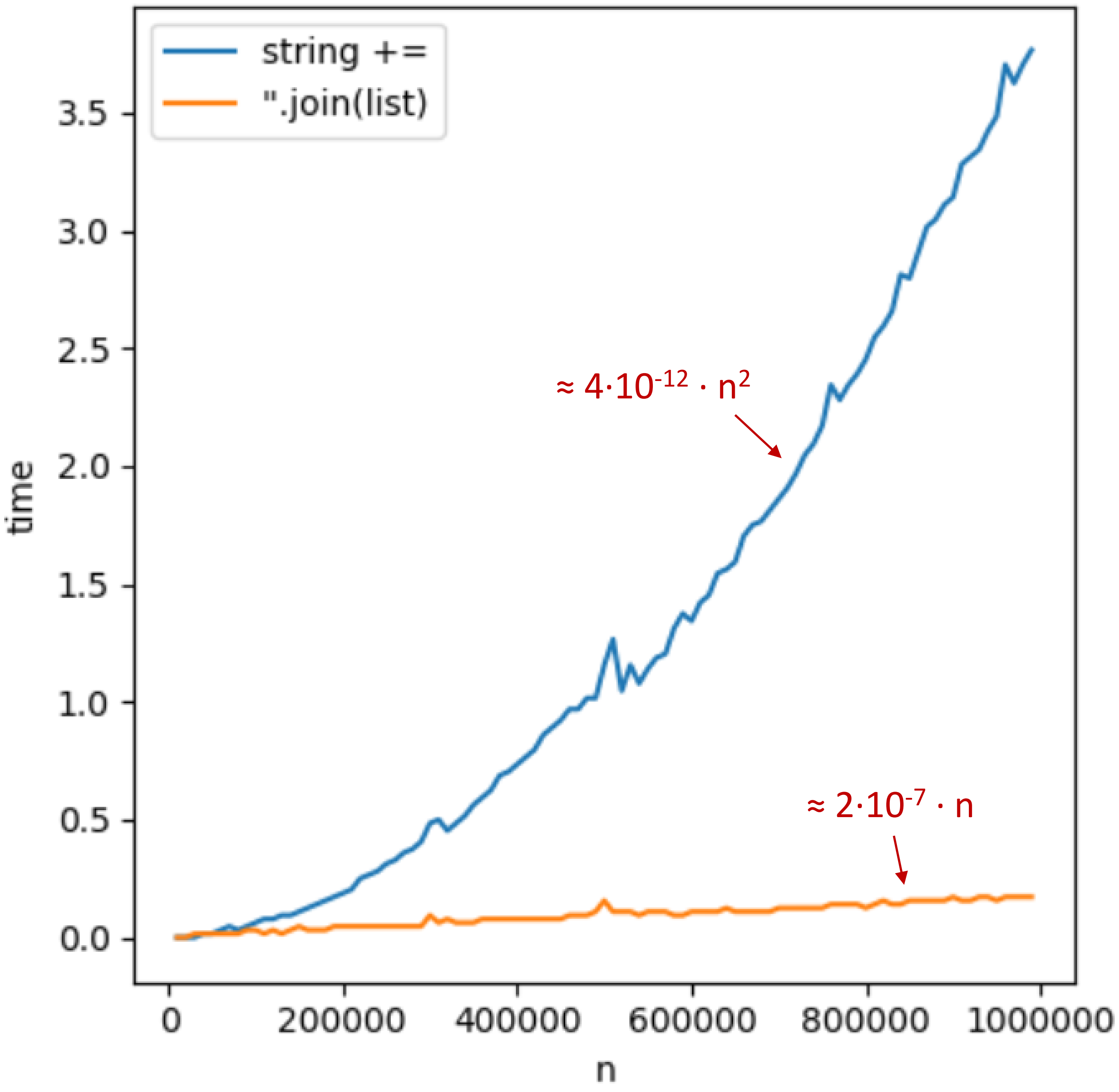
- To concatenate several/many strings use

  `''.join([`str_1, str_2, str_3, ... , str_n`])`

- Concatenating several strings by repeated use of + generates explicitly the longer-and-longer intermediate results; using `join` avoids this slowdown

Python shell

```
> s = 'A' + 'B' + 'C'
> s
| 'ABC'
> 'x'.join(['A', 'B', 'C'])
| 'AxBxC'
> s = ''
> s += 'A'
> s += 'B'
> s += 'C'
> s
| 'ABC'
> L = []
> L.append('A')
> L.append('B')
> L.append('C')
> L
| ['A', 'B', 'C']
> s = ''.join(L)
> s
| 'ABC'
```

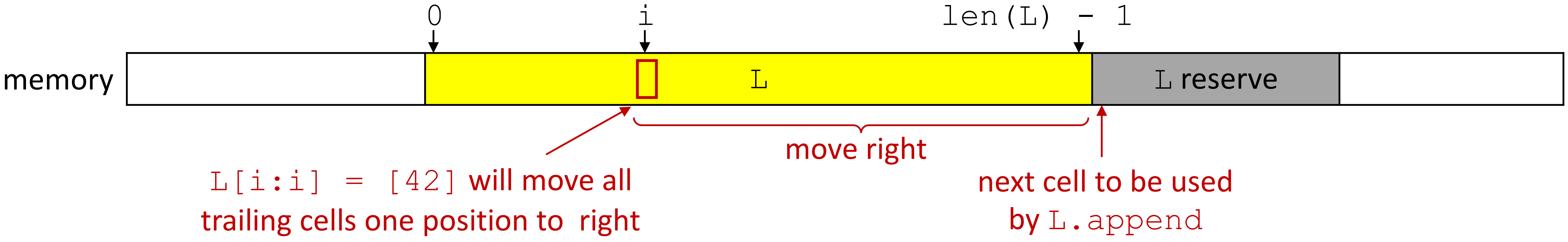


string-concatenation.py

```
from time import time
from matplotlib import pyplot as plt

ns = range(10_000, 1_000_000, 10_000)
time_string = []
time_list = []
for n in ns:
    start = time()
    s = ''
    for _ in range(n):
        s += 'abcdefgh'   # slow
    end = time()
    time_string.append(end - start)

    start = time()
    substrings = []
    for _ in range(n):
        substrings.append('abcdefgh');
    s = ''.join(substrings);
    end = time()
    time_list.append(end - start)

plt.plot(ns, time_string, label='string +=')
plt.plot(ns, time_list, label="''.join(list)")
plt.xlabel('n')
plt.ylabel('time')
plt.legend()
plt.show()
```

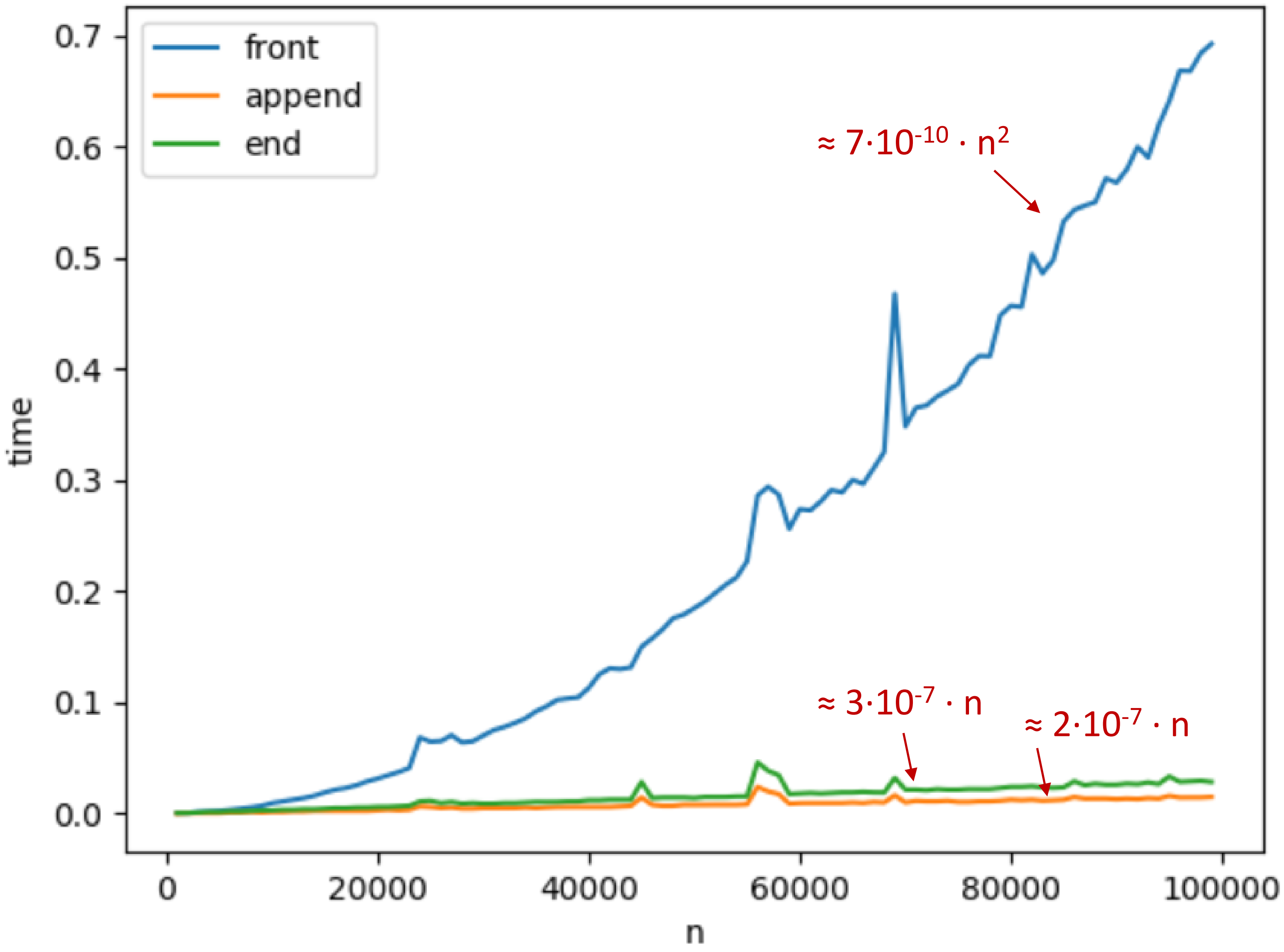
The internal implementation of Python lists

- Accessing and updating list positions take the same time independently of position
- Creating new / deleting entries in a list depends on position, Python optimizes towards updates at the end
- Try to organize your usage of lists to insert / delete elements at the end `L.append(`*element*`)` and `L.pop()`.
- Python lists internally have space for adding ≈ 12.5 % additional entries at the end; when the reserved extra space is exhausted the list is moved to a new chunk of memory with ≈ 12.5 % extra space



github.com/python/cpython/blob/master/Objects/listobject.c

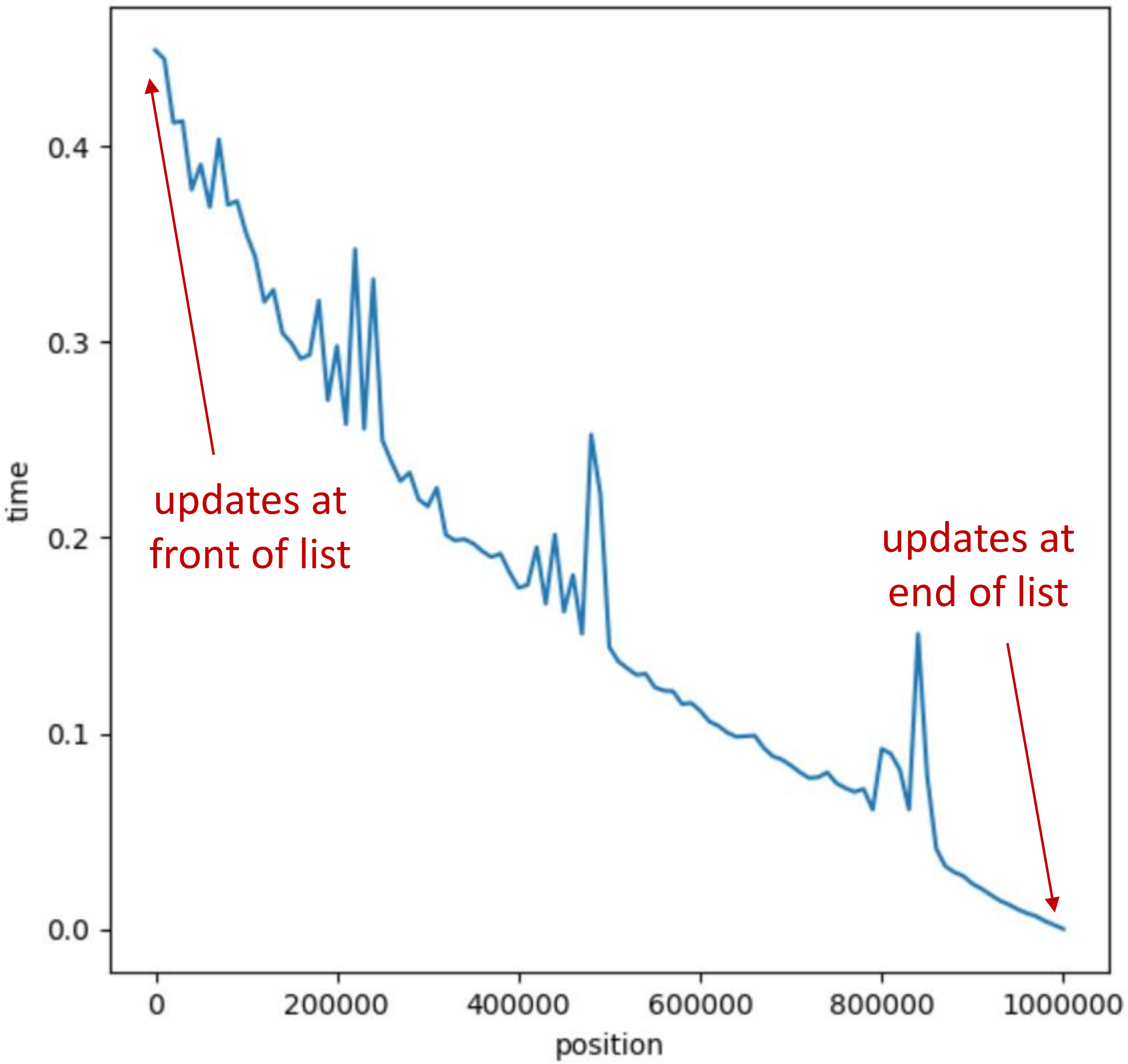
List insertions at begin vs end



list-insertions.py

```python
from time import time
from matplotlib import pyplot as plt

ns = range(1000, 100_000, 1000)
time_end = []
time_append = []
time_front = []
for n in ns:
    start = time()
    L = []
    for i in range(n):
        L[i:i] = [i]   # insert after list
    end = time()
    time_end.append(end - start)

    start = time()
    L = []
    for i in range(n):
        L.append(i)   # append to list
    end = time()
    time_append.append(end - start)

    start = time()
    L = []
    for i in range(n):
        L[0:0] = [i]   # insert at front
    end = time()
    time_front.append(end - start)

plt.plot(ns, time_front, label='front')
plt.plot(ns, time_append, label='append')
plt.plot(ns, time_end, label='end')
plt.xlabel('n')
plt.ylabel('time')
plt.legend()
plt.show()
```

Updates (insertions + deletions) in the middle of a list



list-updates.py

```
from time import time
from matplotlib import pyplot as plt

ns = range(0, 1_000_001, 10_000)
time_pos = []
L = list(range(1_000_000))  # L = [0, ..., 999_999]
for i in ns:
    start = time()
    for _ in range(1000):
        L[i:i] = [42]  # insert element before L[i]
        del L[i]  # remove L[i] from L
    end = time()
    time_pos.append(end - start)

plt.plot(ns, time_pos)
plt.xlabel('position')
plt.ylabel('time')
plt.show()
```

Tuples and lists

- tuples
- lists
- mutability
- list comprehension
- for-if, for-for
- list()
- any(), all()
- enumerate(), zip()

Tuples

$(\text{value}_1,\ \ \text{value}_2,\ \ \ldots\ \ ,\ \ \text{value}_k)$

- Tuples can contain a sequence of zero or more elements, enclosed by " (" and ") "
- Tuples are immutable
- Tuple of length 0: `()`
- Tuple of length 1: `(value, )`

  Note the comma to make a tuple of length one distinctive from an expression in parenthesis
- In many contexts a tuple with ≥ 1 elements can be written without parenthesis
- Accessors to lists also apply to tuples, slices, ...

Python shell

```
> (1, 2, 3)
| (1, 2, 3)
> ()
| ()
> (42)
| 42
> (42,)
| (42,)
> 1, 2
| (1, 2)
> 42,
| (42,)
> x = (3, 7)
> x
| (3, 7)
> x = 4, 6
> x
| (4, 6)
> x[1] = 42
| TypeError: 'tuple' object does
  not support item assignment
```

Question – What value is `((42,))` ?

a) `42`

b) `(42)`

 c) `(42,)`

d) `((42,),)`

e) Don't know

Question – What is `x` ?

```
x = [1, [2, 3], (4, 5)]
x[2][0] = 42
```

a) `[1, [42, 3], (4, 5)]`

b) `[1, [2, 3], (42, 5)]`

c) `[1, [2, 3], 42]`

d) TypeError

e) Don’t know

Question – What tree is `('A',(('B','C'),'D'))` ?

```
          /\
         /  \
        /    \
       /      \
      /\      'D'
     /  \
    /    \
   /\    'C'
  /  \
'A'  'B'
```

a)

```
         /\
        /  \
       /    \
      /      \
     /        \
    /\        'D'
   /  \
  /    \
'A'    /\
      /  \
    'B'  'C'
```

b)

```
        /\
       /  \
      /    \
     /      \
    /        \
   /\        /\
  /  \      /  \
'A'  'B'  'C'  'D'
```

c)

```
      /\
     /  \
    /    \
   /      \
  /        \
'A'        /\
          /  \
         /    \
        /\    'D'
       /  \
     'B'  'C'
```

d)

```
     /\
    /  \
   /    \
  /      \
'A'      /\
        /  \
       /    \
     'B'    /\
           /  \
         'C'  'D'
```

e)

f) Don't know

Tuple assignment

Python shell

```
> point = (10, 25)
> x, y = point
> x
| 10
> y
| 25
```

- Parallel assignments

  `x, y, z = a, b, c`

  is a short hand for a tuple assignment (right side is a single tuple)

  `(x, y, z) = (a, b, c)`

- First the right-hand side is evaluated completely, and then the individual values of the tuple are assigned to `x`, `y`, `z` left-to-right (length must be equal on both sides)

Nested tuple/lists assignments

- Let hand side can be nested (great for unpacking data)

```
(x, (y, (a[0], w)), a[1])
   = 1, (2, (3, 4)), 5
```

- [...] and (...) on left side matches both lists and tuples of equal length (but likely you would like to be consistent with type of parenthesis)

Python shell

```
> two_points = [(10, 25), (30, 40)]
> (x1, y1, x2, y2) = two_points
| ValueError: not enough values to
  unpack (expected 4, got 2)
> ((x1, y1), (x2, y2)) = two_points
> a = [None, None]
> v = ((2, (3, 4)), 5)
> ((y, (a[0], w)), a[1]) = v
> a
| [3, 5]
> [x, y, z] = (3, 5, 7)
> (x, y, z) = [3, 5, 7]
> [x, (y, z), w] = (1, [2, 3], 4)
> [x, (y, z), w] = (1, [2, (5, 6)], 4)
> z
| (5, 6)
```

Tuples vs lists: `a += b`

- Lists
  Extends existing list, i.e. same as `a.extend(b)`
- Tuples
  Must create a new tuple `a + b` and assign to `a`
  (since tuples are immutable)

Python shell

```
> x = [1, 2]
> y = x
> y += [3, 4]
> x
| [1, 2, 3, 4]
> y
| [1, 2, 3, 4]

> x = (1, 2)
> y = x
> y += (3, 4)
> x
| (1, 2)
> y
| (1, 2, 3, 4)
```

*variable assignment

- For a tuple of variable length a single **variable name* on the left side will be assigned a list of the remaining elements not matched by variables preceding/following *
- *Example*

```
a, *b, c = t
```

  is equivalent to

```
a = t[0]
b = t[1:-1]
c = t[-1]
```

- There can be a single * in a left-hand-side tuple (but one new * in each nested tuple)

Python shell

```
> (a,*b,c,d) = (1,2,3,4,5,6)
> b
| [2, 3, 4]
> (a,*b,c,d) = (1,2,3)
> b
| []
> (a,*b,c,d) = (1,2)
| ValueError: not enough values to
  unpack (expected at least 3, got 2)
> v = ((1,2,3),4,5,6,(7,8,9,10))
> ((a,*b),*c,(d,*e)) = v
> b
| [2, 3]
> c
| [4, 5, 6]
> e
| [8, 9, 10]
```

Question – What is `b` ?

```
(*a,(b,),c) = ((1,2),((3,4)),((5,)),(6))
```

a) `(1,2)`

b) `(3,4)`

c) `5`

d) `(5,)`

e) `(6)`

f) Don't know

List comprehension (cool stuff)

- Example:

```
[ x*x for x in [1, 2, 3]]
```

returns

```
[1, 4, 9]
```

- General

[*expression* for *variable* in *sequence*]

returns a list, where *expression* is computed for each element in *sequence* assigned to *variable*

Python shell

```
> [2*x for x in [1,2,3]]
| [2, 4, 6]
> [2*x for x in range(10,15)]
| [20, 22, 24, 26, 28]
> [2*x for x in "abc"]
| ['aa', 'bb', 'cc']
> [(None, None) for _ in range(2)]
| [(None, None), (None, None)]
```

List comprehension (more cool stuff)

- Similarly to the left-hand-side in assignments, the variable part can be a (nested) tuple of variables for unpacking elements:

[*expression* for *tuple of variables* in *sequence*]

Python shell

```
> points = [(3, 4), (2, 5), (4, 7)]
> [(x, y, x*y) for (x, y) in points]
| [(3, 4, 12), (2, 5, 10), (4, 7, 28)]
> [(x, y, x*y) for x, y in points]
| [(3, 4, 12), (2, 5, 10), (4, 7, 28)]
> [x, y, x*y for (x, y) in points]
| SyntexError: invalid syntax
```

parenthesis required for the constructed tuples

List comprehension – for-if and multiple for

- List comprehensions can have nested for-loops

$$[\textit{expression}\ \texttt{for}\ v_1\ \texttt{in}\ s_1\ \texttt{for}\ v_2\ \texttt{in}\ s_2\ \texttt{for}\ v_3\ \texttt{in}\ s_3]$$

- Can select a subset of the elements by adding an if-condition

$$[\textit{expression}\ \texttt{for}\ v_1\ \texttt{in}\ s_1\ \texttt{if}\ \textit{condition}]$$

- and be combined...

Python shell

```
> [(x, y) for x in range(1, 3) for y in range(4, 6)]
| [(1, 4), (1, 5), (2, 4), (2, 5)]
> [x for x in (1, 2) for x in (4, 5)]
| [4, 5, 4, 5]
> [x for x in range(1, 101) if x % 7 == 1 and x % 5 == 2]
| [22, 57, 92]
> [(x, y, x*y) for x in range(1, 11) if 6 <= x <= 7 for y in range(x, 11) if 6 <= y <= 7 and not x == y]
| [(6, 7, 42)]
```

Question – What will print the same?

```
points = [(3,7), (4,10),(12,3), (9,11), (7,5)]
print([(x, y) for x, y in points if x < y])
```

a) `print([x, y for x, y in points if x < y])`

b) `print([(x, y) for p in points if p[0] < p[1]])`

c) `print([p for p in points if p[0] < p[1]])`

d) `print([[x, y] for x, y in points if x < y])`

e) Don't know

any, all

- `any(L)` checks if at least one element in the sequence `L` is true (list, sequence, strings, ranges, ...)

  ```
  any([False, True, False])
  ```

- `all(L)` checks if all elements in the sequence `L` are true

  ```
  all([False, False, True])
  ```

- `any` and `all` returns `True` or `False`

Python shell

```
> any((False, True, False))
| True
> any([False, False, False])
| False
> any([])
| False
> all([False, False, True])
| False
> all((True, True, True))
| True
> all(())
| True
> L = (7, 42, 13)
> any([x == 42 for x in L])
| True
> all([x == 42 for x in L])
| False
```

enumerate

`list(enumerate(L))`

returns

`[(0,L[0]), (1,L[1]), ..., (len(L)-1,L[-1])]`

Python shell

```
> points = [(1,2),(3,4),(5,6)]
> [(idx, x*y) for idx, (x, y) in enumerate(points)]
| [(0, 2), (1, 12), (2,30)]
> L = ('a','b','c')
> list(enumerate(L))
| [(0, 'a'), (1, 'b'), (2, 'c')]
> L_ = []
> for idx in range(len(L)):
>       L_.append((idx, L[idx]))
> print(L_)
| [(0, 'a'), (1, 'b'), (2, 'c')]
> list(enumerate(['a', 'b', 'c'], start=7))
| [(7, 'a'), (8, 'b'), (9, 'c')]
```

zip

$$\texttt{list(zip(}L_1, L_2, \ldots, L_k\texttt{))} = [\,(L_1[0], L_2[0], \ldots, L_k[0]), \ldots, (L_1[n], L_2[n], \ldots, L_k[n])\,]$$

where $n = \min(\mathrm{len}(L_1), \mathrm{len}(L_2), \ldots, \mathrm{len}(L_k))$

- Example ("matrix transpose"):

```
list(zip([1,2,3],
         [4,5,6],
         [7,8,9]))
```

returns

```
[(1, 4, 7),
 (2, 5, 8),
 (3, 6, 9)]
```

Python shell

```
> x = [1, 2, 3]
> y = [4, 5, 6]
| zip(x, y)
> <zip at 0xb02b530>
> points = list(zip(x, y))
> print(points)
| [(1, 4), (2, 5), (3, 6)]
```

Python shell

```
> first = ['Donald', 'Mickey', 'Scrooge']
> last = ['Duck', 'Mouse', 'McDuck']

> for i, (a, b) in enumerate(zip(first, last), start=1):
>     print(i, a, b)

| 1 Donald Duck
| 2 Mickey Mouse
| 3 Scrooge McDuck
```

(Simple) functions

- You can define your own functions using:

  `def` *function-name* `(`*var*$_1$, ..., *var*$_k$`):`

  body code

- If the body code executes

  `return` *expression*

  the result of *expression* will be returned by the function. If expression is omitted or the body code terminates without performing `return`, then `None` is returned.

- When *calling* a function *name* `(`*value*$_1$,..., *value*$_k$`)` body code is executed with *var*$_i$=*value*$_i$

Python shell

```
> def sum3(x, y, z):
      return x + y + z

> sum3(1, 2, 3)
| 6
> sum3(5, 7, 9)
| 21

> def powers(L, power):
      P = [x**power for x in L]
      return P

> powers([2, 3, 4], 3)
| [8, 27, 64]
```

Question – What tuple is printed ?

```
def even(x):
    if x % 2 == 0:
        return True
    else:
        return False

print((even(7), even(6)))
```

a) `(False, False)`

b) `(False, True)`

c) `(True, False)`

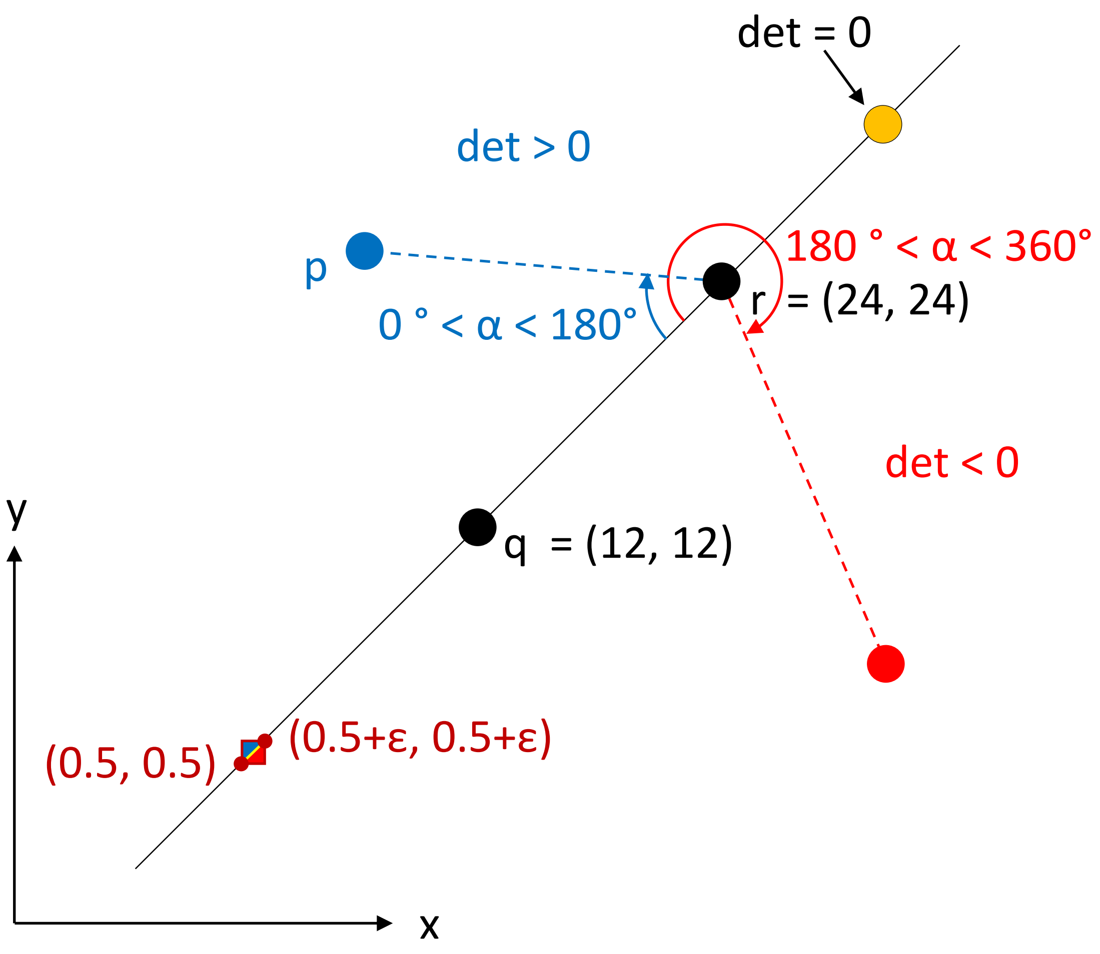
d) `(True, True)`

e) Don’t know

Geometric orientation test

Purpose of example

- illustrate tuples
- list comprehension
- matplotlib.pyplot
- floats are strange ⚠



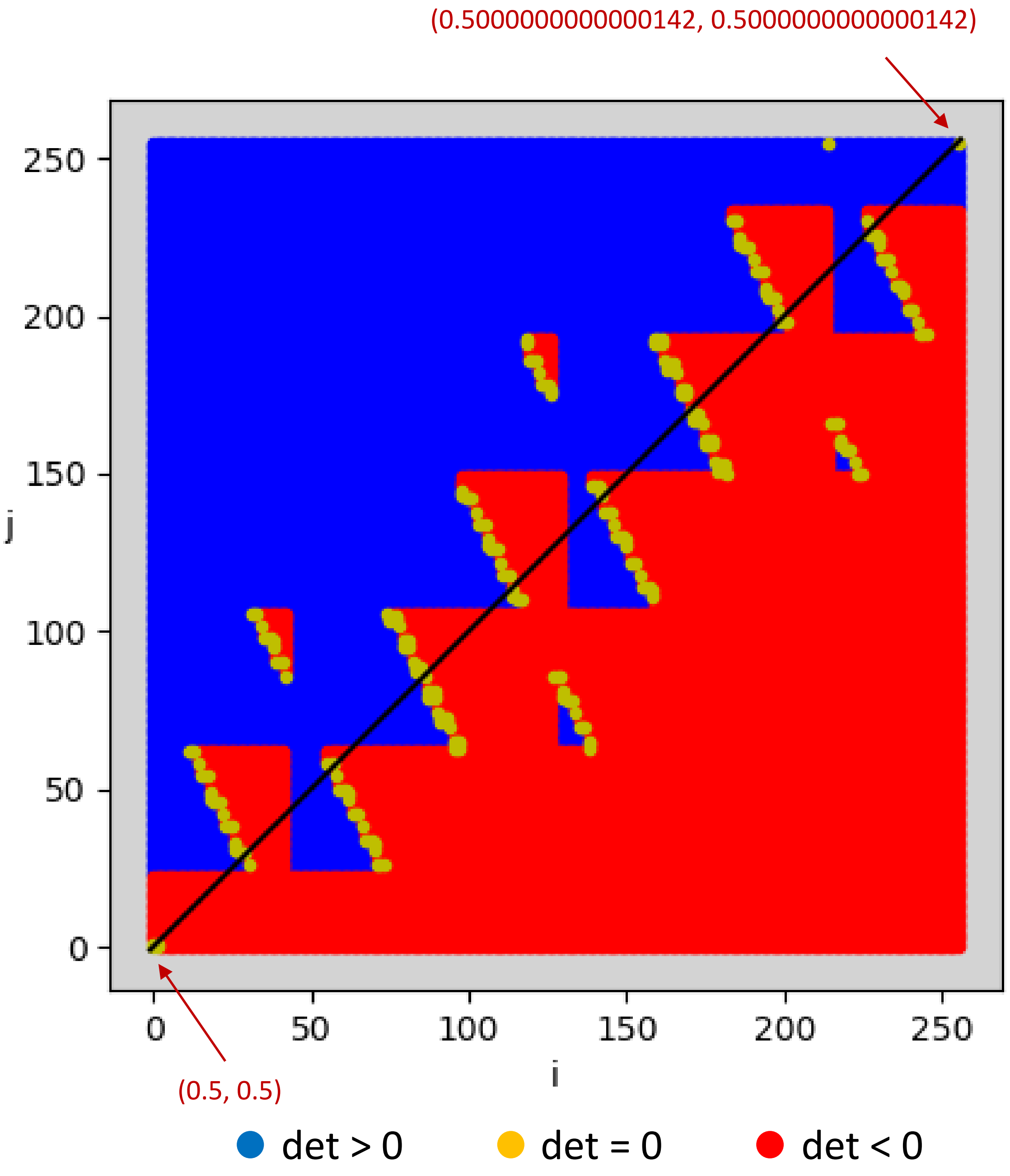
$$\det = \begin{vmatrix} 1 & q_x & q_y \\ 1 & r_x & r_y \\ 1 & p_x & p_y \end{vmatrix} = \underbrace{r_x p_y - p_x r_y - q_x p_y + p_x q_y + q_x r_y - r_x q_y}$$

6 ! = 720 different orders to add ⚠

Kettner, Mehlhorn, Pion, Schirra, Yap:
[Classroom Examples of Robustness Problems in Geometric Computations]

sign-plot.py

```
import matplotlib.pyplot as plt

N = 256
delta = 1 / 2**54
q = (12, 12)
r = (24, 24)
P = []  # points (i, j, det)

for i in range(N):
    for j in range(N):
        p = (1/2 + i * delta, 1/2 + j * delta)
        det = (q[0]*r[1] + r[0]*p[1] + p[0]*q[1]
              - r[0]*q[1] - p[0]*r[1] - q[0]*p[1])
        P.append((i, j, det))

pos = [(i, j) for i, j, det in P if det > 0]
neg = [(i, j) for i, j, det in P if det < 0]
zero = [(i, j) for i, j, det in P if det == 0]

plt.subplot(facecolor='lightgrey', aspect='equal')
plt.xlabel('i')
plt.ylabel('j', rotation=0)

for points, color in [(pos, "b"), (neg, "r"), (zero, "y")]:
    X = [x for x, y in points]
    Y = [y for x, y in points]
    plt.plot(X, Y, color + ".")

plt.plot([-1, N], [-1, N], "k-")
plt.show()
```



Dictionaries and Sets

- dict
- set
- frozenset
- set/dict comprehensions

Dictionaries (type dict)

$$\{\, key_1 : \; value_1 , \; \ldots , \; key_k : \; value_k \,\}$$

- Stores a mutable set of (key, value) pairs, denoted *items*, with distinct keys, i.e. *maps* keys to values
- Constructing empty dictionary: `dict()` or `{}`
- *dict*[*key*] lookup for key in dictionary, and returns associated value. Key must be present otherwise a KeyError is raised.
- *dict*[*key*] = *value* assigns value to *key*, overriding exising value if present.

key	value
'a'	7
'foo'	'42nd'
5	29
'5'	44
5.5	False
False	True
(3, 4)	'abc'

distinct keys,
i.e. not ”==”

https://docs.python.org/3/tutorial/datastructures.html#dictionaries

Dictionaries (type dict)

Python shell

```
> d = {'a': 42, 'b': 57}
> d
| {'a': 42, 'b': 57}

> d.keys()
| dict_keys(['a', 'b'])

> list(d.keys())
| ['a', 'b']

> d.items()
| dict_items([('a', 42), ('b', 57)])

> list(d.items())
| [('a', 42), ('b', 57)]
```

```
> for key in d:
      print(key)
| a
| b

> for key, val in d.items():
      print("Key", key, "has value", val)
| Key a has value 42
| Key b has value 57

> {5: 'a', 5.0: 'b'}
| {5: 'b'}
```

Python shell

```
> surname = dict(zip(['Donald', 'Mickey', 'Scrooge'], ['Duck', 'Mouse', 'McDuck']))
> surname['Mickey']
| 'Mouse'
```

Dictionaries (type dict)

Python shell

```
> gradings = [('A', 7), ('B', 4), ('A', 12), ('C', 10), ('A', 7)]
> grades = {}  # empty dictionary
> for student, grade in gradings:
      if student not in grades:
          grades[student] = []
      grades[student].append(grade)
> grades
| {'A': [7, 12, 7], 'B': [4], 'C': [10]}
> print(grades['A'])
| [7, 12, 7]
> print(grades['E'])  # can only lookup keys in dictionary
| KeyError: 'E'
> print(grades.get('E'))  # .get returns None if key not in dictionary
| None
> print(grades.get('E', []))  # change default return value
| []
> print(grades.get('A', []))
| [7, 12, 7]
```

Dictionary operation	**Description**
len(d)	Items in dictionary
d[key]	Lookup key
d[key] = value	Update value of key
del d[key]	Delete an existing key
key in d	Key membership
key not in d	Key non-membership
clear()	Remove all items
copy()	Shallow copy
get(key), get(key, default)	d[key] if key in dictionary, otherwise `None` or default
items()	*View* of the dictionaries items
keys()	*View* of the dictionaries keys
values()	*View* of the dictionaries values
pop(key)	Remove key and return previous value
popitem()	Remove and return an arbitrary item
update()	Update key/value pairs from another dictionary

https://docs.python.org/3/library/stdtypes.html#mapping-types-dict

Order returned by `list(d.keys())` ?

The Python Standard Library
Mapping Types — dict

"Dictionaries preserve insertion order. Note that updating a key does not affect the order. Keys added after deletion are inserted at the end." (since Python 3.7)

docs.python.org/3/library/stdtypes.html

Python shell

```
> d = {'d': 1, 'c': 2, 'b': 3, 'a':4}
> d['x'] = 5    # new key at end
> d['c'] = 6    # overwrite value
> del d['b']    # remove key 'b'
> d['b'] = 7    # reinsert key 'b' at end
> d
| {'d': 1, 'c': 6, 'a': 4, 'x': 5, 'b': 7}
```

Raymond Hettinger @ Twitter

See also Raymond's talk @ PyCon 2017
Modern Python Dictionaries
A confluence of a dozen great ideas

Dictionary comprehension

- Similarly to creating a list using list comprehension, one can create a set of key-value pairs:

{*key* : *value* `for` *variable* `in` *list*}

Python shell

```
> names = ['Mickey', 'Donald', 'Scrooge']
> list(enumerate(names, start=1))
| [(1, 'Mickey'), (2, 'Donald'), (3, 'Scrooge')]
> dict(enumerate(names, start=1))
| {1: 'Mickey', 2: 'Donald', 3: 'Scrooge'}
> {name: idx for idx, name in enumerate(names, start=1)}
| {'Mickey': 1, 'Donald': 2, 'Scrooge': 3}
```

Sets (set and frozenset)

$$\{\, value_1,\ \ldots,\ value_k \,\}$$

- Values of type `set` represent mutable sets, where "==" elements only appear once
- Do **not** support: indexing, slicing
- `frozenset` is an immutable version of `set`

Python shell

```
> S = {2, 5, 'a', 'c'}
> T = {3, 4, 5, 'c'}
> S | T
| {2, 3, 4, 5, 'a', 'c'}
> S & T
| {5, 'c'}
```

```
> S ^ T
| {2, 3, 4, 'a'}
> S - T
| {2, 'a'}
> {4, 5, 5.0, 5.1}
| {4, 5, 5.1}
```

Operation	Description
S | T	Set union
S & T	Set intersection
S – T	Set difference
S ^ T	Symmetric difference
set()	Empty set ({} = empty dictionary)
set(L)	Create set from list
x in S	Membership
x not in S	Non-membership
S.isdisjoint(T)	Disjoint sets
S <= T	Subset
S < T	Proper subset
S >= T	Superset
S > T	Proper superset
len(S)	Size of S
S.add(x)	Add x to S (not frozenset)

https://docs.python.org/3/tutorial/datastructures.html#sets
https://docs.python.org/3/library/stdtypes.html#set-types-set-frozenset

Question – What value has the expression ?

```
sorted( { 5, 5.5, 5.0, '5' } )
```

a) `{'5', 5, 5.0, 5.5}`
b) `{5, 5.5}`
c) `['5', 5, 5.0, 5.5]`
d) `['5', 5, 5.5]`
e) TypeError
f) Don't know

Set comprehension

- Similarly to creating a list using list comprehension, one can create a set of values (also using nested for- and if-statements):

  `{`*value* `for` *variable* `in` *list*`}`

- A value occurring multiple times as *value* will only be included once

primes_set.py

```
n = 101
not_primes = {m for f in range(2, n) for m in range(2 * f, n, f)}
primes = set(range(2, n)) - not_primes
```

Python shell

```
> L = ['a', 'b', 'c']
> {(x,(y,z)) for x in L for y in L for z in L if x != y and y != z and z != x}
| {('a',('b','c')),('a',('c','b')),('b',('a','c')),...,('c',('b','a'))}
```

Hash, equality, and immutability

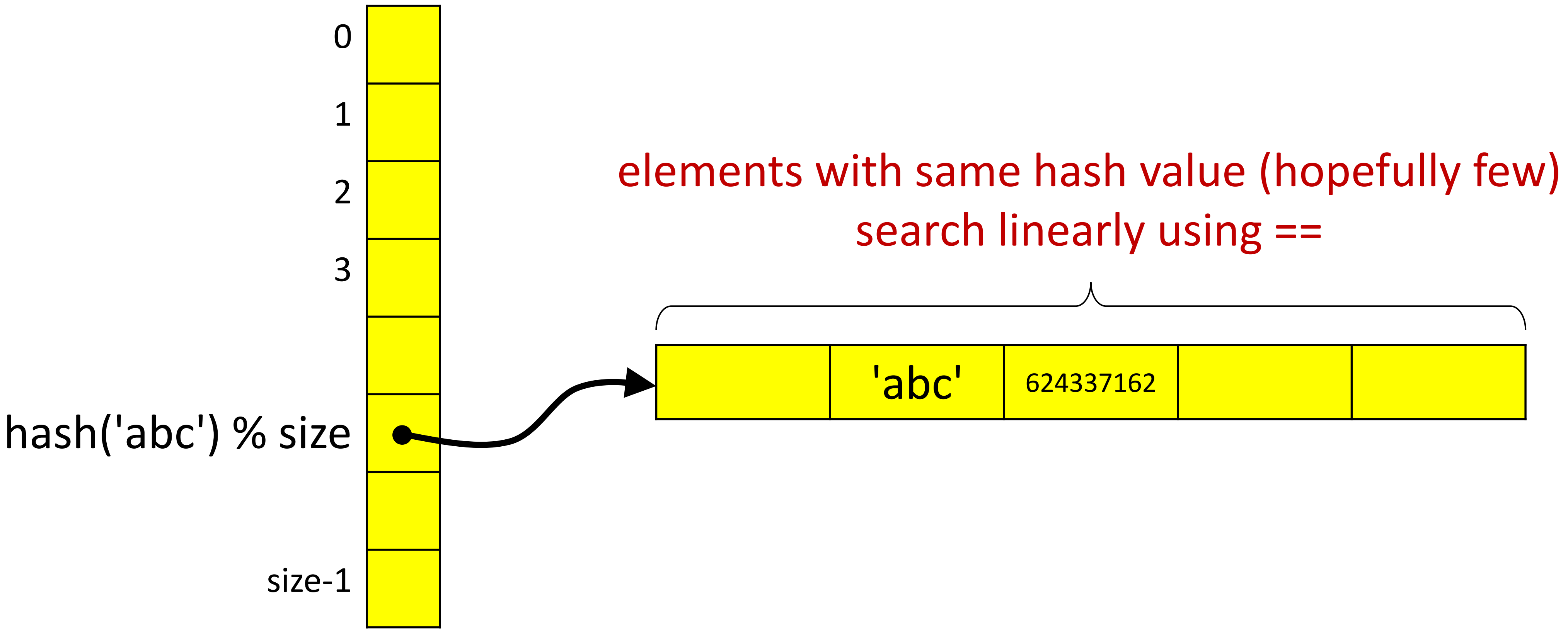
- Keys for dictionaries and sets must be *hashable*, i.e. have a __hash__() method returning an integer that does not change over their lifetime and an __eq__() method to check for equality with “==”.

`'abc'.__hash__()` could e.g. return `624337162`

`(624337162).__hash__()` would also return `624337162`

- All built-in immutable types are hashable. In particular tuples of immutable values are hashable. I.e. trees represented by nested tuples like `((('a'),'b'),('c',('d','e')))` can be used as dictionary keys or stored in a set.

Sketch of internal set implementation



[Raymond Hettinger, *Modern Python Dictionaries - A confluence of a dozen great ideas*]()

Module `collections` (container datatypes)

- Python builtin containers for data: `list`, `tuple`, `dict`, and `set`.
- The module **`collections`** provides further alternatives (but these are not part of the language like the builtin containers)

`deque`	double ended queue
`namedtuple`	tuples allowing access to fields by name
`counter`	special dictionary to count occurrences of elements
…	

https://docs.python.org/3/library/collections.html

deque – double ended queues

- Extends lists with efficient updates at the front
- ⚠ Inserting at the front of a standard Python list takes linear time in the size of the list – very slow for long lists

Python shell

```
> L = list()
> L.append(1)
> L.append(2)
> L.insert(0, 0)   # insert at the front
> L.insert(0, -1)  # slow for long lists
> L.insert(0, -2)
> L
| [-2, -1, 0, 1, 2]
```

```
> from collections import deque
> d = deque()   # create empty deque
> d.append(1)
> d.append(2)
> d.appendleft(0)   # efficient
> d.appendleft(-1)
> d.appendleft(-2)
> d
| deque([2, 1, 0, -1, -2])
> for e in d: print(e, end=', ')
| -2, -1, 0, 1, 2,
```

`namedtuple` – tuples with field names

- Compromise between `tuple` and `dict`, can increase code readability

Python shell

```
> person = ('Donald Duck', 1934, '3 feet')  # as tuple
> person[1]  # not clear what is accessed
| 1934
> person = {'name': 'Donald Duck', 'appeared': 1934, 'height': '3 feet'}  # as dict
> person['appeared']  # more clear what is accessed, but ['...'] overhead
| 1934
> from collections import namedtuple
> Person = namedtuple('Person', ['name', 'appeared', 'height'])  # create new type
> person = Person('Donald Duck', 1934, '3 feet')  # as namedtuple
> person
| Person(name='Donald Duck', appeared=1934, height='3 feet')
> person.appeared  # short and clear
| 1934
> person[1] # still possible
| 1934
```

`Counter` – dictionaries for counting

Python shell

```
> from collections import Counter
> fq = Counter('abracadabra')  # create new counter from a sequence
> fq
| Counter({'a': 5, 'b': 2, 'r': 2, 'c': 1, 'd': 1})  # frequencies of the letters
> fq['a']
| 5
> fq.most_common(3)
| [('a', 5), ('b', 2), ('r', 2)]
> fq['x'] += 5  # increase count of 'x', also valid if 'x' not in Counter yet
> Counter('aaabbbcc') - Counter('aabdd')  # counters can be subtracted
| Counter({'b': 2, 'c': 2, 'a': 1})
> Counter([1, 2, 1, 3, 4, 5]) + Counter([3, 3, 3])  # counters can be added
| Counter({3: 4, 1: 2, 2: 1, 4: 1, 5: 1})
```

Handin 3 & 4 – Triplet distance (Dobson, 1975)

```
((('A','F'),'B'),('D',('C','E')))
```

```
           (a)

            /\
           /  \
          /    \
         /      \
        /        \
       /          \
      /\          /\
     /  \        /  \
    /    \      /    \
   /\    'B'  'D'    /\
  /  \              /  \
'A'  'F'          'C'  'E'
```

```
(((('D','A'),'B'),'F'),('C','E'))
```

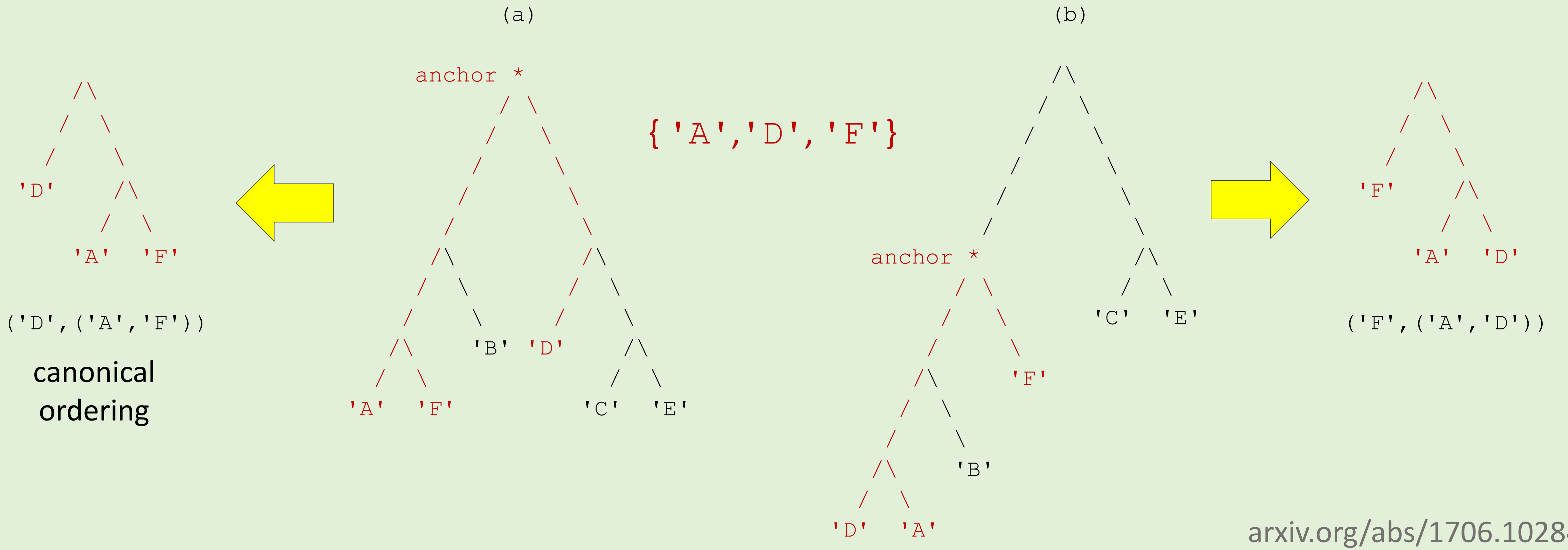
```
               (b)

                /\
               /  \
              /    \
             /      \
            /        \
           /          \
          /\          /\
         /  \        /  \
        /    \     'C'  'E'
       /      \
      /\      'F'
     /  \
    /    \
   /\    'B'
  /  \
'D'  'A'
```

How similar are the trees?

arxiv.org/abs/1706.10284

Handin 3 & 4 – Triplet distance (Dobson, 1975)

Consider all $\binom{n}{3}$ subsets of size three, and count how many do not have identical substructure (topology) in the two trees.



arxiv.org/abs/1706.10284

Functions

- functions
- return
- scoping
- arguments
- keyword arguments
- *, **
- global variables

(Simple) functions

- You can define your own functions using:

  def ***function-name*** **(*var_1*, ..., *var_k*):**
  body code

  var_1, ..., *var_k* are the *formal parameters*

- If the body code executes

  `return` *expression*

  the result of *expression* will be returned by the function. If expression is omitted or the body code terminates without performing `return`, then `None` is returned.

- When *calling* a function ***name*** **(*value*,..., *$value_k$*)** body code is executed with *var_i=$value_i$*

Python shell

```
> def sum3(x, y, z):
      return x+y+z

> sum3(1, 2, 3)
| 6
> sum3(5, 7, 9)
| 21

> def powers(L, power):
      P = [x**power for x in L]
      return P

> powers([2,3,4], 3)
| [8, 27, 64]
```

Questions – `poly(3, "10", '3')` ?

```
def poly(z, x, y):
    return z*x + y
```

a) 33

b) 1010103

c) '33'

d) '1010103'

e) TypeError

f) Don't know

Why functions ?

- Avoid writting the same code multiple times, *re-usability*
- Be able to *name a functionality*
- Clearly state the functionality of a piece of code, *abstraction*: *Input* = arguments, *output* = return value (and/or side effects)
- *Encapsulate* code with clear interface to the dependency to the outside world/code
- Share functionality in modules/libraries/packages with other users, *code sharing*
- Increase *readability* of code, smaller independent blocks of code
- Easier systematically *testing* of code
- ...

Some other Python language features helping structuring programs

- Object orientation
- Modules
- Decorators
- Context managers
- Exceptions
- Doc strings
- doctest

Local variables in functions

- The formal arguments and variables assigned to in the body of a function are created as temporary *local variables*

Global variables		Local variables	
`sum3`	`<function>`	`x`	`4`
`a`	`3`	`y`	`5`
`y`	`42`	`z`	`6`
		`a`	`9`
		`b`	`15`

state just before `return b`

Python shell

```
> def sum3(x, y, z):
      a = x + y
      b = a + z
      return b

> a = 3
> y = 42
> w = sum3(4, 5, 6)
> w
| 15
> a
| 3
> b
| NameError: name 'b' is not defined
> x
| NameError: name 'x' is not defined
> y
| 42
> sum3
| <function sum3 at 0x0356DA98>
```

Global variables

- Variables in function bodies that are only read, are considered access to *global variables*

Python shell

```
> prefix = "The value is"
> def nice_print(x):
      print(prefix, x)
> nice_print(7)
| The value is 7
> prefix = "Value ="
> nice_print(42)
| Value = 42
```

Global variables		Local variables	
nice_print	<function>	x	42
prefix	"Value ="		

state just before returning from 2nd `nice_print`

global

- Global variables that should be updated in the function body must be declared global in the body:

  `global` *variable*, *variable*, ...

- Note: If you only need to read a global variable, it is not required to be declared global (but would be polite to the readers of your code)

Since `counter` assigned in body, `counter` will be considered to be a local variable

Python shell

```
> counter = 1

> def counted_print(x):
      global counter
      print("(%d)" % counter, x)
      counter += 1

> counted_print(7)
| (1) 7
> counted_print(42)
| (2) 42

> def counted_print(x):
      print("(%d)" % counter, x)
      counter += 1

> counted_print(7)
| UnboundLocalError: local variable
  'counter' referenced before
  assignment
```

Question – What value is printed ?

```
x = 1

def f(a):
    global x
    x = x + 1
    return a + x

print(f(2) + f(4))
```

a) 6
b) 7
c) 8
d) 9
e) 10
f) 11
g) 12
h) Don't know

Arbitrary number of arguments

- If you would like your function to be able to take a variable number of additional arguments in addition to the required, add a **variable* as the last argument.
- In a function call *variable* will be assigned a tuple with all the additional arguments.

Python shell

```
> def my_print(x, y, *L):
      print("x =", x)
      print("y =", y)
      print("L =", L)

> my_print(2, 3, 4, 5, 6, 7)
| x = 2
| y = 3
| L = (4, 5, 6, 7)
> my_print(42)
| TypeError: my_print() missing 1
  required positional argument: 'y'
```

Unpacking a list of arguments in a function call

- If you have list `L` (or tuple) containing the arguments to a *function call*, you can unpack them in the function call using `*L`

```
L = [x, y, z]
f(*L)
```

  is equivalent to calling

```
f(L[0], L[1], L[2])
```

  i.e.

```
f(x, y, z)
```

- Note that `f(L)` would pass a single argument to `f`, namely a list
- In a function call several `*` expressions can appear, e.g. `f(*L1, x, *L2, *L3)`

Python shell

```
> import math
> def norm(x, y):
      return math.sqrt(x * x + y * y)
> norm(3, 5)
| 5.830951894845301
> point = (3, 4)
> print(*point, sep=':')
| 3:4
> norm(point)
| TypeError: norm() missing 1 required positional argument: 'y'
> norm(*point)
| 5.0
> def dist(x0, y0, x1, y1):
      return math.sqrt((x1 - x0) ** 2 + (y1 - y0) ** 2)
> p = 3, 7
> q = 7, 4
> dist(p, q)
| TypeError: dist() missing 2 required positional arguments: 'x1' and 'y1'
> dist(*p, *q)
| 5.0
```

Question – How many arguments should `f` take ?

```
a = [1, 2, 3]
b = [4, 5]
c = (6, 7, 8)
d = (9, 10)
f(*a, b, c, *d)
```

a) 4
b) 5
c) 6

d) 7
e) 8
f) 9
g) 10
h) Don’t know

Question – What is `list(zip(*zip(*L)))` ?

```
L = [[1, 2, 3], [4, 5], [6, 7, 8]]
```

a) `[(([1, 2, 3],),), (([4, 5],),), (([6, 7, 8],),)]`
b) `[((1, 4, 6),), ((2, 5, 7),)]`
c) `[(1, 2), (4, 5), (6, 7)]`
d) `[(1, 2, 3), (4, 5), (6, 7, 8)]`
e) `[([1, 2, 3], [4, 5], [6, 7, 8])]`
f) Don't know

Python shell

```
> list(zip((1, 2, 3), (4, 5, 6)))
| [(1, 4), (2, 5), (3, 6)]
```

Keyword arguments

- Previously we have seen the following (strange) function calls

```
print(7, 14, 15, sep=":", end="")
enumerate(my_list, start=1)
```

- *name* = refers to one of the formal arguments, known as a keyword argument.
  A *name* can appear at most once in a function call.

- In function calls, keyword arguments must follow positional arguments.

- Can e.g. be useful if there are many arguments, and the order is not obvious, i.e. improves readability of code:

```
complicated_function(
    name = "Mickey",
    city = "Duckburg",
    state = "Calisota",
    occupation = "Detective",
    gender = "Male")
```

Python shell

```
> def sub(x, y):
      return x - y

> sub(9, 4)
| 5
> sub(y=9, x=4)
| -5
```

Keyword arguments, default values

- When calling a function arguments can be ommitted if the corresponding arguments in the function definition have default values *argument = value*.

Python shell

```
> def my_print(a, b, c=5, d=7):
      print("a=%s, b=%s, c=%s, d=%s" % (a, b, c, d))

> my_print(2, d=3, b=4)
| a=2, b=4, c=5, d=3
```

Question – What is `f(6, z=2)` ?

```
def f(x, y=3, z=7):
    return x + y + z
```

a) `10`
b) `11`
c) `16`
d) `TypeError: f() missing 1 required positional argument: ‘y‘`
e) Don’t know

Keyword arguments, mutable default values ⚠

- Be carefull: Default value will be shared among calls (which can be usefull)

The Python Language Reference
8.6 Function definitions

”**Default parameter values are evaluated from left to right when the function definition is executed**. This means that the expression is evaluated once, when the function is defined, and that the same “pre-computed” value is used for each call. This is especially important to understand when a default parameter is a mutable object, such as a list or a dictionary: if the function modifies the object (e.g. by appending an item to a list), the default value is in effect modified. This is generally not what was intended. A way around this is to use None as the default”

Python shell

```
> def list_append(e, L=[]):
        L.append(e)
        return L
> list_append('x', ['y', 'z'])
| ['y', 'z', 'x']
> list_append("a")
| ['a']
> list_append("b")
| ['a', 'b']
> list_append("c")
| ['a', 'b', 'c']
```

Python shell

```
> def list_append(e, L=None):
      if L == None:
          L = []
      L.append(e)
      return L
> list_append('x', ['y', 'z'])
| ['y', 'z', 'x']
> list_append("a")
| ['a']
> list_append("b")
| ['b']
> list_append("c")
| ['c']
```

Function call, dictionary of keyword arguments

- If you happen to have a *dictionary* containing the keyword arguments you want to pass to function, you can give all dictionary items as arguments using the single argument *****dictionary***

```
Python shell
> print(3, 4, 5, sep=":", end='#\n')
| 3:4:5#
> print_kwarg = {'sep': ':', 'end': '#\n'}
> print(3, 4, 5, **print_kwarg)
| 3:4:5#
```

Function definition, arbitrary keyword arguments

- If you want a function to accept arbitrary keyword arguments, add an argument ** *argument* to the function definition.
- When the function is called *argument* will be assigned a dictionary containing the excess keyword arguments.

Python shell

```
> def my_print(a, b=3, **c):
        print("a =", a)
        print("b =", b)
        print("c =", c)

> my_print(x=27, y=42, a=7)
| a = 7
| b = 3
| c = {'x': 27, 'y': 42}
```

Example

```
Python shell
> L1 = [1, 'a']
> L2 = ['b', 2, 3]
> D1 = {'y':4, 's':10}
> D2 = {'t':11, 'z':5.0}

> def f(a, b, c, d, e, *f, q=0, x=1, y=2, z=3, **kw):
      print("a=%s, b=%s, c=%s, d=%s, e=%s, " % (a, b, c, d, e),
            "f=%s\n" % str(f),
            "q=%s, x=%s, y=%s, z=%s, " % (q, x, y, z),
            "kw=%s" % kw,
            sep="")

> f(7, *L1, 9, *L2, x=7, **D1, w=42, **D2)
| a=7, b=1, c=a, d=9, e=b, f=(2, 3)
| q=0, x=7, y=4, z=5.0, kw={'w': 42, 's': 10, 't': 11}
```

non-keyword arguments must appear before keyword arguments

Forwarding function arguments

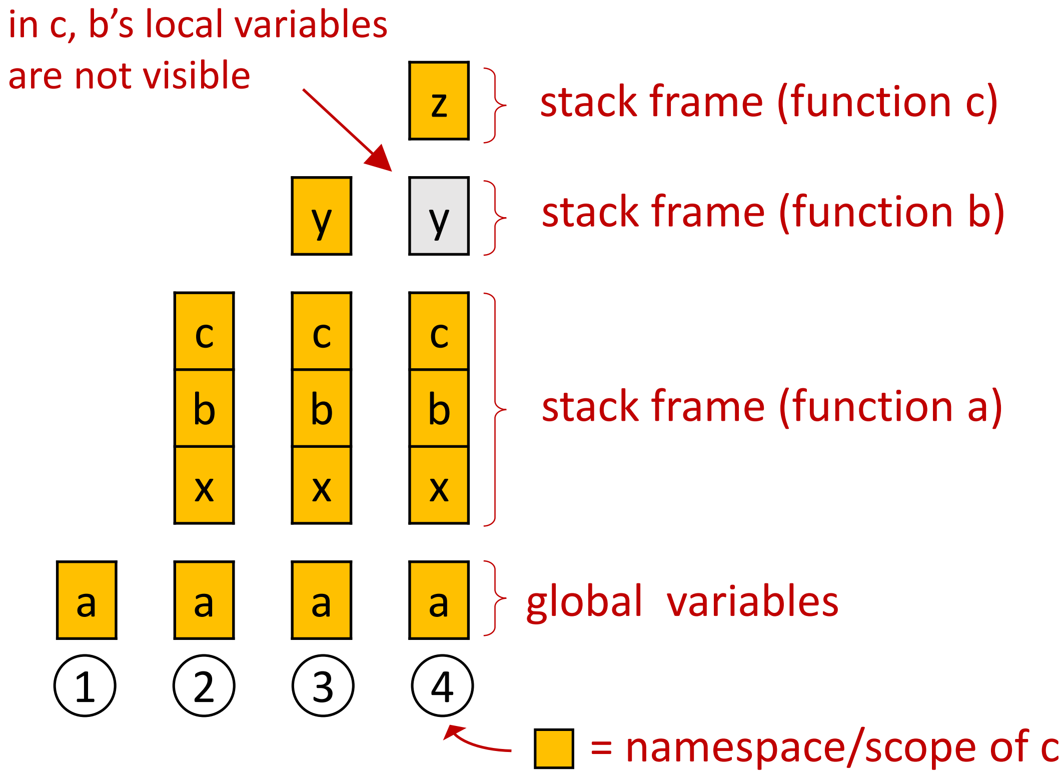
- `*` and `**` can e.g. be used to forward (unknown) arguments to other function calls

Python shell

```
> def my_print(*positional_arguments, sep=":", **keyword_arguments):
      print(*positional_arguments, sep=sep, **keyword_arguments)
> my_print(7, 42)
| 7:42
> my_print("x", "y", end="<")
| x:y<
> my_print("x", "y", sep="_")
| x_y
```

Local function definitions and namespaces

- Function definitions can contain (nested) local function definitions, only accessable inside the function
- *static/lexical scoping*, i.e. can see from the code which variables are in scope



Python shell

```
> def a(x):
      def b(y):
          print("b: y=%s x=%s" % (y, x))
③         c(y + 1)
      def c(z):
④         print("c: z=%s x=%s" % (z, x))
      print("a: x=%s" % x)
②     b(x + 1)
① > a(42)
| a: x=42
| b: y=43 x=42
| c: z=44 x=42
```

Example – nested function definitions

Python shell

```
> def a(x):
      def b(y):
          print("Enter b (y=%s, x=%s)" % (y, x))
          c(y + 1)
          print("leaving b")
      def c(x):  # x hides argument of function a
          def d(z):
              print("Enter d (z=%s, x=%s)" % (z, x))
              print("leaving d")
          print("Enter c (x=%s)" % x)
          d(x + 1)
          print("leaving c")
      print("Enter a (x=%s)" % x)
      b(x + 1)
      print("leaving a")
```

```
> a(5)
| Enter a (x=5)
| Enter b (y=6, x=5)
| Enter c (x=7)
| Enter d (z=8, x=7)
| leaving d
| leaving c
| leaving b
| leaving a
```

Example – nested functions and default values

Python shell

```
> def init_none(var_name):
>     print('initializing', var_name)
>     return None   # redundant line
> def f(a=init_none('a')):
>     def g(b=init_none('b')):
>         print('b =', b)
>     print("a =", a)
>     g(a + 1)
| initializing a
> f(10)
| initializing b
| a = 10
| b = 11
```

nonlocal

- The `nonlocal` statement causes the listed identifiers to refer to previously bound variables in the nearest *enclosing scope excluding globals*.

- `nonlocal` *variable*, *variable*, …

Python shell

```
> x = 0

> def f():
      y = 1
      def f_helper(z):
          global x
          nonlocal y
          print("(%s:%s) %s" % (x, y, z))
          y += 1
          x += 3

      f_helper(7)
      f_helper(42)

> f()
| (0:1) 7
| (3:2) 42
> f()
| (6:1) 7
| (9:2) 42
```

docs.python.org/3/reference/simple_stmts.html#the-nonlocal-statement

A note on Python and functions

- Similarities between Python and other languages:
  - functions are widely supported (sometimes called methods and procedures)
  - scoping rules is present in many languages (but details differ)
- The following are very Python specific (but nice):
  - how to handle global, local and nonlocal variables
  - keyword arguments
  - *, **

Recursion

- symbol table
- stack frames

Recursion

Recursive function
≡
”function that calls itself”

start 5 `recursive_function(5)` end 5
start 4 `recursive_function(4)` end 4
start 3 `recursive_function(3)` end 3
start 2 `recursive_function(2)` end 2
start 1 `recursive_function(1)` end 1
`recursive_function(0)`
done

Python shell

```
> def recursive_function(x):
      if x > 0:
          print("start", x)
          recursive_function(x - 1)
          print("end", x)
      else:
          print("done")

> recursive_function(5)
| start 5
| start 4
| start 3
| start 2
| start 1
| done
| end 1
| end 2
| end 3
| end 4
| end 5
```

Recursion

x	0	stack frame
x	1	
x	2	
x	3	
x	4	
x	5	
recursive_function	<function>	global variables

Recursions stack when x = 0 is reached

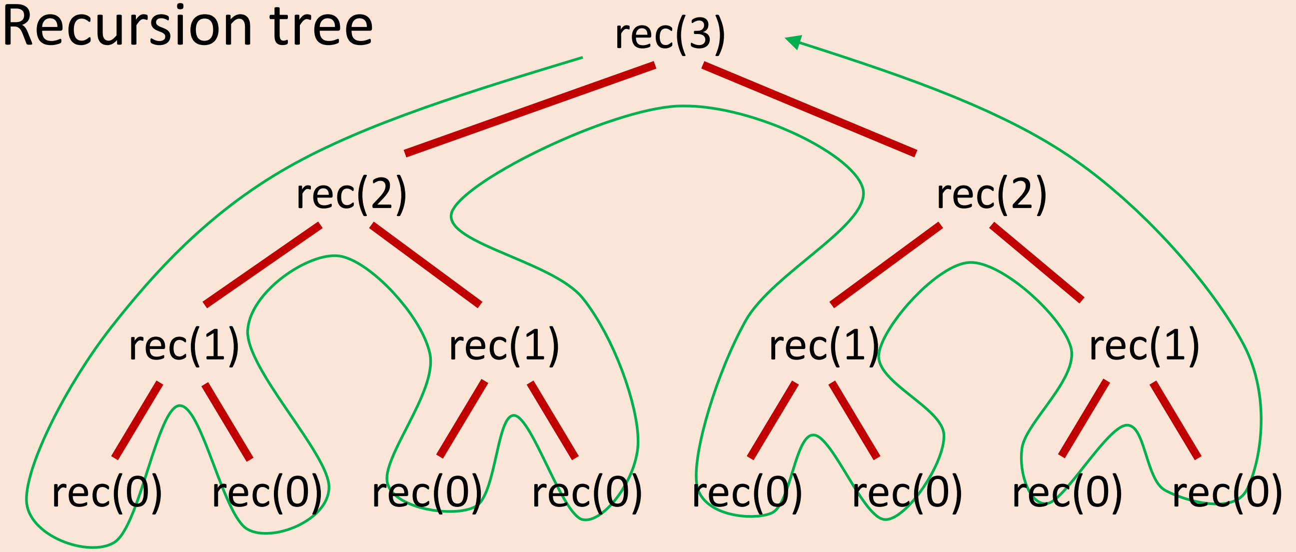
Python shell

```
> def recursive_function(x):
      if x > 0:
          print("start", x)
          recursive_function(x - 1)
          print("end", x)
      else:
          print("done")

> recursive_function(5)
| start 5
| start 4
| start 3
| start 2
| start 1
| done
| end 1
| end 2
| end 3
| end 4
| end 5
```

Python shell

```
> def rec(x):
      if x > 0:
          print("start", x)
          rec(x - 1)
          rec(x - 1)
          print("end", x)
      else:
          print("done")
```

Recursion tree



Python shell

```
> rec(3)
| start 3
| start 2
| start 1
| done
| done
| end 1
| start 1
| done
| done
| end 1
| end 2
| start 2
| start 1
| done
| done
| end 1
| start 1
| done
| done
| end 1
| end 2
| end 3
```

Question – How many times does `rec(5)` print ”`done`”?

Python shell

```
> def rec(x):
      if x > 0:
          print("start", x)
          rec(x - 1)
          rec(x - 1)
          rec(x - 1)
          print("end", x)
      else:
          print("done")
```

a) 3
b) 5
c) 15
d) 81
e) 125
f) 243
g) Don’t know

Factorial

$$n\,! = n \cdot \underbrace{(n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1}_{(n-1)\,!}$$

Observation
(recursive definition)

$$1\,! = 1$$

$$n\,! = n \cdot (n - 1)\,!$$

factorial.py

```
def factorial(n):
    if n <= 1:
        return 1
    return n * factorial(n - 1)
```

factorial.py

```
def factorial(n):
    return n * factorial(n - 1) if n > 1 else 1
```

factorial_iterative.py

```
def factorial(n):
    result = 1
    for i in range(2, n + 1):
        result *= i
    return result
```

Binomial coefficient $\binom{n}{k}$

- $\binom{n}{k}$ = number of ways to pick k elements from a set of size n
- $$\binom{n}{k} = \begin{cases} 1 & \text{if } k = 0 \text{ or } k = n \\ \binom{n-1}{k} + \binom{n-1}{k-1} & \text{otherwise} \end{cases}$$

bionomial_recursive.py

```
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    return binomial(n - 1, k) + binomial(n - 1, k - 1)
```

- Unfolding computation shows $\binom{n}{k}$ 1's are added → slow

Binomial coefficient $\binom{n}{k}$

Observation $\binom{n}{k} = \dfrac{n\,!}{(n-k)\,!\cdot k\,!}$

bionomial_factorial.py

```
def binomial(n, k):
    return factorial(n) // factorial(k) // factorial(n - k)
```

- Unfolding computation shows 2n - 2 multiplications and 2 divisions → fast
- Intermediate value n ! can have significantly more digits than result (bad)

Binomial coefficient $\binom{n}{k}$

Observation $\binom{n}{k} = \dfrac{n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdots 1} = \binom{n-1}{k-1} \cdot \dfrac{n}{k}$

bionomial_recursive_product.py

```
def binomial(n, k):
    if k == 0:
        return 1
    else:
        return binomial(n - 1, k - 1) * n // k
```

- Unfolding computation shows 2k - 2 multiplications and k divisions → fast
- Multiplication with fractions ≥ 1 → intermediate numbers limited size

Questions – Which correctly computes $\binom{n}{k}$?

Observation $\binom{n}{k} = \dfrac{n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdots 1}$

a) `binomial_A`
b) `binomial_B`
c) both
d) none
e) Don't know

bionomial_iterative.py

```
def binomial_A(n, k):
    result = 1
    for i in range(k):
        result = result * (n - i) // (k - i)
    return result

def binomial_B(n, k):
    result = 1
    for i in range(k)[::-1]:
        result = result * (n - i) // (k - i)
    return result
```

Recursively print all leaves of a tree

- Assume a recursively nested tuple represents a tree with strings as leaves

Python shell

```
> def print_leaves(tree):
      if isinstance(tree, str):
          print("Leaf:", tree)
      else:
          for child in tree:
              print_leaves(child)

> print_leaves(('a',('b','c')))
| Leaf: a
| Leaf: b
| Leaf: c
```

Question – How many times is `print_leaves` function called in the example?

Python shell

```
> def print_leaves(tree):
      if isinstance(tree, str):
          print("Leaf:", tree)
      else:
          for child in tree:
              print_leaves(child)

> print_leaves(('a',('b','c')))
| Leaf: a
| Leaf: b
| Leaf: c
```

a) 3
b) 4
c) 5
d) 6
e) Don't know

Collect all leaves of a tree in a set

Python shell

```
> def collect_leaves_slow(tree):
      leaves = set()
      if isinstance(tree, str):
          leaves.add(tree)
      else:
          for child in tree:
              leaves |= collect_leaves_slow(child)
      return leaves

> collect_leaves_slow(('a', ('b', 'c')))
| {'a', 'c', 'b'}
```

copies all labels from child from one set to another set

Python shell

```
> def collect_leaves_wrong(tree, leaves = set()):
      if isinstance(tree, str):
          leaves.add(tree)
      else:
          for child in tree:
              collect_leaves_wrong(child, leaves)
      return leaves

> def collect_leaves_right(tree, leaves = None):
      if leaves == None:
          leaves = set()
      if isinstance(tree, str):
          leaves.add(tree)
      else:
          for child in tree:
              collect_leaves_right(child, leaves)
      return leaves
```

```
> collect_leaves_wrong(('a',('b','c')))
| {'a', 'c', 'b'}
> collect_leaves_wrong(('d',('e','f')))
| {'b', 'e', 'a', 'f', 'c', 'd'}

> collect_leaves_right(('a',('b','c')))
| {'b', 'a', 'c'}
> collect_leaves_right(('d',('e','f')))
| {'f', 'd', 'e'}
```

Python shell

```
> def collect_leaves(tree):
      leaves = set()

      def traverse(tree):
          nonlocal leaves  # can be omitted
          if isinstance(tree, str):
              leaves.add(tree)
          else:
              for child in tree:
                  traverse(child)

      traverse(tree)
      return leaves

> collect_leaves(('a', ('b', 'c')))
| {'b', 'a', 'c'}
> collect_leaves(('d', ('e', 'f')))
| {'f', 'd', 'e'}
```

Maximum recursion depth ?

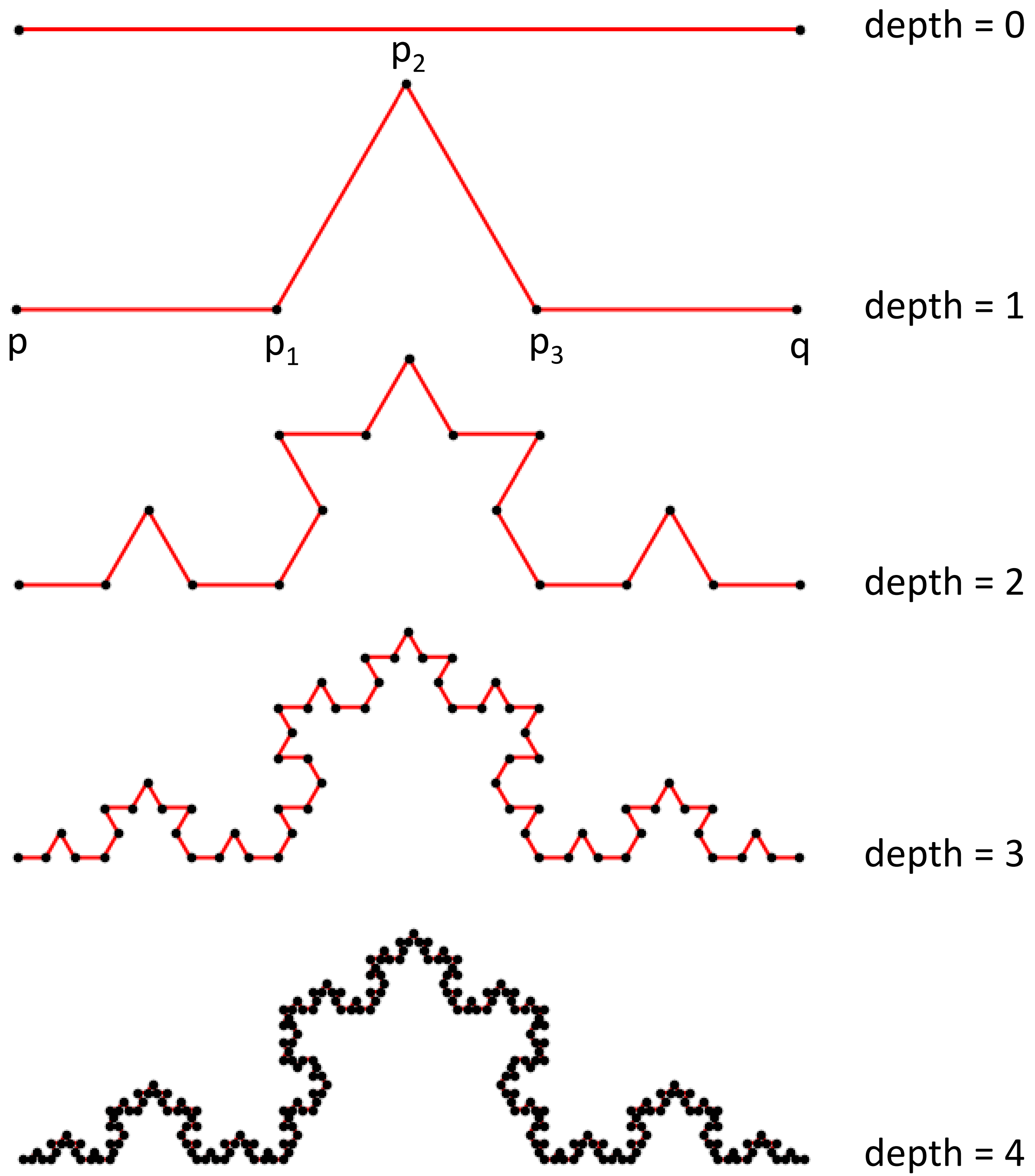
- Pythons maximum allowed recursion depth can be increased by

```
import sys
sys.setrecursionlimit(1500)
```

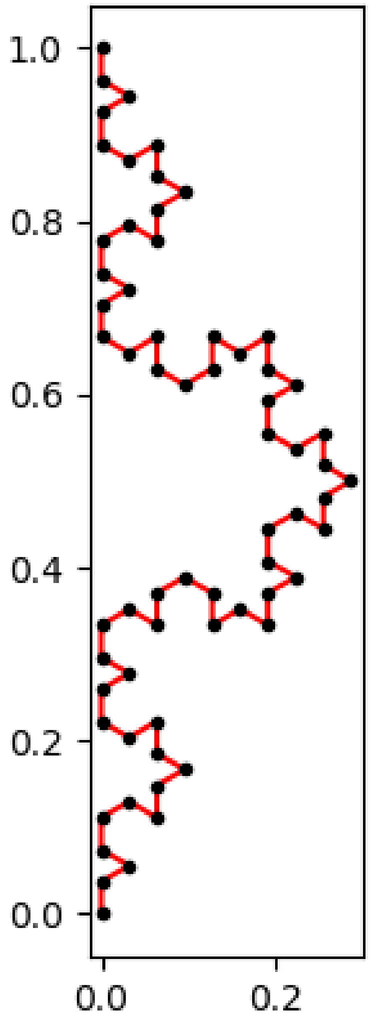
Python shell

```
> def f(x):
      print("#", x)
      f(x + 1)

> f(1)
| # 1
| # 2
| # 3
| ...
| # 975
| # 976
| # 977
| # 978
| RecursionError: maximum
  recursion depth exceeded
  while pickling an object
```

Koch Curves



Koch Curves



koch_curve.py

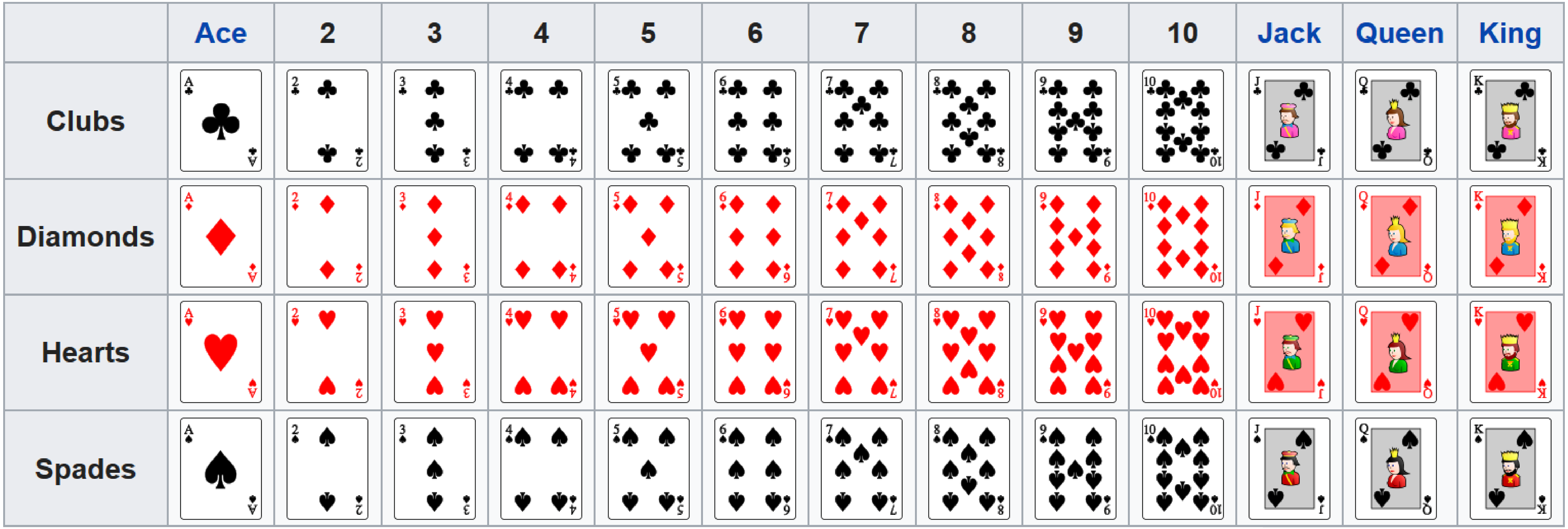
```
import matplotlib.pyplot as plt
from math import sqrt

def koch(p, q, depth=3):
    if depth == 0:
        return [p, q]

    dx, dy = q[0] - p[0], q[1] - p[1]
    h = 1 / sqrt(12)
    p1 = p[0] + dx / 3, p[1] + dy / 3
    p2 = p[0] + dx / 2 - h * dy, p[1] + dy / 2 + h * dx
    p3 = p[0] + dx * 2 / 3, p[1] + dy * 2 / 3
    return (koch(p,  p1, depth - 1)[:-1]
          + koch(p1, p2, depth - 1)[:-1]
          + koch(p2, p3, depth - 1)[:-1]
          + koch(p3, q,  depth - 1))

points = koch((0, 1), (0, 0), depth=3)
X, Y = zip(*points)

plt.subplot(aspect='equal')
plt.plot(X, Y, 'r-')
plt.plot(X, Y, 'k.')
plt.show()
```

remove last point (equal to first point in next recursive call)

Recursion and iteration

- algorithm examples

Standard 52-card deck

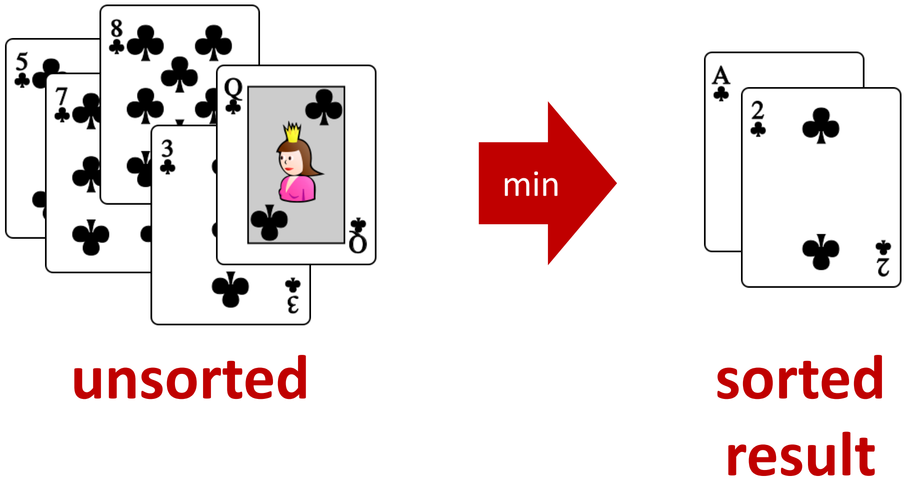


en.wikipedia.org/wiki/Standard_52-card_deck

Selection sort

selection_sort.py

```
def selection_sort(L):
    unsorted = L[:]
    result = []

    while unsorted:
        e = min(unsorted)
        unsorted.remove(e)
        result.append(e)

    return result
```

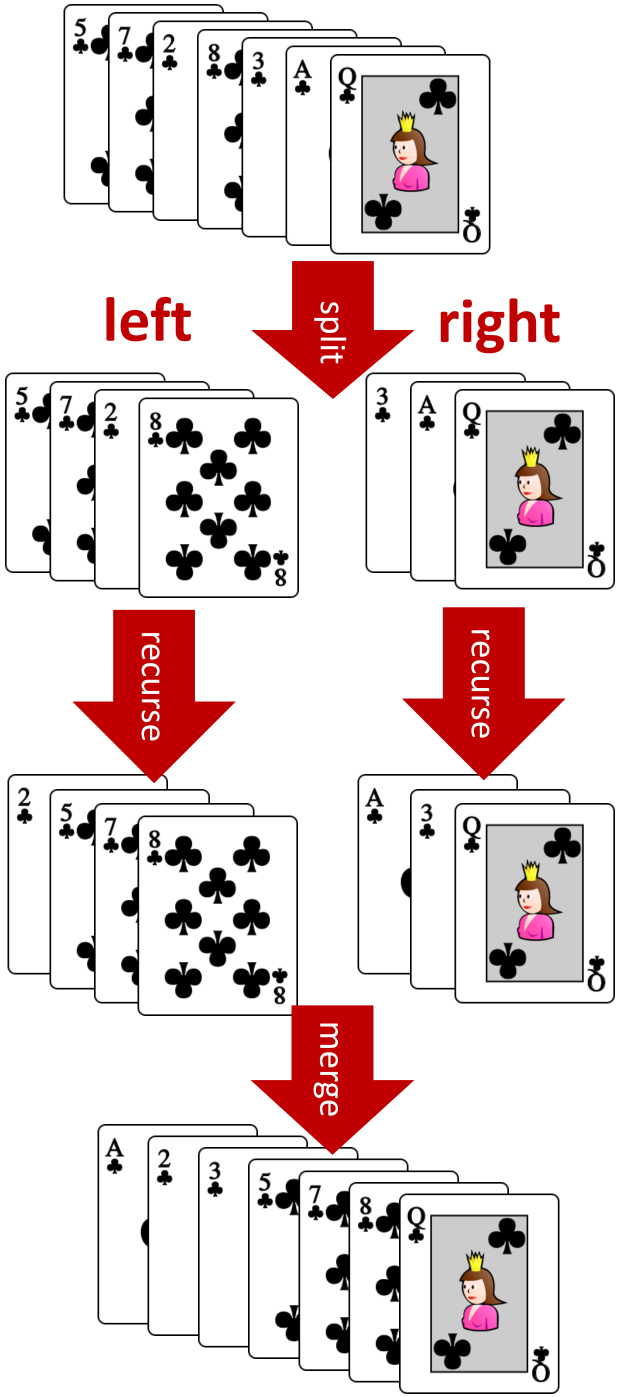


- `min` and `.remove` scan the remaining `unsorted` list for each element moved to `result`
- order $|L|^2$ comparisons

Sorting a pile of cards (Merge sort)

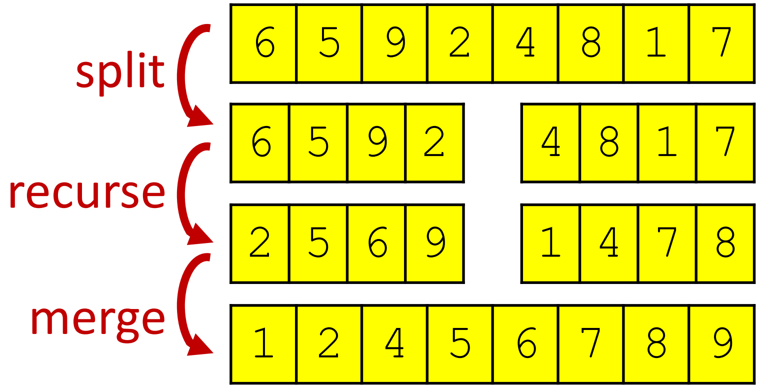
- If one card in pile, i.e. pile is sorted
- Otherwise

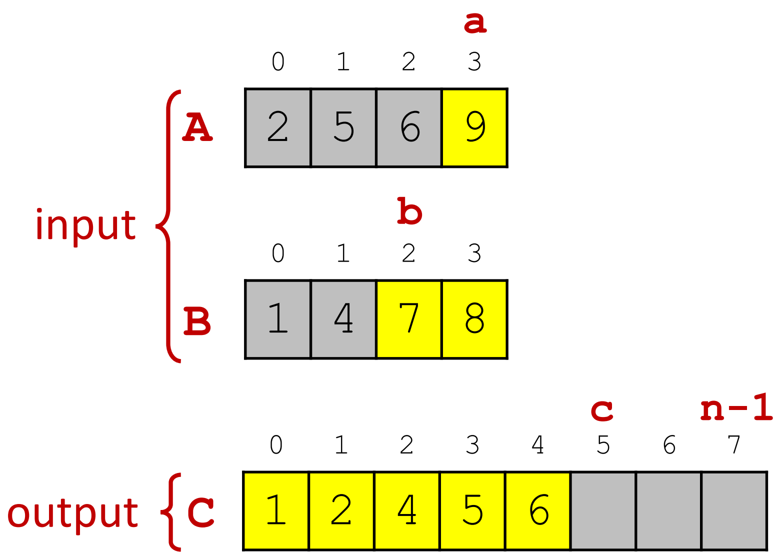
  1) Split pile into two piles, **left** and **right**, of approximately same size
  2) Sort **left** and **right** recursively (independently)
  3) Merge **left** and **right** (which are sorted)



merge_sort.py

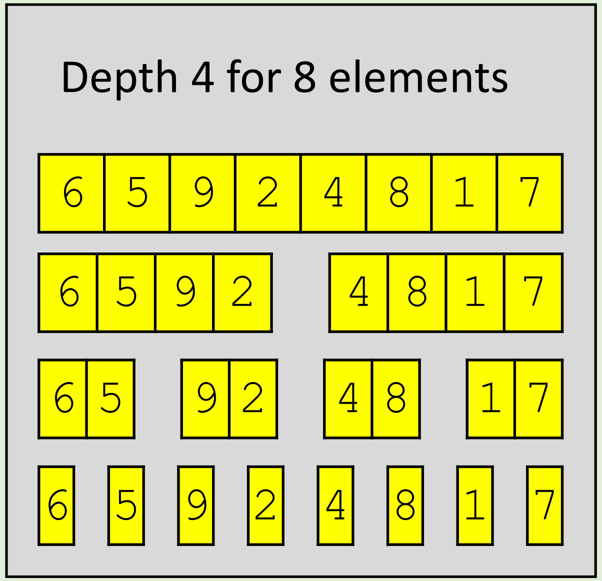
```python
def merge_sort(L):
    n = len(L)
    if n <= 1:
        return L[:]
    else:
        mid = n // 2
        left, right = L[:mid], L[mid:]
        return merge(merge_sort(left), merge_sort(right))

def merge(A, B):
    n = len(A) + len(B)
    C = n * [None]
    a, b = 0, 0
    for c in range(n):
        if a < len(A) and (b == len(B) or A[a] < B[b]):
            C[c] = A[a]
            a = a + 1
        else:
            C[c] = B[b]
            b = b + 1
    return C
```





Question – Depth of recursion for 52 elements

a) 1
b) 2
c) 3
d) 4
e) 5
f) 6
g) 7
h) 8
i) 9
j) 10
k) Don't know



Question – Order of comparisons by Merge sort ?

a) ~ n
b) ~ $n\sqrt{n}$
c) ~ $n \log_2 n$
d) ~ n^2
e) ~ n^3
f) Don't know

merge_sort.py

```
def merge_sort(L):
    n = len(L)
    if n <= 1:
        return L[:]
    else:
        mid = n // 2
        left, right = L[:mid], L[mid:]
        return merge(merge_sort(left), merge_sort(right))

def merge(A, B):
    n = len(A) + len(B)
    C = n * [None]
    a, b = 0, 0
    for c in range(n):
        if a < len(A) and (b == len(B) or A[a] < B[b]):
            C[c] = A[a]
            a = a + 1
        else:
            C[c] = B[b]
            b = b + 1
    return C
```

Merge sort without recursion

- Start with piles of size one
- Repeatedly merge two smallest piles

merge_sort.py

```
def merge_sort_iterative(L):
    Q = [[x] for x in L]
    while len(Q) > 1:
        Q.insert(0, merge(Q.pop(), Q.pop()))
    return Q[0]

from collections import deque

def merge_sort_deque(L):
    Q = deque([[x] for x in L])
    while len(Q) > 1:
        Q.appendleft(merge(Q.pop(), Q.pop()))
    return Q[0]
```

⚠ insert at front of list inefficient

deques are a generalization of lists with efficient updates at both ends

```
merge_sort_iterative([7,1,9,3,-2,5])
```

Values of Q in while-loop

```
[[7], [1], [9], [3], [-2], [5]]
[[-2, 5], [7], [1], [9], [3]]
[[3, 9], [-2, 5], [7], [1]]
[[1, 7], [3, 9], [-2, 5]]
[[-2, 3, 5, 9], [1, 7]]
[[-2, 1, 3, 5, 7, 9]]
```

Note: Lists in Q appear in non-increasing length order, where longest ≤ 2· shortest

Question – Number of iterations of while-loop ?

```
merge_sort_iterative([7, 1, 9, 3, -2, 5])
```

a) 1
b) 2
c) 3
d) 4

e) 5
f) 6
g) 7
h) Don't know

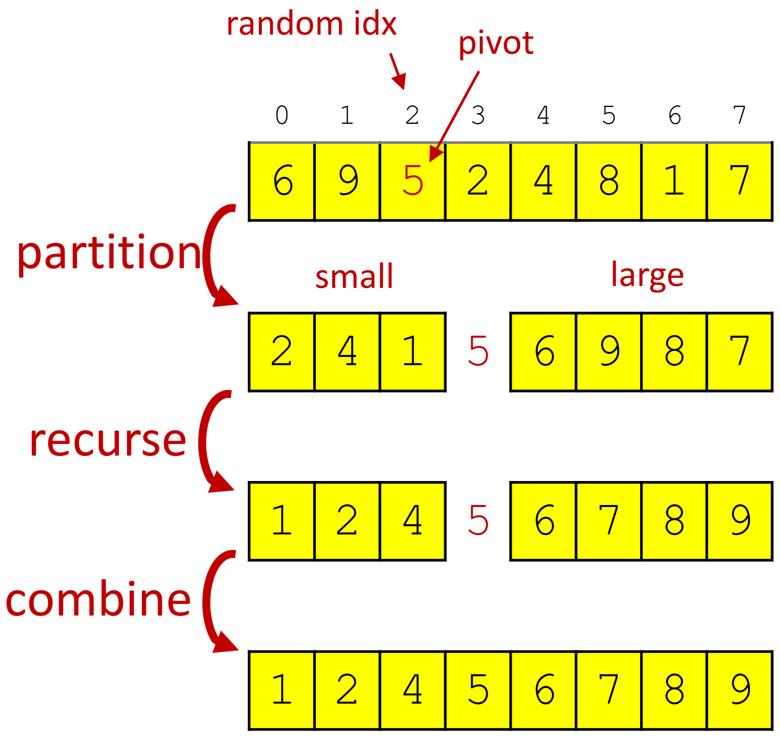
merge_sort.py

```python
def merge_sort_iterative(L):
    Q = [[x] for x in L]
    while len(Q) > 1:
        Q.insert(0, merge(Q.pop(), Q.pop()))
    return Q[0]
```

Quicksort (randomized)

quicksort.py

```python
import random

def quicksort(L):
    if len(L) <= 1:
        return L[:]

    idx = random.randint(0,len(L)-1)
    pivot = L[idx]
    other = L[:idx] + L[idx+1:]

    small = [e for e in other if e < pivot]
    large = [e for e in other if e >= pivot]

    return quicksort(small) + [pivot] + quicksort(large)
```

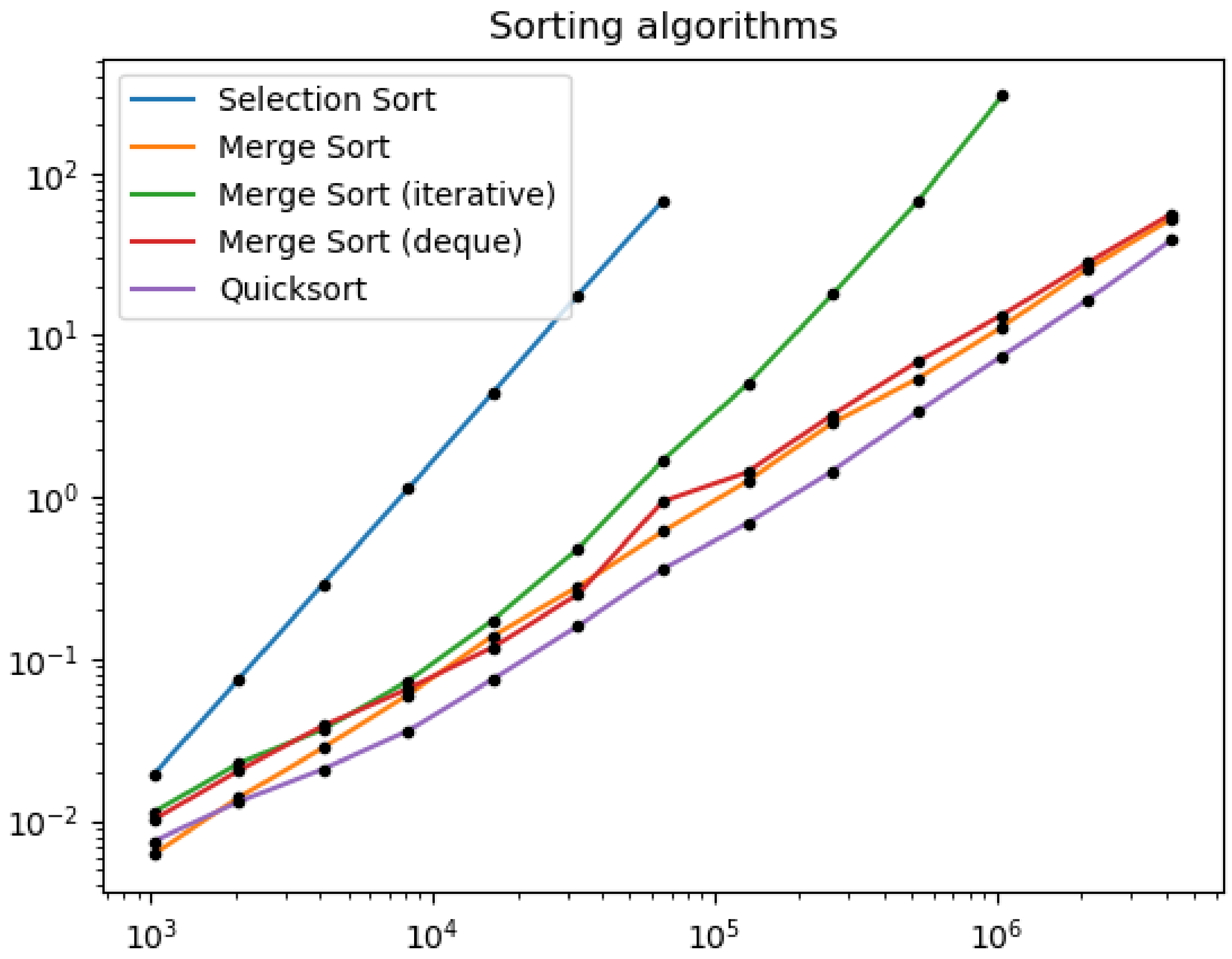


order $|L| \cdot \log_2 |L|$ comparisons, expected

Sorting comparison (single run)

$\lvert L \rvert$	Selection sort	Merge sort Recursive	Merge sort Iterative	Merge sort Deque	Quicksort
2^{10}	0.02	0.00	0.01	0.00	0.00
2^{11}	0.08	0.01	0.02	0.02	0.01
2^{12}	0.29	0.03	0.05	0.04	0.02
2^{13}	1.17	0.07	0.13	0.06	0.04
2^{14}	4.62	0.14	0.28	0.14	0.08
2^{15}	18.78	0.29	0.54	0.28	0.20
2^{16}	74.27 x 4	0.64	1.92	0.89	0.33
2^{17}		1.48	5.74	1.62	0.69
2^{18}		3.11	20.85	3.66	1.49
2^{19}		6.41	79.05 x 4	7.91	3.52
2^{20}		13.52		15.08	7.83
2^{21}		28.30		31.32	17.38
2^{22}		59.60 x 2		63.52 x 2	40.80 x 2

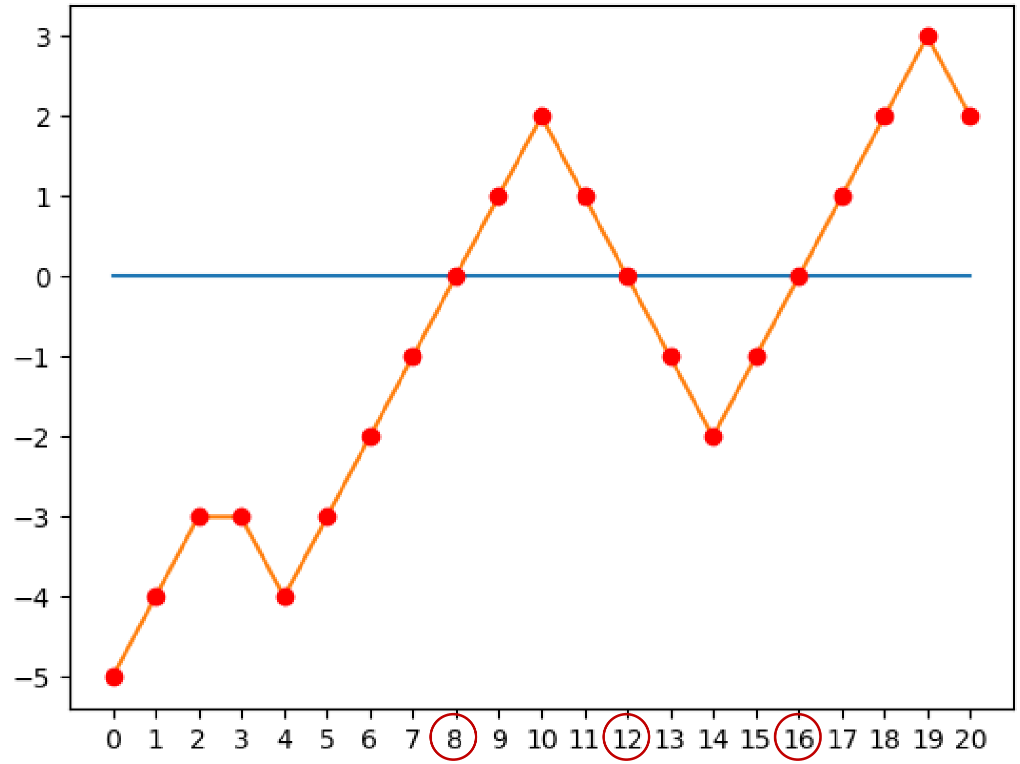
Sorting comparison



Find zero

- Given a list L of integers starting with a negative and ending with a positive integer, and where |L[i+1] - L[i]| ≤ 1, find the position of a zero in L.

L = [-5, -4, -3, -3, -4, -3, -2, -1, 0, 1, 2, 1, 0, -1, -2, -1, 0 , 1, 2, 3, 2]



find_zero.py

```python
def find_zero_loop(L):
    i = 0
    while L[i] != 0:
        i += 1
    return i

def find_zero_enumerate(L):
    for idx, e in enumerate(L):
        if e == 0:
            return idx

def find_zero_index(L):
    return L.index(0)
```

Function ($\vert L\vert = 10^6$)	Time, sec
find_zero_loop	0.13
find_zero_enumerate	0.10
find_zero_index	0.015
find_zero_binary_search	0.000015
find_zero_recursive	0.000088

```python
def find_zero_binary_search(L):
    low = 0
    high = len(L) - 1
    while True:   # L[low] < 0 < L[high]
        mid = (low + high) // 2
        if L[mid] == 0:
            return mid
        elif L[mid] < 0:
            low = mid
        else:
            high = mid

def find_zero_recursive(L):
    def search(low, high):
        mid = (low + high) // 2
        if L[mid] == 0:
            return mid
        elif L[mid] < 0:
            return search(mid, high)
        else:
            return search(low, mid)

    return search(0, len(L)-1)
```

Greatest Common Divisor (GCD)

Notation $x \uparrow y$ denotes y is divisible by x, e.g. $3 \uparrow 12$
i.e. $y = a \cdot x$ for some integer a

Definition $\gcd(m, n) = \max \{ x \mid x \uparrow m \text{ and } x \uparrow n \}$

Fact if $x \uparrow y$ and $x \uparrow z$ then $x \uparrow (y+z)$ and $x \uparrow (y-z)$

Observation
(recursive definition)

$$\gcd(m, n) = \begin{cases} m & \text{if } m = n \\ \gcd(m, n - m) & \text{if } m < n \\ \gcd(m - n, n) & \text{if } m > n \end{cases}$$

gcd(90, 24)

m	n
90	24
66	24
42	24
18	24
18	6
12	6
6	6

Greatest Common Divisor (GCD)

gcd_slow.py

```
def gcd(m, n):
    while m != n:
        if n > m:
            n = n - m
        else
            m = m - n
    return m
```

gcd.py

```
def gcd(m, n):
    while n != 0:
        m, n = n, m % n
    return m
```

gcd_slow_recursive.py

```
def gcd(m, n):
    if m == n:
        return m
    elif m > n:
        return gcd(m - n, n)
    else:
        return gcd(m, n - m)
```

gcd_recursive.py

```
def gcd(m, n):
    if n == 0:
        return m
    else:
        return gcd(n, m % n)
```

gcd_recursive_one_line.py

```
def gcd(m, n):
    return m if n == 0 else gcd(n, m % n)
```

Permutations

- Generate all permutations of a list L as tuples

Python shell

```
> permutations(['a','b','c'])
| [('a', 'b', 'c'), ('b', 'a', 'c'), ('b', 'c', 'a'),
   ('a', 'c', 'b'), ('c', 'a', 'b'), ('c', 'b', 'a')]
```

permutations.py

```
def permutations(L):
    if len(L) == 0:
        return [L[:]]
    else:
        P = permutations(L[1:])
        return [p[:i] + L[:1] + p[i:] for p in P for i in range(len(L))]
```

- An implementation of "permutations" exists in the "itertools" library

Maze solver

Input

- First line #rows and #columns
- Following #rows lines contain strings containing #column characters
- There are exactly one 'A' and one 'B'
- '.' are free cells and '#' are blocked cells

Output

- Print whether there is a path from 'A' to 'B' or not

maze input

```
11 19
#######A###########
#.......#.....#...#
#.###.###...#.#.#.#
#...#.....#.#...#.#
#.#.###.#.#.#.###.#
#.#.....#...#.#...#
#.###########.#.#.#
#.#.#.....#...#.#.#
#.#.#####.#####.#.#
#.........#.....#.#
###############B###
```

Maze solver (recursive)

maze_solver.py

```
def explore(i, j):
    global solution, visited

    if (0 <= i < n and 0 <= j < m and
        maze[i][j] != "#" and not visited[i][j]):

        visited[i][j] = True

        if maze[i][j] == 'B':
            solution = True

        explore(i - 1, j)
        explore(i + 1, j)
        explore(i, j - 1)
        explore(i, j + 1)
```

```
def find(symbol):
    for i in range(n):
        j = maze[i].find(symbol)
        if j >= 0:
            return (i, j)

n, m = [int(x) for x in input().split()]
maze = [input() for i in range(n)]

solution = False
visited = [m*[False] for i in range(n)]

explore(*find('A'))

if solution:
    print("path from A to B exists")
else:
    print("no path")
```

Maze solver (iterative)

maze_solver_iterative.py

```
def explore(i, j):
    global solution, visited

    Q = [(i, j)] # cells to visit

    while Q:
        i, j = Q.pop()
        if (0 <= i < n and 0 <= j < m and
            maze[i][j] != '#' and not visited[i][j]):

            visited[i][j] = True

            if maze[i][j] == 'B':
                solution = True

            Q.append((i - 1, j))
            Q.append((i + 1, j))
            Q.append((i, j - 1))
            Q.append((i, j + 1))
```

```
def find(symbol):
    for i in range(n):
        j = maze[i].find(symbol)
        if j >= 0:
            return (i, j)

n, m = [int(x) for x in input().split()]
maze = [input() for i in range(n)]

solution = False
visited = [m*[False] for i in range(n)]

explore(*find('A'))

if solution:
    print("path from A to B exists")
else:
    print("no path")
```

Question – How difficult is the triplet project on a scale 1 – 10 ?

a) 1 (I’m offended by how trivial the project was)
b) 2 (very easy)
c) 3 (a quite standard review exercise)
d) 4 (not too complicated, got some known concepts repeated)
e) 5 (good exercise to repeat standard programming techniques)
f) 6 (had to use more advanced techniques in a familiar way)
g) 7 (quite complicated, but manageable)
h) 8 (very abstract exercise, using complicated language constructs)
i) 9 (very complicated – barely manageable spending all my time)
j) 10 (this is a research project – could be an MSc thesis/PhD project)
k) 25 (this is wayyy too complicated for a university course)

Functions as objects

- lambda
- higher-order functions
- map, filter, reduce

Aliasing functions – both user defined and builtin

Python shell

```
> def square(x):
      return x * x
> square
| <function square at 0x0329A390>
> square(8)
| 64
> kvadrat = square
> kvadrat(5)
| 25
> len
| <built-in function len>
> length = len
> length([1, 2, 3])
| 3
```

Functions as values

square_or_double.py

```
def square(x):
        return x * x

def double(x):
        return 2 * x

while True:
    answer = input("square or double ? ")
    if answer == "square":
        f = square
        break
    if answer == "double":
        f = double
        break

answer = input("numbers: ")
L_in = [int(x) for x in answer.split()]
L_out = [f(x) for x in L_in]
print(L_out)
```

Python shell

```
| square or double ? square
| numbers: 3 6 7 9
| [9, 36, 49, 81]

| square or double ? double
| numbers: 2 3 4 7 9
| [4, 6, 8, 14, 18]
```

`f` will refer to one of the functions `square` and `double` refer to

call the function `f` is referring to with argument `x`

Functions as values and namespaces

say.py

```python
def what_says(name):
    def say(message):
        print(name, "says:", message)
    return say

alice = what_says("Alice")
peter = what_says("Peter")

alice("Where is Peter?")
peter("I am here")
```

Python shell

```
| Alice says: Where is Peter?
| Peter says: I am here
```

- `what_says` is a function returning a function (`say`)
- Each call to `what_says` with a single string as its argument **creates a new say** function with the current `name` argument in its namespace
- In each call to a an instance of a `say` function, `name` refers to the string in the namespace when the function was created, and `message` is the string given as an argument in the call

Question – What list is printed ?

```
def f(x):
    def g(y):
        nonlocal x
        x = x + 1
        return x + y
    return g
a = f(3)
b = f(6)
print([a(3), b(2), a(4)])
```

a) [7, 7, 10]

b) [7, 9, 8]

c) [7, 9, 9]

d) [7, 9, 12]

e) [7, 10, 10]

f) Don't know

map

- `map(function, list)` applies the function to each element of the sequence `list`
- `map(function, list`$_1$`, ..., list`$_k$`)` requires `function` to take k arguments, and creates a sequence with the i'th element being `function(list`$_1$`[i], ..., list`$_k$`[i])`

Python shell

```
> def square(x):
      return x*x
> list(map(square, [1,2,3,4,5]))
| [1, 4, 9, 16, 25]
> def triple_sum(x, y, z):
      return x + y + z
> list(map(triple_sum, [1,2,3], [4,5,6], [7,8,9]))
| [12, 15, 18]
> list(map(triple_sum, *zip(*[(1,4,7), (2,5,8), (3,6,9)])))
| [12, 15, 18]
```

sorted

- A list `L` can be sorted using `sorted(L)`
- A user defined order on the elements can be defined by providing a function using the keyword argument `key`, that maps elements to values with some default ordering

Python shell

```
> def length_square(p):
      x, y = p
      return x**2 + y**2   # no sqrt

> L = [(5,3), (2,5), (1,9), (2,2), (3,4)]
> list(map(length_square, L))
| [34, 29, 82, 8, 25]
> sorted(L)   # default lexicographical order
| [(1, 9), (2, 2), (2, 5), (3, 4), (5, 3)]
> sorted(L, key=length_square)   # order by length
| [(2, 2), (3, 4), (2, 5), (5, 3), (1, 9)]
```

https://docs.python.org/3/library/functions.html#sorted

Question – What list does sorted produce ?

```
sorted([2, 3, -1, 5, -4, 0, 8, -6], key=abs)
```

key 2 3 1 5 4 0 8 6

a) `[-6, -4, -1, 0, 2, 3, 5, 8]`

b) `[0, 2, 3, 5, 8, -1, -4, -6]`

c) `[0, -1, 2, 3, -4, 5, -6, 8]`

d) `[8, 5, 3, 2, 0, -1, -4, -6]`

e) `[0, 1, 2, 3, 4, 5, 6, 8]`

f) Don’t know

Python shell

```
> abs(7)
| 7
> abs(-42)
| 42
```

filter

- `filter(function, list)` returns the subsequenece of `list` where `function` evaluates to true
- Essentially the same as

```
[x for x in list if function(x)]
```

Python shell

```
> def odd(x):
      return x % 2 == 1

> filter(odd, range(10))
| <filter object at 0x03970FD0>
> list(filter(odd, range(10)))
| [1, 3, 5, 7, 9]
```

reduce (in module functools)

- Python's "reduce" function is in other languages often denoted "foldl"

$$\texttt{reduce(f,[}x_1, x_2, x_3, \ldots, x_k\texttt{])} = \texttt{f(}\cdots\texttt{f(f(}x_1, x_2\texttt{)},x_3\texttt{)}\cdots, x_k\texttt{)}$$

Python shell

```
> from functools import reduce

> def power(x, y):
      return x ** y

> reduce(power, [2, 2, 2, 2, 2])
| 65536
```

lambda (anonymous functions)

- If you need to define a *short* function, that *returns a value*, and the function is only *used once* in your program, then a lambda function might be appropriate:

```
lambda arguments: expression
```

- Creates a function with no name that takes zero or more arguments, and returns the value of the single expression

Python shell

```
> f = lambda x, y : x + y
> f(2, 3)
| 5
> list(filter(lambda x: x % 2, range(10)))
| [1, 3, 5, 7, 9]
```

Examples: sorted using lambda

Python shell

```
> L = [ 'AHA', 'Oasis', 'ABBA', 'Beatles', 'AC/DC', 'B. B. King', 'Bangles', 'Alan Parsons']

> # Sort by length, secondary after input position (default, known as stable)
> sorted(L, key=len)
| ['AHA', 'ABBA', 'Oasis', 'AC/DC', 'Beatles', 'Bangles', 'B. B. King', 'Alan Parsons']

> # Sort by length, secondary alphabetically
> sorted(L, key=lambda s: (len(s), s))
| ['AHA', 'ABBA', 'AC/DC', 'Oasis', 'Bangles', 'Beatles', 'B. B. King', 'Alan Parsons']

> # Sort by most 'a's, if equal by number of 'b's, etc.
> sorted(L, key=lambda s: sorted([a.lower() for a in s if a.isalpha()] + ['~']))
| ['Alan Parsons', 'ABBA', 'AHA', 'Beatles', 'Bangles', 'AC/DC', 'Oasis', 'B. B. King']

> sorted([a.lower() for a in 'Beatles' if a.isalpha()] + ['~'])
| ['a', 'b', 'e', 'e', 'l', 's', 't', '~']
```

min and max

- Similarly to `sorted`, the functions `min` and `max` take a keyword argument `key`, to map elements to values with some default ordering

Python shell

```
> max(['w', 'xyz', 'abcd', 'uv'])
| 'xyz'
> max(['w', 'xyz', 'abcd', 'uv'], key=len)
| 'abcd'
> sorted([210, 13, 1010, 30, 27, 103], key=lambda x: str(x)[::-1])
| [1010, 210, 30, 103, 13, 27]
> min([210, 13, 1010, 30, 27, 103], key=lambda x: str(x)[::-1])
| 1010
```

docs.python.org/3/library/functions.html#min

History of lambda in programming languages

- lambda calculus invented by Alonzo Church in 1930s
- Lisp has had lambdas since 1958
- C++ got lambdas in C++11 in 2011
- Java first got lambdas in Java 8 in 2014
- Python has had lambdas since Version 1.0 in 1994

polynomial.py	Python shell
(code below)	(output below)

polynomial.py

```
def linear_function(a, b):
    return lambda x: a * x + b

def degree_two_polynomial(a, b, c):
    def evaluate(x):
        return a * x**2 + b * x + c
    return evaluate

def polynomial(coefficients):
    return lambda x: sum([c * x**p for p, c in enumerate(coefficients)])

def combine(f, g):
    def evaluate(*args, **kwargs):
        return f(g(*args, **kwargs))
    return evaluate

f = linear_function(2, 3)
for x in [0, 1, 2]:
    print("f(%s) = %s" % (x, f(x)))
p = degree_two_polynomial(1, 2, 3)
for x in [0, 1, 2]:
    print("p(%s) = %s" % (x, p(x)))
print("polynomial([3, 2, 1])(2) =", polynomial([3, 2, 1])(2))
h = combine(abs, lambda x, y: x - y)
print("h(3, 5) =", h(3, 5))
```

Python shell

```
| f(0) = 3
| f(1) = 5
| f(2) = 7
| p(0) = 3
| p(1) = 6
| p(2) = 11
| polynomial([3, 2, 1])(2) = 11
| h(3, 5) = 2
```

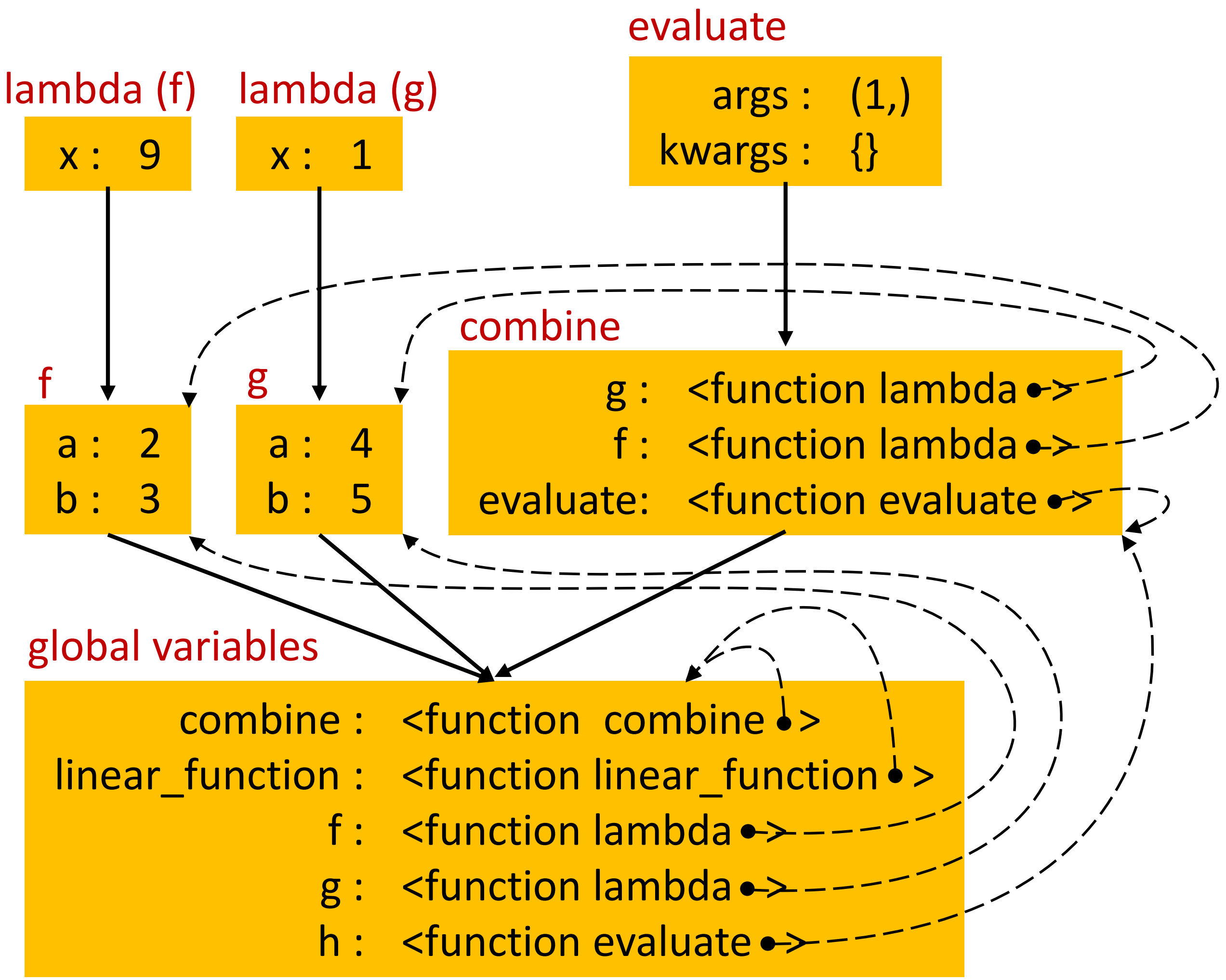
Question – What value is h(1) ?

linear_combine.py

```
def combine(f, g):
    def evaluate(*args, **kwargs):
        return f(g(*args, **kwargs))

    return evaluate

def linear_function(a, b):
    return lambda x: a * x + b

f = linear_function(2, 3)
g = linear_function(4, 5)

h = combine(f, g)

print(h(1))
```

a) 5
b) 9
c) 16
d) 21
e) 25
f) Don't know

Namespace example



linear_combine.py

```
def combine(f, g):
    def evaluate(*args, **kwargs):
        return f(g(*args, **kwargs))

    return evaluate

def linear_function(a, b):
    return lambda x: a * x + b

f = linear_function(2, 3)
g = linear_function(4, 5)

h = combine(f, g)

print(h(1))
```

partial

partial.py

```
def partial(f, *args, **kwargs):
    return lambda *a, **kw : f(*args, *a, **kwargs, **kw)

def f(x, y, z):
    return x + 2 * y + 3 * z

g = partial(f, 7)
h = partial(f, 2, 1 )
k = partial(g, 1, 2)

print(g(2, 1))  # 7 + 2 * 2 + 3 * 1 = 14
print(h(3))     # 2 + 2 * 1 + 3 * 3 = 13
print(k())      # 7 + 2 * 1 + 3 * 2 = 15
```

- The function `partial` from the module `functools` fixes/binds/freezes a subset of the parameters of a function
- Roughly equivalent to the definition in the example

Wikipedia: [Partial application](#)
docs.python.org/3/library/functools.html

partial (trace of computation)

partial_trace.py

```
def partial(fn, *args):
    def new_f(*a):
        print(f'new_f: fn={fn.__name__}, args={args}, a={a})')
        answer = fn(*args, *a)
        print(f'answer={answer}')
        return answer

    return new_f

def f(x, y, z):
    print(f'f({x},{y},{z})')
    return x + 2 * y + 3 * z

g = partial(f, 7)
h = partial(f, 2, 1 )
k = partial(g, 1, 2)

print(f'{g(2, 1)=}\n')   # 7 + 2 * 2 + 3 * 1 = 14
print(f'{h(3)=}\n')      # 2 + 2 * 1 + 3 * 3 = 13
print(f'{k()=}\n')       # 7 + 2 * 1 + 3 * 2 = 15
```

Python shell

```
| new_f: fn=f, args=(7,), a=(2, 1))
| f(7,2,1)
| answer=14
| g(2, 1)=14
|
| new_f: fn=f, args=(2, 1), a=(3,))
| f(2,1,3)
| answer=13
| h(3)=13
|
| new_f: fn=new_f, args=(1, 2), a=())
| new_f: fn=f, args=(7,), a=(1, 2))
| f(7,1,2)
| answer=15
| answer=15
| k()=15
```

Python shell

```
> def f(x): return x
> g = lambda x: x
> f.__name__
| 'f'
> g.__name__
| '<lambda>'
```

Object oriented programming

- classes, objects
- self
- construction
- encapsulation

Object Oriented Programming

- **Programming paradigm**, other paradigms are e.g.
  - *functional programming* where the focus is on functions, lambda's and higher order functions, and
  - *imperative programming* focusing on sequences of statements changing the state of the program
- Core concepts are **objects**, **methods** and **classes**,
  - allowing one to construct *abstract data types*, i.e. user *defined types*
  - objects have states
  - methods manipulate objects, defining the interface of the object to the rest of the program
- OO supported by [many programming languages](), including Python

Object Oriented Programming - History

(selected programming languages)

Mid 1960's **Simular 67**

(Ole-Johan Dahl and Kristen Nygaard, Norsk Regnesentral Oslo)
Introduced classes, objects, virtual procedures

1970's **Smalltalk** (Alan Kay, Dan Ingalls, Adele Goldberg, Xerox PARC)

Object-oriented programming, fully dynamic system
(opposed to the static nature of Simula 67)

1985 **Eiffel** (Bertrand Meyer, Eiffel Software)

Focus on software quality, capturing the full software cycle

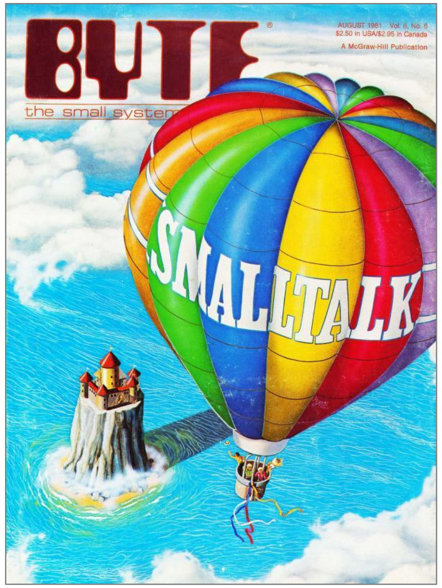
1985 **C++** (Bjarne Stroustrup [MSc Aarhus 1975], AT&T Bell Labs)

1995 **Java** (James Gosling, Sun)

2000 **C#** (Anders Hejlsberg (studied at DTU) et al., Microsoft)

1991 **Python** (Guido van Rossum)

Multi-paradigm programming language, fully dynamic system

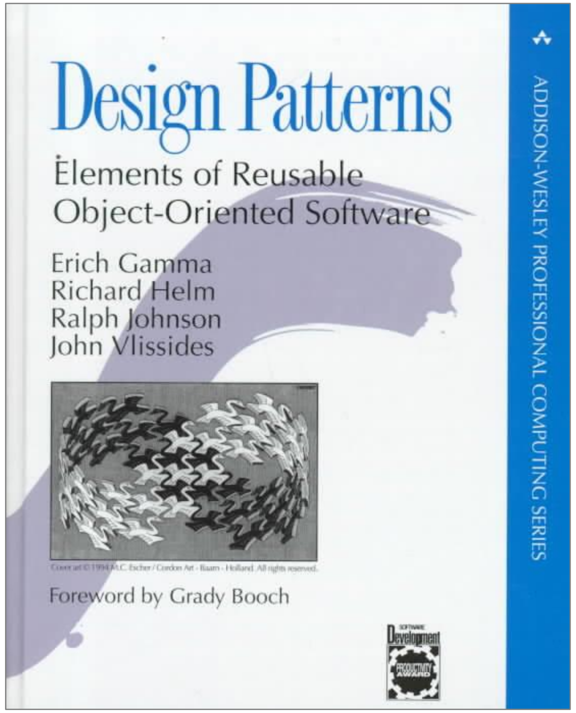


[Byte Magazine, August 1981]

Note: Java, C++, Python, C# are among Top 5 on TIOBE January 2020 index of popular languages (only non OO language among Top 5 was C)

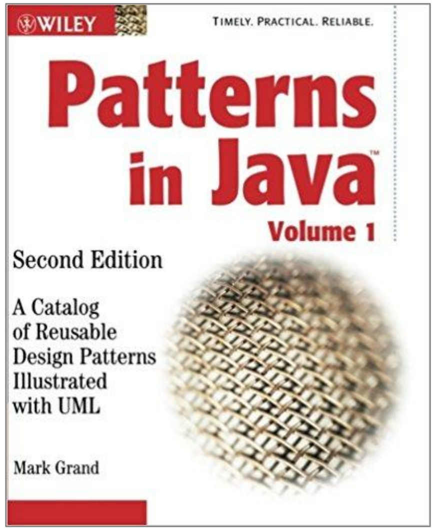
Design Patterns (not part of this course)

reoccuring patterns in software design

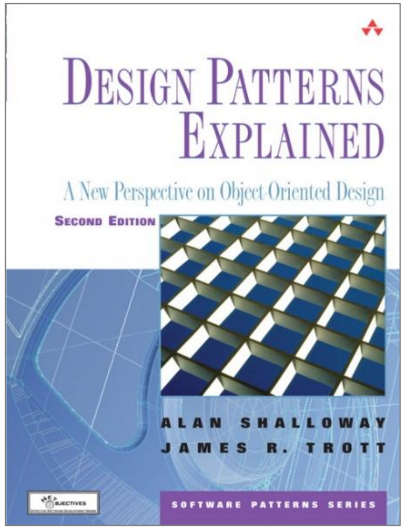


The Classic book 1994
(C++ cookbook)

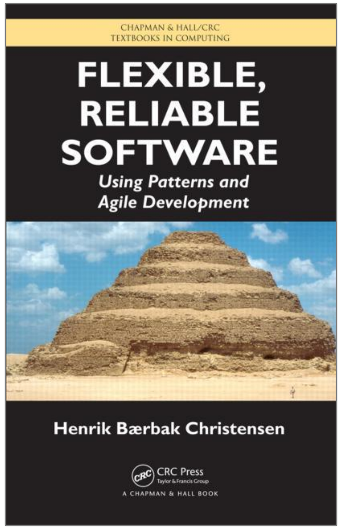
A very alternative book 2004
(Java, very visual)



Java cookbook 2003



Java textbook 2004



Java textbook 2010

...and many more books on the topic of Design Patterns, also with Python

Some known classes, objects, and methods

Type / class	Objects	Methods (examples)
int	0 -7 42 1234567	.__add__(x), .__eq__(x), .__str__()
str	"" 'abc' '12_ a'	.isdigit(), .lower(), .__len__()
list	[] [1,2,3] ['a', 'b', 'c']	.append(x), .clear(), .__mul__(x)
dict	{'foo' : 42, 'bar' : 5}	.keys(), .get(), .__getitem__(x)
NoneType	None	.__str__()

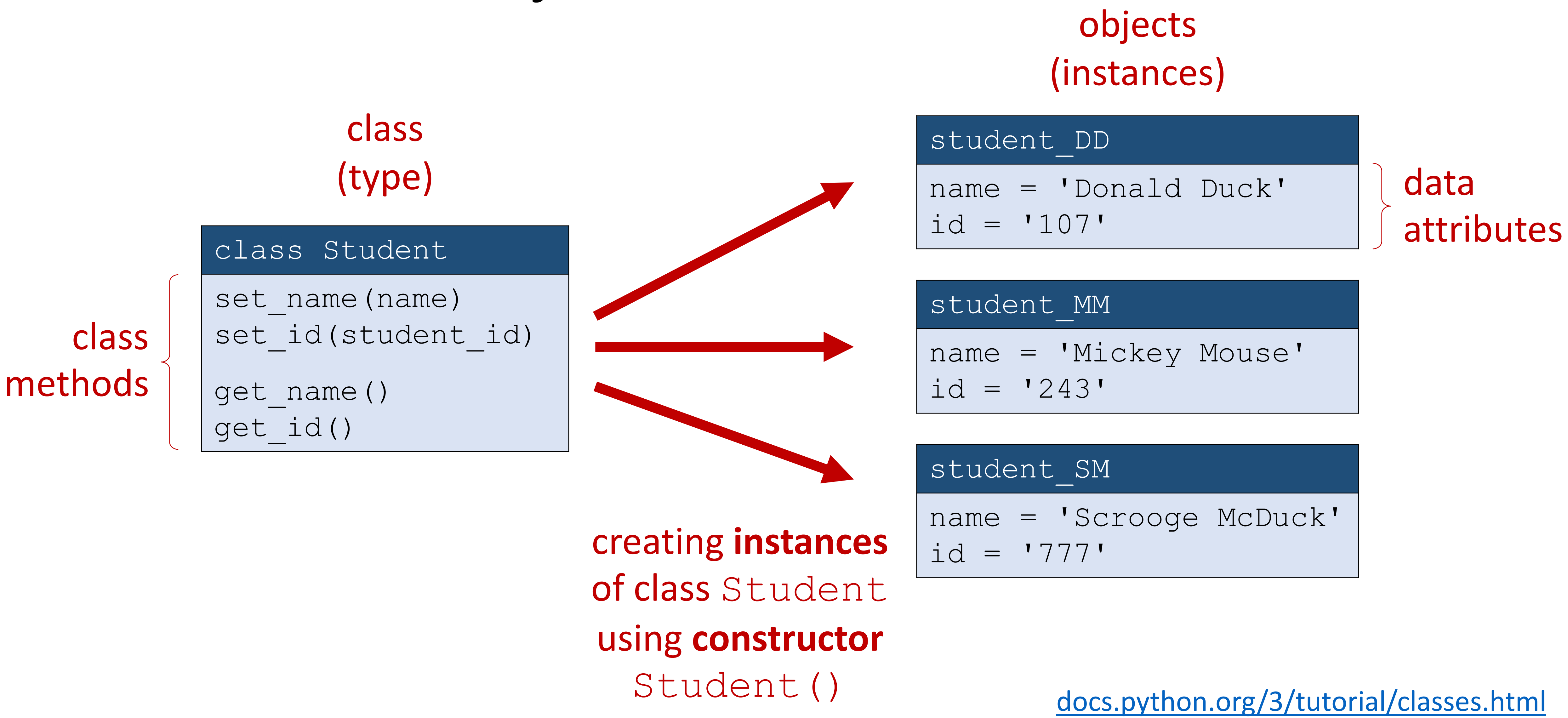
Example:

The function `str(obj)` calls the methods `obj.__str__()` or `obj.__repr__()`, if `obj.__str__` does not exist.

Python shell

```
> 5 + 7   # + calls .__add__(7)
| 12
> (5).__add__(7)   # eq. to 5 + 7
| 12
> (7).__eq__(7)   # eq. to 7 == 7
| True
> 'aBCd'.lower()
| 'abcd'
> 'abcde'.__len__()
  # .__len__() called by len(...)
| 5
> ['x', 'y'].__mul__(2)
| ['x', 'y', 'x', 'y']
> {'foo' : 42}.__getitem__('foo')
  # eq. to {'foo' : 42}['foo']
| 42
> None.__str__()   # used by str(...)
| 'None'
```

Classes and Objects



docs.python.org/3/tutorial/classes.html

Using the `Student` class

student.py

```python
student_DD = Student()
student_MM = Student()
student_SM = Student()

student_DD.set_name('Donald Duck')
student_DD.set_id('107')

student_MM.set_name('Mickey Mouse')
student_MM.set_id('243')

student_SM.set_name('Scrooge McDuck')
student_SM.set_id('777')

students = [student_DD, student_MM, student_SM]

for student in students:
    print(student.get_name(),
          "has id",
          student.get_id())
```

Python shell

```
| Donald Duck has id 107
| Mickey Mouse has id 243
| Scrooge McDuck has id 777
```

Call **constructor** for class `Student`. Each call returns a new `Student` object.

Call class methods to set data attributes

Call class methods to read data attributes

class Student

name of class

class definitions start with the keyword **class**

student.py

```python
class Student:
    def set_name(self, name):
        self.name = name

    def set_id(self, student_id):
        self.id = student_id

    def get_name(self):
        return self.name

    def get_id(self):
        return self.id
```

often called **mutator methods**, since they change the state of an object

often called **accessor methods**, since they only read the state of an object

the first argument to all class methods is a reference to the object called upon, and by convention the first argument should be named **self**.

use **self.** to access an attribute of an object or class method (attribute reference)

class method definitions start with keyword **def** (like normal function definitions)

Note In other OO programming languages the explicit reference to **self** is not required (in Java and C++ **self** is the keyword **this**)

When are object attributes initialized ?

Python shell

```
> x = Student()
> x.set_name("Gladstone Gander")
> x.get_name()
| 'Gladstone Gander'
> x.get_id()
| AttributeError: 'Student' object has no attribute 'id'
```

- Default behaviour of a class is that instances are created with no attributes defined, but has access to the attributes / methods of the class
- In the previous class `Student` both the `name` and `id` attributes were first created when set by `set_name` and `set_id`, respectively

Class construction and `__init__`

- When an object is created using `class_name()` it's initializer metod `__init__` is called.
- To initialize objects to contain default values, (re)define this function.

student.py

```
class Student:
    def __init__(self):
        self.name = None
        self.id = None

    ... previous method definitions ...
```

Question – What is printed ?

Python shell

```
> class C:
        def __init__(self):
              self.v = 0
        def f(self):
              self.v = self.v + 1
              return self.v
> x = C()
> print(x.f() + x.f())
```

a) 1
b) 2
c) 3
d) 4
e) 5
f) Don’t know

`__init__` with arguments

- When creating objects using *`class_name(args)`* the initializer method is called as `__init__(`*`args`*`)`
- To initialize objects to contain default values, (re)define this function to do the appropriate initialization

student.py

```
class Student:
    def __init__(self, name=None, student_id=None):
        self.name = name
        self.id = student_id

   ... previous method definitions ...
```

Python shell

```
> p = Student("Pluto")
> print(p.get_name())
| Pluto
> print(p.get_id())
| None
```

Are accessor and mutator methods necessary ?

No - but good programming style

Python shell

```
> p = Pair(3, 5)
> p.sum()
| 8
> p.set_a(4)
> p.sum()
| 9
> p.a       # access object attribute
| 4
> p.b = 0 # update object attribute
> p.sum()
| 9       # the_sum not updated
```

pair.py

```
class Pair:
    """ invariant: the_sum = a + b """
    def __init__(self, a, b):
        self.a = a
        self.b = b
        self.the_sum = self.a + self.b
    def set_a(self, a):
        self.a = a
        self.the_sum = self.a + self.b
    def set_b(self, b):
        self.b = b
        self.the_sum = self.a + self.b
    def sum(self):
        return self.the_sum
```

constructor

mutator

accessor

Defining order on instances of a class (sorting)

- To define an order on objects, define the “<“ operator by defining `__lt__`
- When ”<” is defined a list `L` of students can be sorted using `sorted(L)` and `L.sort()`

student.py

```
class Student:
    def __lt__(self, other):
        return self.id < other.id

    ... previous method definitions ...
```

Python shell

```
> student_DD < student_MM
| True
> [x.id for x in students]
| ['243', '107', '777']
> [x.id for x in sorted(students)]
| ['107', '243', '777']
```

Converting objects to `str`

- To be able to convert an object to a string using `str(object)`, define the method `__str__`
- `__str__` is e.g. used by `print`

student.py

```
class Student:
    def __str__(self):
        return ("Student['%s', '%s']"
                % (self.name, self.id))

    ... previous method definitions ...
```

Python shell

```
> print(student_DD)  # without __str__
| <__main__.Student object at 0x03AB6B90>
> print(student_DD)  # with __str__
| Student['Donald Duck', '107']
```

Nothing is private in Python

- Python does not support **hiding information** inside objects
- Recommendation is to start attributes with underscore, if these should be used only locally inside a class, i.e. be considered "private"
- **PEP8:** "Use one leading underscore only for non-public methods and instance variables"

private_attributes.py

```
class My_Class:
    def set_xy(self, a, b):
        self._x = a
        self._y = b

    def get_sum(self):
        return self._x + self._y

obj = My_Class()
obj.set_xy(3, 5)

print("Sum =", obj.get_sum())
print("_x =", obj._x)
```

Python shell

```
| Sum = 8
| _x = 3
```

C++ private, public

C++ vs Python

1. argument types
2. return types
3. `void` = `NoneType`
4. `private` / `public` access specifier
5. types of data attributes
6. data attributes must be defined in class
7. object creation
8. no `self` in class methods

private_attributes.cpp

```
#include <iostream>
using namespace std;
class My_Class {
private:④
 ⑤int x, y;⑥
public:④          ⑧①       ①
②③void set_xy(int a, int b) {
     x = a;
     y = b
   };            ⑧
② int get_sum() {
     return x + y;
   };
};
main() {
⑦ My_Class obj;
   obj.set_xy(3, 5);
   cout << "Sum = " << obj.get_sum() << endl;
   cout << "x = " << obj.x << endl;
}
```

invalid reference

Java private, public

Java vs Python

1. argument types
2. return types
3. `void` = `NoneType`
4. `private` / `public` access specifier
5. types of data attributes
6. data attributes must be defined in class
7. object creation
8. no `self` in class methods

private_attributes.java

```
class My_Class {
                 ⑤
④ private int x, y; ⑥
                ②③                 ⑧①          ①
④ public void set_xy(int a, int b) {
     x = a; y = b;
  }
                 ②              ⑧
④ public int get_sum() { return x + y; };
};
class private_attributes {
  public static void main(String args[]){
  ⑦ My_Class obj = new My_Class();
     obj.set_xy(3, 5);
     System.out.println("Sum = " + obj.get_sum());
     System.out.println("x = " + obj.x);
  }
}
```

invalid reference

Name mangling (partial privacy)

- Python handles references to class attributes inside a class definition with *at least two leading underscores and at most one trailing underscore* in a special way: `__attribute` is textually replaced by `_classname__attribute`

- Note that [Guttag, p. 126] states ”that attribute is not visible outside the class” – which only is partially correct (see example)

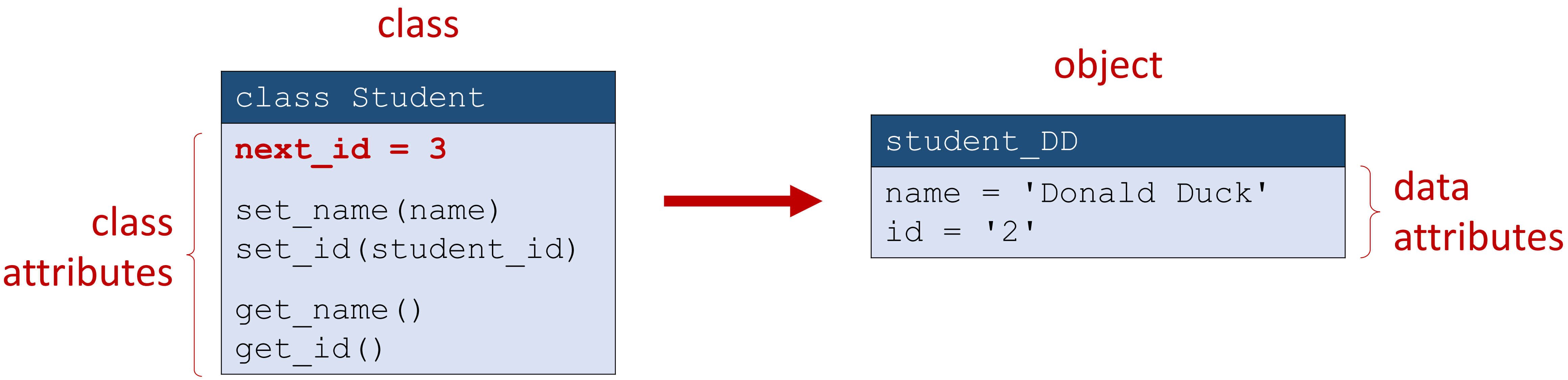
name_mangeling.py

```python
class MySecretBox:
    def __init__(self, secret):
        self.__secret = secret
```

Python shell

```
> x = MySecretBox(42)
> print(x.__secret)
| AttributeError: 'MySecretBox'
  object has no attribute
  '__secret'
> print(x._MySecretBox__secret)
| 42
```

Class attributes



- *obj.attribute* first searches the objects attributes to find a match, if no match, continuous to search the attributes of the class
- Assignments to *obj.attribute* are always to the objects attribute (possibly creating the attribute)
- Class attributes can be accesed directly as *class.attribute* (or *obj.__class__.attribute*)

Class data attribute

- **next_id** is a class attribute
- Accessed using **Student.next_id**
- The lookup ① can be replaced with **self.next_id**, since only the class has this attribute, looking up in the object will be propagated to a lookup in the class attributes
- In the update ② it is crucial that we update the class attribute, since otherwise the incremented value will be assigned as an object attribute

(What will the result be?)

student_auto_id.py

```python
class Student:
    next_id = 1  # class attribute
    def __init__(self, name):
        self.name = name      ①
        self.id = str(Student.next_id)
      ② Student.next_id += 1
    def get_name(self):
        return self.name
    def get_id(self):
        return self.id

students = [Student('Scrooge McDuck'),
            Student('Donald Duck'),
            Student('Mickey Mouse')]
for student in students:
    print(student.get_name(),
          "has student id",
          student.get_id())
```

Python shell

```
| Scrooge McDuck has student id 1
| Donald Duck has student id 2
| Mickey Mouse has student id 3
```

Question – What does `obj.get()` return ?

Python shell

```
> class MyClass:
      x = 2

      def get(self):
          self.x = self.x + 1
          return MyClass.x + self.x

> obj = MyClass()
> print(obj.get())
| ?
```

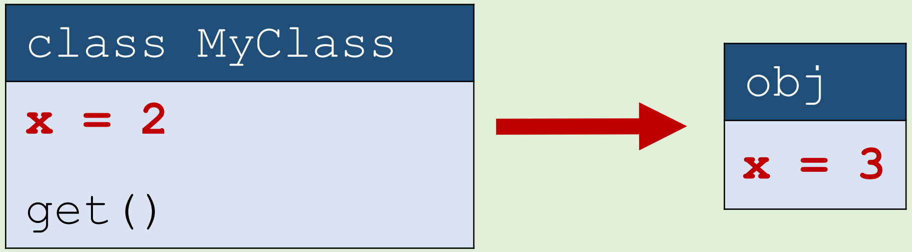
a) 4

b) 5

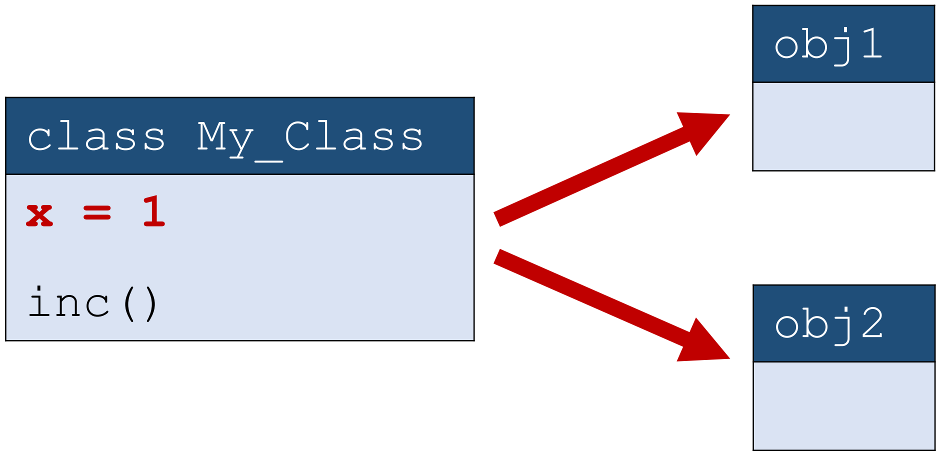
c) 6

d) `UnboundLocalError`

e) Don't know



Class data attribute example (in Python)



class_attributes.py

```
class My_Class:
    x = 1  # class attribute
    def inc(self):
        My_Class.x = self.x + 1
obj1 = My_Class()
obj2 = My_Class()
obj1.inc()
obj2.inc()
print(obj1.x, obj2.x)
```

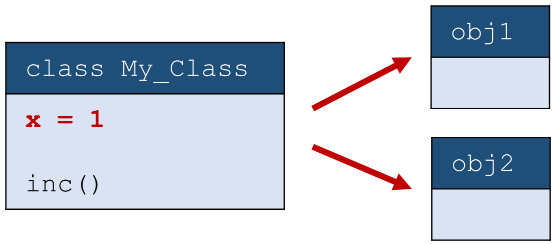
Python shell

```
| 3 3
```

- Note that `My_Class.x` and `self.x` refer to the same class attribute (since `self.x` has never been assigned a value)

Java static

- In Java *class attributes*, i.e. attribute values shared by all instances, are labeled **`static`**
- Python allows both class and instance attributes with the same name – in Java at most one of them can exist



static_attributes.java

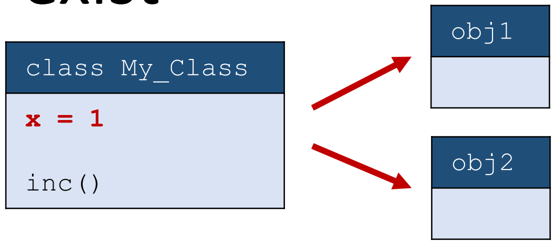
```java
class My_Class {
    public static int x = 1;
    public void inc() { x += 1; };
}

class static_attributes {
    public static void main(String args[]){
        My_Class obj1 = new My_Class();
        My_Class obj2 = new My_Class();
        obj1.inc();
        obj2.inc();
        System.out.println(obj1.x);
        System.out.println(obj2.x);
    }
}
```

Java output

```
| 3
| 3
```

C++ static

- In C++ *class attributes*, i.e. attribute values shared by all instances, are labeled **static**
- ISO C++ forbids in-class initialization of non-const static member
- Python allows both class and instance attributes with the same name – in C++ at most one of them can exist



static_attributes.cpp

```
#include <iostream>
using namespace std;
class My_Class {
public:
   static int x; // "= 1" is not allowed
   void inc() { x += 1; };
};
int My_Class::x = 1; // class initialization
int main(){
   My_Class obj1;
   My_Class obj2;
   obj1.inc();
   obj2.inc();
   cout << obj1.x << endl;
   cout << obj2.x << endl;
}
```

C++ output

```
3
3
```

Constants

- A simple usage of class data attributes is to store a set of constants (but there is nothing preventing anyone to chance these values)

Python shell

```
> class Color:
      RED   = "ff0000"
      GREEN = "00ff00"
      BLUE  = "0000ff"
> Color.RED
| 'ff0000'
```

PEP8 Style Guide for Python Code (some quotes)

- Class names should normally use the CapWords convention.
- Always use self for the first argument to instance methods.
- Use one leading underscore only for non-public methods and instance variables.
- For simple public data attributes, it is best to expose just the attribute name, without complicated accessor/mutator methods.
- Always decide whether a class's methods and instance variables (collectively: "attributes") should be public or non-public. If in doubt, choose non-public; it's easier to make it public later than to make a public attribute non-public.

www.python.org/dev/peps/pep-0008/

Some methods many class have

Method	Description
`__eq__(self, other)`	Used to test if two elements are equal Two elements where __eq__ is true must have equal __hash__
`__str__(self)`	Used by `print`
`__repr__(self)`	Used e.g. for printing to shell (usually something that is a valid Python expression for `eval()`)
`__len__(self)`	Length (integer) of object, e.g. lists, strings, tuples, sets, dictionaries
`__doc__(self)`	The docstring of the class
`__hash__(self)`	Returns hash value (integer) of object Dictionary keys and set values must have a __hash__ method
`__lt__(self, other)`	Comparison (less than, <) used by `sorted` and `sort()`
`__init__(self,...)`	Class initializer

Overloading operators: www.python-course.eu/python3_magic_methods.php

Class hierarcies

- inheritance
- method overriding
- super
- multiple inheritance

Calling methods of a class

- If an object *obj* of class *C* has a method `method`, then usually you call *obj.method()*
- It is possible to call the method in the class directly using *C.method*, where the object is the first argument

  C.method(obj)

X.py

```
class X:
    def set_x(self, x):
        self.x = x

    def get_x(self):
        return self.x

obj = X()

obj.set_x(42)

print("obj.get_x() =", obj.get_x())
print("obj.x =", obj.x)
print("X.get_x(obj) =", X.get_x(obj))
```

Python shell

```
| obj.get_x() = 42
| obj.x = 42
| X.get_x(obj) = 42
```

Classes and Objects

Observation: students and employees are persons with additional attributes

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

instance →

```
Person object
name = 'Mickey Mouse'
address = 'Mouse Street 42, Duckburg'
```

```
class Student
set_name(name)
get_name()

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

instance →

```
Student object
name = 'Donald Duck'
address = 'Duck Steet 13, Duckburg'
id = '1094'
grades = {'programming' : 'A' }
```

```
Employee object
name = 'Goofy'
address = 'Clumsy Road 7, Duckburg'
employer = 'Yarvard University'
```

Classes and Objects

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

```
class Student
set_name(name)
get_name()

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

person attributes

Goal – avoid redefining the 4 methods below from `person` class again in `student` class

person.py

```python
class Person:
    def set_name(self, name):
        self.name = name
    def get_name(self):
        return self.name
    def set_address(self, address):
        self.address = address
    def get_address(self):
        return self.address
```

Classes inheritance

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

```
class Student
set_name(name)
get_name()

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

person
attributes

class `Student` inherits from class `Person`
class `Person` is the base class of `Student`

person.py

```python
class Student(Person):
    def set_id(self, student_id):
        self.id = student_id
    def get_id(self):
        return self.id
    def set_grade(self, course, grade):
        self.grades[course] = grade
    def get_grades(self):
        return self.grades
```

Classes constructors

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

```
class Student
set_name(name)                 person
get_name()                 attributes

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

person.py

```python
class Person:
    def __init__(self):
        self.name = None
        self.address = None
    ...

class Student(Person):
    def __init__(self):
        self.id = None
        self.grades = {}
        Person.__init__(self)
    ...
```

constructor for `Person` class

constructor for `Student` class

Notes

1) If `Student.__init__` is not defined, then `Person.__init__` will be called
2) `Student.__init__` must call `Person.__init__` to initialize the `name` and `address` attributes

super()

class Person

```
set_name(name)
get_name()

set_address(address)
get_address()
```

class Student

person attributes

```
set_name(name)
get_name()

set_address(address)
get_address()
```

```
set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

person.py

```
class Person:
    def __init__(self):
        self.name = None
        self.address = None
    ...

class Student(Person):
    def __init__(self):
        self.id = None
        self.grades = {}
        Person.__init__(self)
        super().__init__()
    ...
```

alternative constructor

Notes

1) Function `super()` searches for attributes in base class
2) `super` is often a keyword in other OO languages, like Java and C++
3) Note `super().__init__()` does not need `self` as argument

Method search order

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

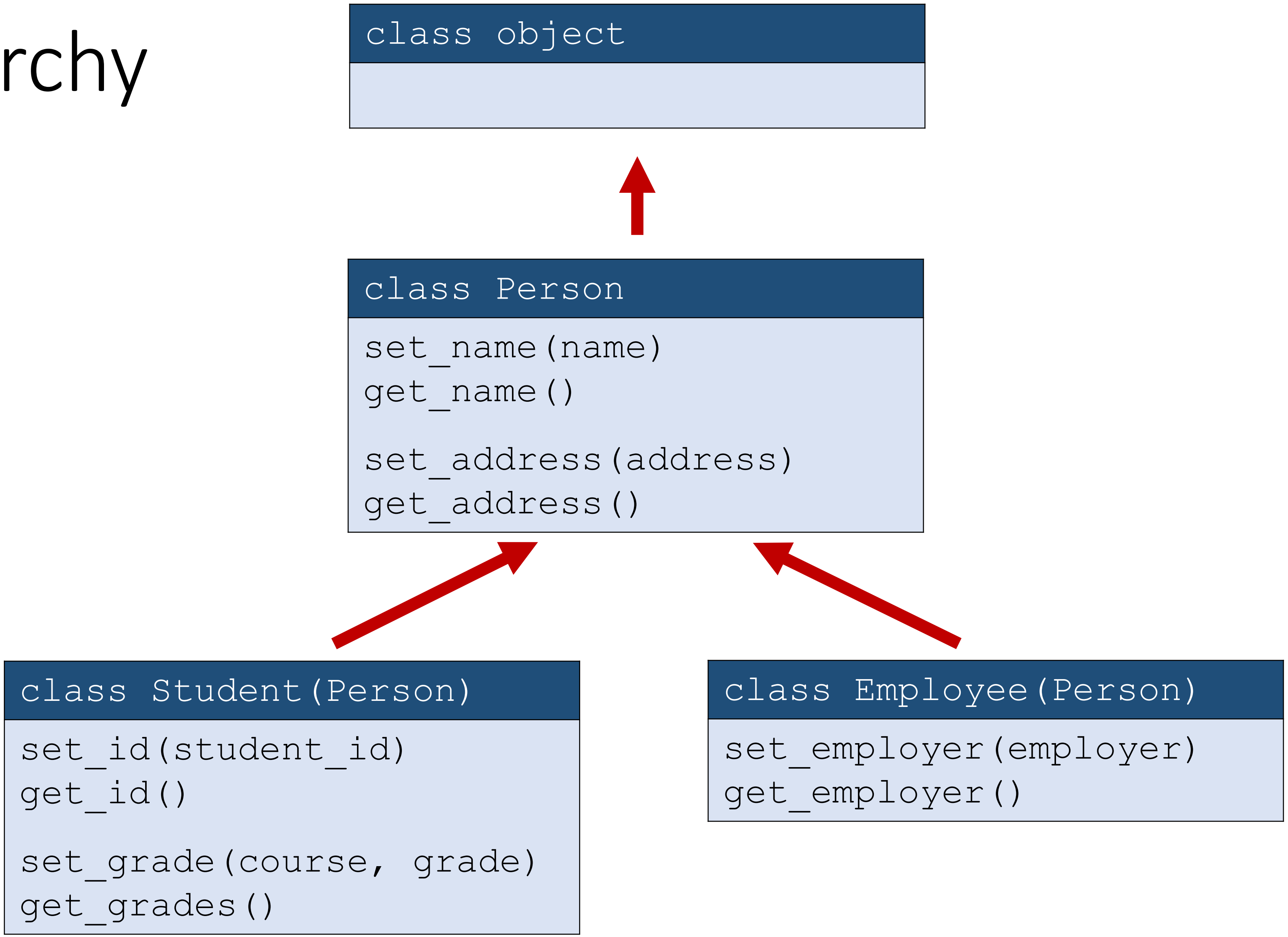
parent class

```
class Student(Person)
set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

instance of

```
Student object
name = 'Donald Duck'
address = 'Duck Steet 13, Duckburg'
id = '1094'
grades = {'programming' : 'A' }
```

Class hierarchy



Method overriding

```
overloading.py
class A:
    def say(self):
        print("A says hello")

class B(A):  # B is a subclass of A
    def say(self):
        print("B says hello")
        super().say()
```

```
Python shell
> B().say()
| B says hello
| A says hello
```

In Java one can use the keyword "`finally`" to prevent any subclass to override a method

Question – What does `b.f()` print ?

Python shell

```
> class A():
      def f(self):
          print("Af")
          self.g()
      def g(self):
          print("Ag")

> class B(A):
      def g(self):
          print("Bg")

> b = B()
> b.f()
| ?
```

a) `AttributeError`

b) `Af Ag`

c) `Af Bg`

d) Don’t know

Undefind methods in superclass ?

Python shell

```
> class A():
      def f(self):
          print("Af")
          self.g()

      def g(self):
          print("Ag")
> class B(A):
      def g(self):
          print("Bg")
> b = B()
> b.f()
| Af
| Bg
> a = A()
> a.f()
| Af
| Ag
```

Python shell

```
> class A():
      def f(self):
          print("Af")
          self.g()
> class B(A):
      def g(self):
          print("Bg")
> b = B()
> b.f()
| Af
| Bg
> a = A()
> a.f()
| Af
| AttributeError: 'A' object has no attribute 'g'
```

method `g` undefined in class `A`; subclasses must implement `g` to be able to call `f`

in Java, `A` would have been required to be declared an *abstract class*

can create instance of `A`

fails since `g` is not defined in class `A`

Name mangling and inheritance ⚠

Python shell

```
> class A():
      def f(self):
          print("Af")
          self.__g()
      def __g(self):
          print("Ag")
> class B(A):
      def __g(self):
          print("Bg")
> b = B()
> b.f()
| Af
| Ag
```

- The call to `A.__g` in `A.f` forces a call to `__g` to stay within `A`
- Recall that due to name mangling, `__g` is accessible as `A._A__g`

Multiple inheritance

- A class can inherit attributes from multiple classes (in example two)
- When calling a method defined in several ancestor classes, Python executes only one of the these (in the example `say_hello`)
- Which one is determined by the so called ”C3 Method Resolution Order” (originating from the Dylan language)

[Raymond Hettinger, *Super considered super!* Conference talk at PyCon 2015]

multiple_inheritance.py

```
class Alice:
    def say_hello(self):
        print("Alice says hello")
    def say_good_night(self):
        print("Alice says good night")
class Bob:
    def say_hello(self):
        print("Bob says hello")
    def say_good_morning(self):
        print("Bob says good morning")
class X(Alice, Bob):  # Multiple inheritance
    def say(self):
        self.say_good_morning()
        self.say_hello()          # C3 resolution
        Alice.say_hello(self)     # from Alice
        Bob.say_hello(self)       # from Bob
        self.say_good_night()
```

Python shell

```
> X().say()
| Bob says good morning
| Alice says hello
| Alice says hello
| Bob says hello
| Alice says good night
```

since `Alice` before `Bob` in list of super classes

C3 Method resolution order

- Use `help(class)` to determine the resolution order for the class
- or access the `__mro__` attribute of the class

Python shell

```
> X.__mro__
| (<class '__main__.X'>, <class '__main__.Alice'>,
  <class '__main__.Bob'>, <class 'object'>)
> help(X)
| Help on class X in module __main__:
| class X(Alice, Bob)
|  |  Method resolution order:
|  |      X
|  |      Alice
|  |      Bob
|  |      builtins.object
|  |  Methods defined here:
|  |  say(self)
|  |  ----------------------------------
|  |  Methods inherited from Alice:
|  |  say_good_night(self)
|  |  say_hello(self)
|  |  ----------------------------------
|  |  ...
|  |  ----------------------------------
|  |  Methods inherited from Bob:
|  |  say_good_morning(self)
```

Question – Who says hello ? Bob says good morning

inheritance.py

```
class Alice:
    def say_hello(self):
        print("Alice says hello")
class Bob:
    def say_hello(self):
        print("Bob says hello")
    def say_good_morning(self):
        self.say_hello()
        print("Bob says good morning")
class X(Alice, Bob):  # Multiple inheritance
    pass

X().say_good_morning()
```

a) Alice

b) Bob

c) Dont' know

Comparing objects and classes

- `id(obj)` returns a unique identifyer for an object (in CPython the memory address)
- `obj1 is obj2` tests if `id(obj1)==id(obj2)`
- `type(obj)` and `obj.__class__` return the class of an object
- `isinstance(object, class)` checks if an object is of a particular class, *or a derived subclass*
- `issubclass(class1, class2)` checks if `class1` is a subclass of `class2`

`is` is not for integers, strings, ... and `is` is not ==

Python shell

```
> 500 + 500 is 1000
| True
> x = 500
> x + x is 1000
| False
> x + x == 1000   # int.__eq__(...)
| True
> for x in range(0, 1000):
      if x - 1 + 1 is not x:
          print(x)
          break
| 257
> for x in range(0, -1000, -1):
      if x + 1 - 1 is not x:
          print(x)
          break
| -6
```

Python shell

```
> "abc" is "abc"
| True
> "abc" is "xabc"[1:]
| False
> x, y = "abc", "xabc"[1:]
> x, y
| ('abc', 'abc')
> x is y
| False
> x == y  # x.__eq__(y)
| True
```

- Only use `is` on objects !
- Even though `isinstance(42, object)` and `isinstance("abc", object)` are true, do not use `is` on integers and strings !

Comparison of OO in Python, Java and C++

- private, public, ... – in Python everything in an object is public
- class inheritance – core concept in OO programming
  - Python and C++ support multiple inheritance
  - Java only allows single inheritance, but Java ”interfaces” allow for something like multiple inheritance
- Python and C++ allows overloading standard operators (+, *, ...).
  In Java it is not possible.
- Overloading methods
  - Python extremely dynamic (hard to say anything about the behaviour of a program in general)
  - Java and C++’s type systems allow several methods with same name in a class, where they are distinguished by the type of the arguments, whereas Python allows only one method that can have * and ** arguments

C++ example

- Multiple methods with identical name (`print`)
- The types distinguish the different methods

printing.py

```python
class MyClass:
    def print(self, value):
        if isinstance(value, int):
            print('An integer', value)
        elif isinstance(value, str):
            print('A string', value)

C = MyClass()
C.print(42)
C.print("abc")
```

printing.cpp

```cpp
#include <iostream>
using namespace std;

class MyClass {
public:
   void print(int x) {
      cout << "An integer " << x << endl;
   };

   void print(string s) {
      cout << "A string " << s << endl;
   };
};

main() {
   MyClass C;

   C.print(42);
   C.print("abc");
}
```

Shell

```
| An integer 42
| A string abc
```

Exceptions and file input/output

- try-raise-except-finally
- Exception
- control flow
- file open/read/write
- sys.stdin, sys.stdout, sys.stderr

Exceptions – Error handling and control flow

- Exceptions is a widespread technique to handle run-time **errors** / abnormal behaviour (e.g. in Python, Java, C++, JavaScript, C#)

- Exceptions can also be used as an **advanced control flow mechanism** (e.g. in Python, Java, JavaScript)

  – *Problem: How to perform a ”break” in a recursive function ?*

Built-in exceptions (class hierarchy)

```
BaseException
 +-- SystemExit
 +-- KeyboardInterrupt
 +-- GeneratorExit
 +-- Exception
      +-- StopIteration
      +-- StopAsyncIteration
      +-- ArithmeticError
      |    +-- FloatingPointError
      |    +-- OverflowError
      |    +-- ZeroDivisionError
      +-- AssertionError
      +-- AttributeError
      +-- BufferError
      +-- EOFError
      +-- ImportError
      |    +-- ModuleNotFoundError
      +-- LookupError
      |    +-- IndexError
      |    +-- KeyError
      +-- MemoryError
      +-- NameError
      |    +-- UnboundLocalError
      +-- TypeError
      +-- ValueError
      |    +-- UnicodeError
      |         +-- UnicodeDecodeError
      |         +-- UnicodeEncodeError
      |         +-- UnicodeTranslateError
      |
      +-- OSError
      |    +-- BlockingIOError
      |    +-- ChildProcessError
      |    +-- ConnectionError
      |    |    +-- BrokenPipeError
      |    |    +-- ConnectionAbortedError
      |    |    +-- ConnectionRefusedError
      |    |    +-- ConnectionResetError
      |    +-- FileExistsError
      |    +-- FileNotFoundError
      |    +-- InterruptedError
      |    +-- IsADirectoryError
      |    +-- NotADirectoryError
      |    +-- PermissionError
      |    +-- ProcessLookupError
      |    +-- TimeoutError
      +-- ReferenceError
      +-- RuntimeError
      |    +-- NotImplementedError
      |    +-- RecursionError
      +-- SyntaxError
      |    +-- IndentationError
      |         +-- TabError
      +-- SystemError
      +-- Warning
           +-- DeprecationWarning
           +-- PendingDeprecationWarning
           +-- RuntimeWarning
           +-- SyntaxWarning
           +-- UserWarning
           +-- FutureWarning
           +-- ImportWarning
           +-- UnicodeWarning
           +-- BytesWarning
           +-- ResourceWarning
```

docs.python.org/3/library/exceptions.html

Typical built-in exceptions

and unhandled behaviour

Python shell

```
> 7 / 0
| ZeroDivisionError: division by zero
> int('42x')
| ValueError: invalid literal for int() with base 10: '42x'
> x = y
| NameError: name 'y' is not defined
> L = list(range(1000000000))
| MemoryError
> 2.5 ** 1000
| OverflowError: (34, 'Result too large')
> t = (3, 4)
> t[0] = 7
| TypeError: 'tuple' object does not support item assignment
> t[3]
| IndexError: tuple index out of range
> t.x
| AttributeError: 'tuple' object has no attribute 'x'
> x = {}
> x['foo']
| KeyError: 'foo'
> def f(x): f(x + 1)
> f(0)
| RecursionError: maximum recursion depth exceeded
> def f(): x = x + 1
> f()
| UnboundLocalError: local variable 'x' referenced before assignment
```

Catching exceptions – Fractions (I)

fraction1.py

```python
while True:
    numerator = int(input('Numerator = '))
    denominator = int(input('Denominator = '))
    result = numerator / denominator
    print('%s / %s = %s' % (numerator, denominator, result))
```

Python shell

```
| Numerator = 10
| Denominator = 3
| 10 / 3 = 3.3333333333333335
| Numerator = 20
| Denominator = 0
| ZeroDivisionError: division by zero
```

Catching exceptions – Fractions (II)

fraction2.py

```
while True:
    numerator = int(input('Numerator = '))
    denominator = int(input('Denominator = '))
    try:
        result = numerator / denominator
    except ZeroDivisionError:
        print('cannot divide by zero')
        continue
    print('%s / %s = %s' % (numerator, denominator, result))
```

catch exception

Python shell

```
| Numerator = 10
| Denominator = 0
| cannot divide by zero
| Numerator = 20
| Denominator = 3
| 20 / 3 = 6.666666666666667
| Numerator = 42x
| ValueError: invalid literal for int() with base 10: '42x'
```

Catching exceptions – Fractions (III)

fraction3.py

```
while True:
    try:
        numerator = int(input('Numerator = '))
        denominator = int(input('Denominator = '))
    except ValueError:
        print('input not a valid integer')
        continue
    try:
        result = numerator / denominator
    except ZeroDivisionError:
        print('cannot divide by zero')
        continue
    print('%s / %s = %s' % (numerator, denominator, result))
```

catch exception

catch exception

Python shell

```
| Numerator = 5
| Denominator = 2x
| input not a valid integer
| Numerator = 5
| Denominator = 2
| 5 / 2 = 2.5
```

fraction3.py

```python
while True:
    try:
        numerator = int(input('Numerator = '))
        denominator = int(input('Denominator = '))
    except ValueError:
        print('input not a valid integer')
        continue
    try:
        result = numerator / denominator
        print('%s / %s = %s' % (numerator, denominator, result))
    except ZeroDivisionError:
        print('cannot divide by zero')
```

Python shell

```
Numerator = 1000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000
Denominator = 1
OverflowError: integer division result too large for a float
```

exception not caught

Catching exceptions – Fractions (IV)

catch exceptions

fraction4.py

```
while True:
    try:
        numerator = int(input('Numerator = '))
        denominator = int(input('Denominator = '))
        result = numerator / denominator
        print('%s / %s = %s' % (numerator, denominator, result))
    except ValueError:
        print('input not a valid integer')
    except ZeroDivisionError:
        print('cannot divide by zero')
```

Python shell

```
| Numerator = 3
| Denominator = 0
| cannot divide by zero
| Numerator = 3x
| input not a valid integer
| Numerator = 4
| Denominator = 2
| 4 / 2 = 2.0
```

Keyboard interrupt (Ctrl-c)

- throws **KeyboardInterrupt** exception

infinite-loop1.py

```
print('starting infinite loop')

x = 0
while True:
    x = x + 1

print('done (x = %s)' % x)
input('type enter to exit')
```

Python shell

```
| starting infinite loop
| Traceback (most recent call last):
|   File 'infinite-loop1.py', line 4, in <module>
|     x = x + 1
| KeyboardInterrupt
```

infinite-loop2.py

```
print('starting infinite loop')

try:
    x = 0
    while True:
        x = x + 1
except KeyboardInterrupt:
    pass

print('done (x = %s)' % x)
input('type enter to exit')
```

Python shell

```
| starting infinite loop
| done (x = 23890363)   # Ctrl-c
| type enter to exit
```

Keyboard interrupt (Ctrl-c)

- Be aware that you likely would like to leave the Ctrl-c generated **`KeyboardInterrupt`** exception unhandled, except when stated explicitly

read-int1.py

```
while True:
    try:
        x = int(input('An integer: '))
        break
    except ValueError:   # only ValueError
        continue

print('The value is:', x)
```

Python shell

```
| An integer:          Ctrl-c
| KeyboardInterrupt
```

read-int2.py

```
while True:
    try:
        x = int(input('An integer: '))
        break
    except:   # all exceptions
        continue

print('The value is:', x)
```

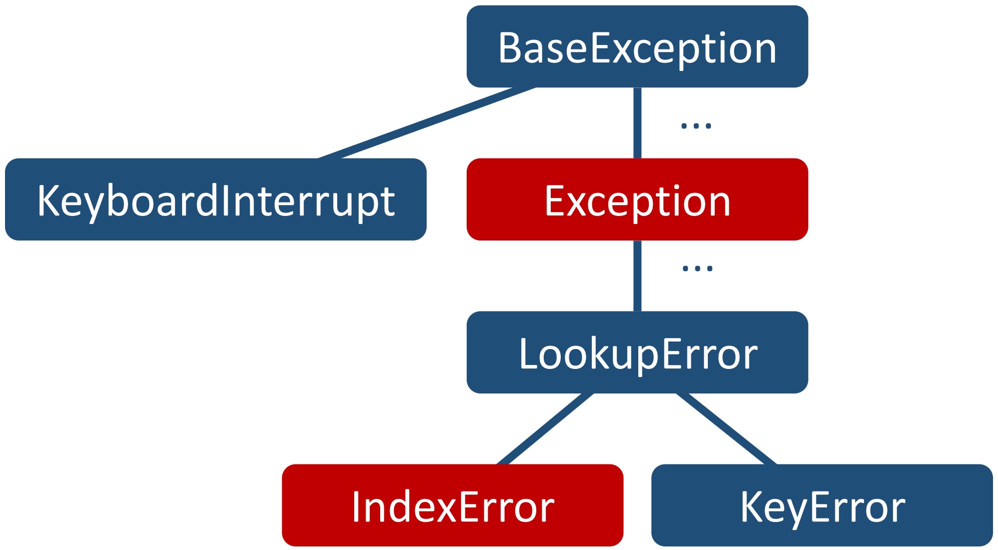
catches KeyboardInterrupt

Python shell

```
| An integer:          Ctrl-c
| An integer:          Ctrl-c
| An integer:
```

- (left) `KeyboardInterrupt` is unhandled (right) it is handled (intentionally?)

Exception class hierarchy



except-twice1.py

```
try:
    L[4]
except IndexError:   # must be before Exception
    print('IndexError')
except Exception:
    print('Fall back exception handler')
```

except-twice2.py

```
try:
    L[4]
except Exception:   # and subclasses of Exception
    print('Fall back exception handler')
except IndexError:
    print('IndexError')   # unreachable
```

`try` statement syntax

arbitrary number of except cases

```
try:
    code
except ExceptionType1:
    code  # executed if raised exception instanceof
          # ExceptionType1 (or subclass of ExceptionType1)
except ExceptionType2:
    code  # executed if exception type matches and none of
          # the previous except statements matched
...
else:
    code  # only executed if no exception was raised
finally:
    code  # always executed independent of exceptions
          # typically used to clean up (like closing files)
```

except variations

```
except:                     # catch all exceptions

except ExceptionType:       # only catch exceptions of class ExceptionType
                            # or subclasses of ExceptionType

except (ExceptionType_1, ExceptionType_2, ..., ExceptionType_k):
                            # catch any of k classes (and subclasses)

except ExceptionType as e:
                            # catch exception and assign exception object to e
                            # e.args contains arguments to the raised exception
```

Raising exceptions

- An exception is raised (or trown) using one of the following (the first being an alias for the second):

```
raise ExceptionType
raise ExceptionType()
raise ExceptionType(args)
```

abstract.py

```python
class A():
    def f(self):
        print('f')
        self.g()
    def g(self):
        raise NotImplementedError
class B(A):
    def g(self):
        print('g')
```

Python shell

```
> B().f()
| f
| g
> A().f()
| f
| NotImplementedError
```

User exceptions

- New exception types are created using class inheritance from an existing exception type (possibly defining __init__)

tree-search.py

```
class SolutionFound(Exception):   # new exception
    pass

def recursive_tree_search(x, t):
    if isinstance(t, tuple):
        for child in t:
            recursive_tree_search(x, child)
    elif x == t:
        raise SolutionFound   # found x in t

def tree_search(x, t):
    try:
        recursive_tree_search(x, t)
    except SolutionFound:
        print('found', x)
    else:
        print('search for', x, 'unsuccessful')
```

Python shell

```
> tree_search(8, ((3,2),5,(7,(4,6))))
| search for 8 unsuccessful
> tree_search(3, ((3,2),5,(7,(4,6))))
| found 3
```

User exception with argument in recursion

- Escape from recursion by raising exception
- Pass result as argument to exception
- Unpack `.args` tuple from result caught by `except`

apx-tree-search.py

```
class SolutionFound(Exception):
    pass

def apx_tree_search(x, tree):
    def search(x, t):
        if isinstance(t, tuple):
            for child in t:
                search(x, child)
        elif abs(x - t) < 1:  # approximate match
            raise SolutionFound(t)

    try:
        search(x, tree)
    except SolutionFound as e:
        result, = e.args  # e.args is a tuple
        print('search for', x, 'found', result)
    else:
        print('search for', x, 'unsuccessful')

tree = ((3.2, 2.1), 5.6, (7.8, (9.3, 6.5)))
apx_tree_search(4.3, tree)
apx_tree_search(5.9, tree)
```

Python shell

```
| search for 4.3 unsuccessful
| search for 5.9 found 5.6
```

3 ways to read lines from a file

Steps

1. Open file using `open`
2. Read lines from file using
   a) *`filehandler`*`.readline`
   b) *`filehandler`*`.readlines`
   c) `for` *`line`* `in` *`filehandler`*`:`
3. Close file using `close`

`open ('filename.txt')` assumes the file to be in the same folder as your Python program, but you can also provide a full path `open('c:/Users/gerth/Documents/filename.txt')`

try to open file for reading

filename

filehandle

reading-file1.py

```python
f = open('reading-file1.py')
for line in f:
    print('>', line[:-1])
f.close()
```

iterate over lines in file

close file when done

reading-file2.py

```python
f = open('reading-file2.py')
lines = f.readlines()
f.close()
for line in lines:
    print('>', line[:-1])
```

read all lines into a list of strings

reading-file3.py

```python
f = open('reading-file3.py')
line = f.readline()
while line != '':
    print('>', line[:-1])
    line = f.readline()
f.close()
```

read single line (terminated by '\n')

3 ways to write lines to a file

- Opening file:
  `open(filename, mode)`
  where *mode* is a string, either `'w'` for opening a new (or truncating an existing file) and `'a'` for appending to an existing file
- Write single string:
  `filehandle.write(string)`
  Returns the number of characters written
- Write list of strings strings:
  `filehandle.writeline(list)`
- Newlines (`'\n'`) must be written explicitly
- `print` can take an optional `file` argument

try to open file for writing

write mode

write single string to file

write list of strings to file

append to existing file

write-file.py

```python
f = open('output-file.txt', 'w')
f.write('Text 1\n')
f.writelines(['Text 2\n', 'Text 3 '])
f.close()

g = open('output-file.txt', 'a')
print('Text 4', file=g)
g.writelines(['Text 5 ', 'Text 6'])
g.close()
```

output-file.txt

```
Text 1
Text 2
Text 3 Text 4
Text 5 Text 6
```

Exceptions while dealing with files

- When dealing with files one should be prepared to handle errors / raised exceptions, e.g. `FileNotFoundError`

reading-file4.py

```
try:
    f = open('reading-file4.py')
except FileNotFoundError:
    print('Could not open file')
else:
    try:
        for line in f:
            print('> ', line[:-1])
    finally:
        f.close()
```

Opening files using `with`

- The Python keyword **`with`** allows to create a *context manager* for handling files
- *Filehandle will automatically be closed, also when exceptions occur*
- Under the hood: filehandles returned by `open()` support `__enter__()` and `__exit__()` methods

f = result of calling `__enter__()` on result of `open` expression, which is the file handle

reading-file5.py

```
with open('reading-file5.py') as f:
    for line in f:
        print('> ', line[:-1])
```

Does a file exist?

- Module `os.path` contains a method `isfile` to check if a file exists

checking-files.py

```
import os.path

filename = input('Filename: ')
if os.path.isfile(filename):
    print('file exists')
else:
    print('file does not exists')
```

docs.python.org/3/library/os.path.html

module `sys`

- Module `sys` contains the three standard file handles

  `sys.stdin` (used by the `input` function)

  `sys.stdout` (used by the `print` function)

  `sys.stderr` (error output from the Python interpreter)

sys-test.py

```
import sys
sys.stdout.write('Input an integer: ')
x = int(sys.stdin.readline())
sys.stdout.write('%s square is %s' % (x, x**2))
```

Python shell

```
| Input an integer: 10
| 10 square is 100
```

docs.python.org/3/library/sys.html

print(..., file=*output file*)

sys-print-file.py

```
import sys

def complicated_function(file):
    print('Hello world', file=file)  # print to file or STDOUT

while True:
    file_name = input('Output file (empty for STDOUT): ')

    if file_name == '':
        file = sys.stdout
        break
    else:
        try:
            file = open(file_name, 'w')
            break
        except Exception:
            pass

complicated_function(file)

if file != sys.stdout:
    file.close()
```

PEP8 on exceptions

- For all try/except clauses, limit the try clause to the absolute minimum amount of code necessary.
- The class naming convention applies (CapWords)
- Use the suffix "Error" on your exception names (if the exception actually is an error)
- A bare **`except:`** clause will catch **`SystemExit`** and **`KeyboardInterrupt`** exceptions, making it harder to interrupt a program with Control-C, and can disguise other problems. If you want to catch all exceptions that signal program errors, use **`except Exception:`**

www.python.org/dev/peps/pep-0008/

Performance of scanning a file

- Python can efficiently scan through quite big files

File	Size	Time
Atom_chem_shift.csv	≈ 750 MB	≈ 8 sec
cano.txt	≈ 3.7 MB	≈ 0.1 sec

The first search finds all lines related to ThrB12-DKP-insulin (Entry ID 6203) in a chemical database available from www.bmrb.wisc.edu

The second search finds all occurrences of “Germany” in Conan Doyle's complete Sherlock Holmes available at sherlock-holm.es

file-scanning.py

```python
from time import time

for filename, query in [
        ('Atom_chem_shift.csv', ',6203,'),
        ('cano.txt', 'Germany')
    ]:
    count = 0
    matches = []
    start = time()
    with open(filename) as f:
        for i, line in enumerate(f, start=1):
            count += 1
            if query in line:
                matches.append((i, line))
    end = time()

    for i, line in matches:
        print(i, ':', line, end='')
    print('Duration:', end - start)
    print(len(matches), 'of', count, 'lines match')
```

Python shell

```
...
3057752 : 195,,2,2,30,30,THR,HB,H,1,4.22,0.02,,1,,,,,,,,,,,,,,,,,,,228896,6203,2
3057753 : 196,,2,2,30,30,THR,HG21,H,1,1.18,0.02,,1,,,,,,,,,,,,,,,,,,,228896,6203,2
3057754 : 197,,2,2,30,30,THR,HG22,H,1,1.18,0.02,,1,,,,,,,,,,,,,,,,,,,228896,6203,2
3057755 : 198,,2,2,30,30,THR,HG23,H,1,1.18,0.02,,1,,,,,,,,,,,,,,,,,,,228896,6203,2
Duration: 7.760039329528809
329 of 9758361 lines match
57557 :      "Well, then, to the West, or to England, or to Germany, where father
66515 :      kind master. He wanted me to go with his wife to Germany yesterday,
66642 :      of business in Germany in the past and my name is probably familiar
73273 :      associates with Germany. This he placed in his instrument cupboard.
Duration: 0.07700657844543457
4 of 76764 lines match
```

sudoku.py

```python
class Sudoku:
  def __init__(self, puzzle):
    self.puzzle = puzzle

  def solve(self):
    def find_free():
      for i in range(9):
        for j in range(9):
          if self.puzzle[i][j] == 0:
            return (i, j)
      return None

    def unused(i, j):
      i_, j_ = i // 3 * 3, j // 3 * 3
      cells = {(i, k) for k in range(9)}
      cells |= {(k, j) for k in range(9)}
      cells |= {(i, j) for i in range(i_, i_ + 3)
                       for j in range(j_, j_ + 3)}
      return set(range(1, 10)) - {self.puzzle[i][j] for i, j in cells}

    class SolutionFound(Exception):
      pass

    def recursive_solve():
      cell = find_free()
      if not cell:
        raise SolutionFound
      i, j = cell
      for value in unused(i, j):
        self.puzzle[i][j] = value
        recursive_solve()
      self.puzzle[i][j] = 0

    try:
      recursive_solve()
    except SolutionFound:
      pass
```

sudoku.py (*continued*)

```python
  def print(self):
    for i, row in enumerate(self.puzzle):
      cells = [' %s ' % c if c else ' . ' for c in row]
      print('|'.join([''.join(cells[j:j+3]) for j in (0,3,6)]))
      if i in (2,5):
        print('---------+---------+---------')

with open('sudoku.txt') as f:
    A = Sudoku([[int(x) for x in line.strip()] for line in f])

A.solve()
A.print()
```

sudoku.txt

```
517600034
289004000
346205090
602000010
038006047
000000000
090000078
703400560
000000000
```

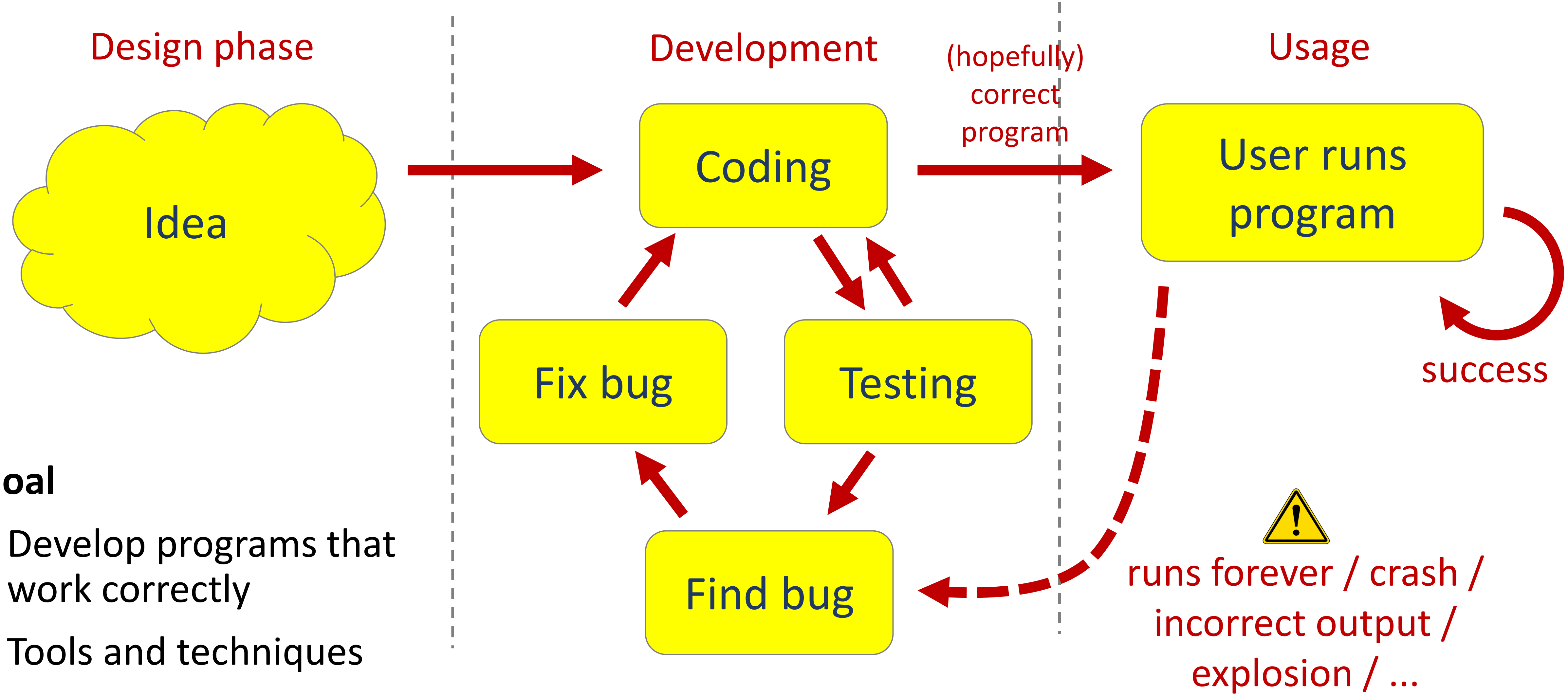
Python shell

```
|  5  1  7 | 6  9  8 | 2  3  4
|  2  8  9 | 1  3  4 | 7  5  6
|  3  4  6 | 2  7  5 | 8  9  1
|  ---------+---------+---------
|  6  7  2 | 8  4  9 | 3  1  5
|  1  3  8 | 5  2  6 | 9  4  7
|  9  5  4 | 7  1  3 | 6  8  2
|  ---------+---------+---------
|  4  9  5 | 3  6  2 | 1  7  8
|  7  2  3 | 4  8  1 | 5  6  9
|  8  6  1 | 9  5  7 | 4  2  3
```

Documentation, testing and debugging

- docstring
- defensive programming
- assert
- test driven developement
- assertions
- testing
- unittest
- debugger
- static type checking (mypy)

Ensuring good quality code ?



Goal

- Develop programs that work correctly
- Tools and techniques

What is good code ?

- Readability
  - well-structured
  - documentation
  - comments
  - follow some standard structure (easy to recognize, follow PEP8 Style Guide)

- Correctness
  - outputs the correct answer on valid input
  - eventually stops with an answer on valid input (should not go in infinite loop)

- Reusable...

Why ?

Documentation

- *specification of functionality*
- docstring
  - *for users of the code*
  - modules
  - methods
  - classes
- comments
  - *for readers of the code*

Testing

- Correct implementation ?
- Try to predict behavior on unknown input ?
- Performance guarantees ?

Debugging

- *Where is the #!¤$ bug ?*

”Program testing can be used to show the presence of bugs, but never to show their absence” – Edsger W. Dijkstra

Built-in exceptions (class hierarchy)

```
BaseException
 +-- SystemExit
 +-- KeyboardInterrupt
 +-- GeneratorExit
 +-- Exception
      +-- StopIteration
      +-- StopAsyncIteration
      +-- ArithmeticError
      |    +-- FloatingPointError
      |    +-- OverflowError
      |    +-- ZeroDivisionError
      +-- AssertionError
      +-- AttributeError
      +-- BufferError
      +-- EOFError
      +-- ImportError
      |    +-- ModuleNotFoundError
      +-- LookupError
      |    +-- IndexError
      |    +-- KeyError
      +-- MemoryError
      +-- NameError
      |    +-- UnboundLocalError
      +-- TypeError
      +-- ValueError
      |    +-- UnicodeError
      |         +-- UnicodeDecodeError
      |         +-- UnicodeEncodeError
      |         +-- UnicodeTranslateError
      |
      +-- OSError
      |    +-- BlockingIOError
      |    +-- ChildProcessError
      |    +-- ConnectionError
      |    |    +-- BrokenPipeError
      |    |    +-- ConnectionAbortedError
      |    |    +-- ConnectionRefusedError
      |    |    +-- ConnectionResetError
      |    +-- FileExistsError
      |    +-- FileNotFoundError
      |    +-- InterruptedError
      |    +-- IsADirectoryError
      |    +-- NotADirectoryError
      |    +-- PermissionError
      |    +-- ProcessLookupError
      |    +-- TimeoutError
      +-- ReferenceError
      +-- RuntimeError
      |    +-- NotImplementedError
      |    +-- RecursionError
      +-- SyntaxError
      |    +-- IndentationError
      |         +-- TabError
      +-- SystemError
      +-- Warning
           +-- DeprecationWarning
           +-- PendingDeprecationWarning
           +-- RuntimeWarning
           +-- SyntaxWarning
           +-- UserWarning
           +-- FutureWarning
           +-- ImportWarning
           +-- UnicodeWarning
           +-- BytesWarning
           +-- ResourceWarning
```

docs.python.org/3/library/exceptions.html

Testing for unexpected behaviour ?

infinite-recursion1.py

```python
def f(depth):
    f(depth + 1)   # infinite recursion ⚠

f(0)
```

Python shell

```
| RecursionError: maximum recursion depth exceeded
```

infinite-recursion2.py

```python
def f(depth):
    if depth > 100:
        print("runaway recursion???")
        raise SystemExit   # raise built-in exception
    f(depth + 1)

f(0)
```

Python shell

```
| runaway recursion???
```

infinite-recursion3.py

```python
import sys

def f(depth):
    if depth > 100:
        print("runaway recursion???")
        sys.exit()   # system function
    f(depth + 1)      raises SystemExit

f(0)
```

Python shell

```
| runaway recursion???
```

- let the program eventually fail
- check and raise exceptions
- check and call `sys.exit`

Catching unexpected behaviour – `assert`

infinite-recursion4.py

```
def f(depth):
    assert depth <= 100  # raise exception if False
    f(depth + 1)

f(0)
```

Python shell

```
| File "...\infinite-recursion4.py", line 2, in f
|     assert depth <= 100
| AssertionError
```

- keyword **`assert`** checks if boolean expression is true, if not, raises exception AssertionError
- optional second parameter passed to the constructor of the exception

infinite-recursion5.py

```
def f(depth):
    assert depth <= 100, "runaway recursion???"
    f(depth + 1)

f(0)
```

Python shell

```
| File "...\infinite-recursion5.py", line 2, in f
|     assert depth <= 100, "runaway recursion???"
| AssertionError: runaway recursion???
```

infinite-recursion6.py

```
def f(depth):
    if not depth <= 100:
        raise AssertionError("runaway recursion???")
    f(depth + 1)

f(0)
```

Python shell

```
| File "...\infinite-recursion6.py", line 3, in f
|     raise AssertionError("runaway recursion???")
| AssertionError: runaway recursion???
```

Disabling `assert` statements

Command Prompt

```
C:\Users\au121\Desktop>python -O infinite-recursion5.py
Traceback (most recent call last):
  File "infinite-recursion5.py", line 5, in <module>
    f(0)
  File "infinite-recursion5.py", line 3, in f
    f(depth + 1)
  File "infinite-recursion5.py", line 3, in f
    f(depth + 1)
  File "infinite-recursion5.py", line 3, in f
    f(depth + 1)
  [Previous line repeated 995 more times]
RecursionError: maximum recursion depth exceeded

C:\Users\au121\Desktop>
```

- **`assert`** statements are good to help check correctness of program – but can slow down program
- invoking Python with option `-O` disables all assertions (by setting `__debug__` to `False`)

docs.python.org/3/reference/simple_stmts.html#assert

Example

$$\left\lfloor \sqrt{x} \right\rfloor$$

First try... (seriously, the bugs were not on purpose)

intsqrt_buggy.py

```python
def int_sqrt(x):
    low = 0
    high = x
    while low < high - 1:
        mid = (low + high) / 2
        if mid ** 2 <= x:
            low = mid
        else:
            high = mid
    return low
```

Python shell

```
> int_sqrt(10)
| 3.125  # 3.125 ** 2 = 9.765625
> int_sqrt(-10)
| 0  # what should the answer be ?
```

Let us add a specification...

intsqrt.py

```
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns integer floor(sqrt(x))"""

    ...
```

docstring

input requirements

output guarantees

Python shell

```
> help(int_sqrt)
| Help on function int_sqrt in module __main__:
|
| int_sqrt(x)
|     Compute the integer square root of an integer x.
|     Assumes that x is an integer and x >= 0
|     Returns integer floor(sqrt(x))
```

- all methods, classes, and modules can have a **`docstring`** (ideally have) as a **specification**
- for methods: summarize purpose in first line, followed by input requirements and ouput guarantees
- the docstring is assigned to the object's `__doc__` attribute

PEP 257 -- Docstring Conventions
www.python.org/dev/peps/pep-0257/

Let us check input requirements...

intsqrt.py

```
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns integer floor(sqrt(x))"""

    assert isinstance(x, int)
    assert 0 <= x
    ...
```

check input requirements

Python shell

```
> int_sqrt(-10)
| File "...\int_sqrt.py", line 7, in int_sqrt
|     assert 0 <= x
| AssertionError
```

- doing explicit checks for valid input arguments is part of **defensive programming** and helps spotting errors early

  (instead of continuing using likely wrong values... resulting in a final meaningless error)

Let us check if output correct...

intsqrt.py

```
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns integer floor(sqrt(x))"""

    assert isinstance(x, int)
    assert 0 <= x
    ...
    assert isinstance(result, int)
    assert result ** 2 <= x < (result + 1) ** 2
    return result
```

check output

Python shell

```
> int_sqrt(10)
|   File "...\int_sqrt.py", line 20, in int_sqrt
|     assert isinstance(result, int)
| AssertionError
```

- output check identifies the error

  `mid = (low+high) / 2`

- should have been

  `mid = (low+high) // 2`

- The output check helps us to ensure that function specifications are satisfied in applications

Let us test some input values...

intsqrt.py

```
def int_sqrt(x):
    ...

assert int_sqrt(0) == 0
assert int_sqrt(1) == 1
assert int_sqrt(2) == 1
assert int_sqrt(3) == 1
assert int_sqrt(4) == 2
assert int_sqrt(5) == 2
assert int_sqrt(200) == 14
```

Python shell

```
| Traceback (most recent call last):
|   File "...\int_sqrt.py", line 28, in <module>
|     assert int_sqrt(1) == 1
|   File "...\int_sqrt.py", line 21, in int_sqrt
|     assert result ** 2 <= x < (result + 1) ** 2
| AssertionError
```

- test identifies
  wrong output for x = 1

Let us check progress of algorithm...

intsqrt.py

```
...
low, high = 0, x
while low < high - 1:   # low <= floor(sqrt(x)) < high
    assert low ** 2 <= x < high ** 2     } check invariant
    mid = (low + high) // 2              } for loop
    if mid ** 2 <= x:
        low = mid
    else:
        high = mid
result = low
...
```

Python shell

```
| Traceback (most recent call last):
|   File "...\int_sqrt.py", line 28, in <module>
|     assert int_sqrt(1) == 1
|   File "...\int_sqrt.py", line 21, in int_sqrt
|     assert result ** 2 <= x < (result + 1) ** 2
| AssertionError
```

- test identifies
  wrong output for x = 1
- but invariant apparently correct ???
- problem

  `low == result == 0`
  `high == 1`

  implies loop never entered
- output check identifies the error

  `high = x`
- should have been

  `high = x + 1`

Final program

We have used assertions to:

- Test if input arguments / usage is valid (defensive programming)
- Test if computed result is correct
- Test if an internal invariant in the computation is satisfied
- Perform a final test for a set of test cases (should be run whenever we change anything in the implementation)

intsqrt.py

```python
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns the integer floor(sqrt(x))"""

    assert isinstance(x, int)
    assert 0 <= x

    low, high = 0, x + 1
    while low < high - 1:   # low <= floor(sqrt(x)) < high
        assert low ** 2 <= x < high ** 2
        mid = (low + high) // 2
        if mid ** 2 <= x:
            low = mid
        else:
            high = mid
    result = low

    assert isinstance(result, int)
    assert result ** 2 <= x < (result + 1) ** 2

    return result

assert int_sqrt(0) == 0
assert int_sqrt(1) == 1
assert int_sqrt(2) == 1
assert int_sqrt(3) == 1
assert int_sqrt(4) == 2
assert int_sqrt(5) == 2
assert int_sqrt(200) == 14
```

Which checks would you add to the below code?

binary-search.py

```
def binary_search(x, L):
    """Binary search for x in sorted list L

    Assumes x is an integer, and L a non-decreasing list of integers

    Returns index i, -1 <= i < len(L), where L[i] <= x < L[i+1],
    assuming L[-1] = -infty and L[len(L)] = +infty"""

    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    result = low

    return result
```

binary-search-assertions.py

```python
def binary_search(x, L):
    """Binary search for x in sorted list L

    Assumes x is an integer, and L a non-decreasing list of integers

    Returns index i, -1 <= i < len(L), where L[i] <= x < L[i+1],
    assuming L[-1] = -infty and L[len(L)] = +infty"""
    assert isinstance(x, int)
    assert isinstance(L, list)
    assert all([isinstance(e, int) for e in L])
    assert all([L[i] <= L[i + 1] for i in range(len(L) - 1)])
    low, high = -1, len(L)
    while low + 1 < high:  # L[low] <= x < L[high]
        assert (low == -1 or L[low] <= x) and (high == len(L) or x < L[high])
        mid = (low + high) // 2
        assert isinstance(L[mid], int)
        assert (low == -1 or L[low] <= L[mid]) and (high == len(L) or L[mid] <= L[high])
        if x < L[mid]:
            high = mid
        else:
            low = mid
    result = low
    assert (isinstance(result, int) and -1 <= result < len(L) and
            ((result == -1 and (len(L) == 0 or x < L[0])) or
             (result == len(L) - 1 and x >= L[-1]) or
             (0 <= result < len(L) - 1 and L[result] <= x < L[result + 1])))
    return result

assert binary_search(42, []) == -1
assert binary_search(42, [7]) == 0
assert binary_search(7, [42]) == -1
assert binary_search(7, [42,42,42]) == -1
assert binary_search(42, [7,7,7]) == 2
assert binary_search(42, [7,7,7,56,81]) == 2
assert binary_search(8, [1,3,5,7,9]) == 3
```

input

loop

input

output

① inefficient

②

test cases

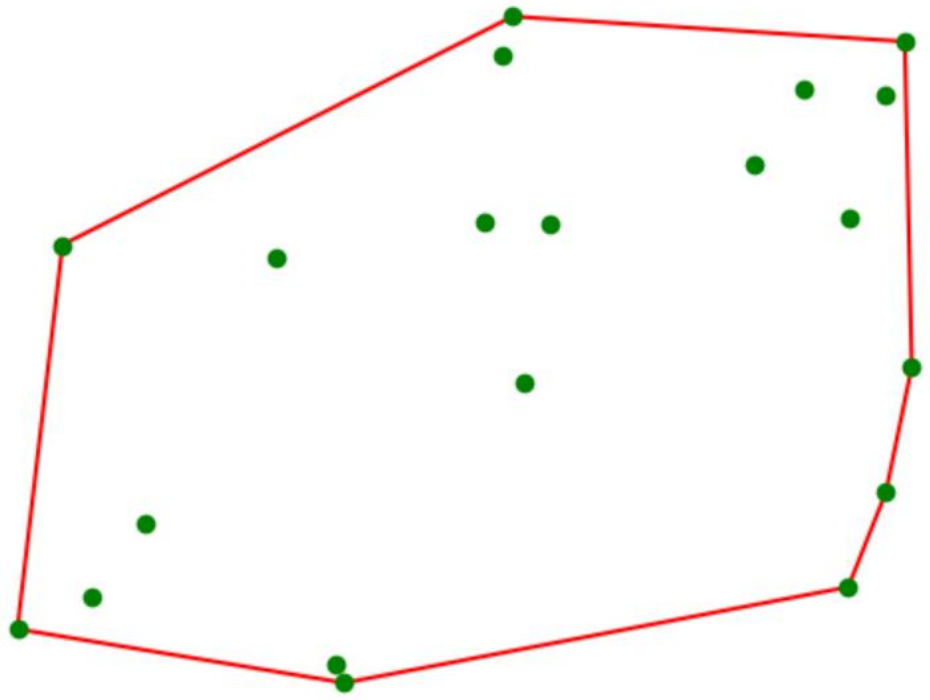
① Verifying if L is a sorted list of integers can slow down the program significantly

② Alternative is to only verify if the part of L visited is a sorted subsequence

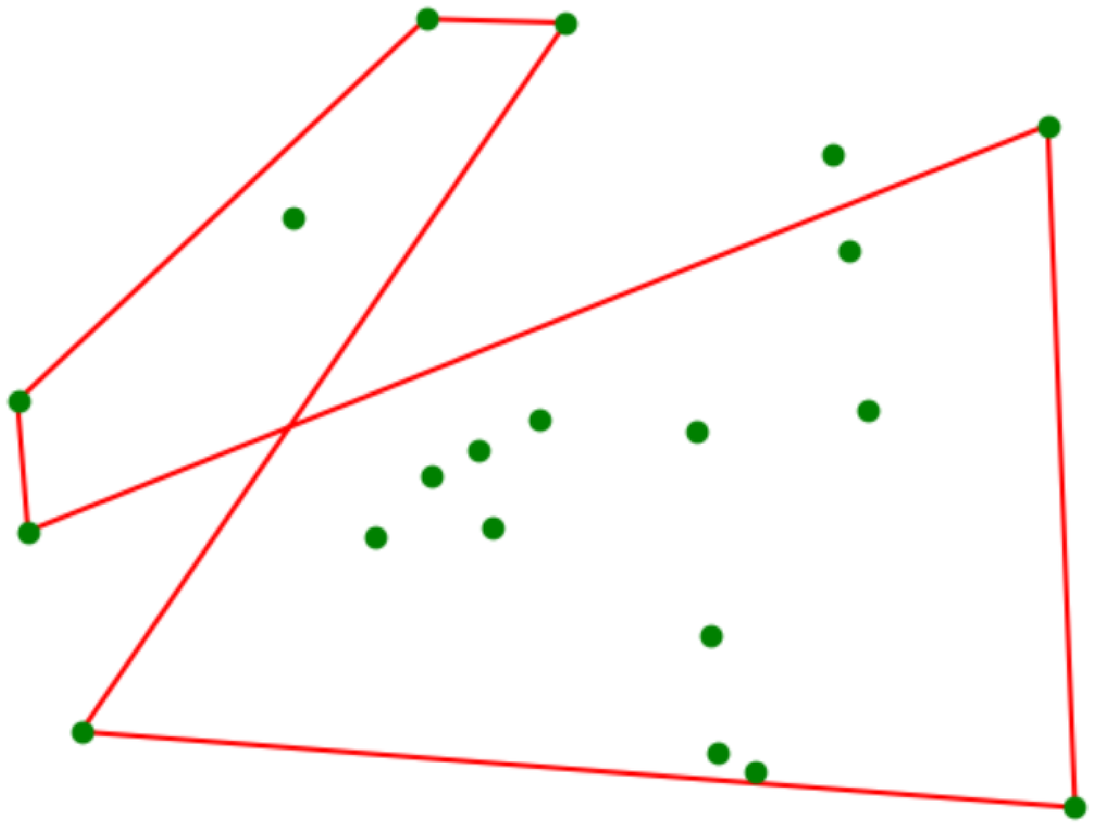
Testing – how ?

- Run set of test cases
  - test all cases in input/output specification **(black box testing)**
  - test all special cases **(black box testing)**
  - set of tests should force all lines of code to be tested **(glass box testing)**
- Visual test
- Automatic testing
  - Systematically / randomly generate input instances
  - Create function to **validate** if output is correct
    (hopefully easier than finding the solution)
- Formal verification
  - Use computer programs to do formal proofs of correctness, like using Coq

Visual testing – Convex hull computation



Correct



Bug !
(not convex)

doctest

- Python module
- Test instances (pairs of input and corresponding output) are written in the doc strings, formatted as in an interactive Python session

binary-search-doctest.py

```python
def binary_search(x, L):
    """Binary search for x in sorted list L

    Examples:
    >>> binary_search(42, [])
    -1
    >>> binary_search(42, [7])
    0
    >>> binary_search(42, [7,7,7,56,81])
    2
    >>> binary_search(8, [1,3,5,7,9])
    3
    """

    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    return low

import doctest
doctest.testmod(verbose=True)
```

Python shell

```
| Trying:
|     binary_search(42, [])
| Expecting:
|     -1
| ok
| Trying:
|     binary_search(42, [7])
| Expecting:
|     0
| ok
| Trying:
|     binary_search(42, [7,7,7,56,81])
| Expecting:
|     2
| ok
| Trying:
|     binary_search(8, [1,3,5,7,9])
| Expecting:
|     3
| ok
| 1 items had no tests:
|     __main__
| 1 items passed all tests:
|    4 tests in __main__.binary_search
| 4 tests in 2 items.
| 4 passed and 0 failed.
| Test passed.
```

docs.python.org/3/library/doctest.html

pytest

- Run all tests stored in functions prefixed by `test_` or `test_` prefixed test methods inside `Test` prefixed test classes
- `pip install pytest`
- Run the `pytest` program from a shell

pytest.org

binary-search-pytest.py

```python
def binary_search(x, L):
    """Binary search for x in sorted list L"""
    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    return low

def test_binary_search():
    assert binary_search(42, []) == -1
    assert binary_search(42, [7]) == 0
    assert binary_search(42, [7,7,7,56,81]) == 2
    assert binary_search(8, [1,3,5,7,9]) == 3

import pytest

def test_types():
    with pytest.raises(TypeError):
        _ = binary_search(5, ['a', 'b', 'c'])
```

Shell

```
> pytest binary-search-pytest.py
| ============= test session starts =============
| platform win32 -- Python 3.7.2, pytest-4.3.1, ... 0.9.0
| collected 2 items
| binary-search-pytest.py .. [100%]
| ============= 2 passed in 0.06 seconds =============
```

unittest

- Python module
- A comprehensive object-oriented test framework, inspired by the corresponding JUnit test framework for Java

binary-search-unittest.py

```
def binary_search(x, L):
    """Binary search for x in sorted list L"""

    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    return low

import unittest

class TestBinarySearch(unittest.TestCase):
    def test_search(self):
        self.assertEqual(binary_search(42, []), -1)
        self.assertEqual(binary_search(42, [7]), 0)
        self.assertEqual(binary_search(42, [7,7,7,56,81]), 2)
        self.assertEqual(binary_search(8, [1,3,5,7,9]), 3)

    def test_types(self):
        self.assertRaises(TypeError, binary_search, 5, ['a', 'b', 'c'])

unittest.main(verbosity=2)
```

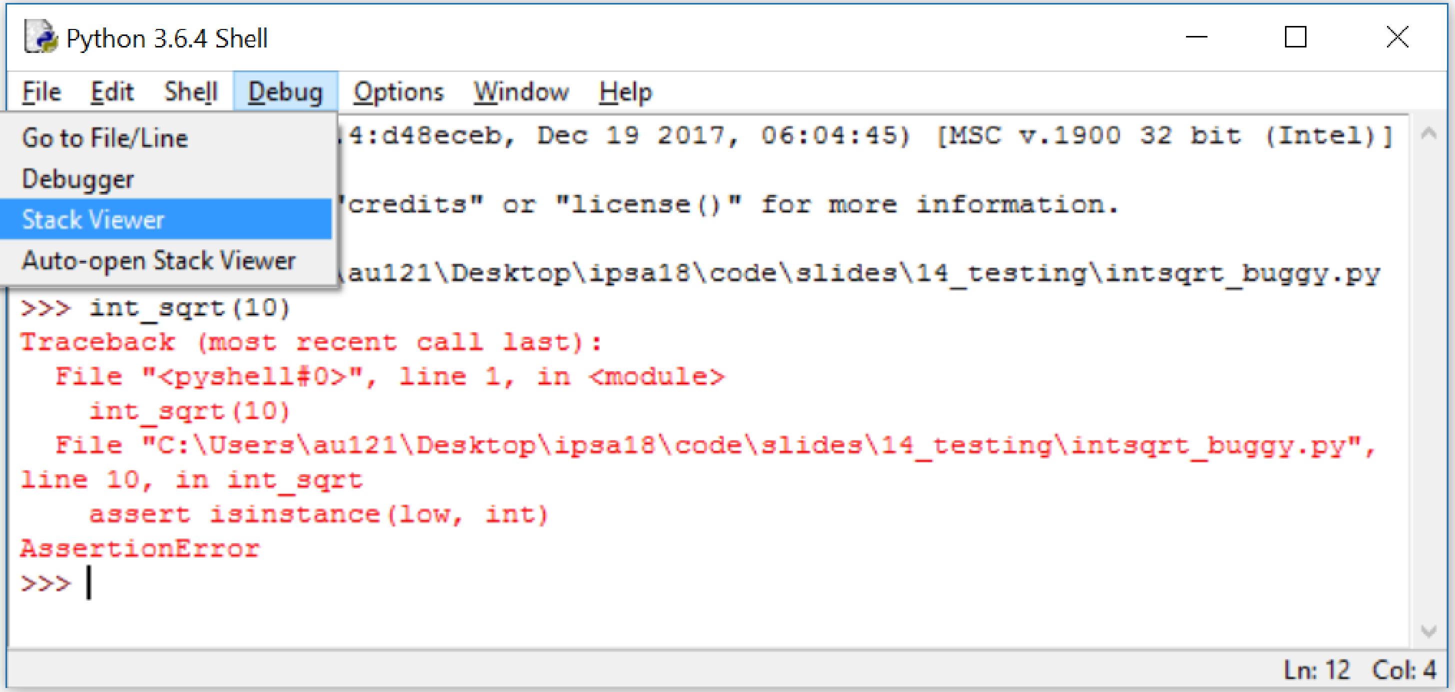
Python shell

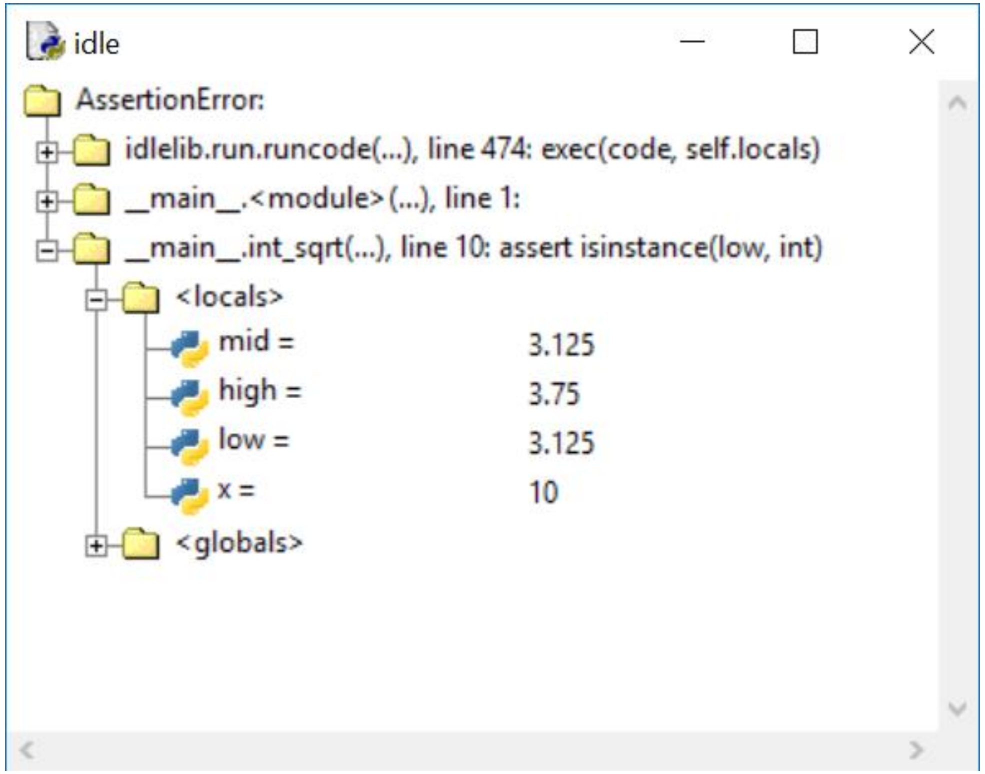
```
| test_search (__main__.TestBinarySearch) ... ok
| test_types (__main__.TestBinarySearch) ... ok
| ----------------------------------------------------------------------
| Ran 2 tests in 0.051s
| OK
```

docs.python.org/3/library/unittest.html

Debugger (IDLE)

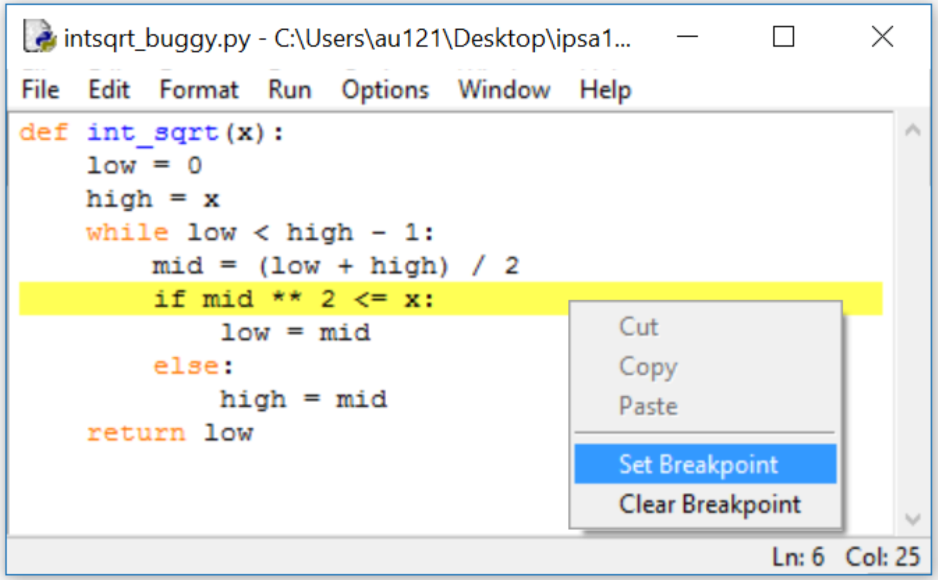
- When an exception has stopped the program, you can examine the state of the variables using **Debug > Stack Viewer** in the Python shell

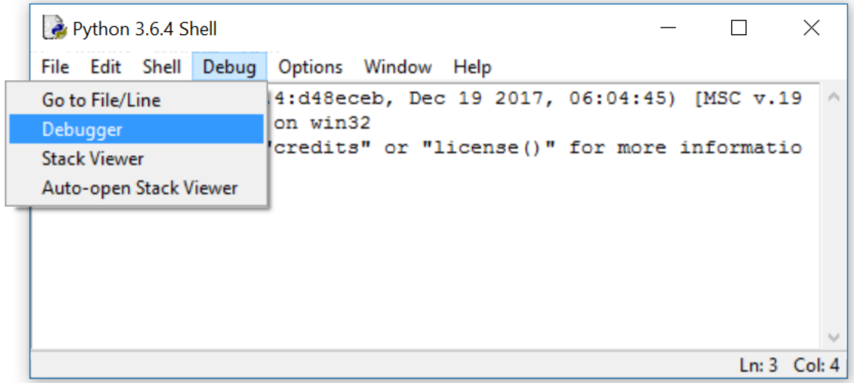


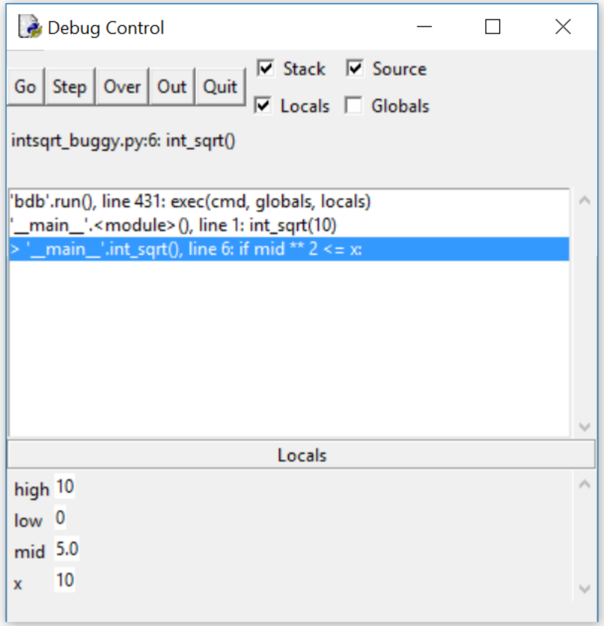


Stepping through a program (IDLE debugger)

- **Debug > Debugger** in the Python shell opens Debug Control window
- **Right click** on a code line in editor to set a “breakpoint” in your code
- **Debug Control:** Go → run until next breakpoint is encountered;
  Step → execute one line of code; Over → run function call without details;
  Out → finish current function call; Quit → Stop program;







Concluding remarks

- Simple debugging: add print statements
- **Test driven development** → Strategy for code development, where tests are written before the code
- **Defensive programming** → add tests (assertions) to check if input/arguments are valid according to specification
- When designing tests, ensure **coverage** (the set of test cases should make sure all code lines get executed)
- **Python testing frameworks: doctest, unittest, pytest, ...**

Mypy – a static type checker for Python

Experimental ⚠

- **Static type checking** tries to analyze a program for potential type errors without executing the program
- Installing:

  `pip install mypy`

- Running Python will cause an error during execution, whereas using mypy the error will be found without executing the program
- Standard (and required) in statically typed languages like Java, C, C++

mypy-simple.py

```python
print("start")
print(42 + "abc")   # error
print("end")
```

Shell

```
> python mypy-simple.py
| start
| TypeError: unsupported operand type(s)
  for +: 'int' and 'str'
> mypy mypy-simple.py
| mypy-simple.py:2: error: Unsupported
  operand types for + ("int" and "str")
```

mypy-lang.org
PEP 484 - Type Hints

Type hints (PEP 484)

- Python allows type hints in programs
- They are ignored at run-time by Python, but useful for static type analysis (mypy)
- Syntax

```
variable : type
variable : type = value
```

mypy-basic-types.py

```python
x : int   # type hint
x = 42
x = "abc"   # type error
y : int = 42   # type hint
y = "abc"   # type error
z = 42
z = "abc"   # type changed from int to str
print(x, y, z)
```

Shell

```
> python mypy-basic-types.py
| abc abc abc
> mypy mypy-basic-types.py
| mypy-basic-types.py:3: error: Incompatible
  types in assignment (expression has type
  "str", variable has type "int")
| mypy-basic-types.py:6: error: ...
| mypy-basic-types.py:9: error: ...
```

Type hints – functions

Syntax

```
def name(variable : type, ...) -> return type:
```

mypy-function.py	Shell

`def f(x: int, units: str) -> str:`
`    return str(x) + " " + units`
 	
`def g(x, units: str) -> str:`
`    return str(x) + " " + units`
 	
`print(f(3, "cm"))`
`print(f("one", "meter"))`
 	
`print(g(3, "cm"))`
`print(g("one", "meter"))`
	`> python mypy-function.py`
	`| 3 cm`
	`| one meter`
	`| 3 cm`
	`| one meter`
	`> mypy mypy-function.py`
	`| mypy-function.py:8: error: Argument 1 to "f" has incompatible type "str"; expected "int"`

- Note: for functions and methods *function*.`__annotations__` is a dictionary with the annotation

Type hints – objects

mypy-classes.py

```python
class A:
    pass
class B(A):
    pass
class C:
    pass
a : A
b : B
c : C
```

```python
a = A()
a = B()  # valid, B subclass of A
a = C()  # error
b = A()  # error
b = B()
b = C()  # error
c = A()  # error
c = B()  # error
c = C()
```

Shell

```
> mypy mypy-classes.py
| mypy-classes.py:15: error: Incompatible types in assignment (expression has type "C", variable has type "A")
| mypy-classes.py:16: error: Incompatible types in assignment (expression has type "A", variable has type "B")
| mypy-classes.py:18: error: Incompatible types in assignment (expression has type "C", variable has type "B")
| mypy-classes.py:19: error: Incompatible types in assignment (expression has type "A", variable has type "C")
| mypy-classes.py:20: error: Incompatible types in assignment (expression has type "B", variable has type "C")
```

More type hints... see PEP 484 for even more...

mypy-typing.py

```python
from typing import Mapping, Set, List, Tuple, Union, Optional

S : Set = {}                                      # error {} dictionary
S2 : Set[int] = {1, 2, "abc"}                     # error "abc" is not int
D : Mapping[int, int] = {1: 42, 'a': 1}           # error 'a' is not int
T : Tuple[int, str] = (42, 7)                     # error 7 is not str
L : List[Union[int, str]] = [42, 'a', None]       # list can only contain int and str
L2 : List[Optional[str]] = ['abc', None, 42]      # list can only contain str og None
```

Shell

```
> mypy mypy-function.py
| mypy-typing.py:3: error: Incompatible types in assignment (expression has type "Dict[<nothing>,
  <nothing>]", variable has type "Set[Any]")
| mypy-typing.py:4: error: Argument 3 to <set> has incompatible type "str"; expected "int"
| mypy-typing.py:5: error: Dict entry 1 has incompatible type "str": "int"; expected "int": "int"
| mypy-typing.py:6: error: Incompatible types in assignment (expression has type "Tuple[int, int]",
  variable has type "Tuple[int, str]")
| mypy-typing.py:7: error: List item 2 has incompatible type "None"; expected "Union[int, str]"
| mypy-typing.py:8: error: List item 2 has incompatible type "int"; expected "Optional[str]"
```

[PEP 484 - Type Hints]()

Decorators

- @

www.python.org/dev/peps/pep-0318/

Course overview

Basic programming
Advanced / specific python
Libraries & applications

1. Introduction to Python	10. Functions as objects	19. Linear programming
2. Python basics / if	11. Object oriented programming	20. Generators, iterators, with
3. Basic operations	12. Class hierarchies	21. Modules and packages
4. Lists / while / for	13. Exceptions and files	22. Working with text
5. Tuples / comprehensions	14. Doc, testing, debugging	23. Relational data
6. Dictionaries and sets	15. Decorators	24. Clustering
7. Functions	16. Dynamic programming	25. Graphical user interfaces (GUI)
8. Recursion	17. Visualization and optimization	26. Java vs Python
9. Recursion and Iteration	18. Multi-dimensional data	27. Final lecture

10 handins
1 final project (last 1 month)

Python decorators are just syntatic sugar

```
Python
@dec2
@dec1
def func(arg1, arg2, ...):
    pass
```

≡

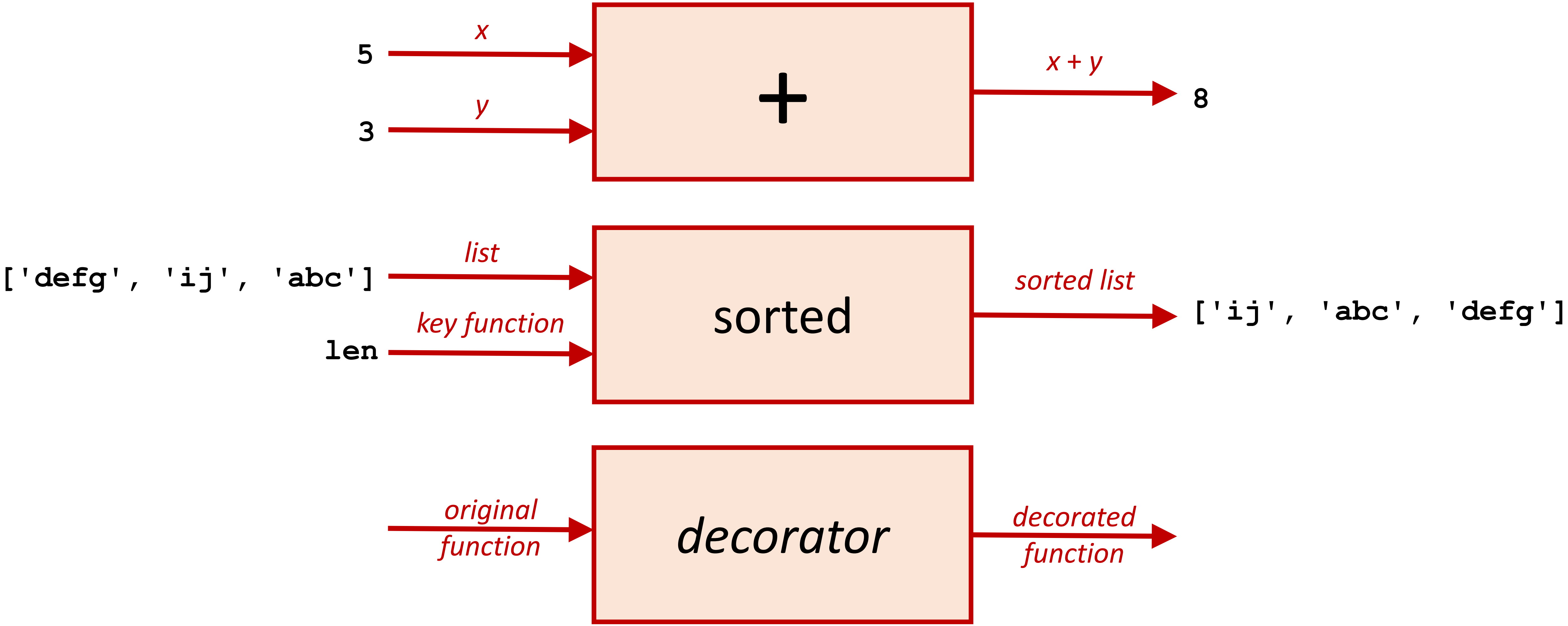
```
Python
def func(arg1, arg2, ...):
    pass

func = dec2(dec1(func))
```

'pie-decorator' syntax

dec1, dec2, ... are functions (decorators) taking a *function as an argument* and *returning a new function*

Note: decorators are listed bottom up in order of execution

Recap functions



Contrived example : Plus one (I-II)

plus_one1.py

```
def plus_one(x):
    return x + 1

def square(x):
    return x ** 2

def cube(x):
    return x ** 3

print(plus_one(square(5)))
print(plus_one(cube(5)))
```

Python shell

```
| 26
| 126
```

Assume we *always* need to call `plus_one` on the result of `square` and `cube` (don’t ask why!)

plus_one2.py

```
def plus_one(x):
    return x + 1

def square(x):
    return plus_one(x ** 2)

def cube(x):
    return plus_one(x ** 3)

print(square(5))
print(cube(5))
```

Python shell

```
| 26
| 126
```

We could call `plus_one` inside functions (but could be more `return` statements in functions)

Contrived example : Plus one (III-IV)

plus_one3.py

```
def plus_one(x):
    return x + 1

def square(x):
    return x ** 2

def cube(x):
    return x ** 3

square_original = square
cube_original = cube

square = lambda x: plus_one(square_original(x))
cube = lambda x: plus_one(cube_original(x))

print(square(5))
print(cube(5))
```

Python shell

```
| 26
| 126
```

Overwrite `square` and `cube` with decorated versions

plus_one4.py

```
def plus_one(x):
    return x + 1

def plus_one_decorator(f):
    return lambda x: plus_one(f(x))

def square(x):
    return x ** 2

def cube(x):
    return x ** 3

square = plus_one_decorator(square)
cube = plus_one_decorator(cube)

print(square(5))
print(cube(5))
```

Python shell

```
| 26
| 126
```

Create a decorator function `plus_one_decorator`

Contrived example : Plus one (V-VI)

plus_one5.py

```
def plus_one(x):
    return x + 1

def plus_one_decorator(f):
    return lambda x: plus_one(f(x))

@plus_one_decorator
def square(x):
    return x ** 2

@plus_one_decorator
def cube(x):
    return x ** 3

print(square(5))
print(cube(5))
```

Python shell

```
| 26
| 126
```

Use Python decorator syntax

plus_one6.py

```
def plus_one_decorator(f):
    def plus_one(x):
        return f(x) + 1

    return plus_one

@plus_one_decorator
def square(x):
    return x ** 2

@plus_one_decorator
def cube(x):
    return x ** 3

print(square(5))
print(cube(5))
```

Python shell

```
| 26
| 126
```

Create local function instead of using `lambda`

Contrived example : Plus one (VII)

plus_one7.py

```python
def plus_one_decorator(f):
    def plus_one(x):
        return f(x) + 1
    return plus_one

@plus_one_decorator
@plus_one_decorator
def square(x):
    return x ** 2

@plus_one_decorator
@plus_one_decorator
@plus_one_decorator
def cube(x):
    return x ** 3

print(square(5))
print(cube(5))
```

Python shell

```
| 27
| 128
```

- A function can have an arbitrary number of decorators (also the same repeated)
- Decorators are listed bottom up in order of execution

Handling arguments

run_twice1.py

```
def run_twice(f):
    def wrapper():
        f()
        f()
    return wrapper

@run_twice
def hello_world():
    print("Hello world")

hello_world()
```

Python shell

```
| Hello world
| Hello world
```

"`wrapper`" is a common name for the function returned by a decorator

run_twice2.py

```
def run_twice(f):
    def wrapper(*args):
        f(*args)
        f(*args)
    return wrapper

@run_twice
def hello_world():
    print("Hello world")

@run_twice
def hello(txt):
    print("Hello", txt)

hello_world()
hello("Mars")
```

Python shell

```
| Hello world
| Hello world
| Hello Mars
| Hello Mars
```

`args` holds the arguments in a tuple given to the function to be decorated

Question – What does the decorated program print ?

decorator_quizz.py

```
def double(f):
    def wrapper(*args):
        return 2 * f(*args)
    return wrapper

def add_three(f):
    def wrapper(*args):
        return 3 + f(*args)
    return wrapper

@double
@add_three
def seven():
    return 7

print(seven())
```

- 7
- 10
- 14
- 17
- 20
- Don't know

Example: Enforcing argument types

- Defining decorators can be (slightly) complicated
- Using decorators is easy

integer_sum1.py

```
def integer_sum(*args):
    assert all([isinstance(x, int) for x in args]),\
           "all arguments most be int"
    return sum(args)
```

Python shell

```
> integer_sum(1, 2, 3, 4)
| 10
> integer_sum(1, 2, 3.2, 4)
| AssertionError: all arguments most be int
```

integer_sum2.py

```
def enforce_integer(f):    # decorator function
    def wrapper(*args):
        assert all([isinstance(x, int) for x in args]),\
               "all arguments most be int"
        return f(*args)
    return wrapper

@enforce_integer
def integer_sum(*args):
    return sum(args)
```

Python shell

```
> integer_sum(1, 2, 3, 4)
| 10
> integer_sum(1, 2, 3.2, 4)
| AssertionError: all arguments most be int
```

Decorators can take arguments

```python
@dec(argA, argB, ...)
def func(arg1, arg2, ...):
    pass
```

≡

```python
def func(arg1, arg2, ...):
    pass
func = dec(argA, argB, ...)(func)
```

`dec` is a function (decorator) that takes a *list of arguments* and *returns a function* (to decorate `func`) that takes a *function as an argument* and *returns a new function*

Example: Generic type enforcing

print_repeated.py

```python
def enforce_types(*decorator_args):
    def decorator(f):
        def wrapper(*args):
            assert len(args) == len(decorator_args),\
                   ("got %s arguments, expected %s" % (len(args), len(decorator_args)))
            assert all([isinstance(x, t) for x, t in zip(args, decorator_args)]),\
                   "unexpected types"
            return f(*args)
        return wrapper
    return decorator

@enforce_types(str, int)  # decorator with arguments
def print_repeated(txt, n):
    print(txt * n)

print_repeated("Hello ", 3)
print_repeated("Hello ", "world")
```

Python

```python
@dec(argA, argB, ...)
def func(arg1, arg2, ...):
    pass
```

≡

Python

```python
def func(arg1, arg2, ...):
    pass
func = dec(argA, argB, ...)(func)
```

Python shell

```
| Hello Hello Hello
| AssertionError: unexpected types
```

Example: A timer decorator

time_it.py

```
import time

def time_it(f):
    def wrapper(*args, **kwargs):
        t_start = time.time()
        result = f(*args, **kwargs)
        t_end = time.time()
        t = t_end - t_start
        print("%s took %.2f sec" % (f.__name__, t))
        return result

    return wrapper

@time_it
def slow_function(n):
    sum_ = 0
    for x in range(n):
        sum_ += x
    print("The sum is:", sum_)

for i in range(6):
    slow_function(1_000_000 * 2**i)
```

Python shell

```
| The sum is: 499999500000
| slow_function took 0.27 sec
| The sum is: 1999999000000
| slow_function took 0.23 sec
| The sum is: 7999998000000
| slow_function took 0.41 sec
| The sum is: 31999996000000
| slow_function took 0.81 sec
| The sum is: 127999992000000
| slow_function took 1.52 sec
| The sum is: 511999984000000
| slow_function took 3.12 sec
```

Built-in `@property`

- decorator specific for class methods
- allows accessing `x.attribute()` as `x.attribute`, convenient if `attribute` does not take any arguments (also readonly)

rectangle1.py

```
class Rectangle:
    def __init__(self, width, height):
        self.width = width
        self.height = height

#   @property
    def area(self):
        return self.width * self.height
```

Python shell

```
> r = Rectangle(3, 4)
> print(r.area())
| 12
```

rectangle2.py

```
class Rectangle:
    def __init__(self, width, height):
        self.width = width
        self.height = height

    @property
    def area(self):
        return self.width * self.height
```

Python shell

```
> r = Rectangle(3, 4)
> print(r.area)
| 12
```

Class decorators

Python

```python
@dec2
@dec1
class A:
  pass
```

≡

Python

```python
class A:
  pass

A = dec2(dec1(A))
```

Module `dataclasses` (Since Python 3.7)

- New (and more configurable) alternative to `namedtuple`

Python shell

```
> from dataclasses import dataclass
> @dataclass  # uses a decorator to add methods to the class
  class Person:
      name: str  # uses type annotation to define fields
      appeared: int
      height: str = 'unknown height'  # field with default value
> person = Person('Donald Duck', 1934, '3 feet')
> person
| Person(name='Donald Duck', appeared=1934, height='3 feet')
> person.name
| 'Donald Duck'
> Person('Mickey Mouse', 1928)
| Person(name='Mickey Mouse', appeared=1928, height='unknown height')
```

docs.python.org/3/library/dataclasses.html#module-dataclasses

[Raymond Hettinger - Dataclasses: The code generator to end all code generators - PyCon 2018]()

@functools.total_ordering (class decorator)

time_it.py

```python
import functools

@functools.total_ordering
class Student():
    def __init__(self,name, student_id):
        self.name = name
        self.id = student_id
    def __eq__(self, other):
        return (self.name == other.name
                and self.id == other.id)
    def __lt__(self, other):
        my_name = ', '.join(reversed(self.name.split()))
        other_name = ', '.join(reversed(other.name.split()))
        return (my_name < other_name
                or (my_name == other_name and self.id < other.id))
donald = Student('Donald Duck', 7)
gladstone = Student('Gladstone Gander', 42)
grandma = Student('Grandma Duck', 1)
```

Automatically creates <, <=, >, >= if at least one of the functions is implemented and == is implemented

Python shell

```
> donald < grandma
| True
> grandma >= gladstone
| False
> grandma <= gladstone
| True
> donald > gladstone
| False
```

Summary

- *@decorator_name*
- Pyton decorators are just syntatic sugar
- Adds functionality to a function without having to augment each call to the function or each return statement in the function
- There are decorators for functions, class methods, and classes
- There are many decorators in the Python Standard Library
- Decorators are easy to use
- ...and (slightly) harder to write

Dynamic programming

- memoization
- decorator memoized
- systematic subproblem computation

```
--(7, 5)
  |--(6, 5)
  |  |--(5, 5)
  |   --(5, 4)
  |     |--(4, 4)
  |      --(4, 3)
  |        |--(3, 3)
  |         --(3, 2)
  |           |--(2, 2)
  |            --(2, 1)
  |              |--(1, 1)
  |               --(1, 0)
   --(6, 4)
     |--(5, 4)
     |  |--(4, 4)
     |   --(4, 3)
     |     |--(3, 3)
     |      --(3, 2)
     |        |--(2, 2)
     |         --(2, 1)
     |           |--(1, 1)
     |            --(1, 0)
      --(5, 3)
        |--(4, 3)
        |  |--(3, 3)
        |   --(3, 2)
        |     |--(2, 2)
        |      --(2, 1)
        |        |--(1, 1)
        |         --(1, 0)
         --(4, 2)
           |--(3, 2)
           |  |--(2, 2)
           |   --(2, 1)
           |     |--(1, 1)
           |      --(1, 0)
            --(3, 1)
              |--(2, 1)
              |  |--(1, 1)
              |   --(1, 0)
               --(2, 0)
```

identical computations

Binomial coefficient

$$\binom{n}{k} = \begin{cases} 1 & \text{if } k = 0 \text{ or } k = n \\ \binom{n-1}{k} + \binom{n-1}{k-1} & \text{otherwise} \end{cases}$$

binomial_recursive.py

```python
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    return binomial(n - 1, k) + binomial(n - 1, k - 1)
```

recursion tree for `binomial(7,5)`

Dynamic Programming

$\equiv$

Remember solutions already found (memoization)

- Technique sometimes applicable when running time otherwise becomes exponential
- Only applicable if stuff to be remembered is manageable

Binomial Coefficient
Dynamic programming using a dictionary

recursion tree for
`binomial(7,5)`

```
--(7, 5)
   |--(6, 5)
   |   |--(5, 5)
   |    --(5, 4)
   |       |--(4, 4)
   |        --(4, 3)
   |           |--(3, 3)
   |            --(3, 2)
   |               |--(2, 2)
   |                --(2, 1)
   |                   |--(1, 1)
   |                    --(1, 0)
    --(6, 4)
       |--(5, 4)
        --(5, 3)
           |--(4, 3)
            --(4, 2)
               |--(3, 2)
                --(3, 1)
                   |--(2, 1)
                    --(2, 0)
```

reuse value stored in dictionary `answers`

binomial_dictionary.py

```python
answers = {}  # answers[(n, k)] = binomial(n,k)

def binomial(n, k):
    if (n, k) not in answers:
        if k==0 or k==n:
            answer = 1
        else:
            answer = binomial(n-1, k) + binomial(n-1, k-1)
        answers[(n, k)] = answer
    return answers[(n,k)]
```

Python shell

```
> binomial(6, 3)
| 20
> answers
| {(3, 3): 1, (2, 2): 1, (1, 1): 1, (1, 0): 1, (2, 1): 2, (3, 2):
  3, (4, 3): 4, (2, 0): 1, (3, 1): 3, (4, 2): 6, (5, 3): 10, (3,
  0): 1, (4, 1): 4, (5, 2): 10, (6, 3): 20}
```

- Use a dictionary `answers` to store already computed values

Question – What is the order of the size of the dictionary `answers` after **calling** `binomial(n,k)` ?

binomial_dictionary.py

```
answers = {}  # answers[(n, k)] = binomial(n,k)
def binomial(n, k):
    if (n, k) not in answers:
        if k==0 or k==n:
            answer = 1
        else:
            answer = binomial(n-1, k) + binomial(n-1, k-1)
        answers[(n, k)] = answer
    return answers[(n,k)]
```

a) max(n, k)
b) n + k
c) n * k
d) n^k
e) k^n
f) Don't know

Binomial Coefficient
Dynamic programming using decorator

- Use a decorator (@memoize) that implements the functionality of remembering the results of previous function calls

binomial_decorator.py

```python
def memoize(f):
    # answers[args] = f(*args)
    answers = {}

    def wrapper(*args):
        if args not in answers:
            answers[args] = f(*args)
        return answers[args]

    return wrapper
```

```python
@memoize
def binomial(n, k):
    if k==0 or k==n:
        return 1
    else:
        return binomial(n-1, k) + binomial(n-1, k-1)
```

www.python-course.eu/python3_memoization.php

binomial_decorator_trace.py

```
def trace(f):  # decorator to trace recursive calls
    indent = 0

    def wrapper(*args):
        nonlocal indent

        spaces = '|  ' * indent
        arg_str = ', '.join(map(repr, args))
        print(spaces + f'{f.__name__}({arg_str})')
        indent += 1
        result = f(*args)
        indent -= 1
        print(spaces + f'> {result}')
        return result

    return wrapper

def memoize(f):
    answers = {}

    def wrapper(*args):
        if args not in answers:
            answers[args] = f(*args)
        return answers[args]

    wrapper.__name__ = f.__name__ + '_memoize'

    return wrapper

@trace
@memoize
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    return binomial(n - 1, k) + binomial(n-1, k-1)

print(binomial(5, 2))
```

Python shell (without @memoize)

```
binomial(5, 2)
|  binomial(4, 2)
|  |  binomial(3, 2)
|  |  |  binomial(2, 2)
|  |  |  > 1
|  |  |  binomial(2, 1)
|  |  |  |  binomial(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  > 3
|  |  binomial(3, 1)
|  |  |  binomial(2, 1)
|  |  |  |  binomial(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  |  binomial(2, 0)
|  |  |  > 1
|  |  > 3
|  > 6
|  binomial(4, 1)
|  |  binomial(3, 1)
|  |  |  binomial(2, 1)
|  |  |  |  binomial(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  |  binomial(2, 0)
|  |  |  > 1
|  |  > 3
|  |  binomial(3, 0)
|  |  > 1
|  > 4
> 10
10
```

saved recursive calls when using memoization

Python shell (with @memoize)

```
binomial_memoize(5, 2)
|  binomial_memoize(4, 2)
|  |  binomial_memoize(3, 2)
|  |  |  binomial_memoize(2, 2)
|  |  |  > 1
|  |  |  binomial_memoize(2, 1)
|  |  |  |  binomial_memoize(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial_memoize(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  > 3
|  |  binomial_memoize(3, 1)
|  |  |  binomial_memoize(2, 1)
|  |  |  > 2
|  |  |  binomial_memoize(2, 0)
|  |  |  > 1
|  |  > 3
|  > 6
|  binomial_memoize(4, 1)
|  |  binomial_memoize(3, 1)
|  |  > 3
|  |  binomial_memoize(3, 0)
|  |  > 1
|  > 4
> 10
10
```

without assigning `wrapper.__name__` the name shown would be `wrapper`

Dynamic programming using lru_cache decorator

```
bionomial_lru_cache.py
```

```python
from functools import lru_cache

@lru_cache(maxsize=None)
def binomial(n, k):
    if k==0 or k==n:
        return 1
    else:
        return binomial(n-1, k) + binomial(n-1, k-1)
```

- The decorator `@lru_cache` in the standard library `functools` supports the same as the decorator `@memoize`
- By default it at most remembers (caches) 128 previous function calls, always evicting **L**east **R**ecently **U**sed entries from its dictionary

docs.python.org/3/library/functools.html#functools.lru_cache

Subset sum using dynamic programming

- In the subset sum problem (Exercise 13.4) we are given a number `x` and a list of numbers `L`, and want to determine if a subset of `L` has sum `x`

```
L = [3, 7, 2, 11, 13, 4, 8]    x = 22 = 7 + 11 + 4
```

- Let S(v, k) denote if it is possible to achieve value v with a subset of `L[:k]`, i.e. S(v, k) = True if and only if a subset of the first k values in `L` has sum v
- S(v, k) can be computed from the following recurrence:

$$S(v, k) = \begin{cases} \text{True} & \text{if } k = 0 \text{ and } v = 0 \\ \text{False} & \text{if } k = 0 \text{ and } v \neq 0 \\ S(v, k-1) \ \text{ or } \ S(v - \mathtt{L}[k-1], k-1) & \text{otherwise} \end{cases}$$

Subset sum using dynamic programming

subset_sum_dp.py

```python
def subset_sum(x, L):
    @memoize
    def solve(value, k):
        if k == 0:
            return value == 0
        return solve(value, k-1) or solve(value - L[k-1], k-1)
    return solve(x, len(L))
```

Python shell

```
> subset_sum(11, [2, 3, 8, 11, -1])
| True
> subset_sum(6, [2, 3, 8, 11, -1])
| False
```

Question – What is a bound on the size order of the memoization table if all values are possitive integers?

subset_sum_dp.py

```python
def subset_sum(x, L):
    @memoize
    def solve(value, k):
        if k == 0:
            return value == 0
        return solve(value, k-1) or solve(value - L[k-1], k-1)
    return solve(x, len(L))
```

Python shell

```
> subset_sum(11, [2, 3, 8, 11, -1])
| True
> subset_sum(6, [2, 3, 8, 11, -1])
| False
```

a) `len(L)`

b) `sum(L)`

c) `x`

d) $2^{\texttt{len(L)}}$

e) `len(L)`

f) `len(L)*sum(L)`

g) Don't know

Subset sum using dynamic programming

subset_sum_dp.py

```
def subset_sum_solution(x, L):
    @memoize
    def solve(value, k):
        if k == 0:
            if value == 0:
                return []
            else:
                return None
        solution = solve(value, k-1)
        if solution != None:
            return solution
        solution = solve(value - L[k-1], k-1)
        if solution != None:
            return solution + [L[k-1]]
        return None
    return solve(x, len(L))
```

Python shell

```
> subset_sum_solution(11, [2, 3, 8, 11, -1])
| [3, 8]
> subset_sum_solution(6, [2, 3, 8, 11, -1])
| None
```

Knapsack problem

	Volume	Value
0	3	6
1	3	7
2	2	8
3	5	9

- Given a **knapsack** with volume **capacity C**, and set of **objects** with different **volumes and value**.
- **Objective**: Find a subset of the objects that fits in the knapsack (sum of volume ≤ capacity) and has maximal value.
- **Example**: If C = 5 and the volume and weights are given by the table, then the maximal value 15 can be achieved by the 2nd and 3rd object.
- Let V(c, k) denote the maximum value achievable by a subset of the first k objects within capacity c.

$$V(c, k) = \begin{cases} 0 & \text{if } k = 0 \\ V(c, k-1) & \text{volume}[k-1] > c \\ \max\{V(c, k-1), \text{value}[k-1] + V(c - \text{volume}[k-1], k-1)\} & \text{otherwise} \end{cases}$$

Knapsack – maximum value

knapsack.py

```
def knapsack_value(volume, value, capacity):
    @memoize
    def solve(c, k):  # solve with capacity c and objects 0..k-1
        if k == 0:  # no objects to put in knapsack
            return 0
        v = solve(c, k - 1)  # try without object k-1
        if volume[k - 1] <= c:  # try also with object k-1 if space
            v = max(v, value[k - 1] + solve(c - volume[k - 1], k - 1))
        return v

    return solve(capacity, len(volume))
```

Python shell

```
> volumes = [3, 3, 2, 5]
> values = [6, 7, 8, 9]
> knapsack_value(volumes, values, 5)
| 15
```

Knapsack – maximum value and objects

knapsack.py

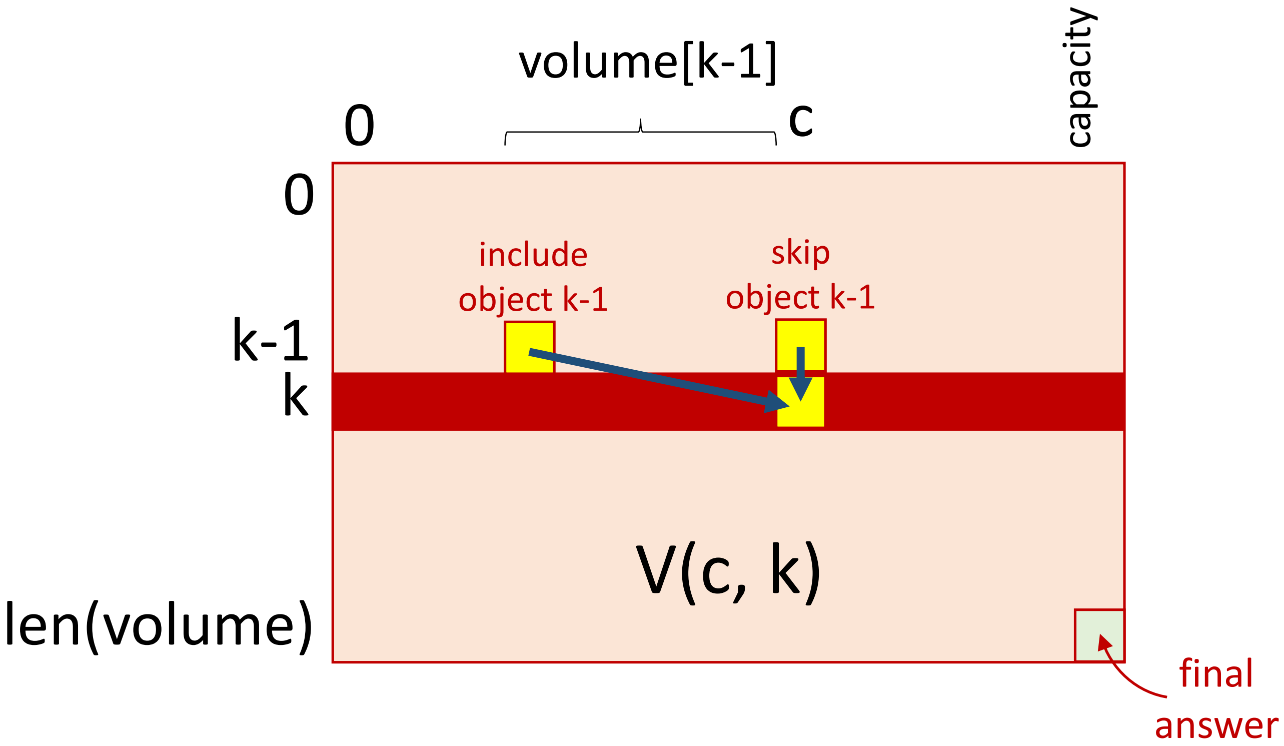
```
def knapsack(volume, value, capacity):
    @memoize
    def solve(c, k):  # solve with capacity c and objects 0..k-1
        if k == 0:  # no objects to put in knapsack
            return 0, []
        v, solution = solve(c, k-1)  # try without object k-1
        if volume[k - 1] <= c:  # try also with object k-1 if space
            v2, sol2 = solve(c - volume[k - 1], k - 1)
            v2 = v2 + value[k - 1]
            if v2 > v:
                v = v2
                solution = sol2 + [k - 1]
        return v, solution

    return solve(capacity, len(volume))
```

Python shell

```
> volumes = [3, 3, 2, 5]
> values = [6, 7, 8, 9]
> knapsack(volumes, values, 5)
| (15, [1, 2])
```

Knapsack - Table

$$V(c, k) = \begin{cases} 0 & \text{if } k = 0 \\ V(c, k-1) & \text{value}[k-1] > c \\ \max\{V(c, k-1), \text{value}[k-1] + V(c - \text{volume}[k-1], k-1)\} & \text{otherwise} \end{cases}$$



- systematic fill out table
- only need to remember two rows

Knapsack – Systematic table fill out

knapsack_systematic.py

```
def knapsack(volume, value, capacity):
  ① solutions = [(0, [])] * (capacity + 1)
  ② for obj in range(len(volume)): ⑤
          for c in reversed(range(volume[obj], capacity + 1)): ④
              prev_v, prev_solution = solutions[c - volume[obj]]
              v = value[obj] + prev_v
              if solutions[c][0] < v:
               ③ solutions[c] = v, prev_solution + [obj]

      return solutions[capacity]
```

Python shell

```
> volumes = [3, 3, 2, 5]
> values = [6, 7, 8, 9]
> knapsack(volumes, values, 5)
| (15, [1, 2])
```

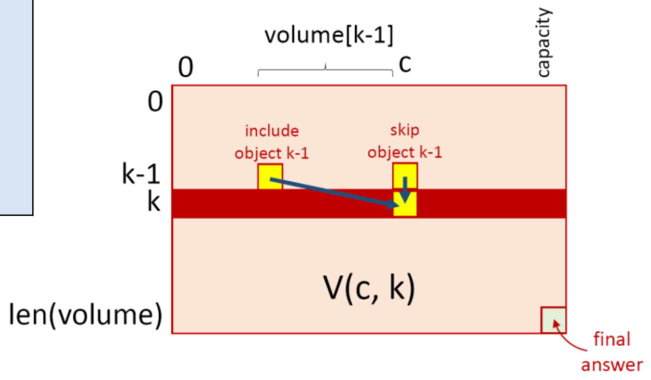
① base case k = 0

② consider each object

③ `solutions[c:]` current row
`solutions[:c]` previous row

④ compute next row right-to-left

⑤ `solutions[:volume[obj]]`
unchanged from previous row



Summary

- Dynamic programming is a general approach for recursive problems where one tries to avoid recomputing the same expresions repeatedly

- **Solution 1: Memoization**
  - add dictionary to function to remember previous results
  - decorate with a @memoize decorator
- **Solution 2: Systematic table fill out**
  - can need to compute more values than when using memoization
  - can discard results not needed any longer (reduced memory usage)

Visualization and optimization

- Matplotlib
- Jupyter
- scipy.optimize.minimize

Matplotlib is a Python 2D plotting library which produces publication quality figures in a variety of hardcopy formats and interactive environments across platforms. Matplotlib can be used in Python scripts, the Python and IPython shells, the Jupyter notebook, web application servers, and four graphical user interface toolkits.

Matplotlib tries to make easy things easy and hard things possible. You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code. For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython. For the power user, you have full control of line styles, font properties, axes properties, etc, via an object oriented interface or via a set of functions familiar to MATLAB users.

pip install matplotlib

matplotlib.org

Plot

pyplot module ≈ MATLAB-like plotting framework

matplotlib-simple.py

```python
import matplotlib.pyplot as plt

plt.plot([1, 2, 3], [5, 2, 7], 'bo:')

plt.show()
```

add plot to figure

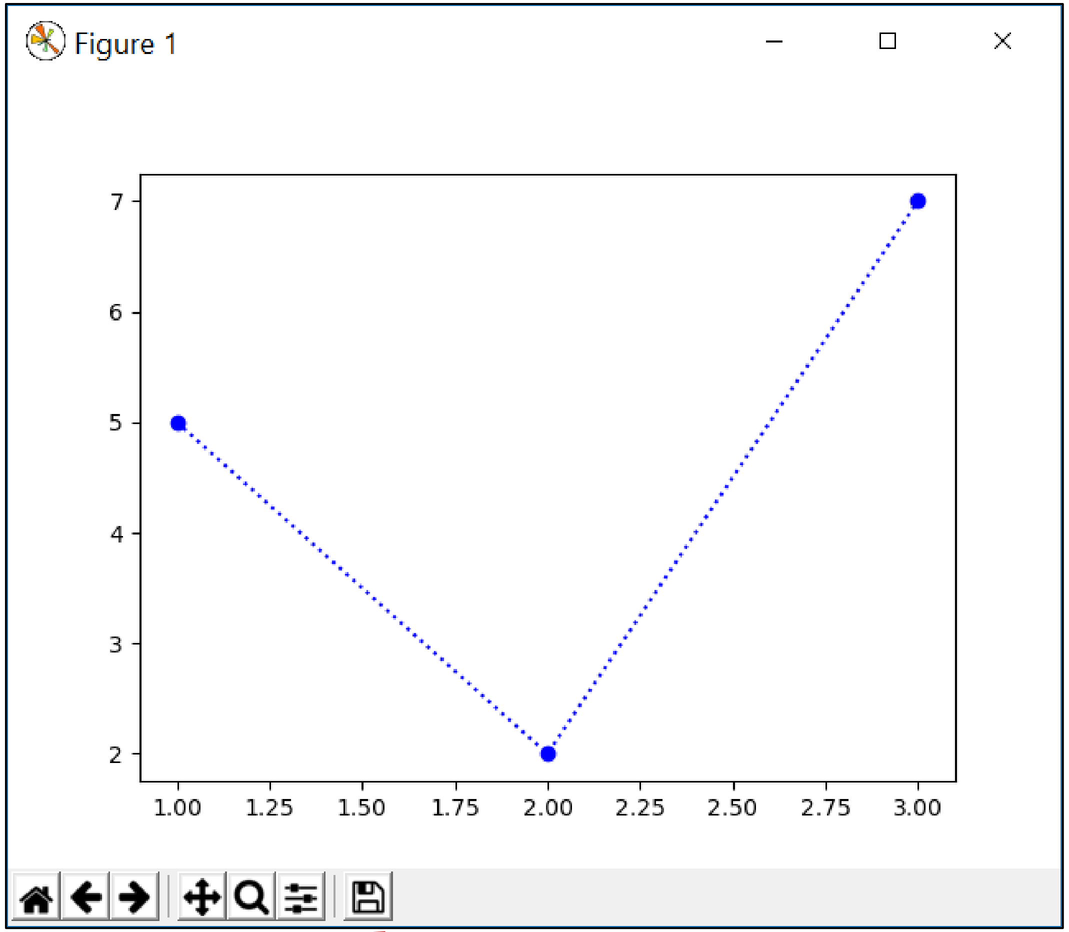
x coordinates

y coordinates

format string

figure is first shown when show is called

Colors	Line styles	Marker styles		
b	-	.	2	+
g	--	,	3	x
r	-.	o	4	D
c	:	v	s	d
m		^	p	|
y		<	*	_
k		>	h	
w		1	H	



save current view as picture

adjust margins

zoom rectangle

pan and zoom

navigate view history

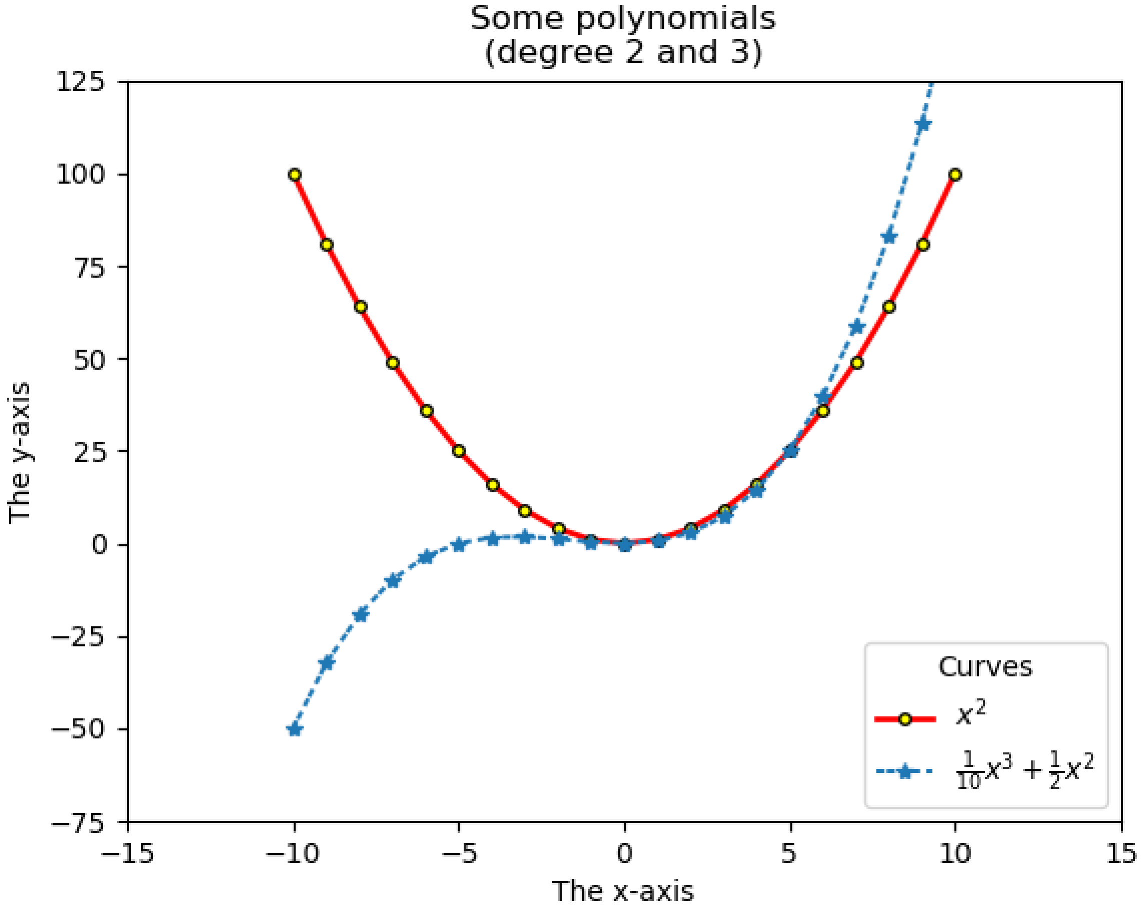
reset view

Plot – some keyword arguments

matplotlib-plot.py

```
import matplotlib.pyplot as plt
X = range(-10, 11)
Y1 = [x ** 2 for x in X]
Y2 = [x ** 3 / 10 + x ** 2 / 2 for x in X]
plt.plot(X, Y1, color='red', label='$x^2$',
    linestyle='-', linewidth=2,
    marker='o', markersize=4,
    markeredgewidth=1,
    markeredgecolor='black',
    markerfacecolor='yellow')
plt.plot(X, Y2, '*', dashes=(2, 0.5, 2, 1.5),
    label=r'$\frac{1}{10}x^3+\frac{1}{2}x^2$')
plt.xlim(-15, 15)
plt.ylim(-75, 125)
plt.title('Some polynomials\n(degree 2 and 3)')
plt.xlabel('The x-axis')
plt.ylabel('The y-axis')
plt.legend(title='Curves')
plt.show()  # finally show figure
```

LaTeX



matplotlib.org/api/_as_gen/matplotlib.pyplot.plot.html

Colors: matplotlib.org/gallery/color/named_colors.html

Scatter (points with individual size and color)

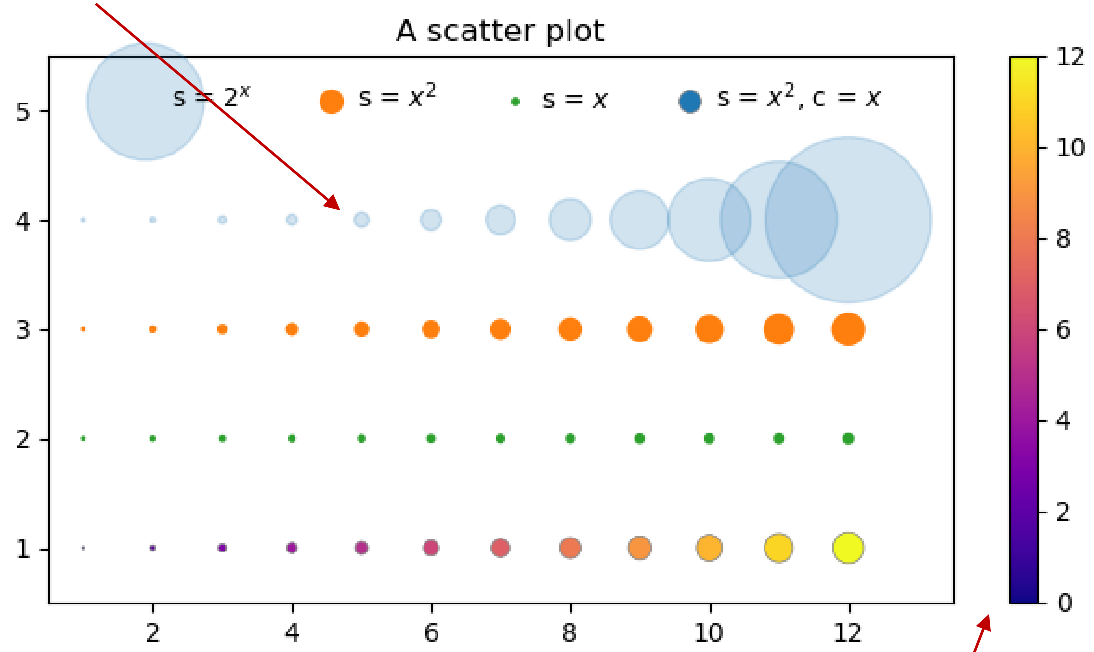
matplotlib-scatter.py

```python
import matplotlib.pyplot as plt

n = 13
X = range(n)
S = [x ** 2 for x in X]
E = [2 ** x for x in X]

plt.scatter(X, [4] * n, s=E, label='s = $2^x$', alpha=.2)
plt.scatter(X, [3] * n, s=S, label='s = $x^2$')
plt.scatter(X, [2] * n, s=X, label='s = $x$')
plt.scatter(X, [1] * n, s=S, c=X, cmap='plasma',
    label='s = $x^2$, c = $x$',
    edgecolors='gray', linewidth=0.5)
plt.colorbar()

plt.ylim(0.5, 5.5)
plt.xlim(0.5, 13.5)
plt.title('A scatter plot')
plt.legend(loc='upper center', frameon=False, ncol=4,
           handletextpad=0)
plt.show()
```

colormap (predefined)
color of each point
size ≈ area of each point
point boundary width
point boundary color

manual placement of legend box (default automatic); remove frame; place legends in 4 columns (default 1); reduce space between marks and label

transparency



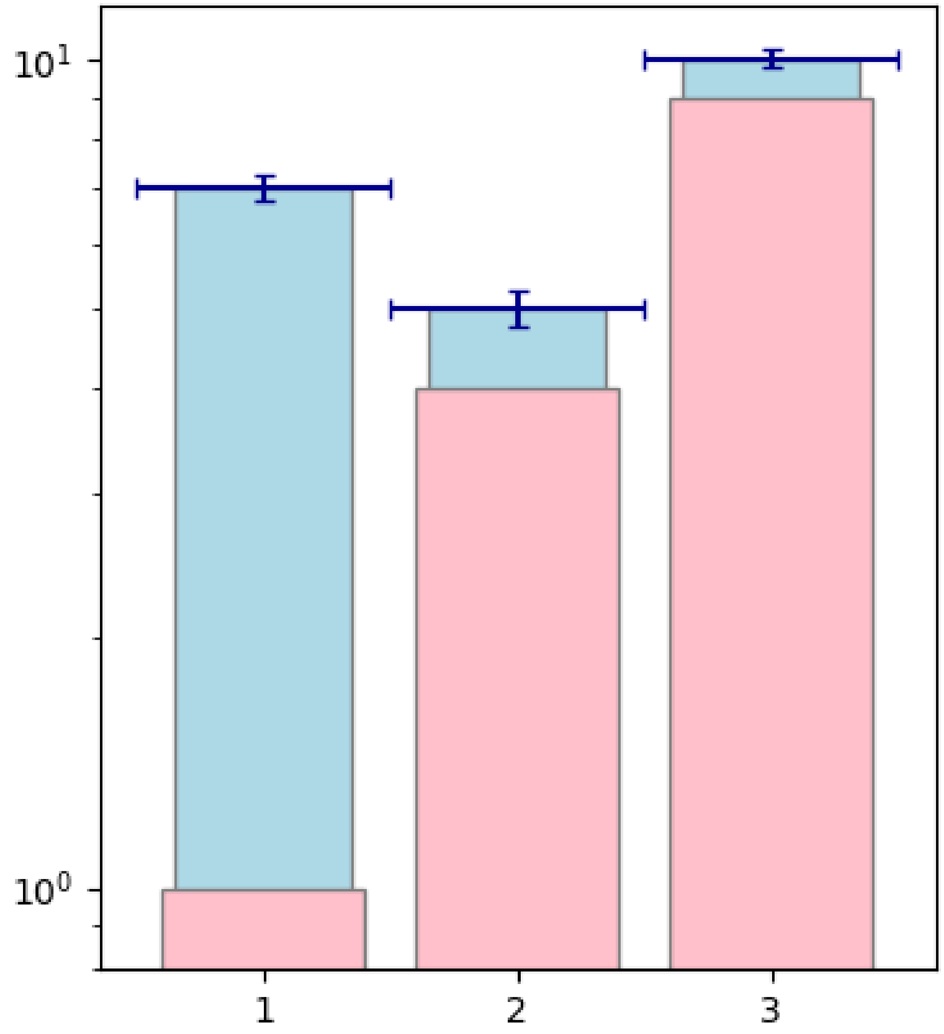
colorbar
(of most recently used colormap)

matplotlib.org/api/_as_gen/matplotlib.pyplot.scatter.html
matplotlib.org/tutorials/colors/colormaps.html

Bars

matplotlib-bars.py

```
import matplotlib.pyplot as plt

x = [1, 2, 3]
y = [7, 5, 10]

plt.bar(x, y,
        color='lightblue',    # bar background color
        linewidth=1,          # bar boundary width
        edgecolor='gray',     # bar boundary color
        tick_label=x,         # ticks on x-axis
        width=0.7,            # width, default 0.8
        yerr=0.25,            # Error bar: y length
        xerr=0.5,             #   x length
        capsize=3,            #   capsize in points
        ecolor='darkblue',    #   error bar color
        log=True)             # y-axis log scale
plt.bar(x, [v**2 for v in x],
        color='pink',
        linewidth=1,
        edgecolor='gray')

plt.show()
```

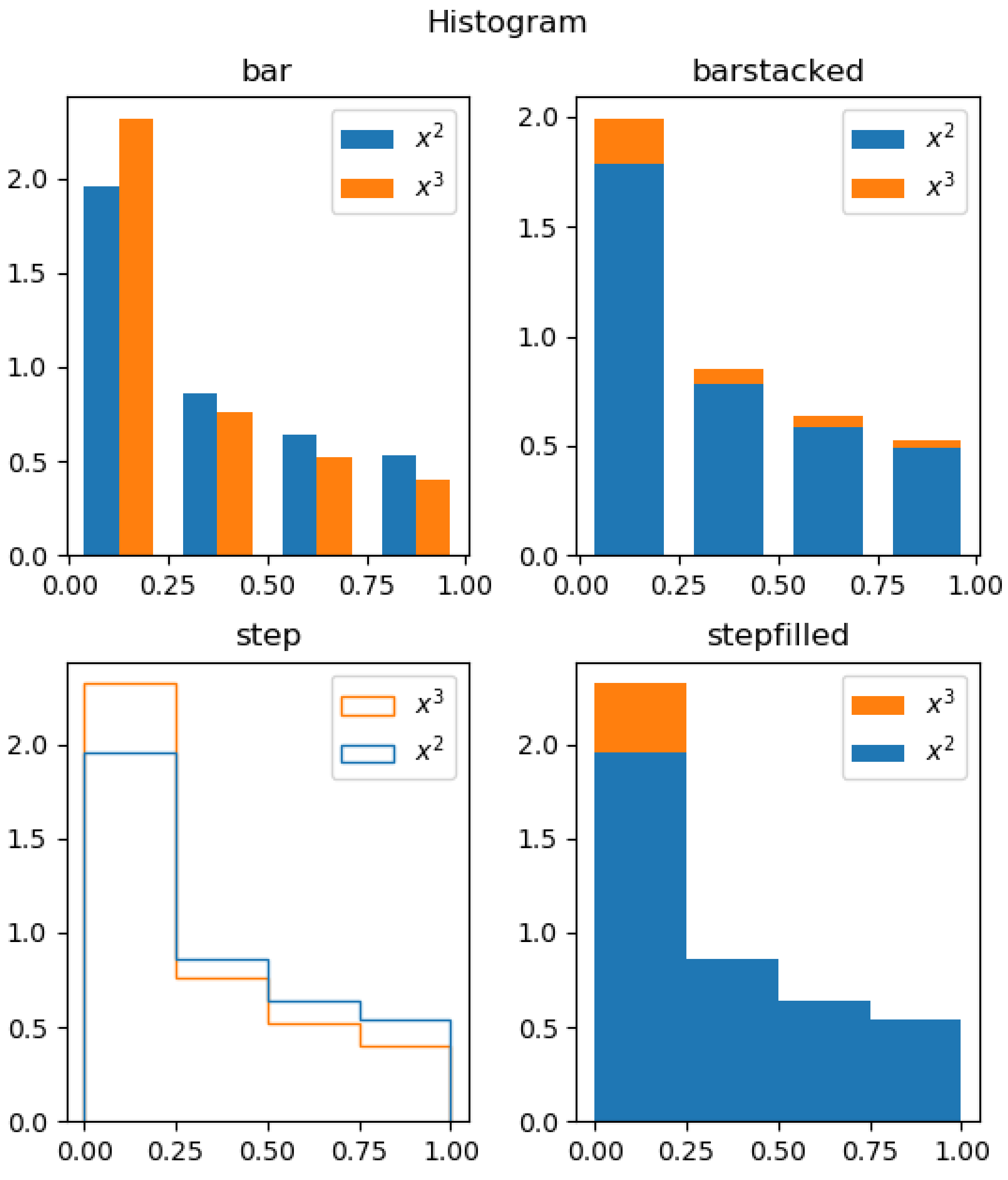


matplotlib.org/api/_as_gen/matplotlib.pyplot.bar.html

Histogram

matplotlib-histogram.py

```
import matplotlib.pyplot as plt
from random import random

values1 = [random()**2 for _ in range(1000)]
values2 = [random()**3 for _ in range(100)]

bins = [0.0, 0.25, 0.5, 0.75, 1.0]

for i, ht in enumerate(
        ['bar', 'barstacked', 'step', 'stepfilled'],
        start=1):
    plt.subplot(2, 2, i)  # start new plot
    plt.hist([values1, values2],  # data sets
             bins,          # bucket boundaries
             histtype=ht, # default ht='bar'
             rwidth=0.7,  # fraction of bucket width
             label=['$x^2$', '$x^3$'],  # labels
             density=True)  # norm. prob. density
    plt.title(ht)           # plot title
    plt.xticks(bins)        # ticks on x-axis
    plt.legend()

plt.suptitle('Histogram') # figure title
plt.show()
```

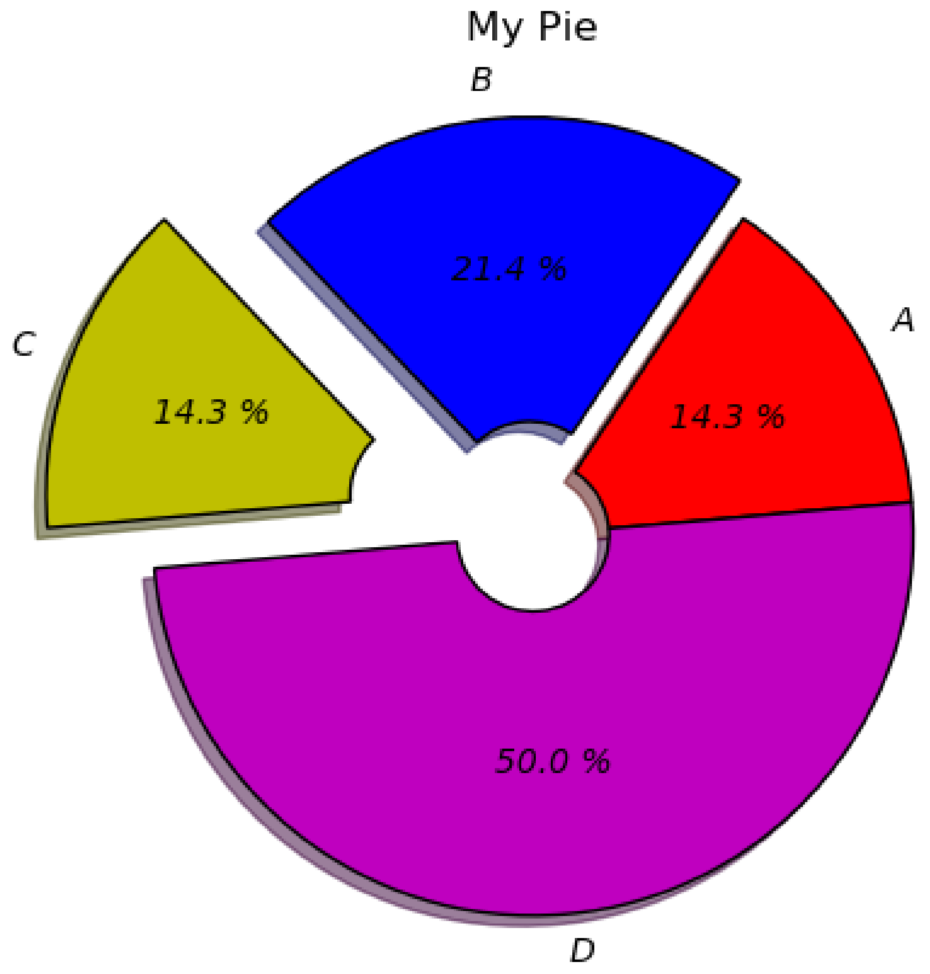


matplotlib.org/api/_as_gen/matplotlib.pyplot.hist.html

Pie

matplotlib-pie.py

```python
import matplotlib.pyplot as plt

plt.title('My Pie')
plt.pie([2, 3, 2, 7],          # relative wedge sizes
        labels=['A','B','C','D'],
        colors=['r', 'b', 'y', 'm'],
        explode=(0, 0.1, 0.3, 0),  # radius fraction
        startangle=5,          # angle above horizontal
        counterclock=True,     # default True
        rotatelabels=False,    # default False
        shadow=True,           # default False
        textprops=dict(        # text properties, dict
            color='black',     # text color
            style='italic'),   # text style
        wedgeprops=dict(       # wedge properties, dict
            width=0.8,         # width (missing center)
            linewidth=1,       # wedge boundary width
            edgecolor='black'),  # boundary color
        autopct='%1.1f %%')  # percent formatting

plt.show()
```



matplotlib.org/api/_as_gen/matplotlib.pyplot.pie.html

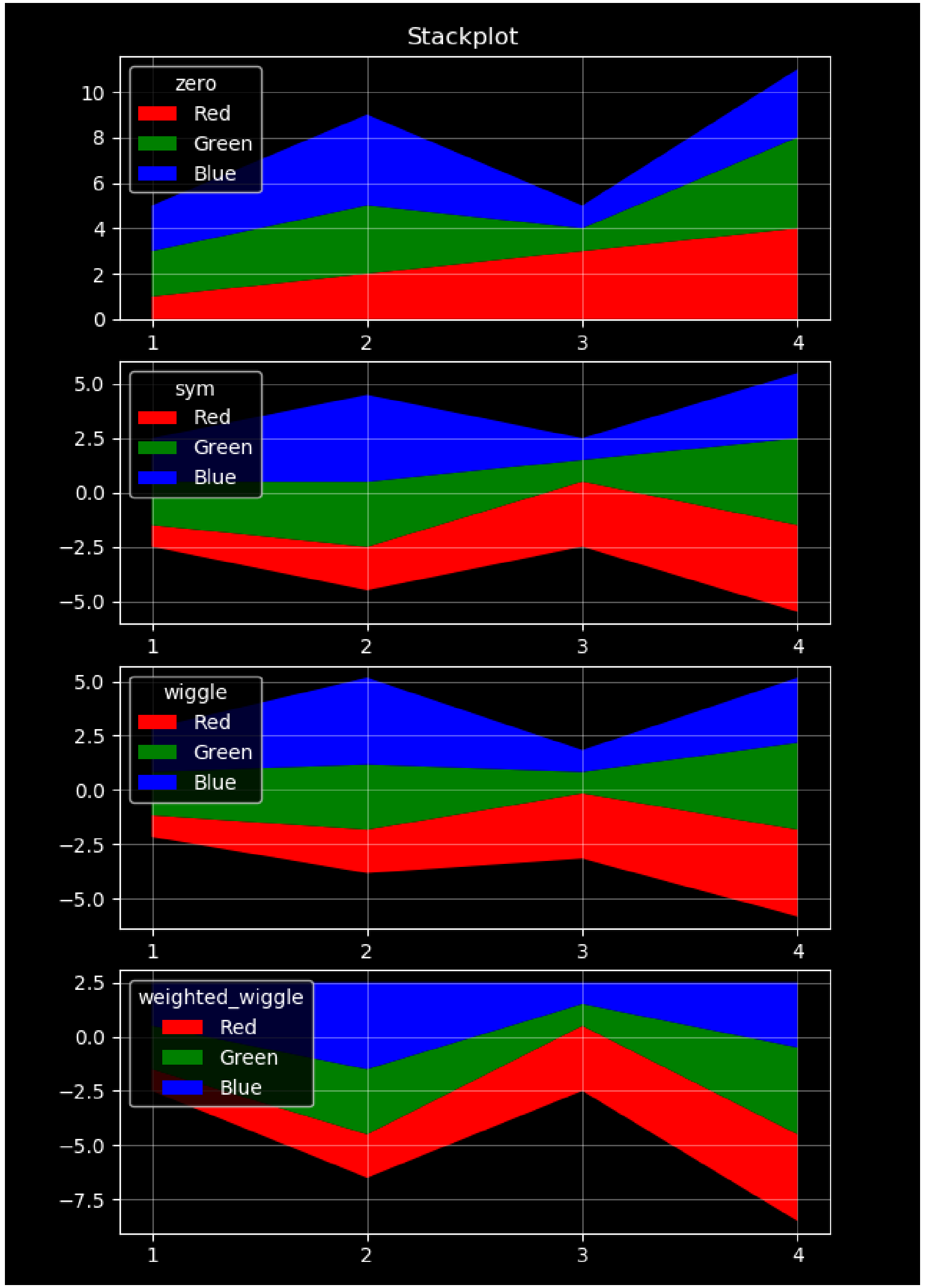
Stackplot

matplotlib-stackplot.py

```
import matplotlib.pyplot as plt
x = [1, 2, 3, 4]
y1 = [1, 2, 3, 4]
y2 = [2, 3, 1, 4]
y3 = [2, 4, 1, 3]
plt.style.use('dark_background')
for i, base in enumerate(
        ['zero', 'sym', 'wiggle','weighted_wiggle'],
        start=1):
    plt.subplot(4, 1, i)
    plt.stackplot(x, y1, y2, y3,
                  colors=['r', 'g', 'b'],
                  labels=['Red', 'Green', 'Blue'],
                  baseline=base)
    plt.grid(axis='both',  # 'x', 'y', or 'both'
        linewidth=0.5, linestyle='-', alpha=0.5)
    plt.legend(title=base, loc='upper left')
    plt.xticks(x)  # a tick for each value in x
plt.suptitle('Stackplot')
plt.show()
```

[Stacked Graphs – Geometry & Aesthetics](#)
Lee Byron & Martin Wattenberg, 2008

To list all available styles:
`print(plt.style.available)`



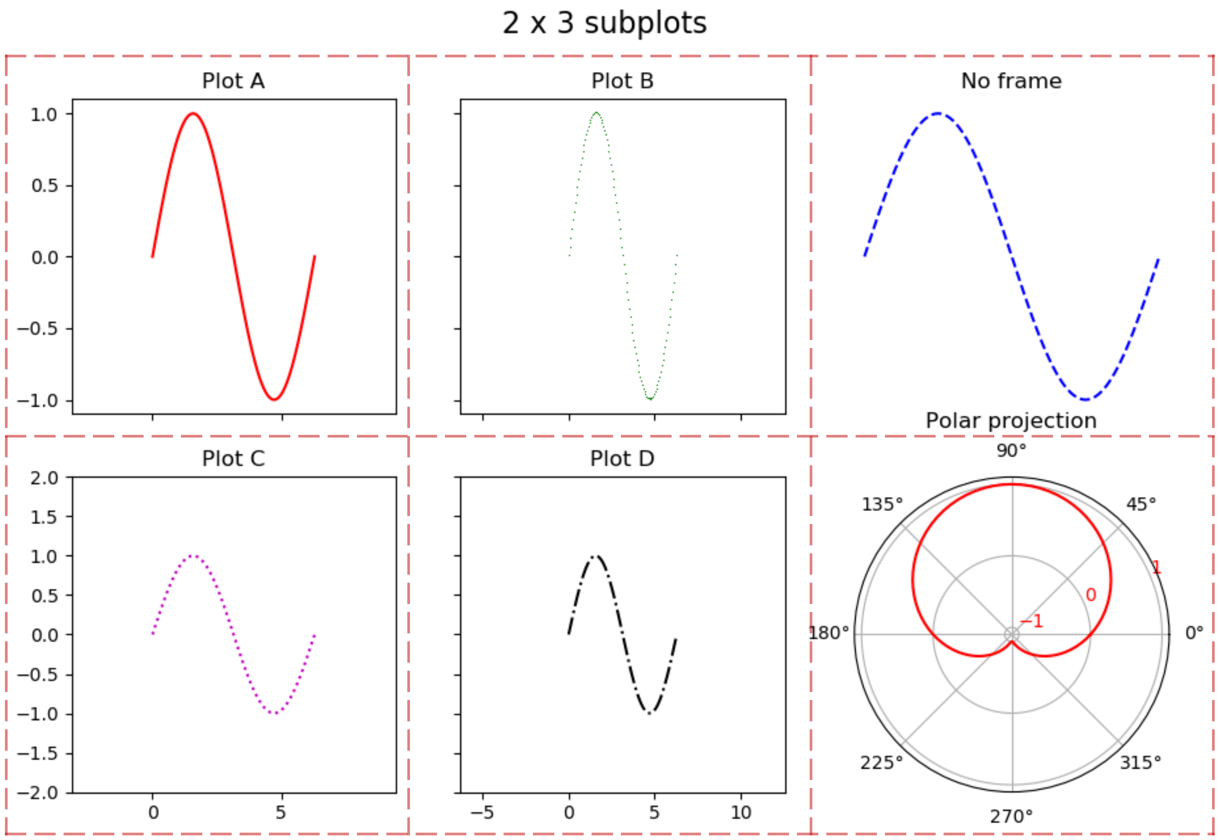
matplotlib.org/api/_as_gen/matplotlib.pyplot.stackplot.html

matplotlib-subplot.py

```python
import matplotlib.pyplot as plt
from math import pi, sin

x_min, x_max, n = 0, 2 * pi, 100
x = [x_min + (x_max - x_min) * i / n for i in range(n + 1)]
y = [sin(v) for v in x]

ax1 = plt.subplot(2, 3, 1)  # 2 rows, 3 columns
ax1.label_outer()           # removes x-axis labels
plt.xlim(-pi, 3 * pi)       # increase x-axis range
plt.plot(x, y, 'r-')
plt.title('Plot A')

ax2 = plt.subplot(2, 3, 2)
ax2.label_outer()           # removes x- and y-axis labels
plt.xlim(-2 * pi, 4 * pi)   # increase x-axis range
plt.plot(x, y, 'g,')
plt.title('Plot B')

ax3 = plt.subplot(2, 3, 3, frameon=False)  # remove frame
ax3.set_xticks([])          # remove x-axis ticks & labels
ax3.set_yticks([])          # remove x-axis ticks & labels
plt.plot(x, y, 'b--')
plt.title('No frame')

ax4 = plt.subplot(2, 3, 4, sharex=ax1)  # share x-axis range
plt.ylim(-2, 2)             # increase y-axis range
plt.plot(x, y, 'm:')
plt.title('Plot C')

ax5 = plt.subplot(2, 3, 5, sharex=ax2, sharey=ax4) # share ranges
ax5.set_xticks(range(-5, 15, 5))  # specific x-ticks & x-labels
ax5.label_outer()           # removes y-axis labels
plt.plot(x, y, 'k-.')
plt.title('Plot D')

ax6 = plt.subplot(2, 3, 6, projection='polar')  # polar projection
ax6.set_yticks([-1, 0, 1])                       # y-labels
ax6.tick_params(axis='y', labelcolor='red')      # color of y-labels
plt.plot(x, y, 'r')
plt.title('Polar projection\n')  # \n to avoid overlap with 90°

plt.suptitle('2 x 3 subplots', fontsize=16)

plt.show()
```

Subplot

(2 rows, 3 columns)



- Subplots are numbered 1..6 row-by-row, starting top-left
- subplot returns an axes to access the plot in the figure

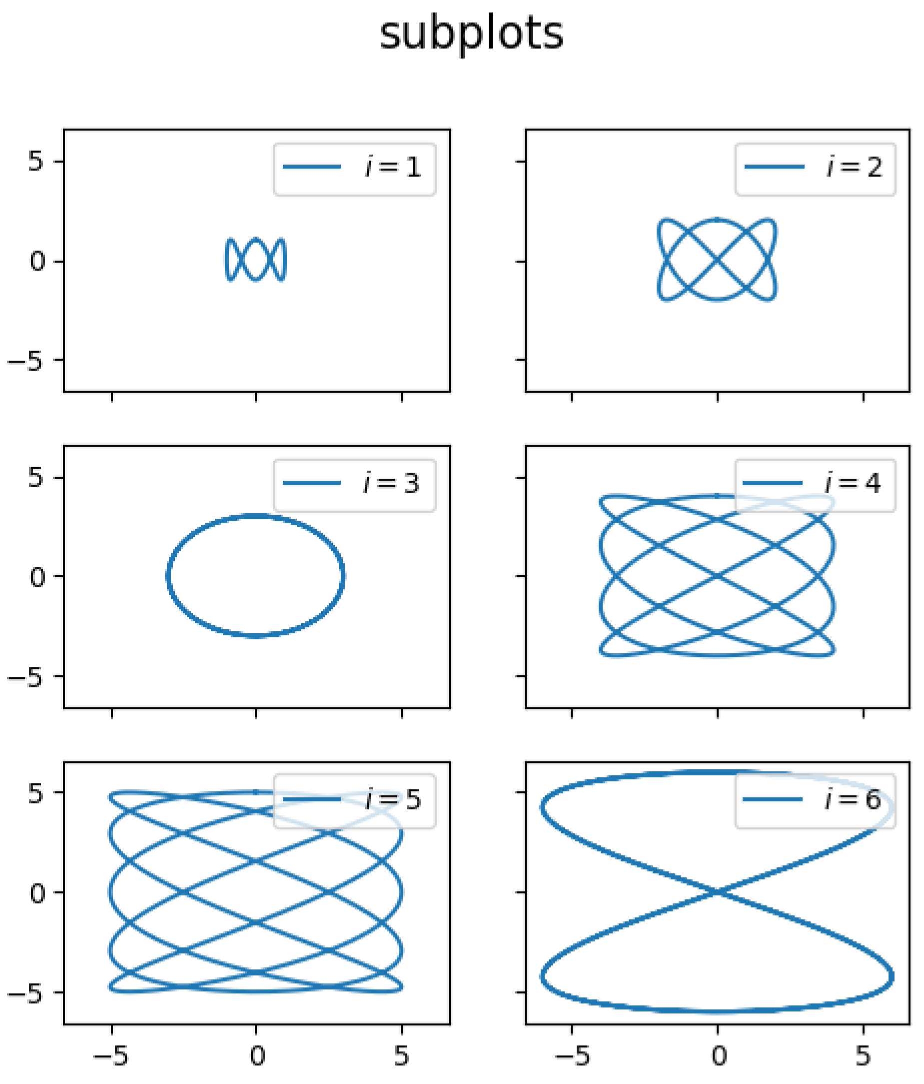
matplotlib.org/api/_as_gen/matplotlib.pyplot.subplot.html

Subplots

matplotlib-subplots.py

```
import matplotlib.pyplot as plt
from math import pi, sin, cos
times = [2 * pi * t / 1000 for t in range(1001)]

fig, ((ax1, ax2), (ax3, ax4), (ax5, ax6)) = \
    plt.subplots(3, 2, sharex=True, sharey=True)

for i, ax in enumerate([ax1, ax2, ax3, ax4, ax5, ax6],
                       start=1):
    x = [i * sin(i * t) for t in times]
    y = [i * cos(3 * t) for t in times]
    ax.plot(x, y, label=f'$i = {i}$')   # plot to axes
    ax.legend(loc='upper right')        # axes legend
fig.suptitle('subplots', fontsize=16)   # figure title
plt.show()
```

create 6 axes in 3 rows with 2 colums
share the x- and y-axis ranges (automatically applies label_outer to created axes)
returns a pair (figure, axes)



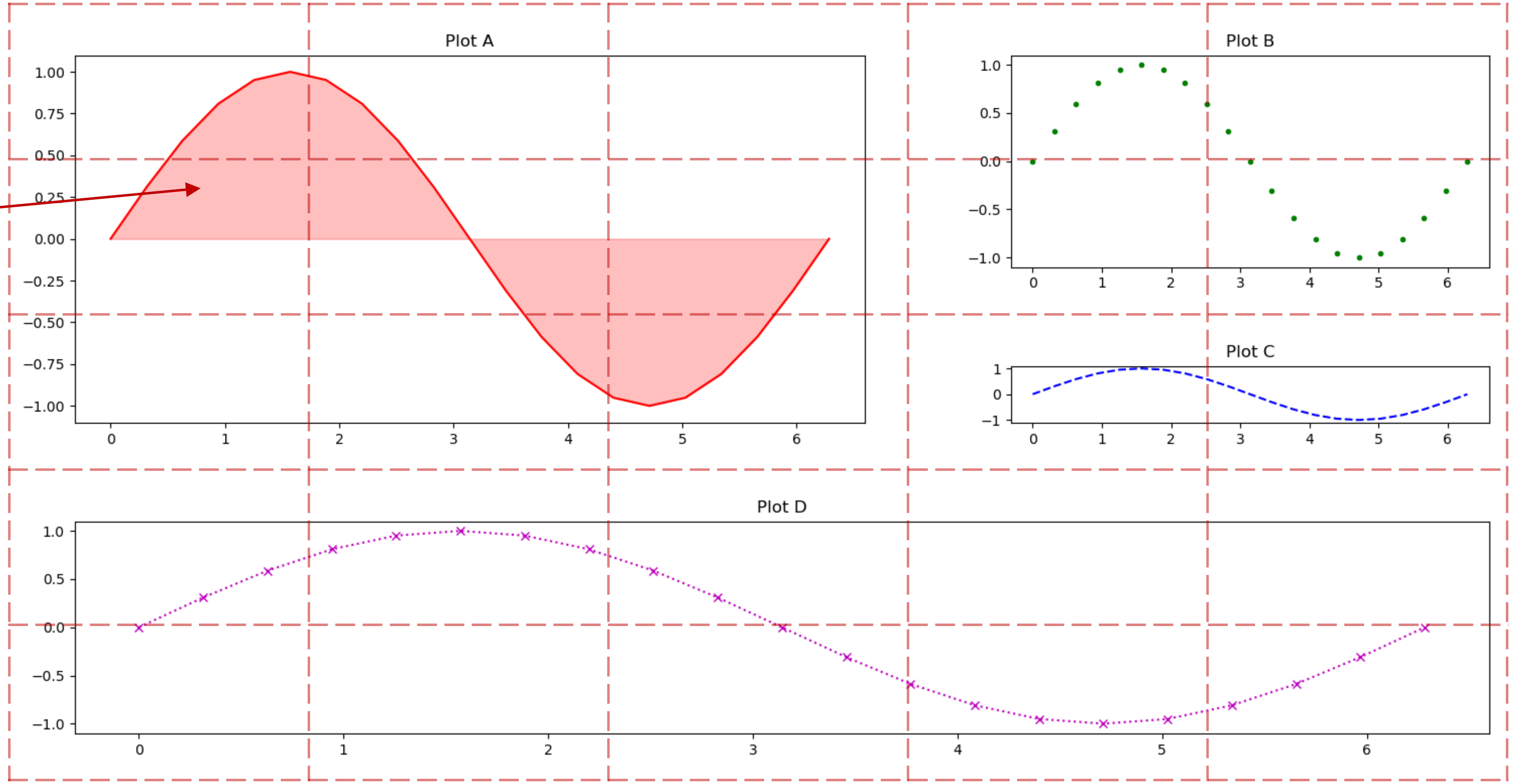
matplotlib.org/api/_as_gen/matplotlib.pyplot.subplots.html

```
matplotlib-subplot2grid.py
```

```python
import matplotlib.pyplot as plt
import math

x_min, x_max, n = 0, 2 * math.pi, 20

x = [x_min + (x_max - x_min) * i / n
     for i in range(n + 1)]
y = [math.sin(v) for v in x]

plt.subplot2grid((5, 5), (0,0),
                 rowspan=3, colspan=3)
plt.fill_between(x, 0.0, y,
                 alpha=0.25, color='r')
plt.plot(x, y, 'r-')
plt.title('Plot A')

plt.subplot2grid((5, 5), (0,3),
                 rowspan=2, colspan=2)
plt.plot(x, y, 'g.')
plt.title('Plot B')

plt.subplot2grid((5, 5), (2,3),
                 rowspan=1, colspan=2)
plt.plot(x, y, 'b--')
plt.title('Plot C')

plt.subplot2grid((5, 5), (3,0),
                 rowspan=2, colspan=5)
plt.plot(x, y, 'mx:')
plt.title('Plot D')

plt.tight_layout()  # adjust padding
plt.show()
```

subplot2grid (5 x 5)

upper left corner (row, column)

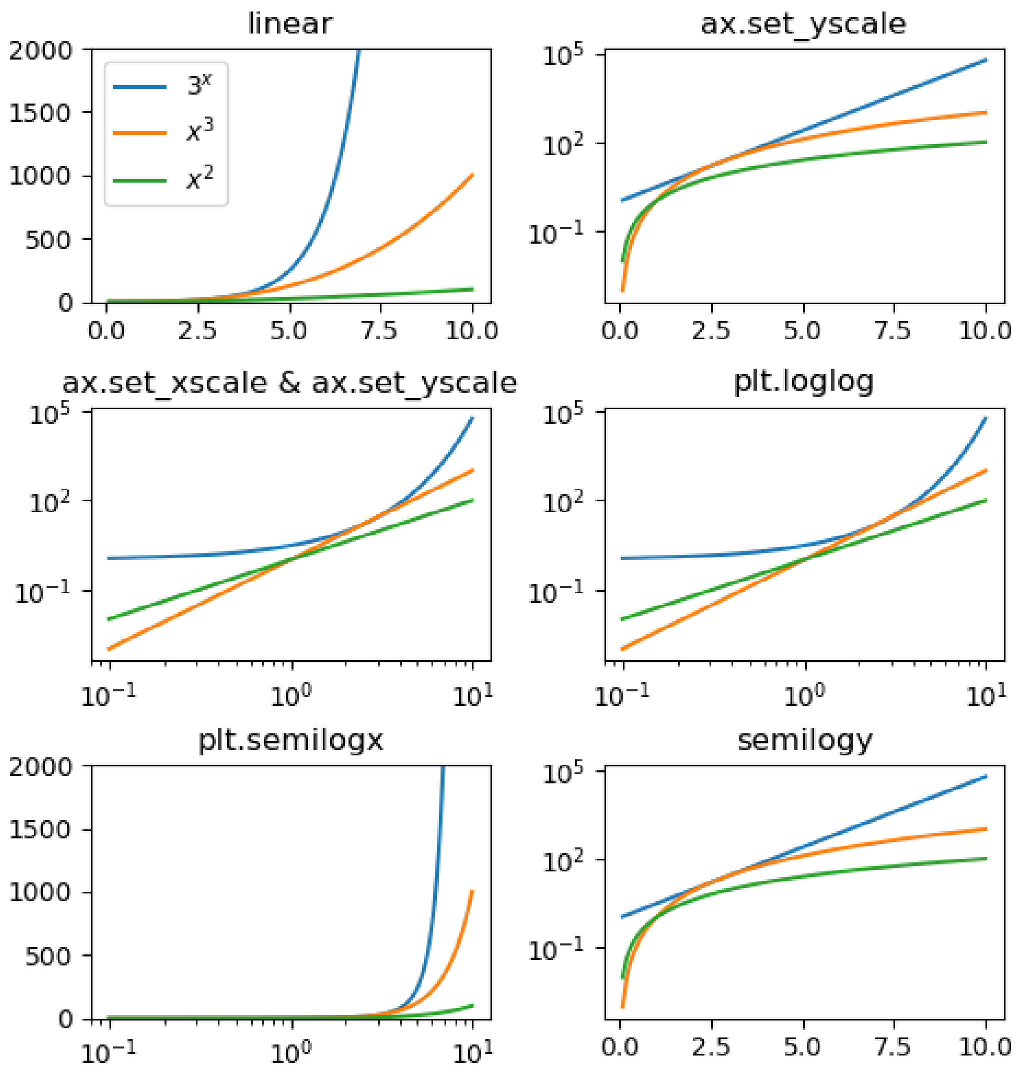


matplotlib.org/api/_as_gen/matplotlib.pyplot.subplot2grid.html

log scales

matplotlib-log.py

```
import matplotlib.pyplot as plt
x = [i / 10 for i in range(1, 101)]
y1 = [i ** 2 for i in x]
y2 = [i ** 3 for i in x]
y3 = [3 ** i for i in x]
for i in range(1, 7):
    ax = plt.subplot(3, 2, i)
    plt.plot(x, y3, label='$3^x$')
    plt.plot(x, y2, label='$x^3$')
    plt.plot(x, y1, label='$x^2$')
    if i == 1:
        plt.ylim(0, 2000)
        plt.xscale('linear')  # default
        plt.yscale('linear')  # default
        plt.legend()
        plt.title('linear')
    if i == 2:
        plt.yscale('log')
        plt.title('ax.yscale')
    if i == 3:
        ax.set_xscale('log')
        ax.set_yscale('log')
        plt.title('ax.set_xscale & ax.set_yscale')
    if i == 4:
        plt.loglog()
        plt.title('plt.loglog')
    if i == 5:
        plt.ylim(0, 2000)
        plt.semilogx()
        plt.title('plt.semilogx')
    if i == 6:
        plt.semilogy()
        plt.title('semilogy')
plt.show()
```

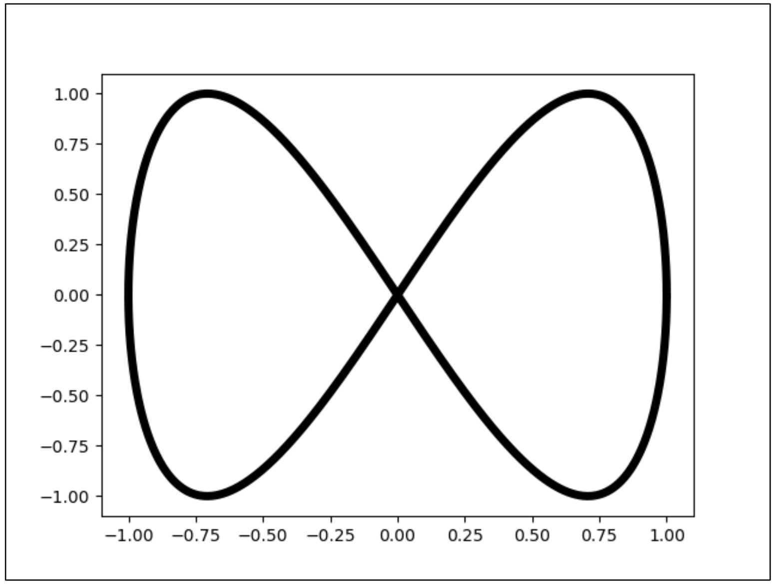


- There are many ways to make the x- and/or y-axis logarithmic with pyplot

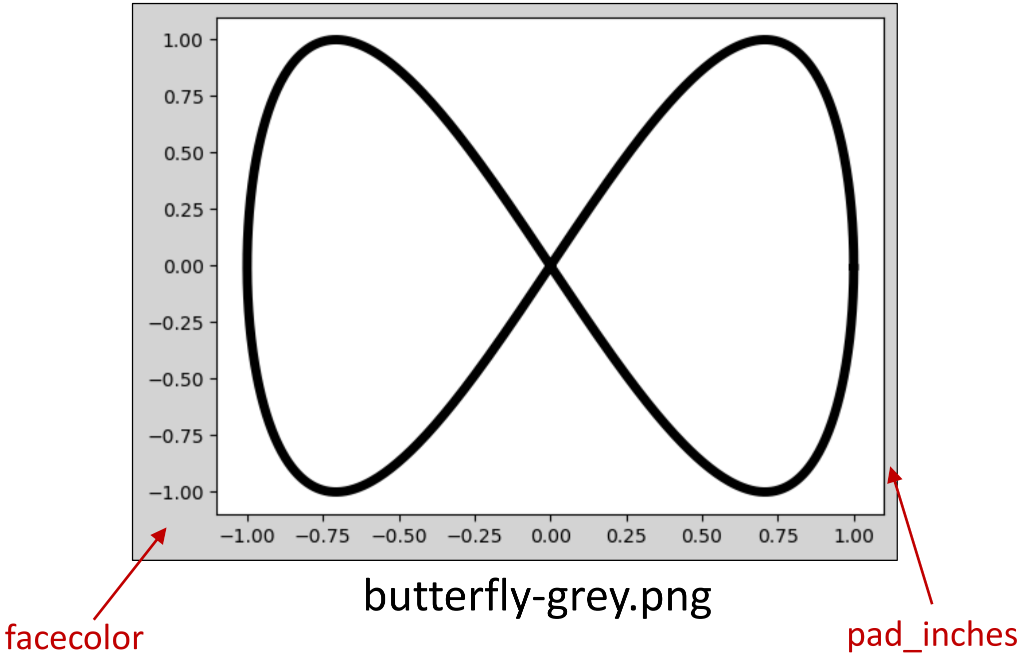
Saving figures

matplotlib-savefig.py

```
import matplotlib.pyplot as plt
from math import pi, sin, cos

n = 1000
points = [(cos(2 * pi * i / n),
           sin(4 * pi * i / n)) for i in range(n)]
x, y = zip(*points)
plt.plot(x, y, 'k-', linewidth=5)

plt.savefig('butterfly.png')  # save plot as PNG

plt.savefig('butterfly-grey.png',
    dpi=100,                  # dots per inch
    bbox_inches='tight',      # crop to bounding box
    pad_inches=0.1,           # space around figure
    facecolor='lightgrey',    # background color
    format='png')             # optional if file extension

plt.savefig('butterfly.pdf')  # save plot as PDF

plt.show()                    # interactive viewer
```



butterfly.png



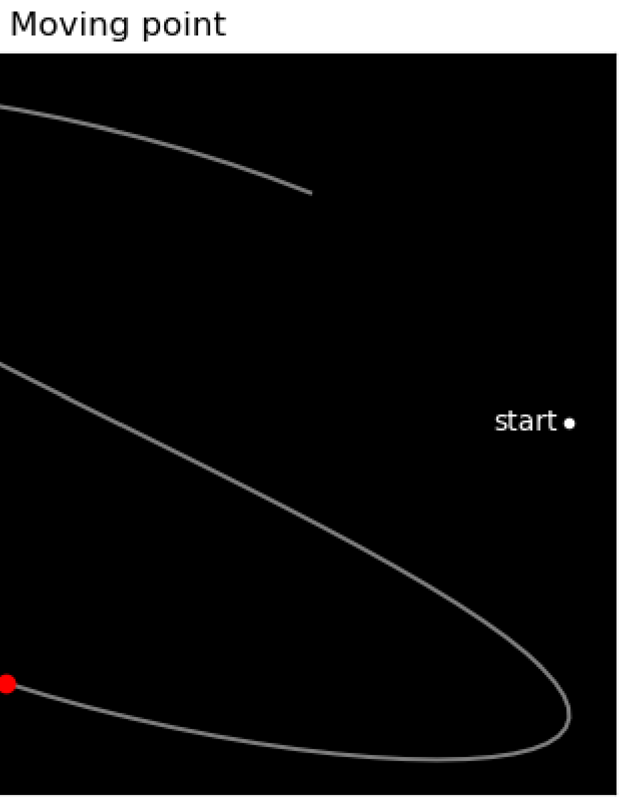
butterfly-grey.png

matplotlib.org/api/_as_gen/matplotlib.pyplot.savefig.html

matplotlib-animation.py

```
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from math import pi, cos, sin

n, tail_length = 200, 75
points = []                  # tail_length recent poins

def point(i):
    t = 2 * pi * i / n
    return (cos(3 * t), sin(2 * t))

fig = plt.figure()           # new figure
ax = plt.gca()               # get current axes
ax.set_facecolor('black')    # set background color
plt.xlim(-1.1, 1.1)          # set x-axis range
plt.ylim(-1.1, 1.1)          # set y-axis range
plt.xticks([])               # remove x-ticks & labels
plt.yticks([])               # remove y-ticks & labels
plt.title('Moving point')    # plot title

x, y = point(0)
plt.plot(x, y, 'w.')         # start point
plt.text(x - 0.025, y, 'start', color='w', # text label
    ha='right', va='center')               # alignment
tail, = plt.plot([], [], 'w-', alpha=0.5)  # init. tail
head, = plt.plot([], [], 'ro')     # init. current point

def move(frame):        # frame = value from frames
    points.append(point(frame))
    del points[:-tail_length]      # limit tail
    tail.set_data(*zip(*points))   # update tail points
    head.set_data(*points[-1])     # update head point

animation = FuncAnimation(fig,     # figure to animate
    func=move,          # function called for each frame
    frames=range(n),    # array like to iterate over
    interval=25,        # milliseconds between frames
    repeat=True,        # repeat frames when done
    repeat_delay=0)     # wait milliseconds before repeat

plt.show()
```

matplotlib.animation.FuncAnimation



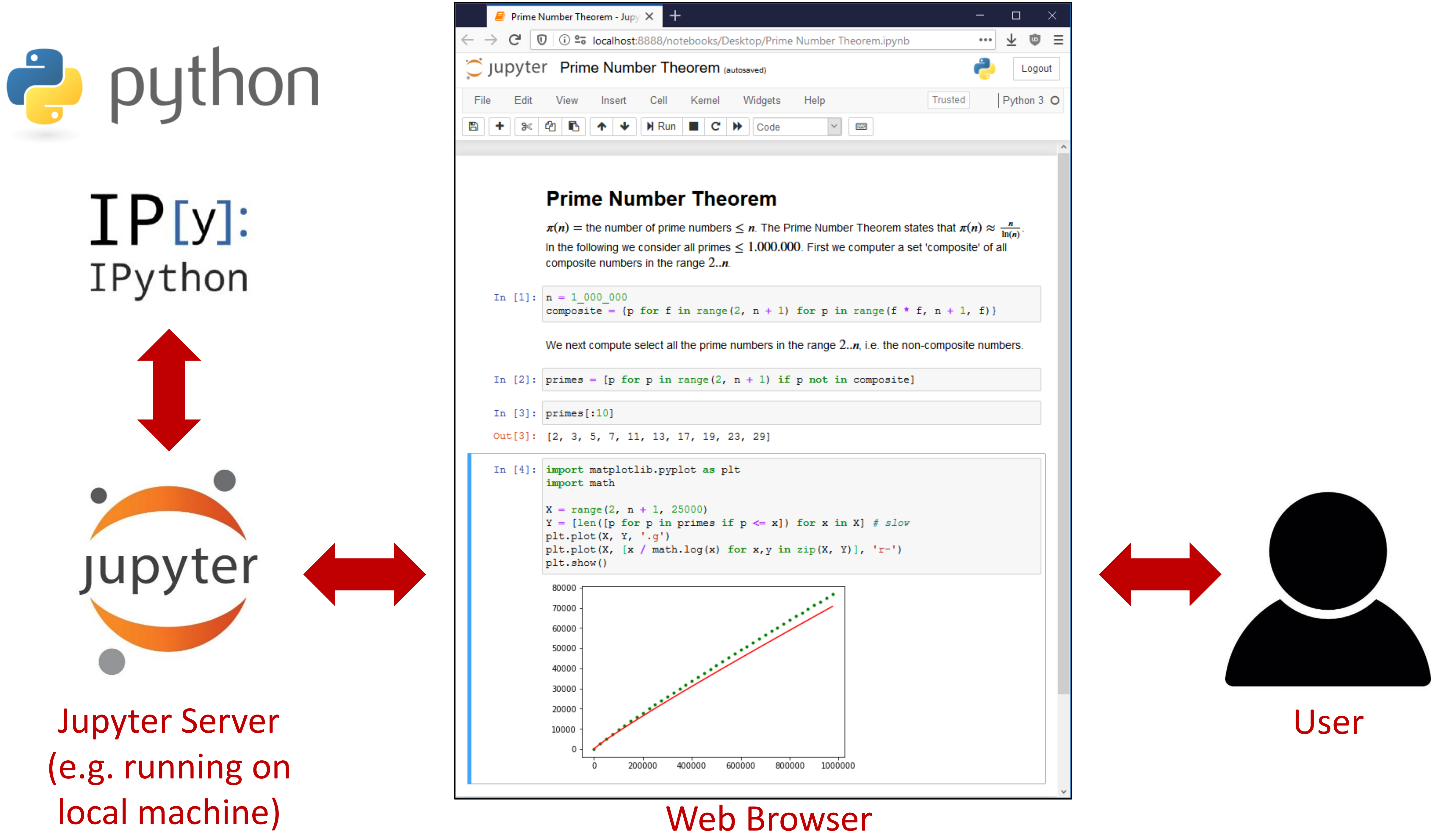
- `plot` returns "Line2D" objects representing the plotted data
- "Line2D" objects can be updated using `set_data`
- To make an animation you need to repeatedly update the "line2D" objects
- `FuncAnimation` repeatedly calls func in regular intervals interval, each time with the next value from frames (if frames is None, then the frame values provided to func will be the infinite sequence 0,1,2,3,...)

matplotlib.org/api/_as_gen/matplotlib.animation.FuncAnimation.html

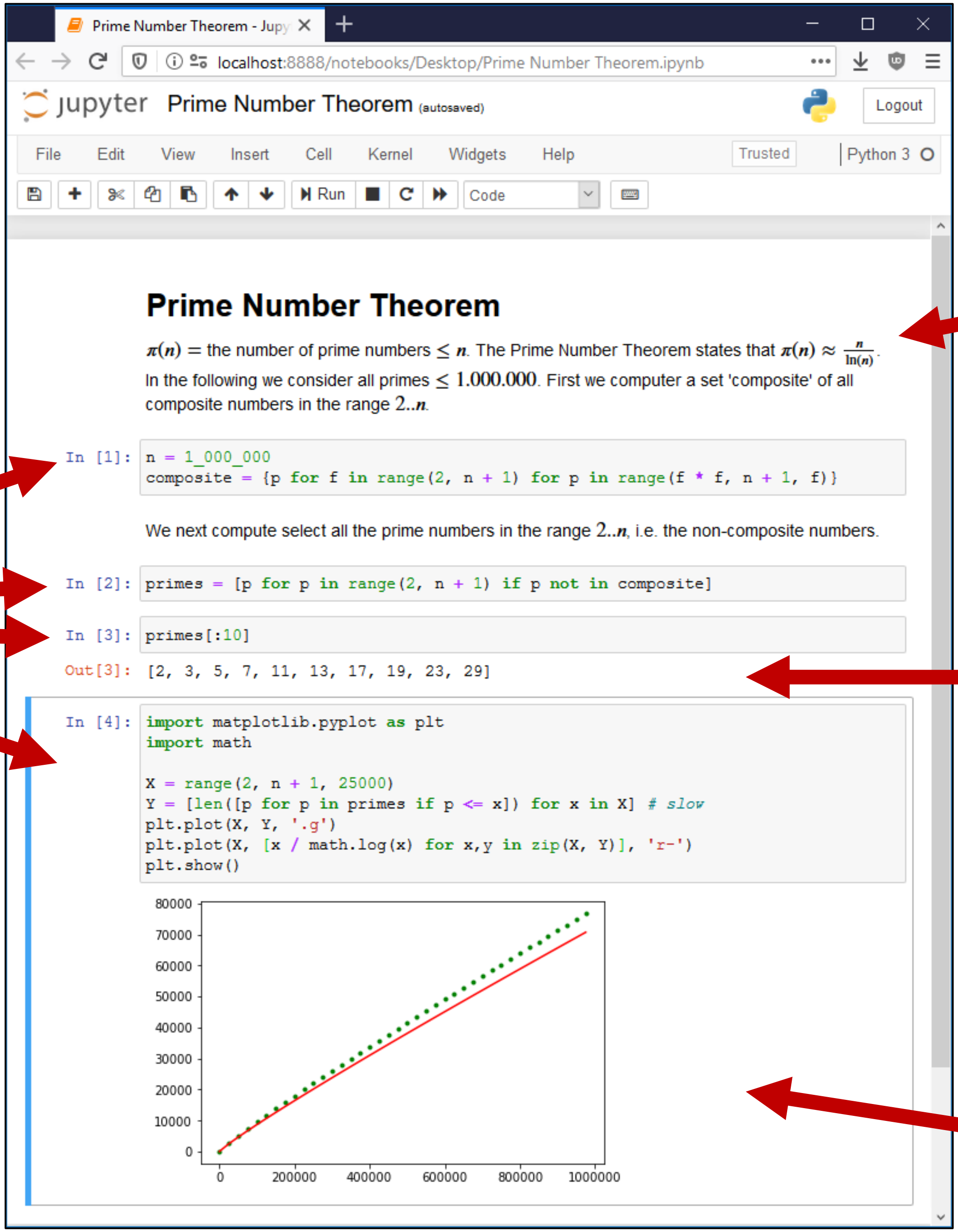
The Jupyter Notebook

The Jupyter Notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations and narrative text. Uses include: data cleaning and transformation, numerical simulation, statistical modeling, data visualization, machine learning, and much more.

jupyter.org


python
IP[y]:
IPython
jupyter
Jupyter Server
(e.g. running on
local machine)
Prime Number Theorem - Jupy
localhost:8888/notebooks/Desktop/Prime Number Theorem.ipynb
jupyter Prime Number Theorem (autosaved)
Logout
File Edit View Insert Cell Kernel Widgets Help
Trusted
Python 3
Run
Code
Prime Number Theorem
π(n) = the number of prime numbers ≤ n. The Prime Number Theorem states that π(n) ≈ n/ln(n).
In the following we consider all primes ≤ 1.000.000. First we computer a set 'composite' of all composite numbers in the range 2..n.
In [1]: n = 1_000_000
composite = {p for f in range(2, n + 1) for p in range(f * f, n + 1, f)}
We next compute select all the prime numbers in the range 2..n, i.e. the non-composite numbers.
In [2]: primes = [p for p in range(2, n + 1) if p not in composite]
In [3]: primes[:10]
Out[3]: [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
In [4]: import matplotlib.pyplot as plt
import math
X = range(2, n + 1, 25000)
Y = [len([p for p in primes if p <= x]) for x in X] # slow
plt.plot(X, Y, '.g')
plt.plot(X, [x / math.log(x) for x,y in zip(X, Y)], 'r-')
plt.show()
80000
70000
60000
50000
40000
30000
20000
10000
0
0
200000
400000
600000
800000
1000000
Web Browser
User

cells



formatted text: Markdown / LaTeX / HTML / ...

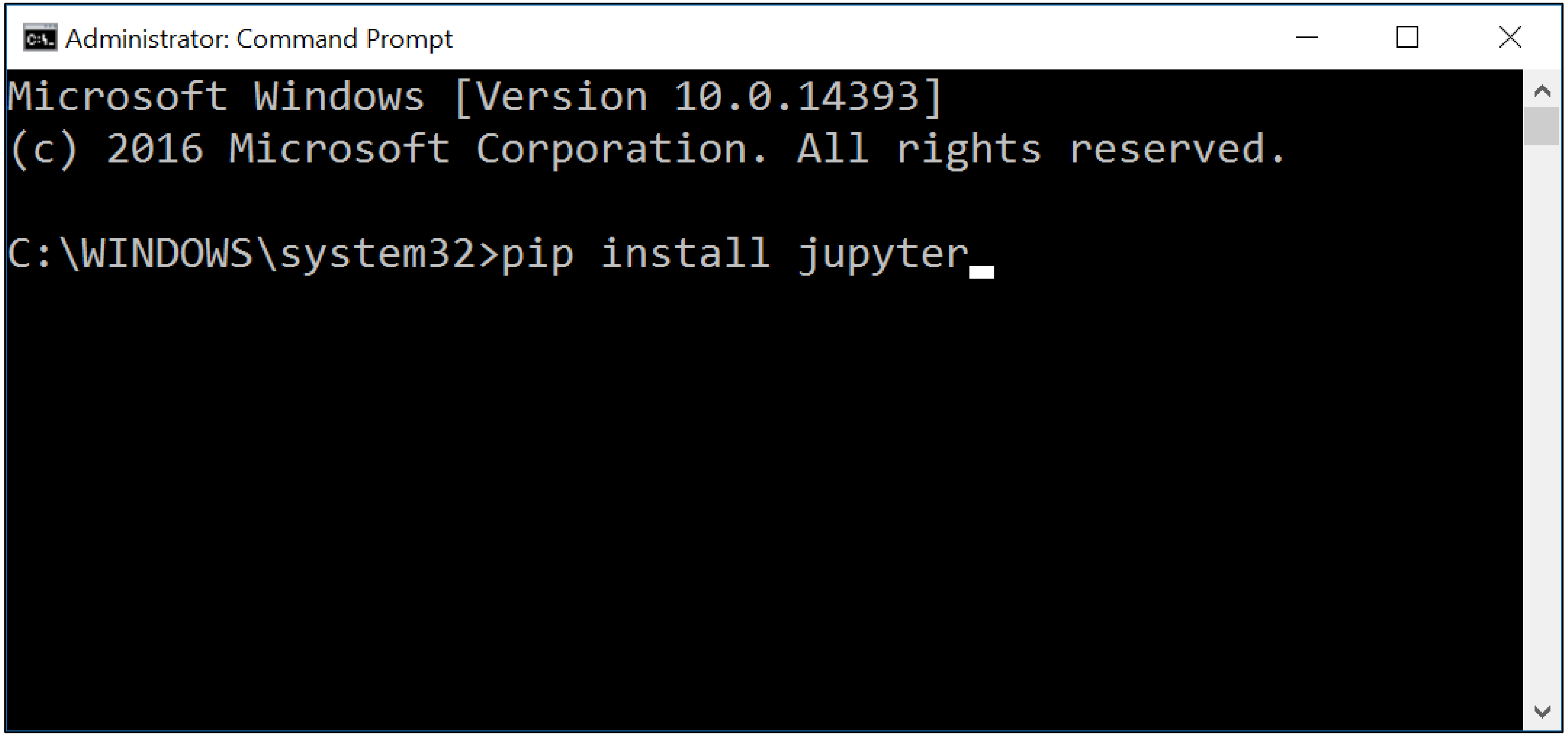
python code

python shell output

matplotlib / numpy / ... output

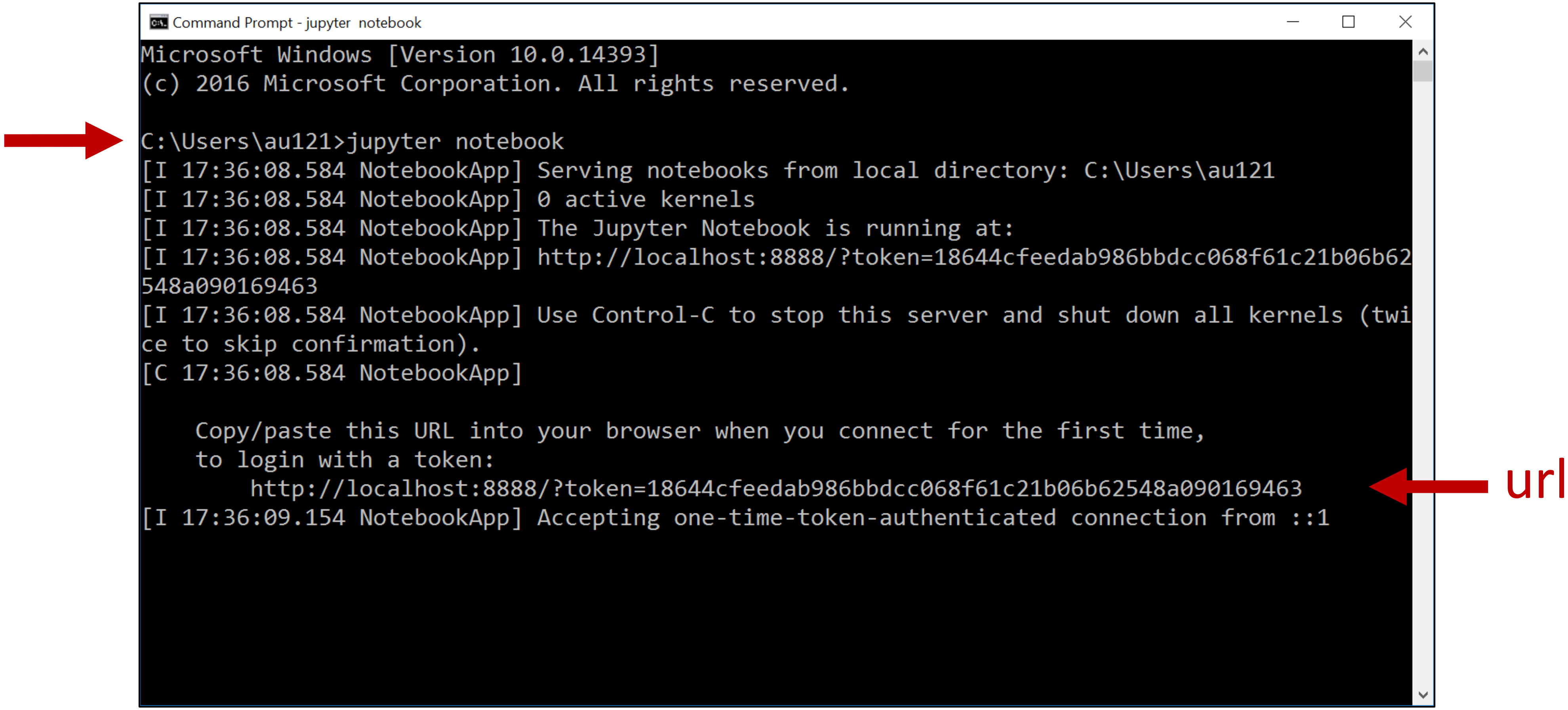
Jupyter - installing

- Open a windows shell and run: `pip install jupyter`

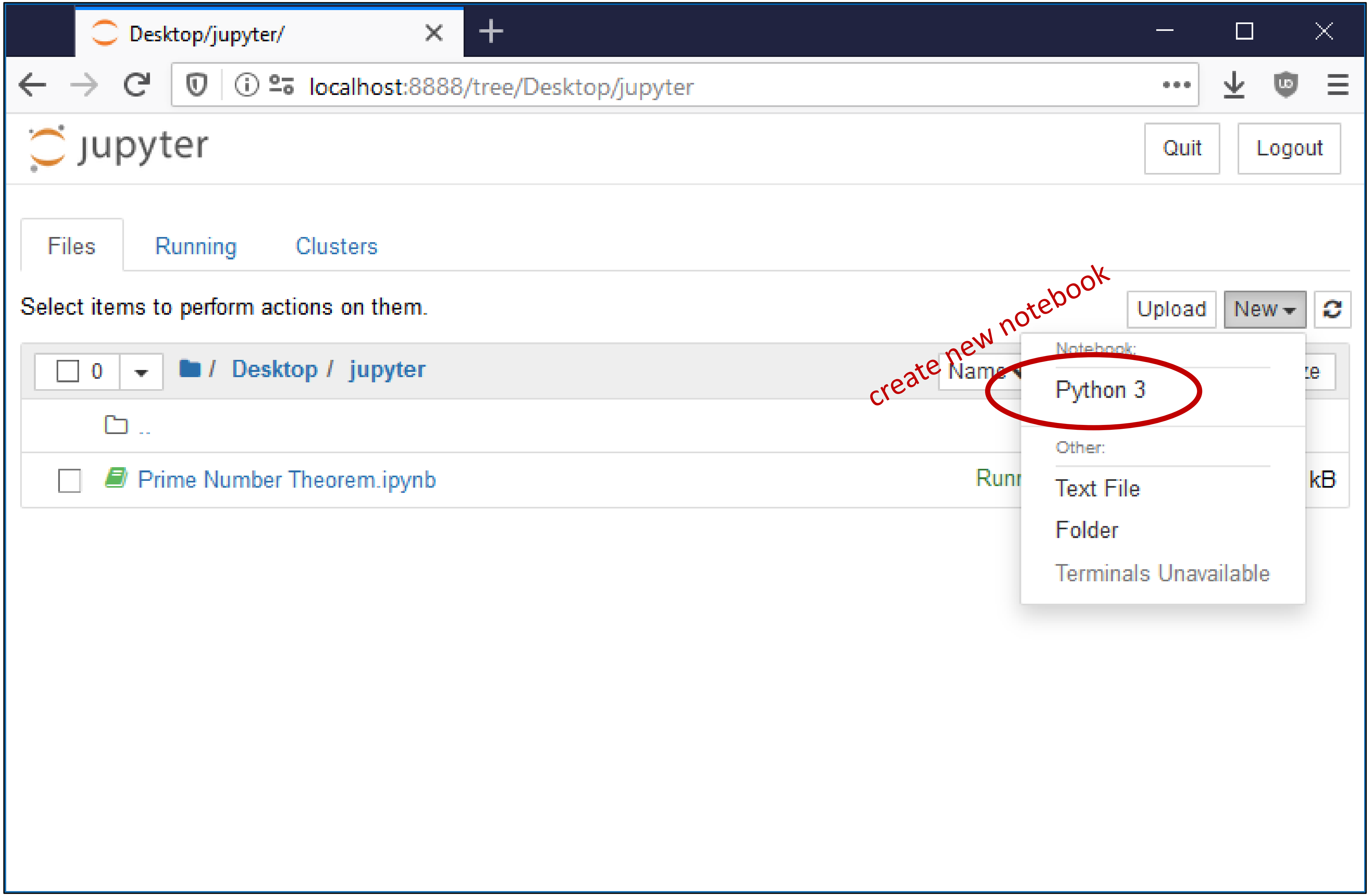


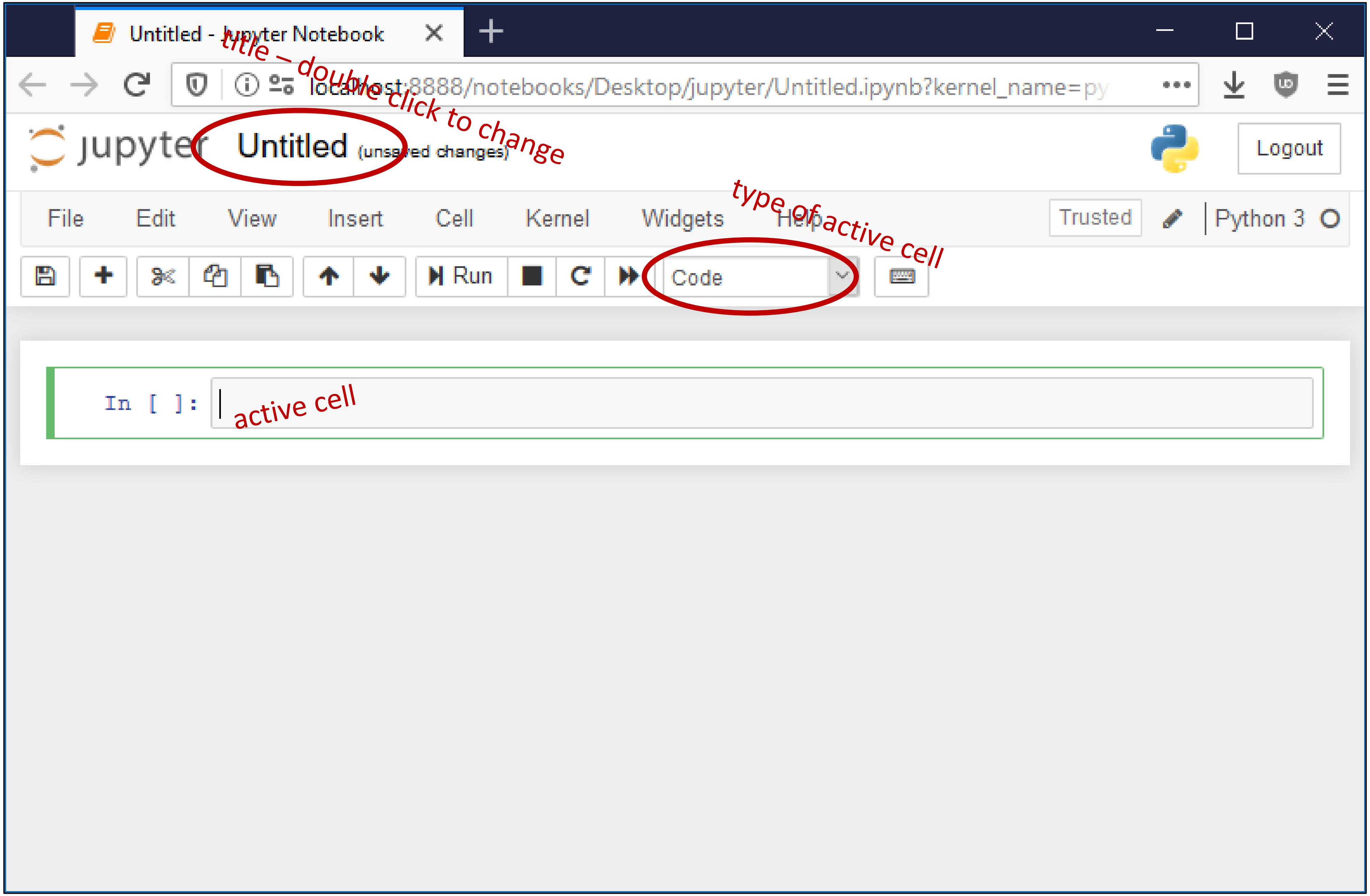
Jupyter – launching the jupyter server

- Open a windows shell and run: `jupyter notebook`



- If this does not work, then try `python -m notebook`


Desktop/jupyter/
localhost:8888/tree/Desktop/jupyter
jupyter
Quit
Logout
Files
Running
Clusters
Select items to perform actions on them.
Upload
New
create new notebook
Notebook:
Python 3
Other:
Text File
Folder
Terminals Unavailable
0
Desktop / jupyter
..
Prime Number Theorem.ipynb
kB

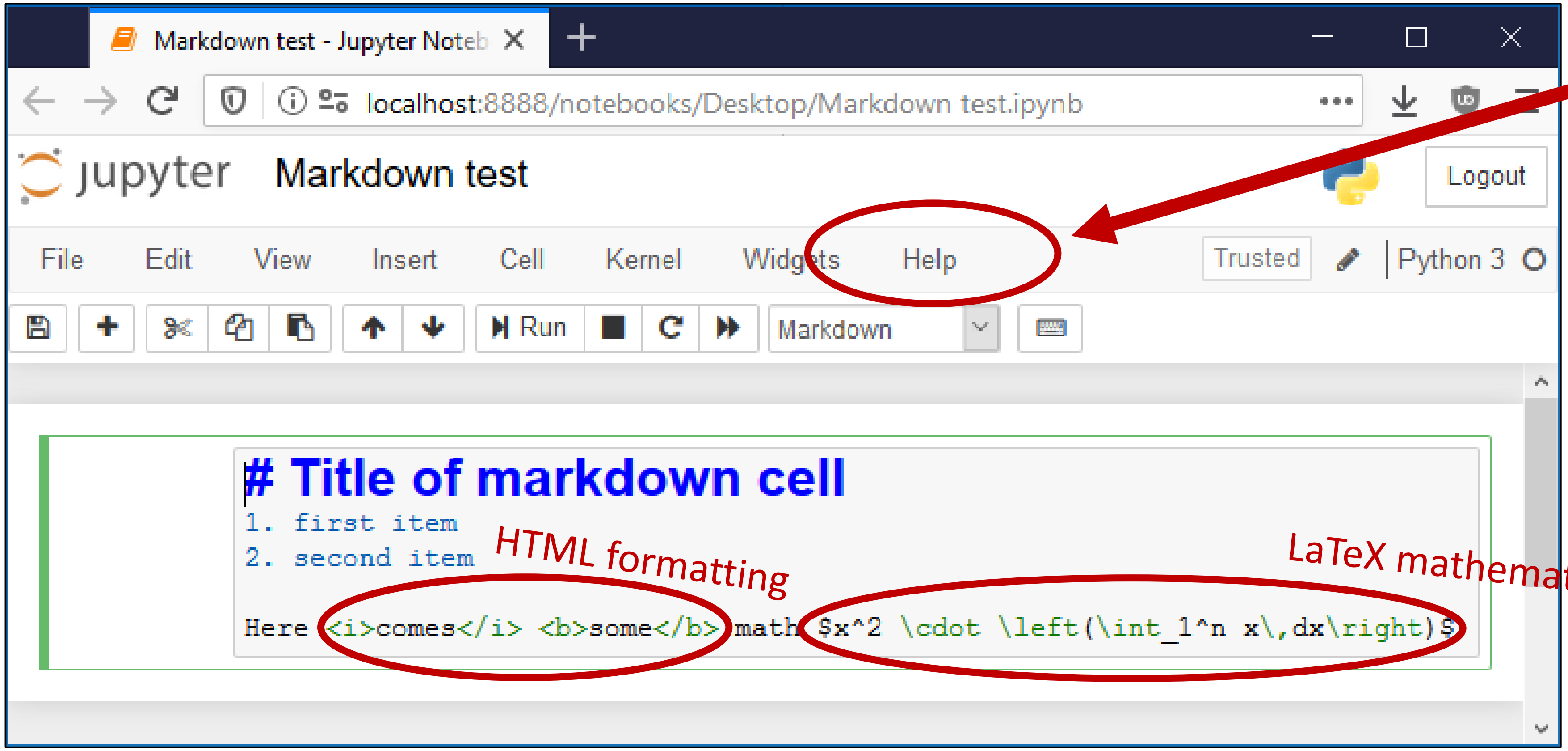


Untitled - Jupyter Notebook
localhost:8888/notebooks/Desktop/jupyter/Untitled.ipynb?kernel_name=py
jupyter
Untitled (unsaved changes)
title – double click to change
Logout
File
Edit
View
Insert
Cell
Kernel
Widgets
Help
Trusted
Python 3
type of active cell
Run
Code
In [ ]:
active cell

Try:

Help > User Interface Tour

Help > Markdown



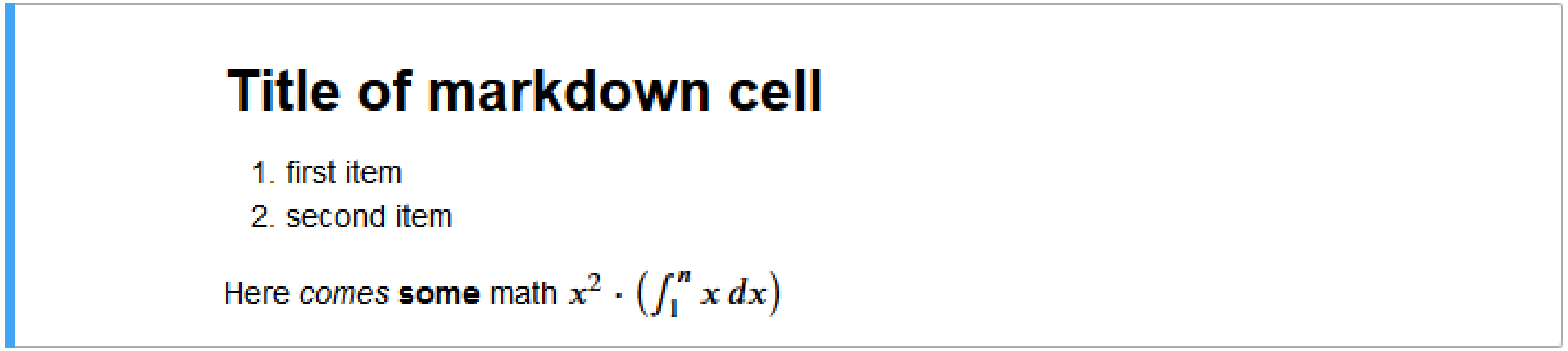
HTML formatting

LaTeX mathematics

after pressing

Ctrl + Enter (evaluates)

Alt + Enter (evaluates + new cell)

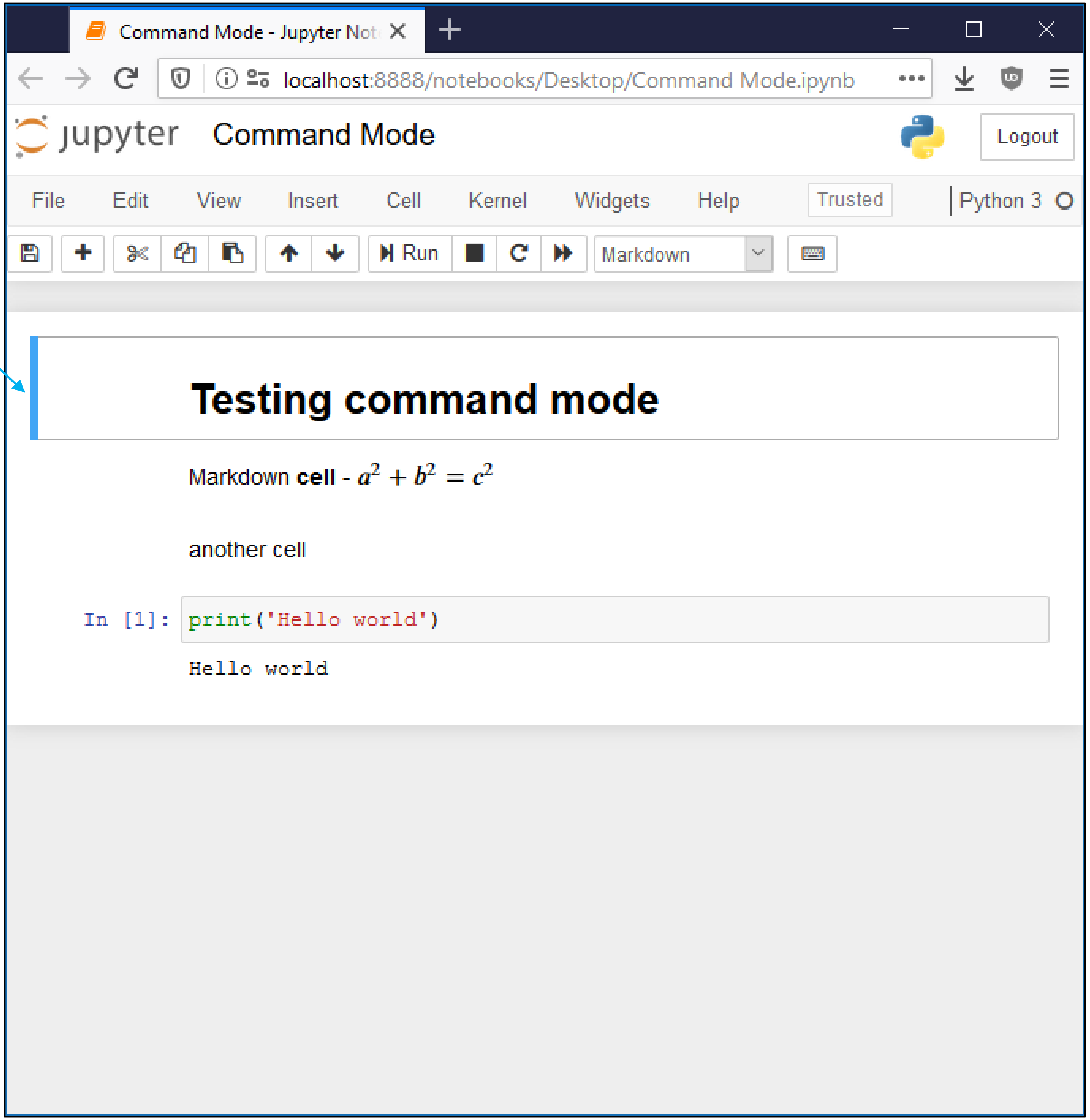


github.com/adam-p/markdown-here/wiki/Markdown-Cheatsheet

Command Mode

- Used to naviagte between cells
- Current cell is marked with blue bar
- Keyboard shortcuts

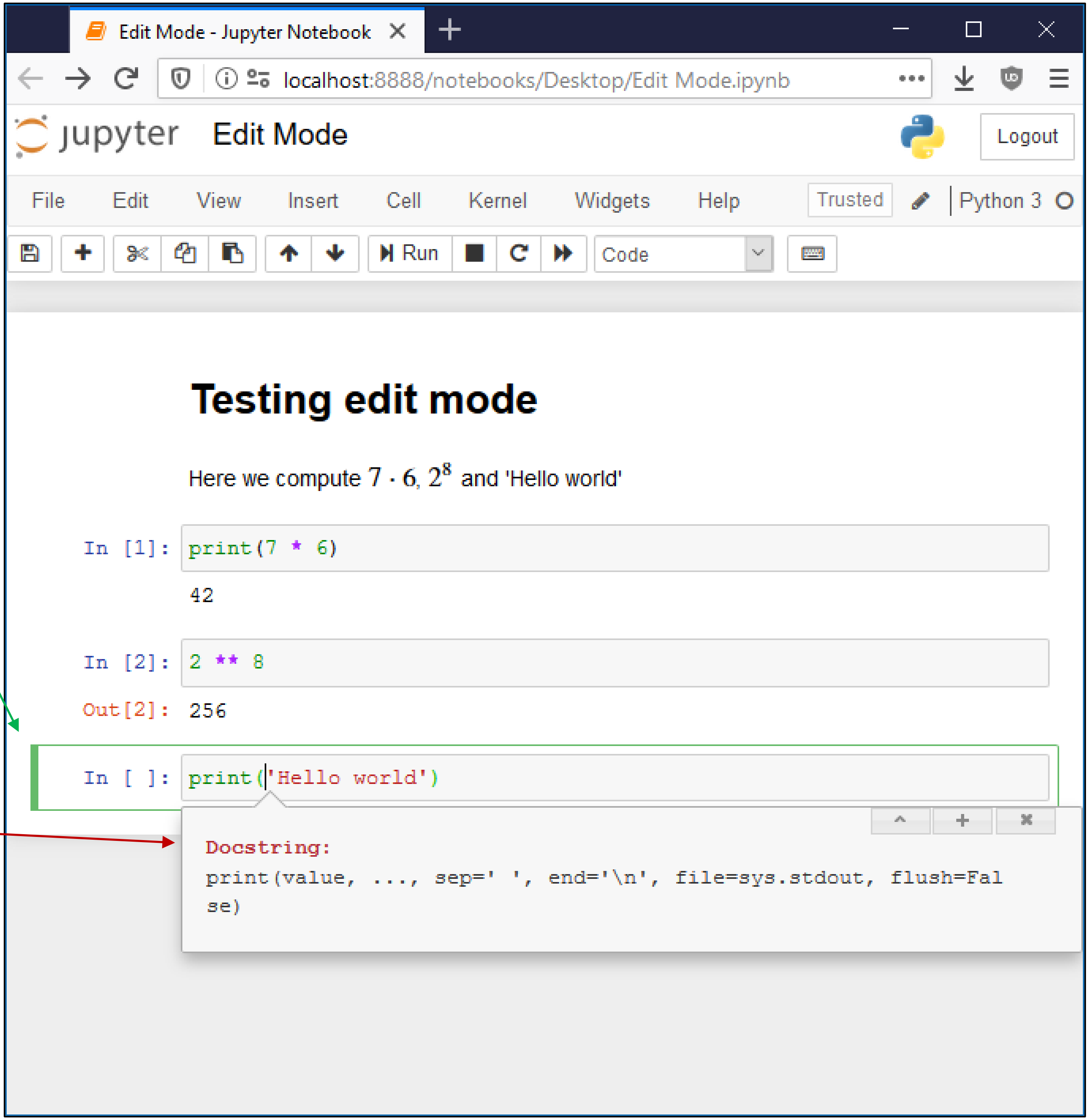
h	show keyboard shortcuts
enter	enter Edit Mode on current cell
shift-enter	run cell + select below
ctrl-enter	run selected cells
alt-enter	run cell and insert below
Y M R	change cell type (code, markdown, raw text)
1 2 3 4 5 6	change heading level
ctrl-A	select all cells
down up	move to next/previous cell
space shift-space	scroll down/up
shift-up shift-down	extend selected cells
A B	insert cell above/below
X C V shift-V Z DD	cut, copy, paste below/above, undo, delete cells
shift-L	toggle line numbers in cells
shift-M	merge selected cells (or with cell below)
O	toggle output of selected cells
shift-O	toggle scrollbar on selected cells (long output)



Edit Mode

- Used to edit current cell
- Current cell is marked with green bar
- Keyboard shortcuts

esc	enter Command Mode
shift-enter	run cell + select below
ctrl-enter	run selected cells
alt-enter	run cell and insert below
ctrl-shift- -	split cell at cursor
ctrl-shift-f	command palette
tab	indent or code completion
shift-tab	show docstring
ctrl-a -x -c -v -z -y	select all, cut, copy, paste, undo, redo
ctrl-d	delete line



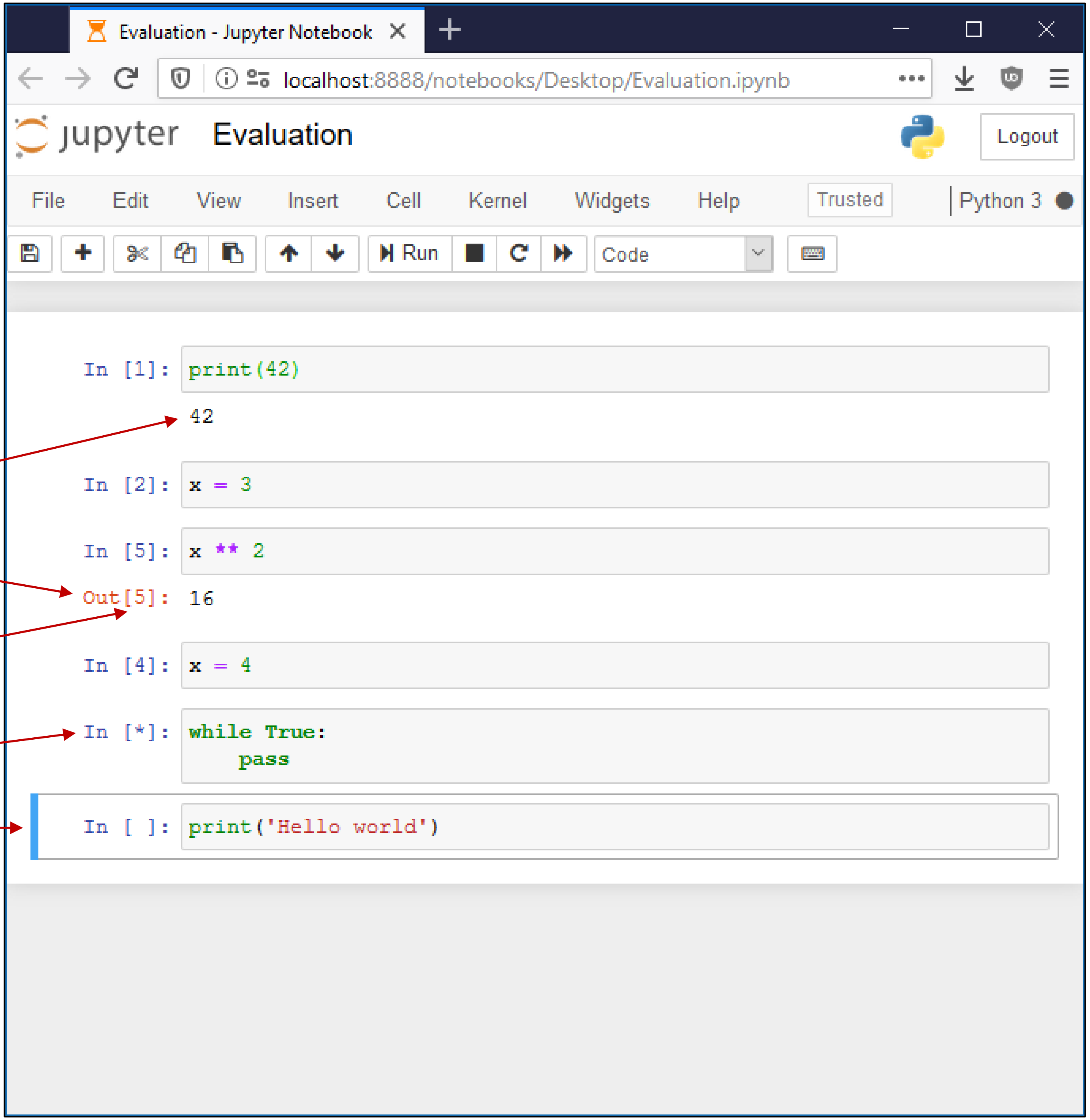
Evaluating cells

- To evaluate cell

  ctrl-enter, alt-enter, shift-enter
- Output from program shown below cell
- Result of last evaluated line
- Order of code cells evaluated

  Note ”x ** 2” computed after ”x = 4”
- [*] are cells being evaluated / waiting
- [ ] not yet evaluated
- Recompute all cells top-down

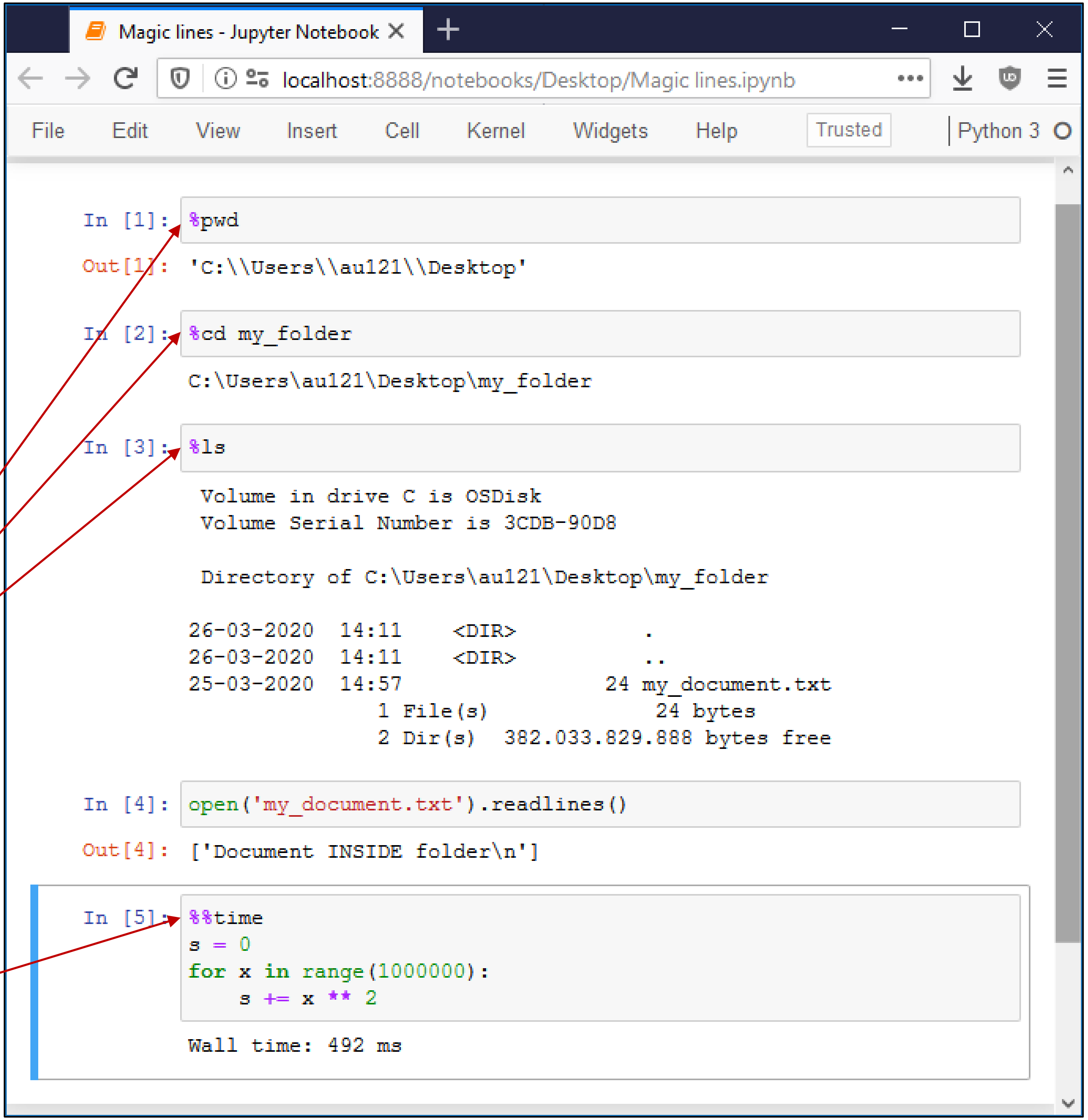
  or Kernel > Restart & Run all



Magic lines

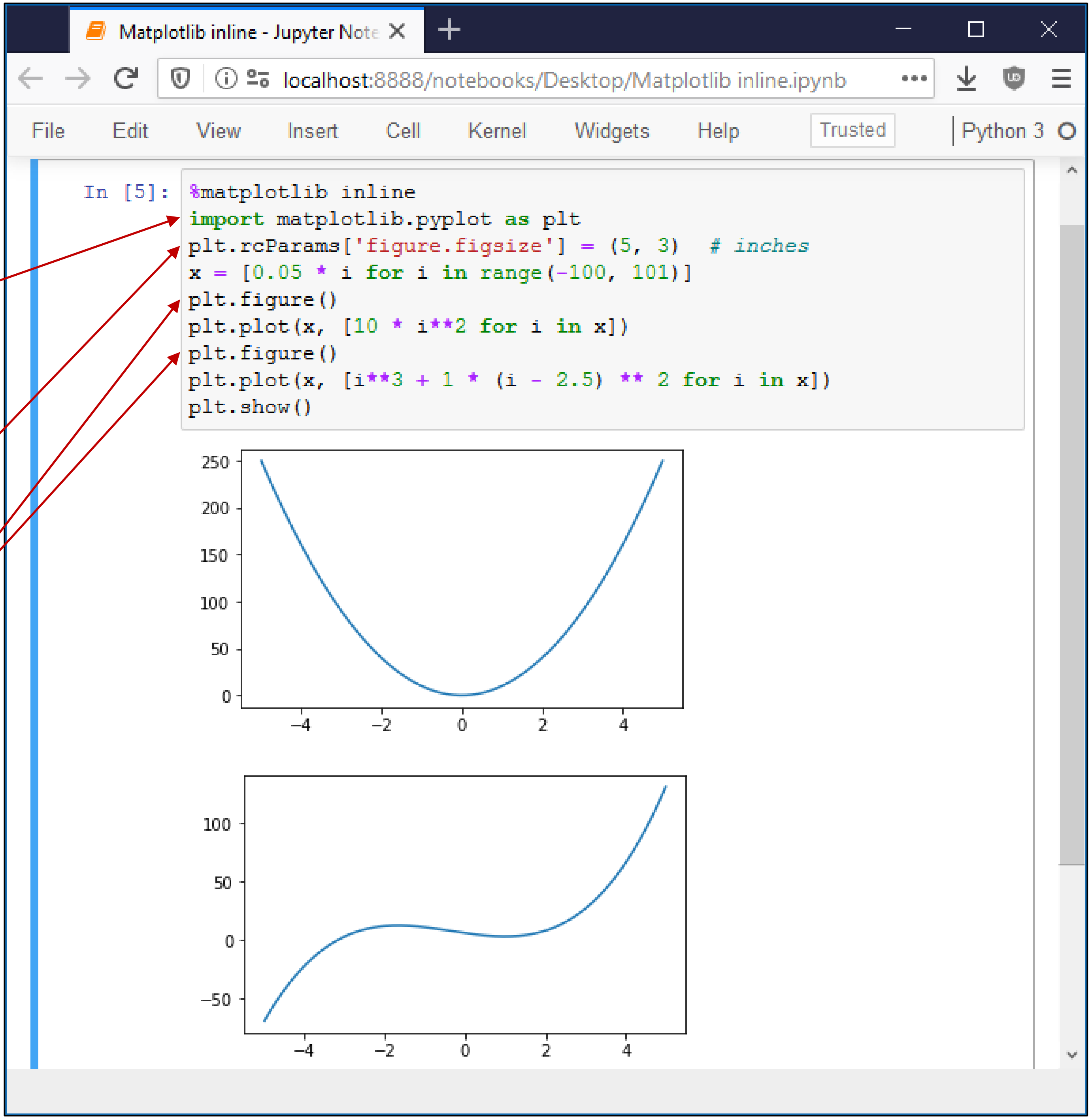
- Jupyter code cells support *magic commands* (actually it is IPython)
- % is a *line magic*
- %% is a *cell magic*

%lsmagic	list magic commands
%quickref	quick reference sheet to IPython
%pwd	print working directory (current folder)
%cd *directory*	change directory (absolut or relative)
%ls	list content of current directory
%pip *or* %conda	run pip or conda from jupyter
%load *script*	insert external script into cell
%run *program*	run external program and show output
%automagic	toggle if %-prefix is required
%matplotlib inline	no zoom & resize, allows multiple plots
%matplotlib notebook	a single plot can be zoomed & resized
%%writefile *file*	write content of cell to a file
%%time	measure time for cell execution
%timeit *expression*	time for simple expression



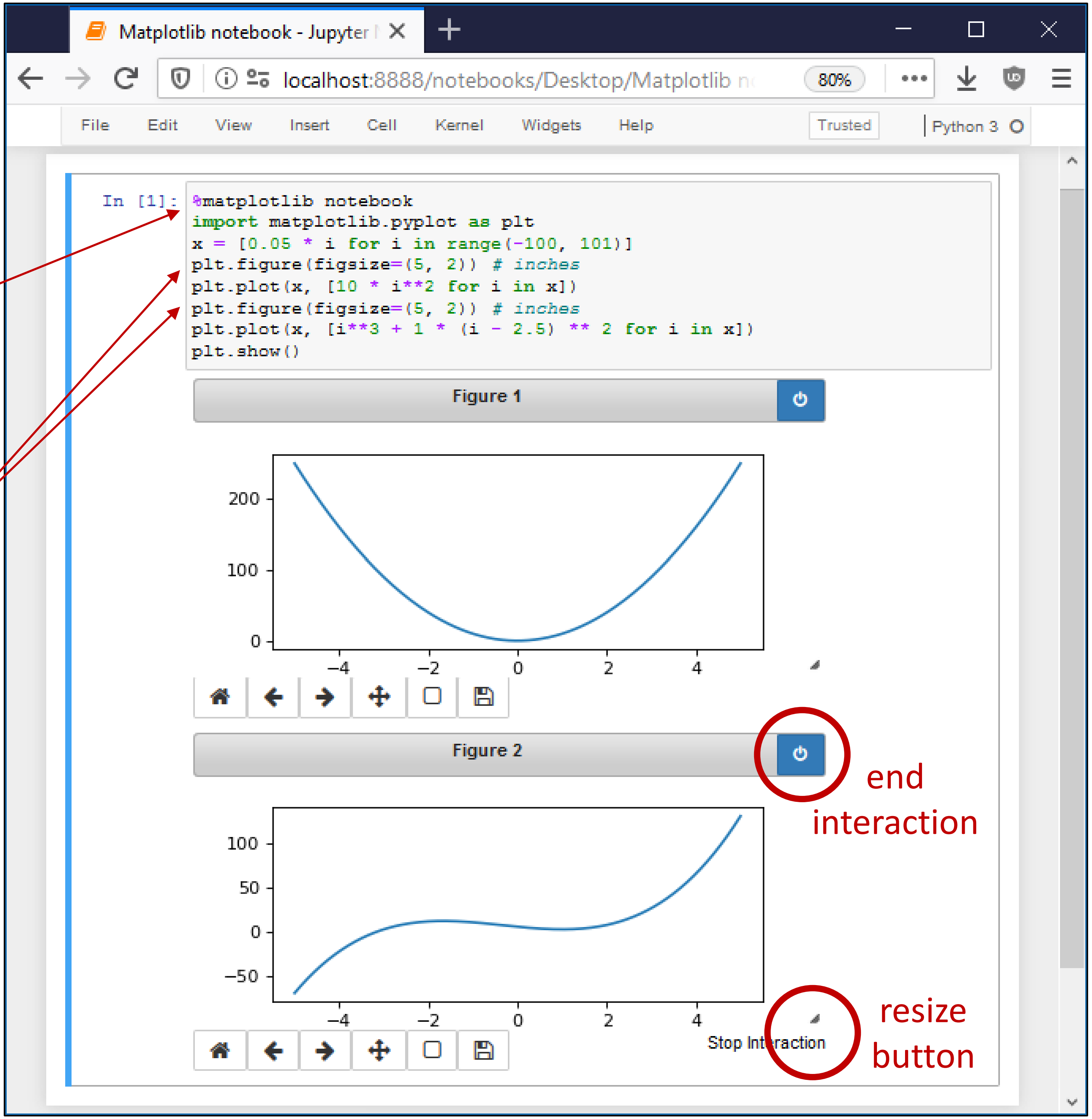
Jupyter
and matplotlib

- `%matplotlib inline`
  pyplot figures are shown *without* interactive zoom and pan (default)
- Consider changing default figure size
  `plt.rcParams['figure.figsize']`
- Start each figure with `plt.figure`
- Final call to `show` can be omitted



Jupyter and matplotlib

- `%matplotlib notebook` pyplot figures are shown *with* interactive zoom and pan
- Start each figure with `plt.figure` (also allows setting figure size)
- Final call to `show` can be omitted



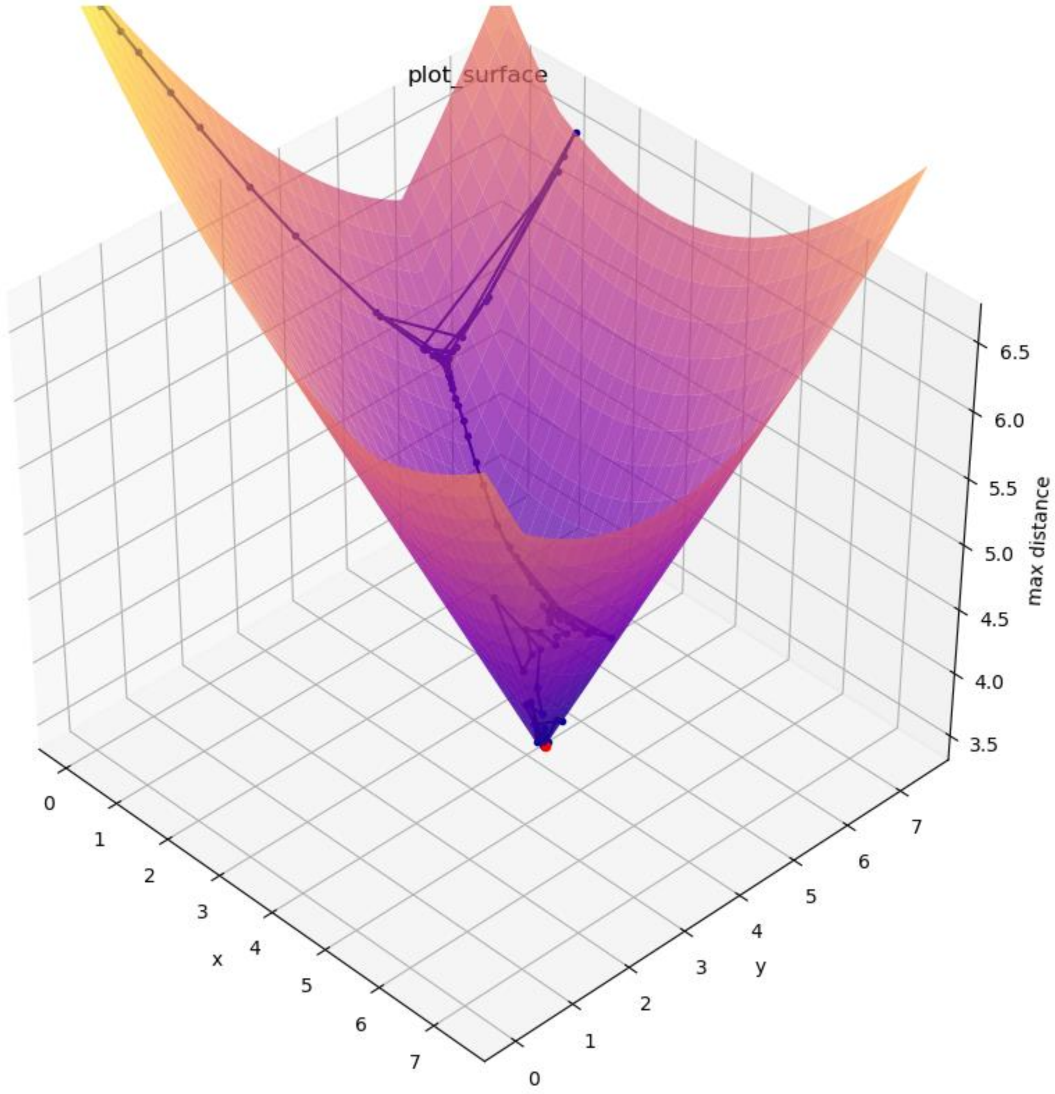
- Widespread tool used for data science applications
- Documentation, code for data analysis, and resulting visualizations are stored in one common format
- Easy to update visualizations
- Works with about 100 different programming languages (not only Python 3), many special features, ...
- Easy to share data analysis

- *Many online tutorials and examples are available*

https://www.youtube.com/results?search_query=jupyter+python

scipy.optimize.minimize

- Find point p minimizing function f
- Supports 13 algorithms – but no guarantee that result is correct
- Knowledge about optimization will help you know what optimization algorithm to select and what parameters to provide for better results
- ⚠WARNING ⚠
  Many solvers return the wrong value when used as a black box



pip install scipy

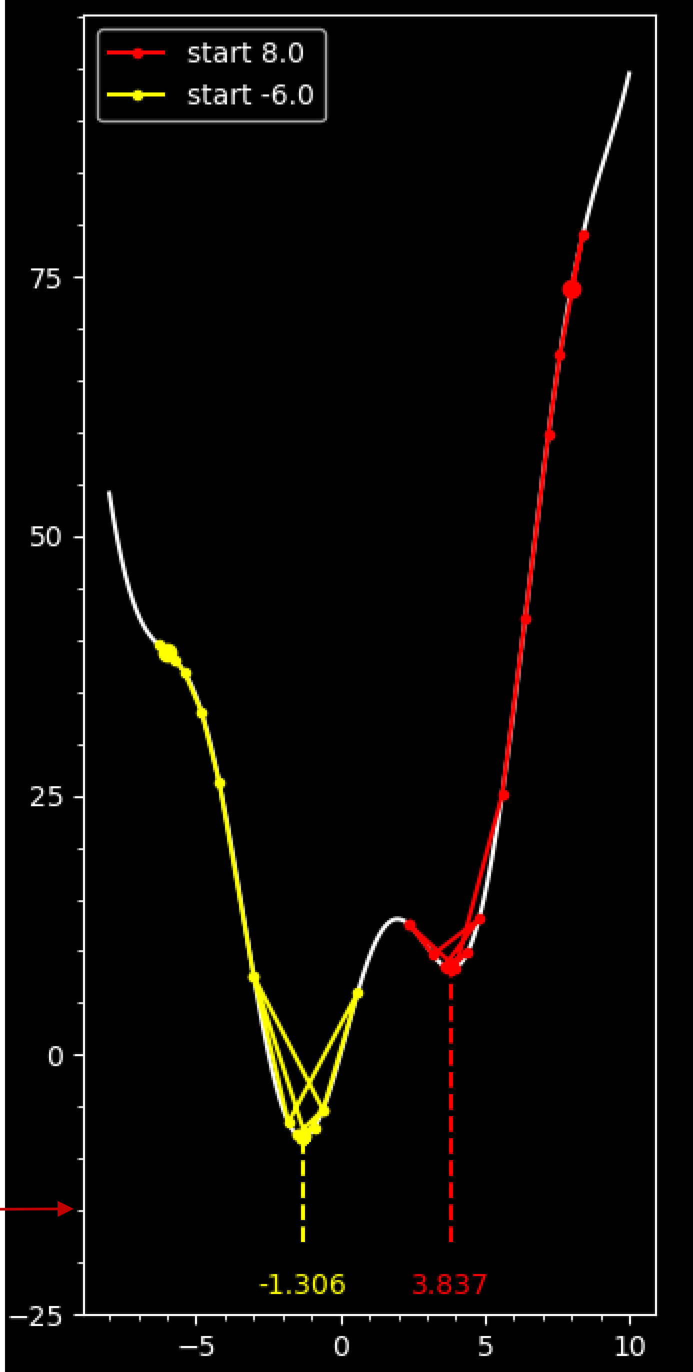
docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html

minimize.py

```python
from math import sin
import matplotlib.pyplot as plt
from scipy.optimize import minimize

trace = []  # remember calls to f

def f(x):
    value = x ** 2 + 10 * sin(x)
    trace.append((x, value))
    return value

X = [-8 + 18 * i / 9999 for i in range(1000)]
Y = [f(x) for x in X]

plt.style.use('dark_background')
plt.plot(X, Y, 'w-')
for start, color in [(8, 'red'), (-6, 'yellow')]:
    trace = []
    solution = minimize(f, [start], method='nelder-mead')

    x, y = solution.x[0], solution.fun
    plt.plot(*zip(*trace), '.-', c=color, label=f'start {start:.1f}') # trace
    plt.plot(*trace[0], 'o', c=color)                    # first trace point
    plt.text(x, -23, f'{x:.3f}', c=color, ha='center')   # show minimum x
    plt.plot([x, x], [-18, y], '--', c=color)            # dash to minimum
plt.xticks(range(-5, 15, 5))
plt.yticks(range(-25, 100, 25))
plt.minorticks_on()
plt.legend()
plt.show()
```

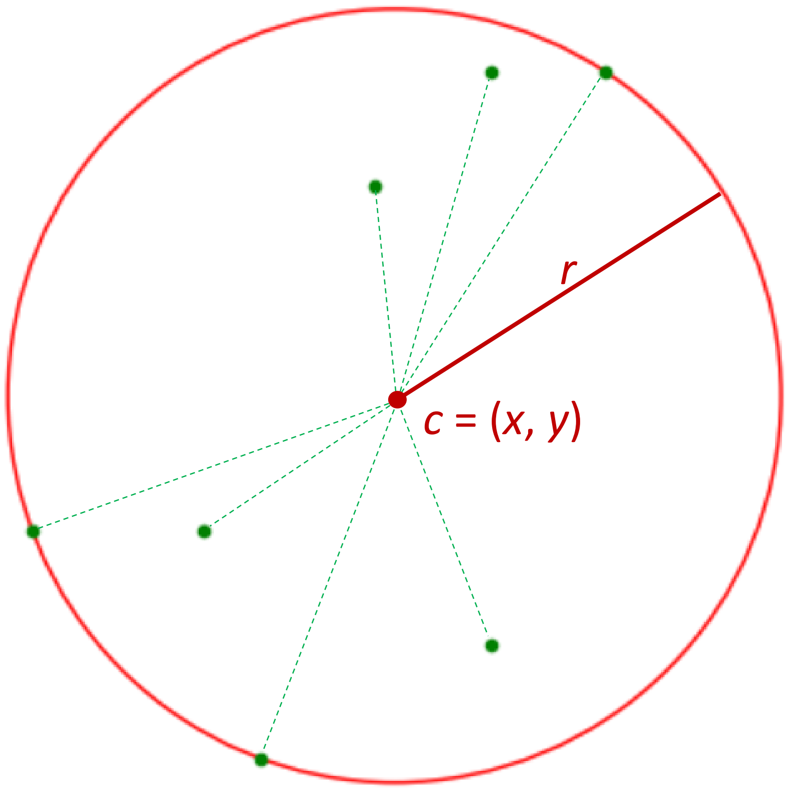
Python shell

```
> print(solution)
| final_simplex: (array([[-1.3064209 ],
|   [-1.30649414]]), array([-7.94582337, -7.94582336]))
|            fun: -7.94582337348758
|        message: 'Optimization terminated successfully.'
|           nfev: 38
|            nit: 19
|         status: 0
|        success: True
|              x: array([-1.3064209])
```



minimize tries to find a local minimum for **f** by repeatedly evaluating **f** for different **x** values

Example: Minimum enclosing circle



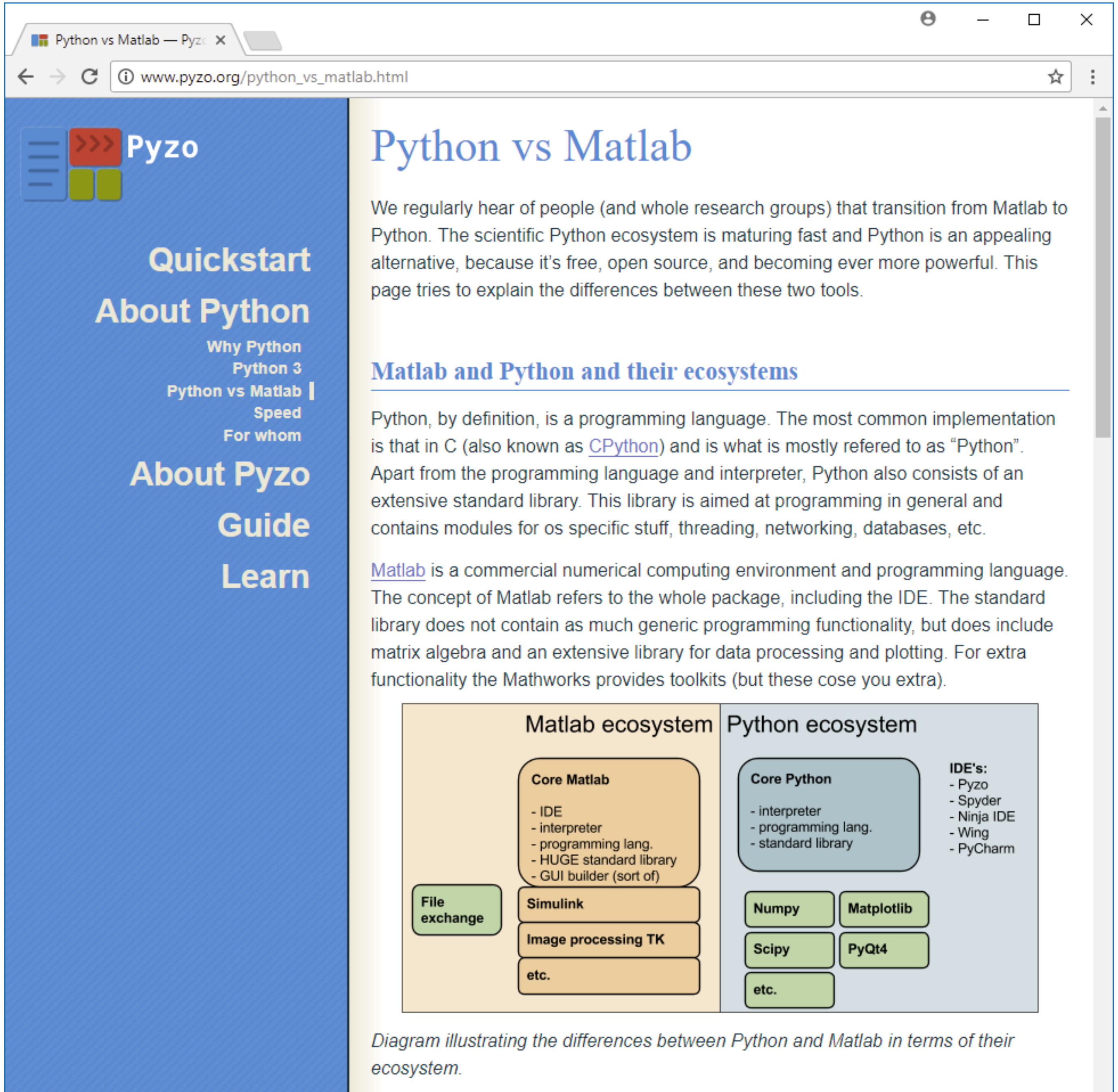
- Find c such that $r = \max_p |p - c|$ is minimized
- A solution is characterized by either
  1) three points on circle, where the triangle contains the circle center
  2) two opposite points on diagonal
- Try a standard numeric minimization solver
- ⚠ Computation involves max and $\sqrt{x}$, which can be hard for numeric optimization solvers

Python/scipy vs MATLAB

Some basic differences

- "**end**" closes a MATLAB block
- "**;**" at end of command avoids command output
- a(i) instead a[i]
- 1^{st} element of a list a(1)
- a(i:j) includes both a(i) and a(j)

like R, Mathematica, Julia, AWK, Smalltalk, ...



Python vs Matlab — Pyzo

www.pyzo.org/python_vs_matlab.html

Pyzo

Quickstart

About Python

Why Python

Python 3

Python vs Matlab

Speed

For whom

About Pyzo

Guide

Learn

Python vs Matlab

We regularly hear of people (and whole research groups) that transition from Matlab to Python. The scientific Python ecosystem is maturing fast and Python is an appealing alternative, because it's free, open source, and becoming ever more powerful. This page tries to explain the differences between these two tools.

Matlab and Python and their ecosystems

Python, by definition, is a programming language. The most common implementation is that in C (also known as CPython) and is what is mostly refered to as "Python". Apart from the programming language and interpreter, Python also consists of an extensive standard library. This library is aimed at programming in general and contains modules for os specific stuff, threading, networking, databases, etc.

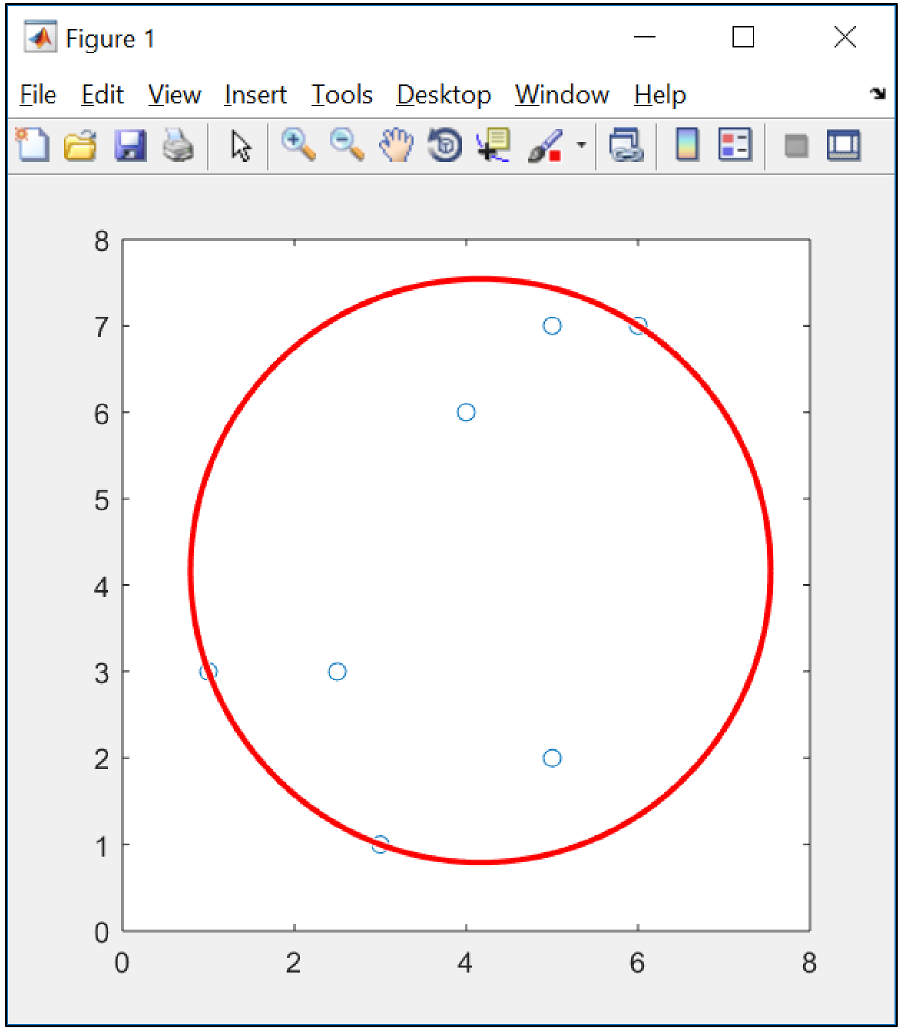
Matlab is a commercial numerical computing environment and programming language. The concept of Matlab refers to the whole package, including the IDE. The standard library does not contain as much generic programming functionality, but does include matrix algebra and an extensive library for data processing and plotting. For extra functionality the Mathworks provides toolkits (but these cose you extra).

Diagram illustrating the differences between Python and Matlab in terms of their ecosystem.

Minimum enclosing circle in MATLAB

enclosing_circle.m

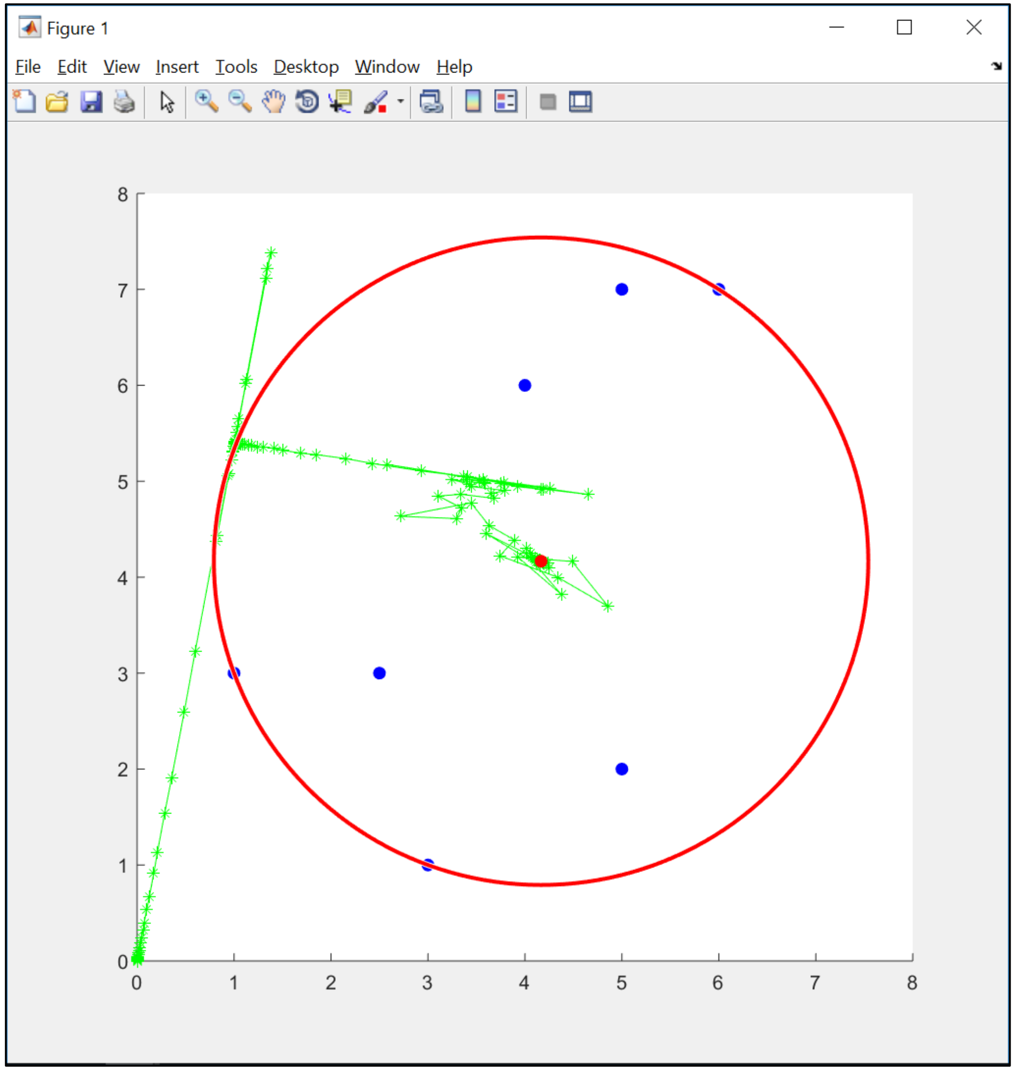
```
% Minimum enclosing circle of a point set
% fminsearch uses the Nelder-Mead algorithm

global x y
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
c = fminsearch(@(x) max_distance(x), [0,0]);
plot(x, y, "o");
viscircles(c, max_distance(c));

function dist = max_distance(p)
    global x y
    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)],
                                'euclidean'));
    end
end
```



Minimum enclosing circle in MATLAB (trace)

enclosing_circle_trace.m

```
global x y trace_x trace_y

x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
trace_x = [];
trace_y = [];

c = fminsearch(@(x) max_distance(x), [0,0]);

hold on
plot(x, y, "o", 'color', 'b', 'MarkerFaceColor', 'b');
plot(trace_x, trace_y, "*-", "color", "g");
plot(c(1), c(2), "o", 'color', 'r', 'MarkerFaceColor', 'r');
viscircles(c, max_distance(c),"color","red");

function dist = max_distance(p)
    global x y trace_x trace_y
    trace_x = [trace_x, p(1)];
    trace_y = [trace_y, p(2)];

    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)], 'euclidean' ));
    end
end
```



Minimum enclosing circle in Python

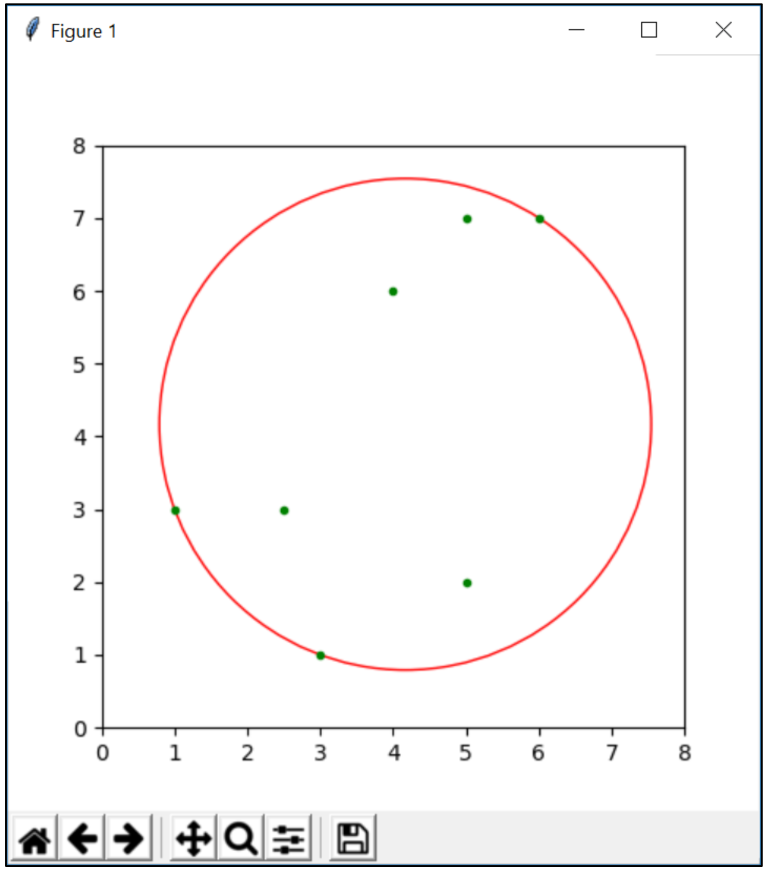
enclosing_circle.py

```
from math import sqrt
from scipy.optimize import minimize
import matplotlib.pyplot as plt

x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]

def dist(p, q):
    return sqrt((p[0]-q[0])**2 + (p[1]-q[1])**2)
def max_distance(c):
    return max(dist(p, c) for p in zip(x, y))

c = minimize(max_distance, [0.0, 0.0],
             method="nelder-mead").x
ax = plt.gca()
ax.set_xlim((0, 8))
ax.set_ylim((0, 8))
ax.set_aspect("equal")
plt.plot(x, y, "g.")
ax.add_artist(plt.Circle(c, max_distance(c),
                         color="r", fill=False))
plt.show()
```

import modules

force optimization method

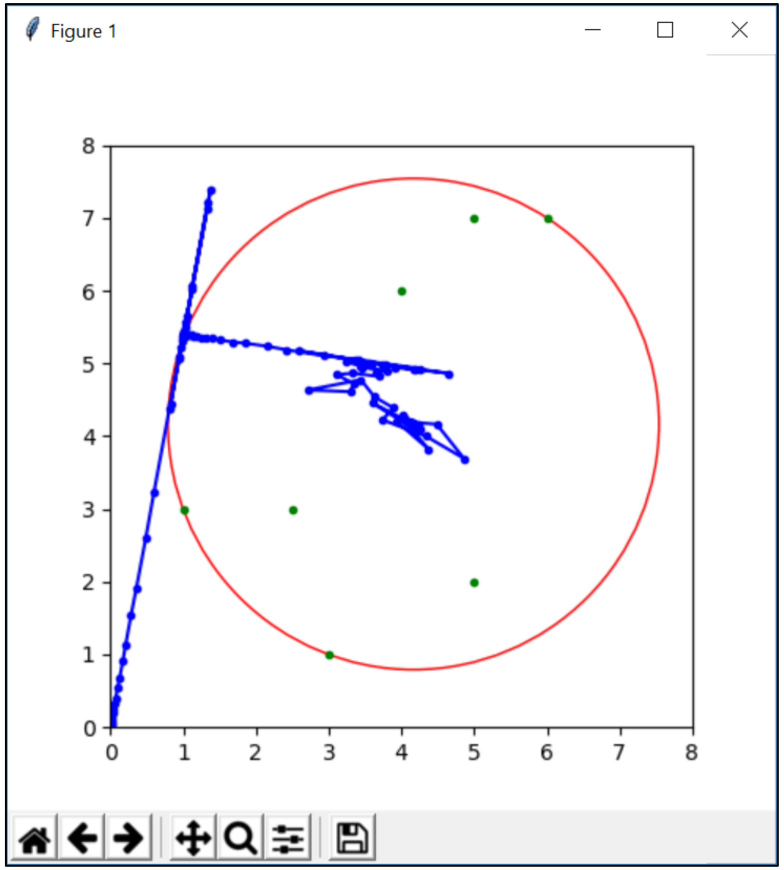
manually set axis (force circle inside plot)



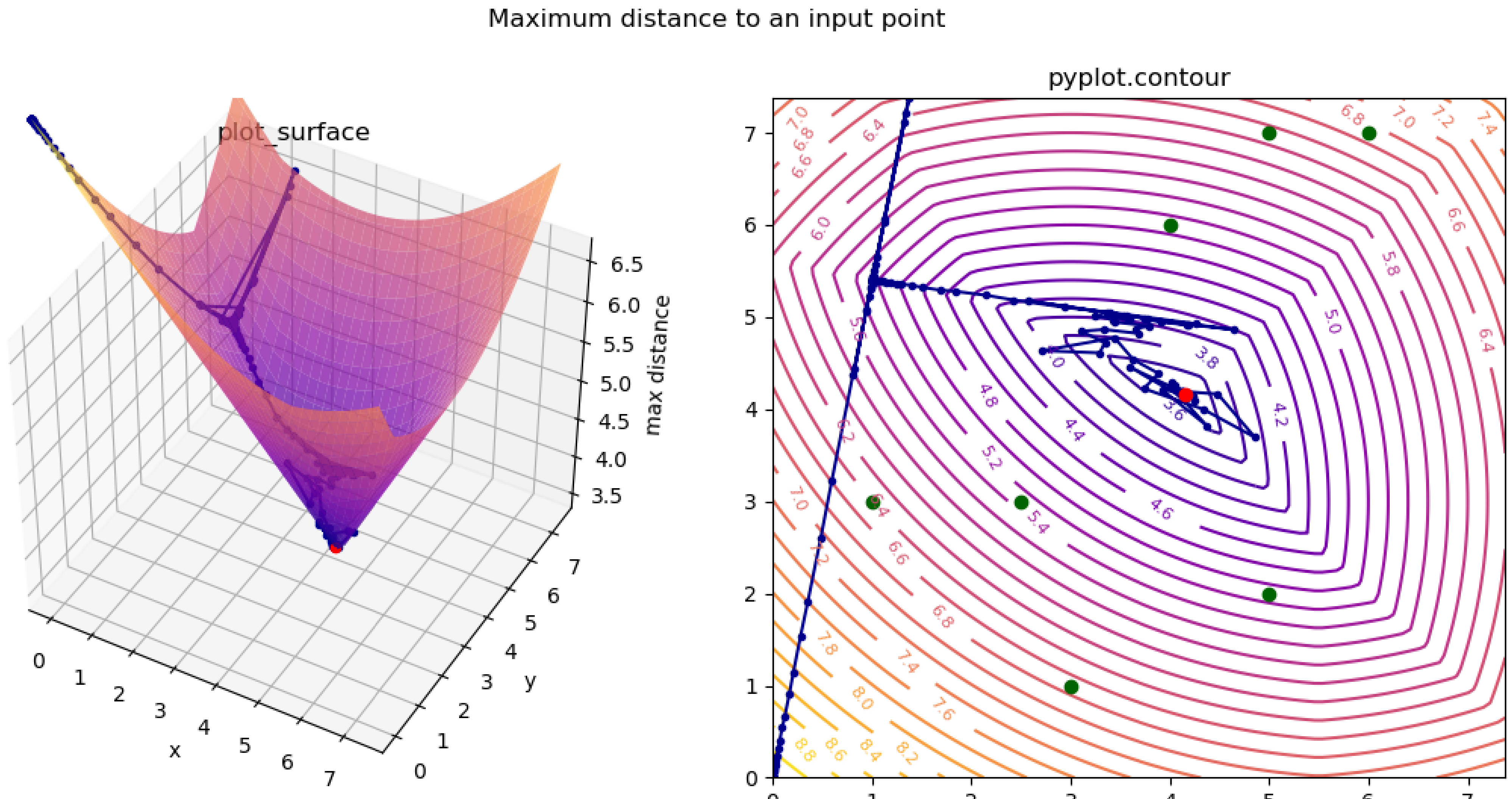
Minimum enclosing circle in Python (trace)

enclosing_circle_trace.py

```
from math import sqrt
from scipy.optimize import minimize
import matplotlib.pyplot as plt

x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]
trace = []

def dist(p, q):
    return sqrt((p[0]-q[0])**2 + (p[1]-q[1])**2)
def max_distance(c):
    trace.append(c)
    return max(dist(p, c) for p in zip(x, y))

c = minimize(max_distance, [0.0, 0.0],
             method="nelder-mead").x
ax = plt.gca()
ax.set_xlim((0, 8))
ax.set_ylim((0, 8))
ax.set_aspect("equal")
plt.plot(x, y, "g.")
plt.plot(*zip(*trace), "b.-")
ax.add_artist(plt.Circle(c, max_distance(c),
                         color="r", fill=False))
plt.show()
```



Minimum enclosing circle – search space



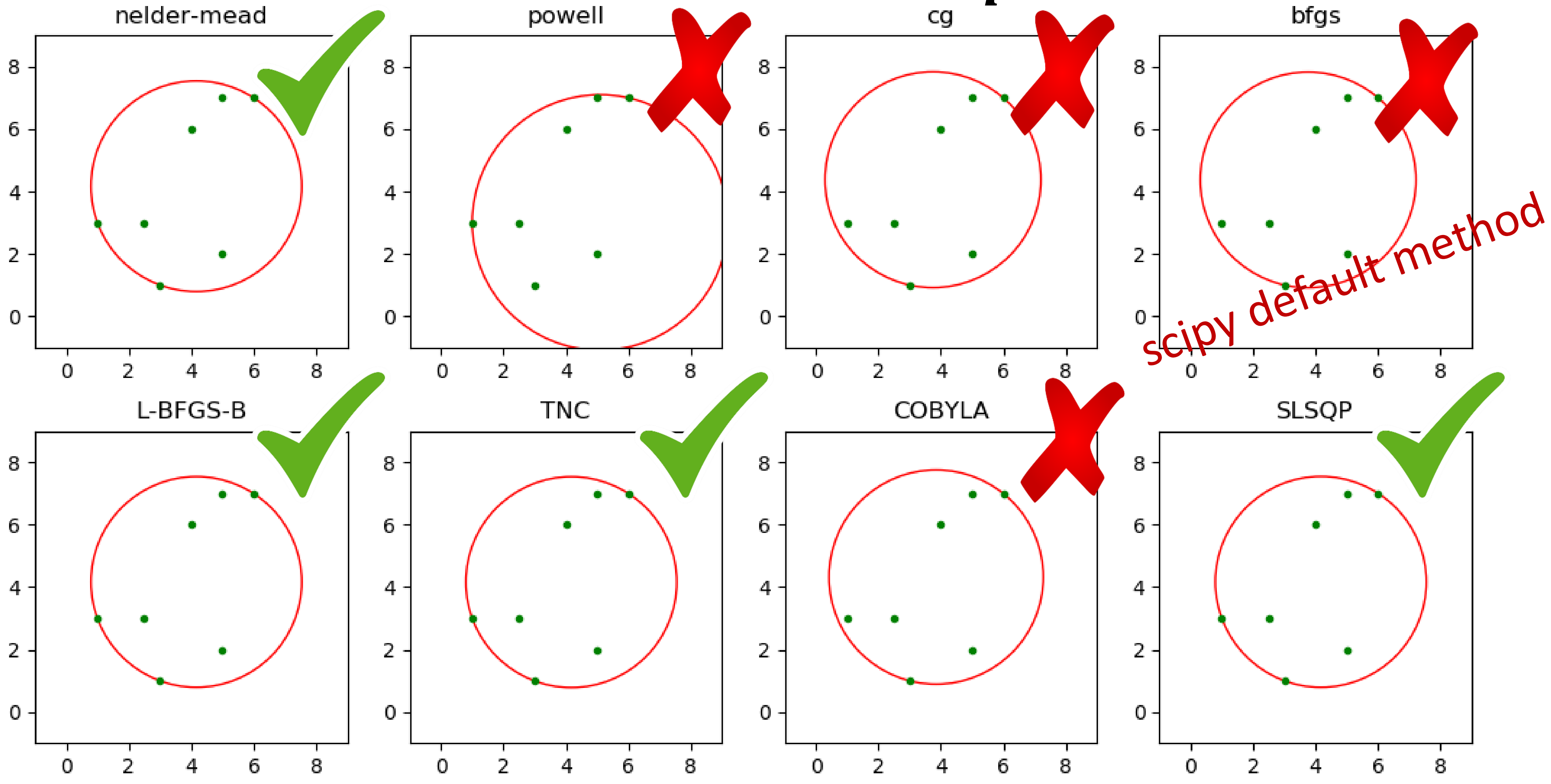
enclosing_circle_search_space.py (previous slide)

```python
from math import sqrt
from scipy.optimize import minimize
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D

points = [(1.0, 3.0), (3.0, 1.0), (2.5, 3.0),
  (4.0, 6.0), (5.0, 7.0), (6.0, 7.0), (5.0, 2.0)]

#  Minimum enclosing circle solver

trace = []

def distance(p, q):
    return sqrt((p[0]-q[0])**2 + (p[1]-q[1])**2)

def distance_max(q):
    dist = max(distance(p, q) for p in points)
    trace.append((*q, dist))
    return dist

solution = minimize(distance_max, [0.0, 0.0],
                    method='nelder-mead')
center = solution.x
radius = solution.fun

#  unzip point coordinates

points_x, points_y = zip(*points)
trace_x, trace_y, trace_z = zip(*trace)

#  Bounding box [x_min, x_max] x [y_min, y_max]

xs, ys = points_x + trace_x, points_y + trace_y
x_min, x_max = min(xs), max(xs)
y_min, y_max = min(ys), max(ys)
# enforce apsect ratio
x_max = max(x_max, x_min + y_max - y_min)
y_max = max(y_max, y_min + x_max - x_min)
```

```python
#  Minimum enclosing circle - 3D surface plot
#  (plot_surface requires X, Y, Z are 2D numpy.arrays)

X, Y = np.meshgrid(np.linspace(x_min, x_max, 100),
                   np.linspace(y_min, y_max, 100))
Z = np.zeros(X.shape)
for px, py in points:
    Z = np.maximum(Z, (X - px)**2 + (Y - py)**2)
Z = np.sqrt(Z)

ax = plt.subplot(1, 2, 1, projection='3d')
ax.plot_surface(X, Y, Z, cmap='plasma', alpha=0.7)
ax.plot(trace_x, trace_y, trace_z, '.-', c='darkblue')
ax.scatter(*center, radius, 'o', c='red')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('max distance')
ax.set_title('plot_surface')

#  Minimum enclosing circle - contour plot

plt.subplot(1, 2, 2)
plt.title('pyplot.contour')
plt.plot(trace_x, trace_y, '.-', color='darkblue')
plt.plot(points_x, points_y, 'o', color='darkgreen')
plt.plot(*center, 'o', c='red')
qcs = plt.contour(X, Y, Z, levels=30, cmap='plasma')
plt.clabel(qcs, inline=1, fontsize=8, fmt='%.1f')

plt.suptitle('Maximum distance to an input point')
plt.tight_layout()
plt.show()
```

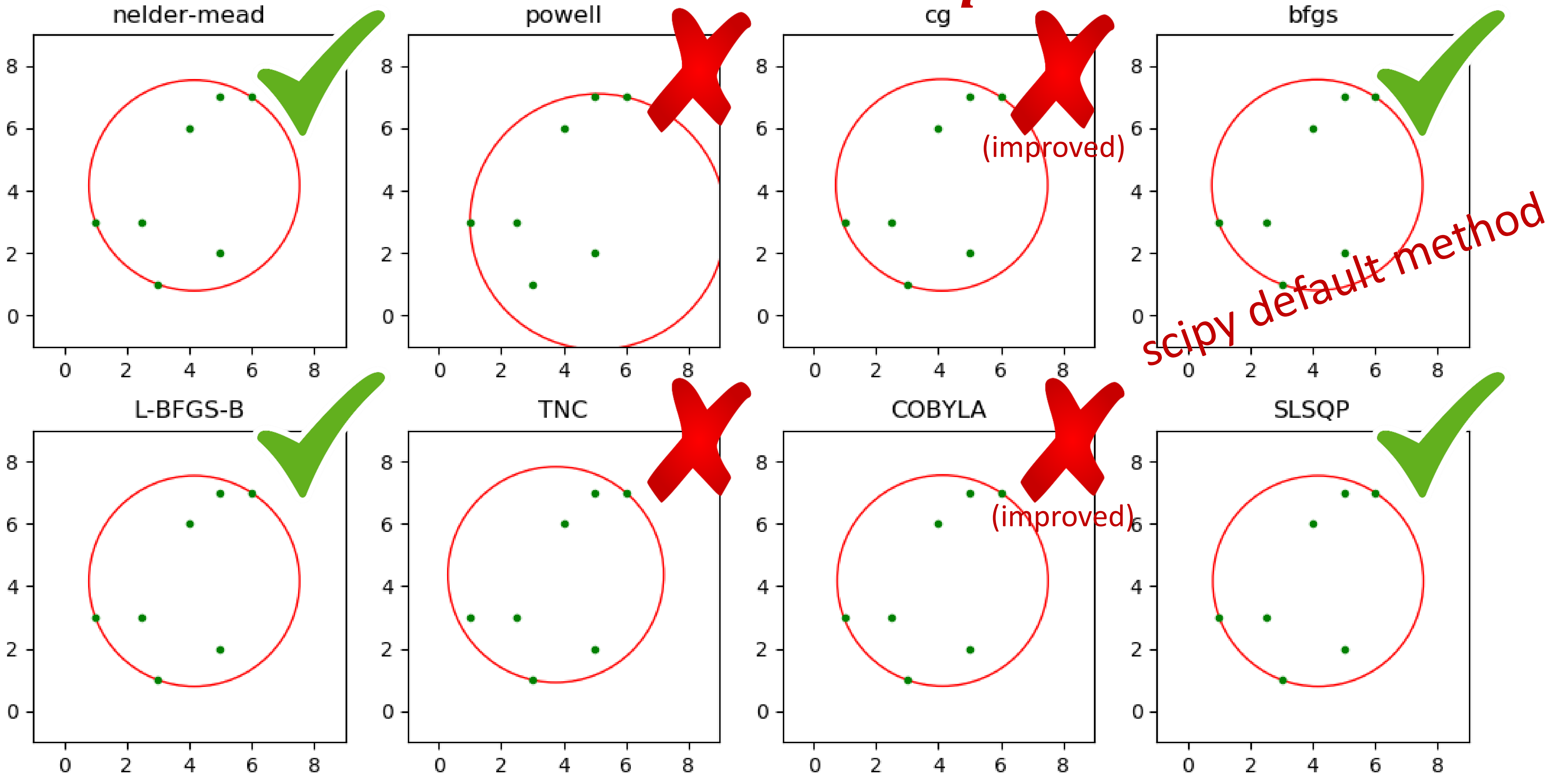
numpy arrays

scipy.minimize $\boldsymbol{f}(\boldsymbol{c}) = \max_{\boldsymbol{p}} |\boldsymbol{p} - \boldsymbol{c}|$



docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html scipy.minimize $f(c) = \max_p |p - c|^2$

avoids $\sqrt{\ }$



Multi-dimensional data

- NumPy
- matrix multiplication, @
- numpy.linalg.solve, numpy.polyfit

- NumPy is a Python package for dealing with multi-dimensional data

www.numpy.org

docs.scipy.org/doc/numpy/user/quickstart.html

pylab ?

- Guttag uses pylab in the examples, but...

> *"pylab is a convenience module that bulk imports matplotlib.pyplot (for plotting) and numpy (for mathematics and working with arrays) in a single name space. Although many examples use pylab, it is no longer recommended."*

matplotlib.org/faq/usage_faq.html

NumPy arrays (example)

Python shell

```
> range(0, 1, .3)
| TypeError: 'float' object cannot be
  interpreted as an integer
> [1 + i / 4 for i in range(5)]
| [1.0, 1.25, 1.5, 1.75, 2.0]
```

python only supports ranges of int

generate 5 uniform values in range [1,2]

Python shell

```
> import numpy as np
> np.arange(0, 1, 0.3)
| array([0. , 0.3, 0.6, 0.9])
> type(np.arange(0, 1, 0.3))
| <class 'numpy.ndarray'>
> help(numpy.ndarray)
| +2000 lines of text
> np.linspace(1, 2, 5)
| array([1.  , 1.25, 1.5 , 1.75, 2.  ])
```

numpy can generate ranges with float

returns a ”NumPy array” (not a list)

”arange” ≡ ”array range” and generates the array explicitly

generate n uniformly spaced values

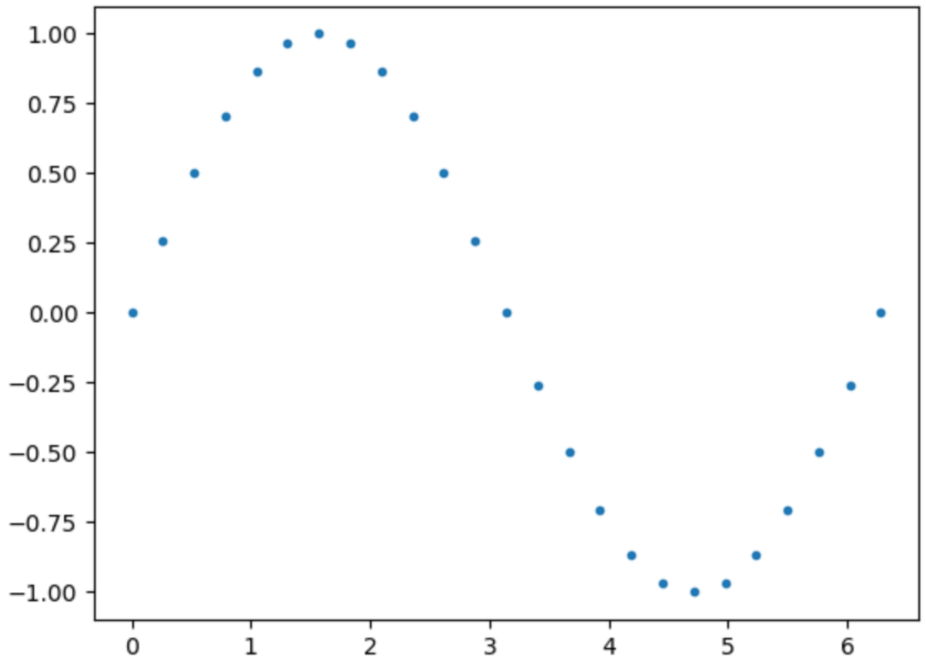
Plotting a function (example)

sin.py

```
import matplotlib.pyplot as plt
import math
n = 25
x = [2*math.pi * i / (n-1) for i in range(n)]
y = [math.sin(v) for v in x]
plt.plot(x, y, '.')
plt.show()
```

sin_numpy.py

```
import matplotlib.pyplot as plt
import numpy as np
x = np.linspace(0, 2 * np.pi, 25)
y = np.sin(x)
plt.plot(x, y, '.')
plt.show()
```

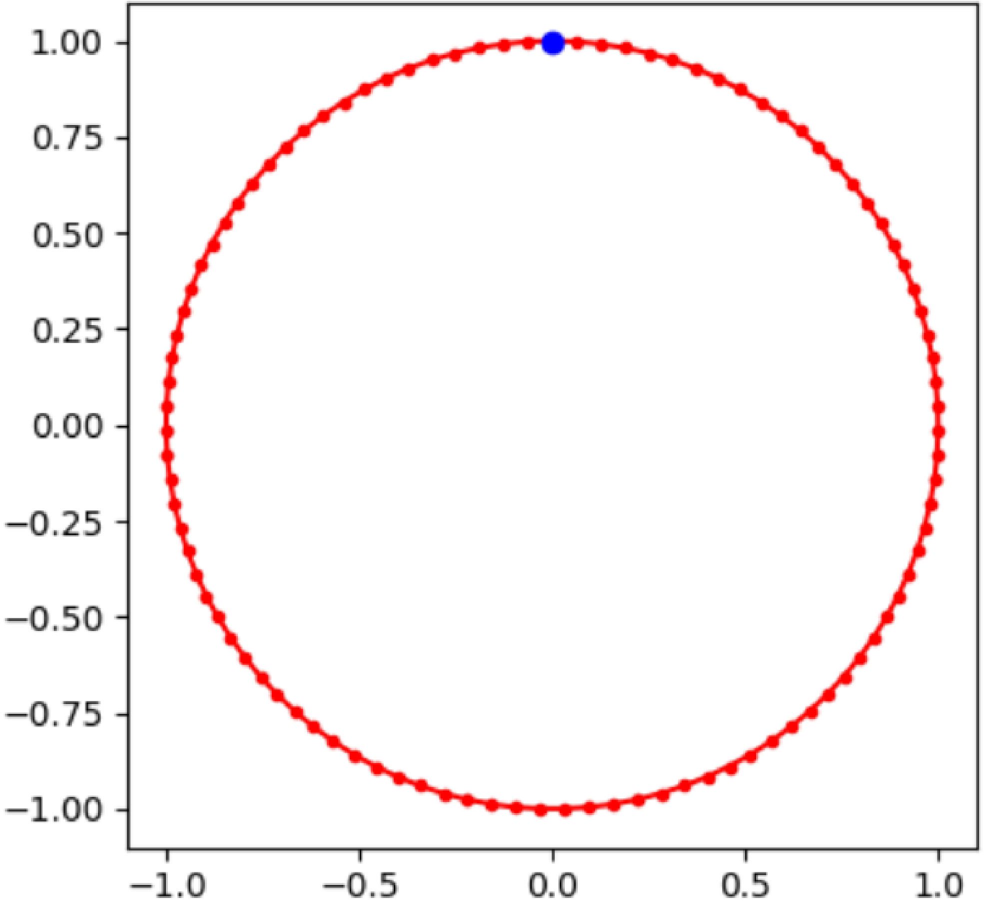


- `np.sin` applies the `sin` function *to each* element of `x`
- `pyplot` accepts NumPy arrays
- `math.pi == np.pi` $\approx \frac{22}{7}$

A circle

circle.py

```
import matplotlib.pyplot as plt
import numpy as np

a = np.linspace(0, 2 * np.pi, 100)
x = np.sin(a)
y = np.cos(a)

plt.plot(x, y, 'r.-')
plt.plot(x[0], y[0], 'bo')
plt.show()
```

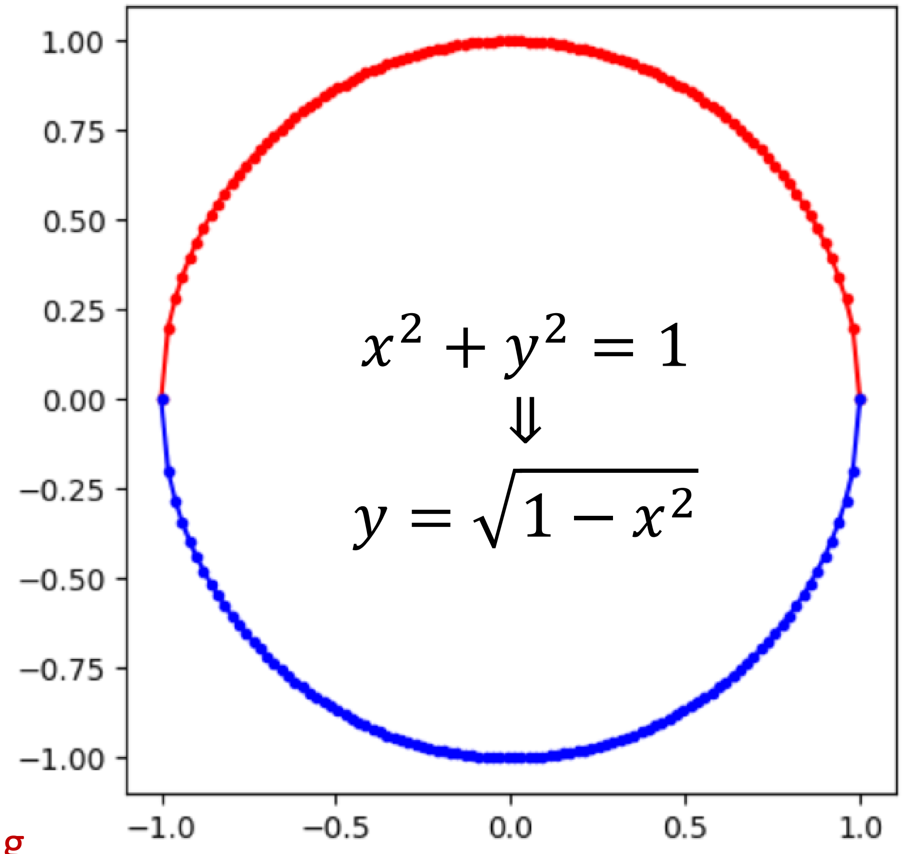


Two half circles

half_circles.py

```
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-1, 1, 100)
plt.plot(x, np.sqrt(1 - x**2), 'r.-')
plt.plot(x, -np.sqrt(1 - x**2), 'b.-')
plt.show()
```

compact expression computing something quite comlicated



- `x` is a NumPy array
- `**` NumPy method `__pow__` squaring each element in `x`
- binary – NumPy method `__rsub__` that for each element e in `x` computes 1 - e
- `np.sqrt` NumPy method computing the square root of each element in `x`
- unary – NumPy method `__neg__` that negates each element in `x`

Creating one-dimensional NumPy arrays

Python shell

```
> np.array([1, 2, 3])
| array([1, 2, 3])
> np.array((1, 2, 3))
| array([1, 2, 3])
> np.array(range(1, 4))
| array([1, 2, 3])
> np.arange(1., 4., 1.)
| array([1., 2., 3.])
> np.linspace(1, 3, 3)
| array([1., 2., 3.])
> np.zeros(3)
| array([0., 0., 0.])
> np.ones(3)
| array([1., 1., 1.])
> np.full(3, 7)
| array([7, 7, 7])
> np.random.random(3)
| array([0.73761651,
  0.60607355, 0.3614118 ])
```

```
> np.array([1, 2, 3]).dtype    # type of all values
| dtype('int32')
> np.arange(3, dtype='float')
| array([0., 1., 2.])
> np.arange(3, dtype='int16') # 16 bit integers
| array([0, 1, 2], dtype=int16)
> np.arange(3, dtype='int32') # 32 bit integers
| array([0, 1, 2])
> 1000 ** np.arange(5)
| array([1, 1000, 1000000, 1000000000,
  -727379968], dtype=int32)    # OOPS.. overflow
> 1000 ** np.arange(5, dtype='O')
| array([1, 1000, 1000000, 1000000000,
  1000000000000], dtype=object) # Python integer
> np.arange(3, dtype='complex')
| array([0.+0.j, 1.+0.j, 2.+0.j])
```

Elements of a NumPy array are not arbitrary precision integers by default – you can select between +25 number representations

docs.scipy.org/doc/numpy-1.13.0/user/basics.types.html

Mantissa size in various numpy floats

Python shell

```
> for data_type in ['half', 'float', 'single', 'double', 'longdouble', 'float32', 'float64']:
    x = np.array([1], dtype=data_type)
    for i in range(100):
      if x == x + (x / 2) ** i:
        break
    print(data_type, i - 1, 'bits mantissa')
| half 10 bits mantissa
| float 52 bits mantissa
| single 23 bits mantissa
| double 52 bits mantissa
| longdouble 52 bits mantissa
| float32 23 bits mantissa          # platform independent
| float64 52 bits mantissa          # platform independent
```

$$1.\overbrace{\underbrace{00001}_{i}00000}^{\text{mantissa}}{}_2$$

docs.scipy.org/doc/numpy-1.13.0/user/basics.types.html

Creating multi-dimensional NumPy arrays

Python shell

```
> np.array([[1, 2, 3], [4, 5, 6]])
| array([[1, 2, 3],
|        [4, 5, 6]])
> np.arange(1, 7).reshape(2, 3)
| array([[1, 2, 3],
|        [4, 5, 6]])
> x = np.arange(12).reshape(2, 2, 3)
> x
| array([[[ 0,  1,  2],
|         [ 3,  4,  5]],
|
|        [[ 6,  7,  8],
|         [ 9, 10, 11]]])
> numpy.zeros((2, 5), dtype='int32')
| array([[0, 0, 0, 0, 0],
|        [0, 0, 0, 0, 0]])
```

```
> x.size
| 12
> x.ndim
| 3
> x.shape
| (2, 2, 3)
> x.dtype
| dtype('int32')
> x.flatten()
| array([0,1,2,3,4,5,6,7,8,9,10,11])
> list(x.flat)   # flat is an iterator
| [0,1,2,3,4,5,6,7,8,9,10,11]
> np.eye(3)      # diagonal 2D array
| array([[1., 0., 0.],
|        [0., 1., 0.],
|        [0., 0., 1.]])
```

View vs Copy

- Numpy is optimized to handle big multi-dimensional arrays
- To *avoid copying* data, **views** allows one to look at the underlying data in different ways (data can be shared by multiple views)
- `reshape`, `ravel` and slicing are examples creating views
- `flatten` and `ravel` both turn multiple dimensional arrays into one dimensional arrays but `flatten` creates an explicit copy whereas `ravel` creates a space efficient view

Python shell

```
> x = np.arange(6)
> y = x.reshape(2, 3)   # view
> y[0][0] = 42          # updates x
> x
| array([42, 1, 2, 3, 4, 5])
> y
| array([[42, 1, 2],
|        [ 3, 4, 5]])
> z = y.flatten()       # copy
> z[5] = 0
> z
| array([42, 1, 2, 3, 4, 0])
> x
| array([42, 1, 2, 3, 4, 5])
> w = y.ravel()        # view
> w[5] = -1
> w
| array([42, 1, 2, 3, 4, -1])
> x
| array([42, 1, 2, 3, 4, -1])
```

NumPy operations

Python shell

```
> x = numpy.arange(3)
> x
| array([0, 1, 2])
> x + x  # elementwise addition
| array([0, 2, 4])
> 1 + x  # add integer to each element
| array([1, 2, 3])
> x * x  # elementwise multiplication
| array([0, 1, 4])
> np.dot(x, x)  # dot product
| 5
> np.cross([1, 2, 3], [3, 2, 1]) # cross product
| array([-4,  8, -4])
```

```
> a = np.arange(6).reshape(2,3)
> a
| array([[0, 1, 2],
|        [3, 4, 5]])
> a.T  # matrix transposition, view
| array([[0, 3],
|        [1, 4],
|        [2, 5]])
> a @ a.T  # matrix multiplication
| array([[ 5, 14],
|        [14, 50]])
> a += 1
> a
| array([[1, 2, 3],
|        [4, 5, 6]])
```

[PEP 465 - A dedicated infix operator for matrix multiplication]()

Universal functions (apply to each entry)

Python shell

```
> x = np.array([[1, 2], [3, 4]])
> np.sin(x)   # also: cos, exp, sqrt, log, ceil, floor, abs
| array([[ 0.84147098,  0.90929743],
|        [ 0.14112001, -0.7568025 ]])
> np.sign(np.sin(x))
| array([[ 1.,  1.],
|        [ 1., -1.]])
> np.mod(np.arange(10), 3)  # same as: np.arange(10) % 3
| array([0, 1, 2, 0, 1, 2, 0, 1, 2, 0], dtype=int32)
```

Axis

Python shell

```
> x = np.arange(1, 7).reshape(2, 3)
> x
| array([[1, 2, 3],
|        [4, 5, 6]])
> x.sum()  # = x.sum(axis=(0, 1))
| 21
> x.sum(axis=0)
| array([5, 7, 9])
> x.sum(axis=1)
| array([6, 15])
> x.min() # = x.min(axis=(0, 1))
| 1
```

Python shell

```
> x.min(axis=0)
| array([1, 2, 3])
> x.min(axis=1)
| array([1, 4])
> x.cumsum()
| array([ 1,  3,  6, 10, 15, 21], dtype=int32)
> x.cumsum(axis=0)
| array([[1, 2, 3],
|        [5, 7, 9]], dtype=int32)
> x.cumsum(axis=1)
| array([[ 1,  3,  6],
|        [ 4,  9, 15]], dtype=int32)
```

Slicing

Python shell

```
> x = numpy.arange(20).reshape(4,5)
> x
| array([[ 0,  1,  2,  3,  4],
|        [ 5,  6,  7,  8,  9],
|        [10, 11, 12, 13, 14],
|        [15, 16, 17, 18, 19]])
> x[2, 3]  # = x[(2, 3)]
| 13
> x[1:4:2, 2:4:1]  # rows 1 and 3, and columns 2 and 3, view
| array([[ 7,  8],
|        [17, 18]])
> x[:,3]
| array([ 3,  8, 13, 18])
> x[...,3]  # ... is placeholder for ':' for all missing dimensions
| array([ 3,  8, 13, 18])
> type(...)
| <class 'ellipsis'>
```

Broadcasting (stretching arrays to get same size)

- Numpy tries to apply *broadcasting*, if array shapes do not match, i.e. adds missing leading dimensions and repeats a dimension with only one element:

```
  [[1],[2]] + [10,20]        column + row vector
≡ [[1],[2]] + [[10,20]]      both ndim = 2
≡ [[1,1],[2,2]] + [[10,20]]
≡ [[1,1],[2,2]] + [[10,20],[10,20]]
≡ [[11,21],[12,22]]
```

- To prevent unexpected broadcasting, add an assertion to your program:

```
assert x.shape == y.shape
```

docs.scipy.org/doc/numpy/user/basics.broadcasting.html

Python shell

```
> x = np.array([[1, 2, 3], [4, 5, 6]])
> y = np.array([1, 2, 3])  # one row
> z = np.array([[1], [2]]) # one column
> x + 3  # add 3 to each entry
| array([[4, 5, 6],
|        [7, 8, 9]])
> x + y  # add y to each row
| array([[2, 4, 6],
|        [5, 7, 9]])
> x + z  # add z to each column
| array([[2, 3, 4],
|        [6, 7, 8]])
> y + z  # 2 rows with y + 3 columns with z
| array([[2, 3, 4],
|        [3, 4, 5]])
> z == z.T  # [[1,1],[2,2]] == [[1,2],[1,2]]
| array([[ True, False],
|        [False,  True]])
```

Masking

Python shell

```
> x = np.arange(1, 11).reshape(2, 5)
> x
| array([[ 1,  2,  3,  4,  5],
|        [ 6,  7,  8,  9, 10]])
> x % 3
| array([[1, 2, 0, 1, 2],
|        [0, 1, 2, 0, 1]], dtype=int32)
> x % 3 == 0
| array([[False, False,  True, False, False],
|        [ True, False, False,  True, False]])
> x[x % 3 == 0]  # use Boolean matrix to select entries
| array([3, 6, 9])
> x[:, x.sum(axis=0) % 3 == 0]  # columns with sum divisible by 3
| array([[ 2,  5],
|        [ 7, 10]])
```

Numpy is fast... but be aware of dtype ⚠

Python shell

```
> sum([x**2 for x in range(1000000)])
| 333332833333500000
> (np.arange(1000000)**2).sum()
| 584144992    # wrong since overflow when default dtype='int32'
> (np.arange(1000000, dtype="int64")**2).sum()
| 333332833333500000  # 64 bit integers do not overflow
> import timeit from timeit
> timeit('sum([x**2 for x in range(1000000)])', number=1)
| 0.5614346340007614
> timeit('(np.arange(1000000)**2).sum()', setup='import numpy as np', number=1)
| 0.014362967000124627  # ridiculous fast but also wrong result...
> timeit('(np.arange(1000000, dtype="int64")**2).sum()',
  setup='import numpy as np', number=1)
| 0.048017077999247704  # fast and correct
> np.iinfo(np.int32).min
| -2147483648
> np.iinfo(np.int32).max
| 2147483647
```

docs.scipy.org/doc/numpy/reference/routines.linalg.html

Linear algebra

Python shell

```
> x = np.arange(1, 5, dtype=float).reshape(2, 2)
> x
| array([[1., 2.],
|        [3., 4.]])
> x.T  # matrix transpose
| array([[1., 3.],
|        [2., 4.]])
> np.linalg.det(x)  # matrix determinant
| -2.0000000000000004
> np.linalg.inv(x)  # matrix inverse
| array([[-2. ,  1. ],
|        [ 1.5, -0.5]])
> np.linalg.eig(x)  # eigenvalues and eigenvectors
| (array([-0.37228132,  5.37228132]),
   array([[-0.82456484, -0.41597356], [0.56576746, -0.90937671]]))
> y = np.array([[5.], [7.]])
> np.linalg.solve(x, y)  # solve linear matrix equations
| array([[-3.],   # z1
|        [ 4.]])  # z2
```

numpy.matrix

"It is no longer recommended to use this class, even for linear algebra. Instead use regular arrays. The class may be removed in the future."

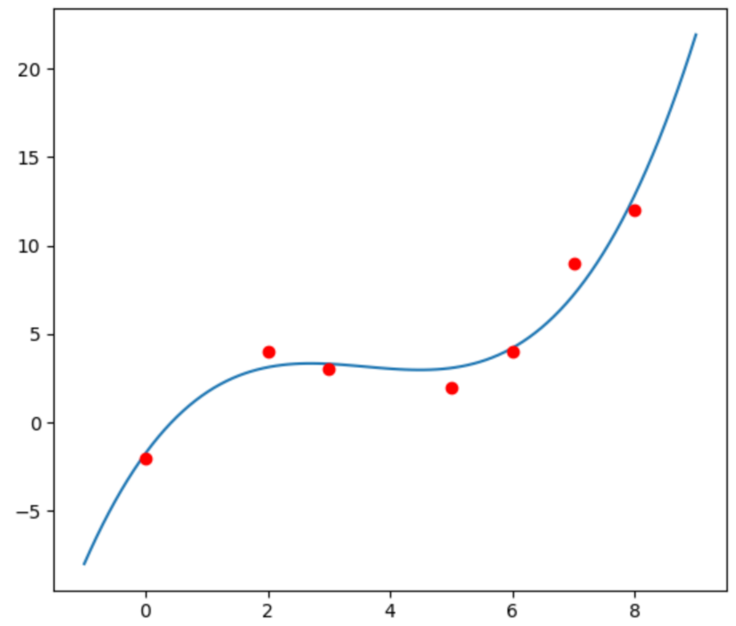
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

numpy.polyfit

- Given n points with $(x_0, y_0), \ldots, (x_{n-1}, y_{n-1})$
- Find polynomial p of degree d that minimizes

$$\sum_{i=0}^{n-1} (y_i - p(x_i))^2$$

- know as least squares fit / linear regression / polynomial regression

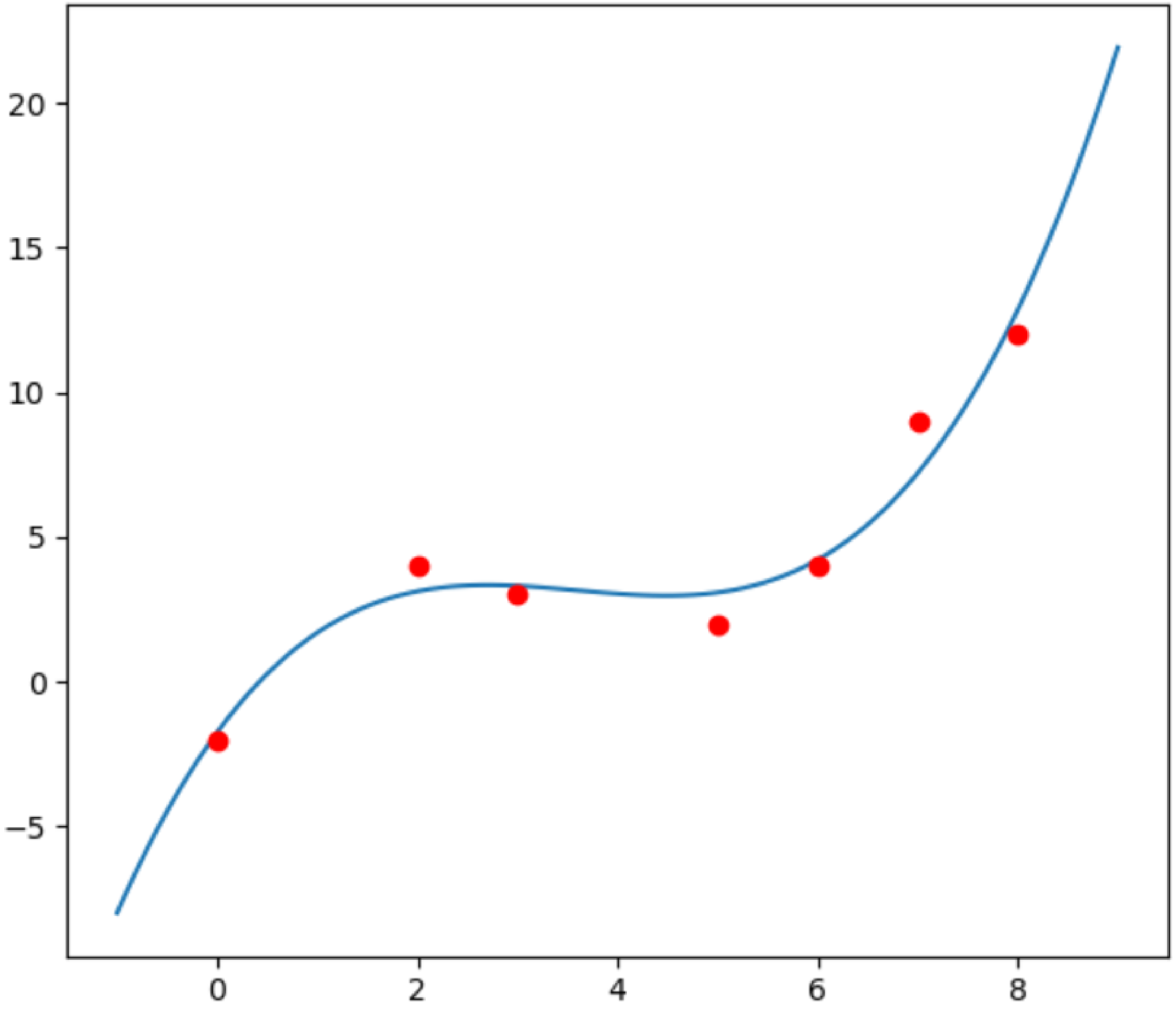


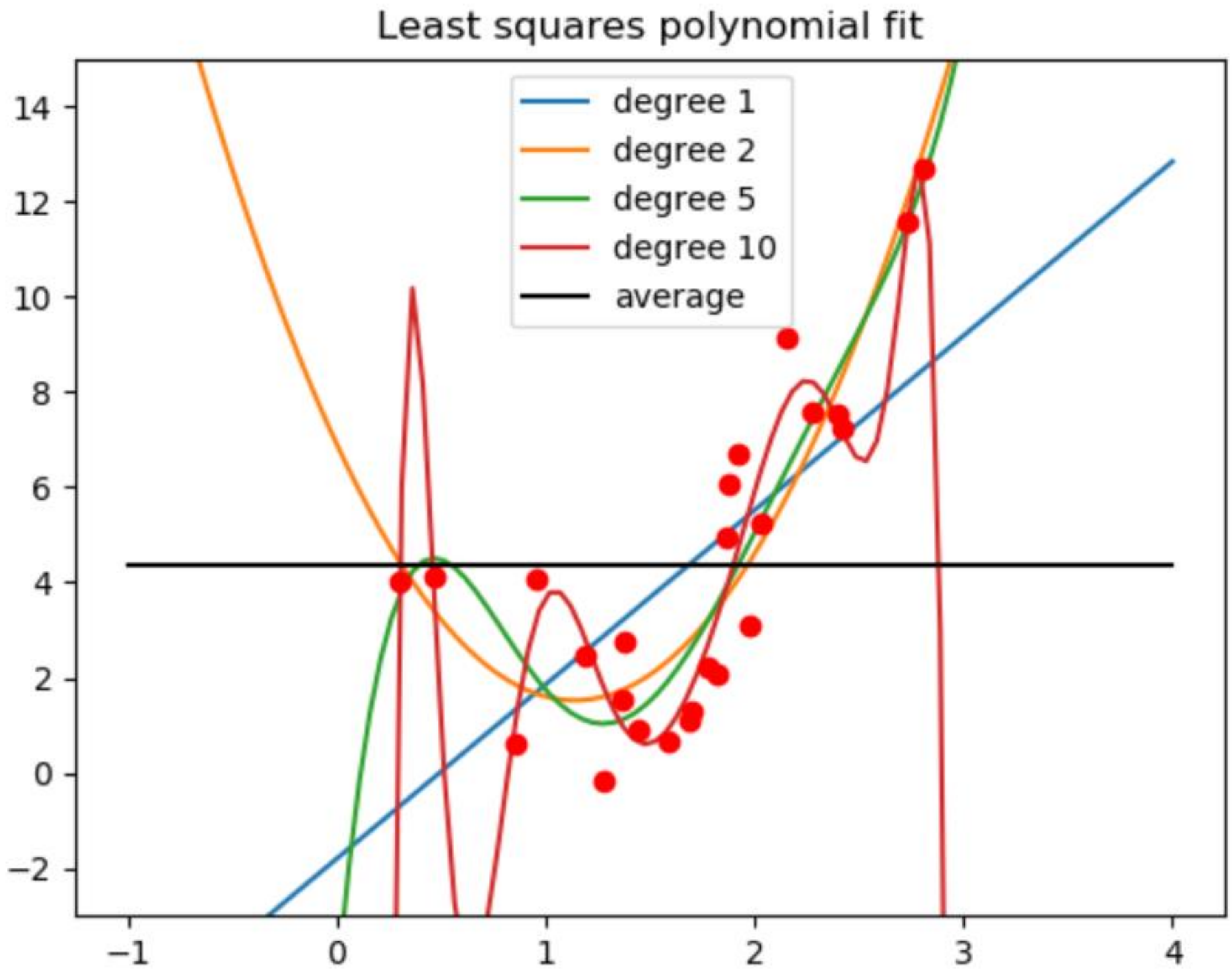
docs.scipy.org/doc/numpy/reference/generated/numpy.polyfit.html

fit.py

```
import matplotlib.pyplot as plt
import numpy as np

x = [0, 2, 3, 5, 6, 7, 8]
y = [-2, 4, 3, 2, 4, 9, 12]

coefficients = np.polyfit(x, y, 3)

fx = np.linspace(-1, 9, 100)
fy = np.polyval(coefficients, fx)

plt.plot(fx, fy, '-')
plt.plot(x, y, 'ro')

plt.show()
```

degree

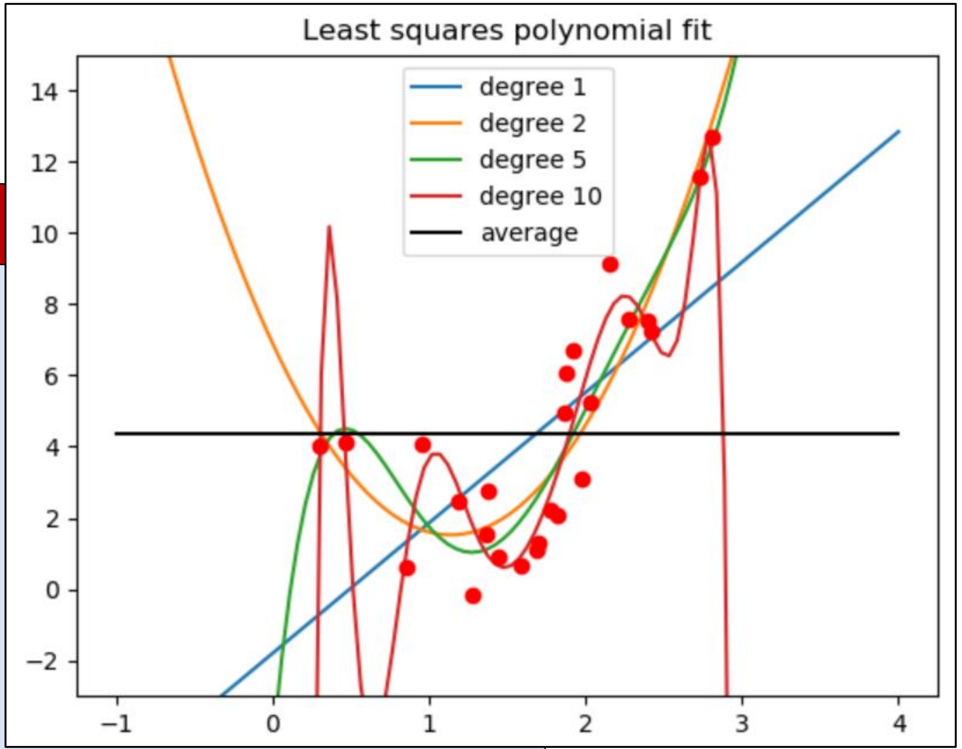




Least squares polynomial fit
degree 1
degree 2
degree 5
degree 10
average
14
12
10
8
6
4
2
0
−2
−1
0
1
2
3
4

polyfit.py

```
import matplotlib.pyplot as plt
import numpy as np

x = 3 * np.random.random(25)
noise = np.random.random(x.size) ** 2
y = 5 * x ** 2 - 12 * x + 7 + 5 * noise

for degree in [1, 2, 5, 10]:
    coefficients = np.polyfit(x, y, degree)
    fx = np.linspace(-1, 4, 100)
    fy = np.polyval(coefficients, fx)
    plt.plot(fx, fy, '-', label="degree %s" % degree)

avg = np.average(y)
plt.plot(x, y, 'ro')
plt.plot([-1, 4], [avg, avg], 'k-', label="average")
plt.ylim(-3, 15)
plt.title('Least squares polynomial fit')
plt.legend()
plt.show()
```

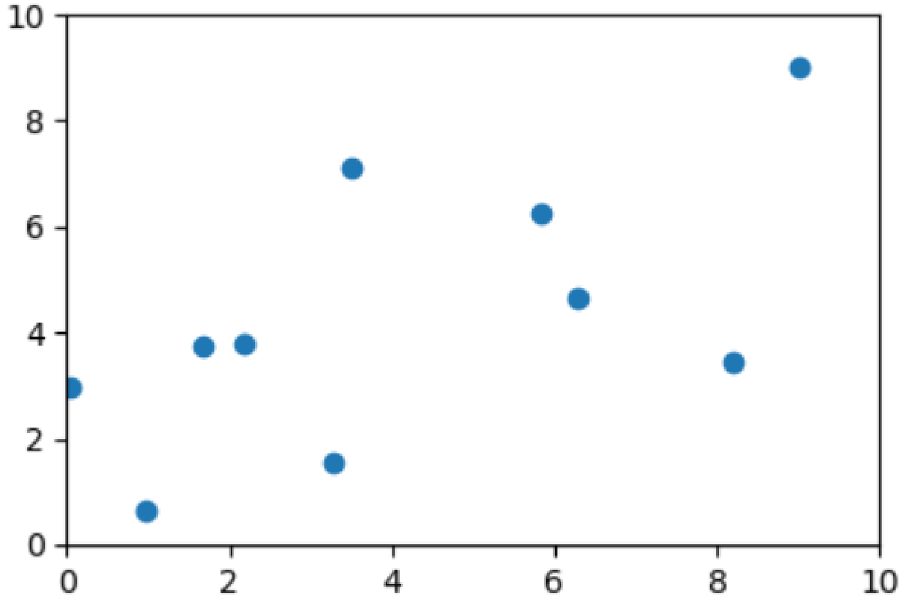


Animating bouncing balls

- matplotlib figures can be animated using

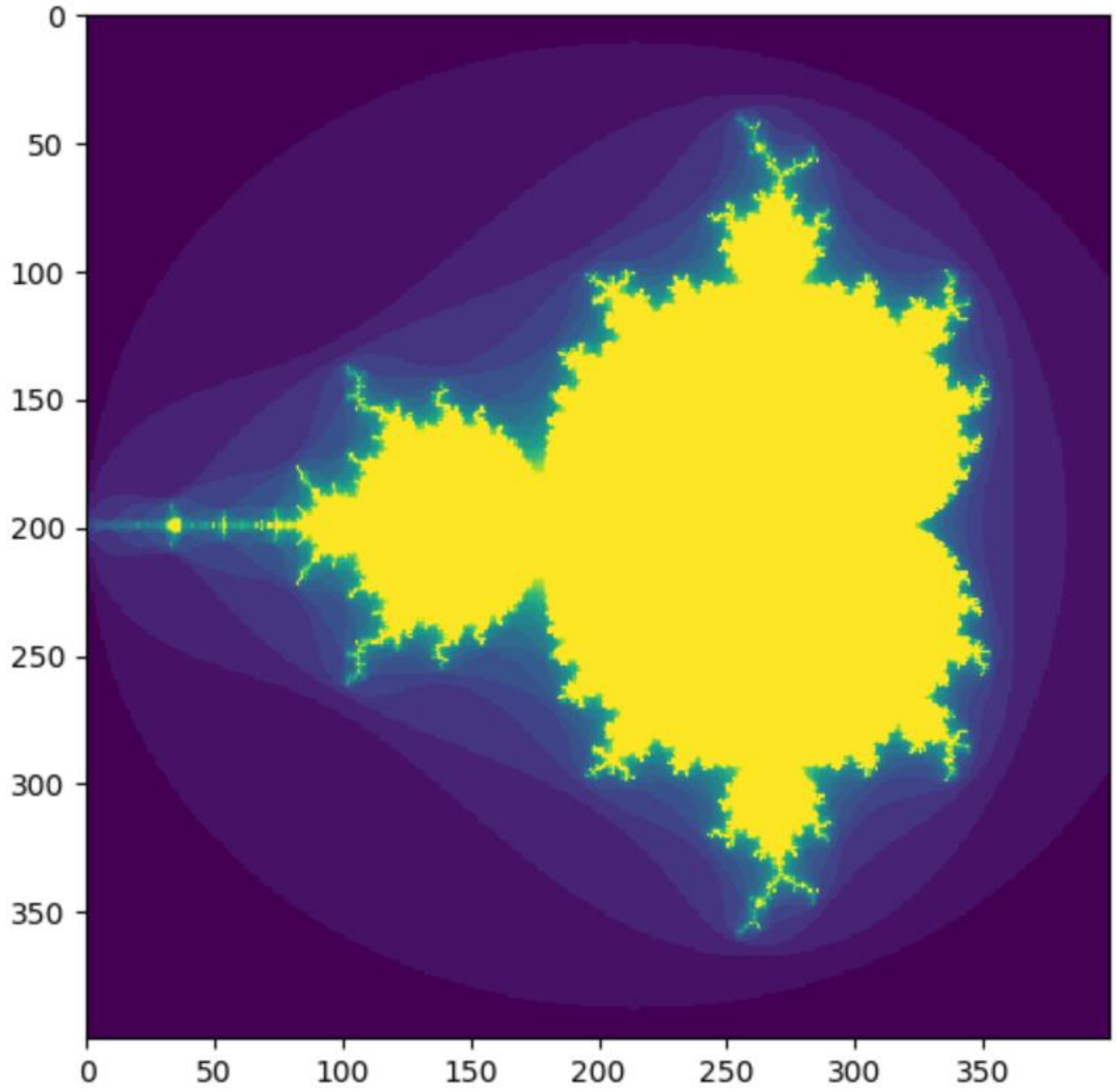
  `matplotlib.animation.FuncAnimation`

  that as arguments take the figure to me updated/ redrawn, a function to call for each update, and an interval in milliseconds between updates



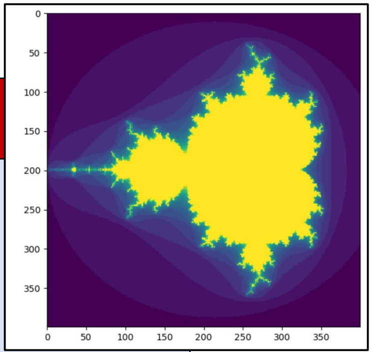
balls.py

```
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from numpy import zeros, maximum, minimum
from numpy.random import random

g = 0.01
N = 10
x, y = 10.0 * random(N), 1.0 + 9.0 * random(N)
dx, dy = random(N) / 5, zeros(N)

fig = plt.figure()
plt.xlim(0, 10)
plt.ylim(0, 10)
balls, = plt.plot(x, y, 'o') # returns Line2D obj

def move(frame):
    global x, y, dx, dy

    x += dx
    bounce = (x > 10.0) | (x < 0.0) # numpy mask
    dx[bounce] = -dx[bounce]
    x = minimum(10.0, maximum(0.0, x))

    y += dy
    bounce = y < 0.0  # numpy mask
    y[bounce] -= dy[bounce]
    dy[bounce] = -dy[bounce]
    dy -= g

    balls.set_data(x, y)  # update positions

# removing 'ani =' causes program to fail...
ani = FuncAnimation(fig, move, interval=25)
plt.show()
```



0
50
100
150
200
250
300
350
0
50
100
150
200
250
300
350

mandelbrot.py




```python
import numpy as np
import matplotlib.pyplot as plt
def mandelbrot(h, w, maxit=20):
    """Returns an image of the Mandelbrot fractal of size (h, w)."""
    x = np.linspace(-2.0, 0.8, w).reshape(1, w)    # row vector
    y = np.linspace(-1.4, 1.4, h).reshape(h, 1)    # column vector
    c = x + y * 1j                                 # broadcast & complex
    z = c
    divtime = np.full(z.shape, maxit, dtype=int)   # all values = maxit
    for i in range(maxit):
        z = z * z + c                              # elementwise
        diverge = z * np.conj(z) > 4               # who is diverging
        div_now = diverge & (divtime == maxit)     # who is diverging now
        divtime[div_now] = i                       # note when
        z[diverge] = 2                             # limit divergence
    return divtime                                 #   (avoids overflows)
plt.imshow(mandelbrot(400, 400))
plt.show()
```

code from docs.scipy.org/doc/numpy/user/quickstart.html

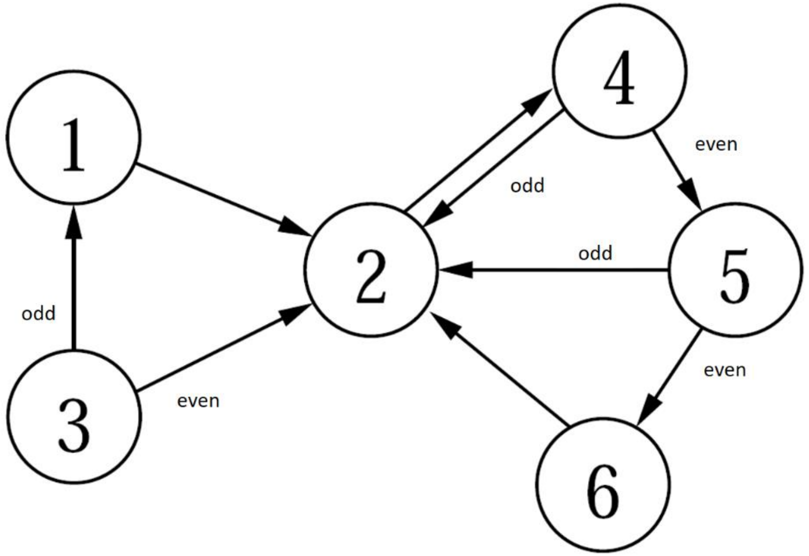
Linear programming

- Example Numpy: PageRank
- scipy.optimize.linprog
- Example linear programming: Maximum flow

PageRank

PageRank - A NumPy / Jupyter / matplotlib example

- Google's original search engine ranked webpages using PageRank
- View the internet as a graph where nodes correspond to webpages and directed edges to links from one webpage to another webpage
- Google's PageRank algorithm was described in (ilpubs.stanford.edu:8090/361/, 1998)



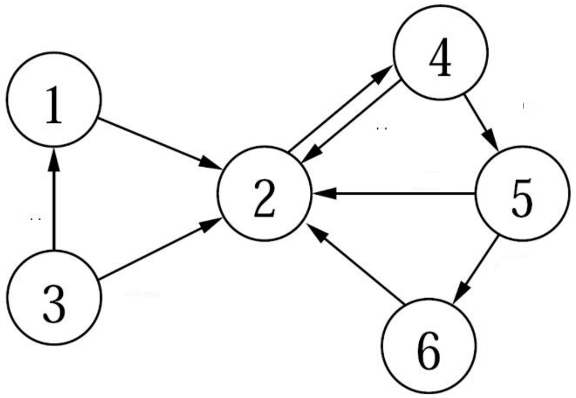
The Anatomy of a Large-Scale Hypertextual Web Search Engine

Sergey Brin and Lawrence Page

Computer Science Department,
Stanford University, Stanford, CA 94305, USA
sergey@cs.stanford.edu and page@cs.stanford.edu

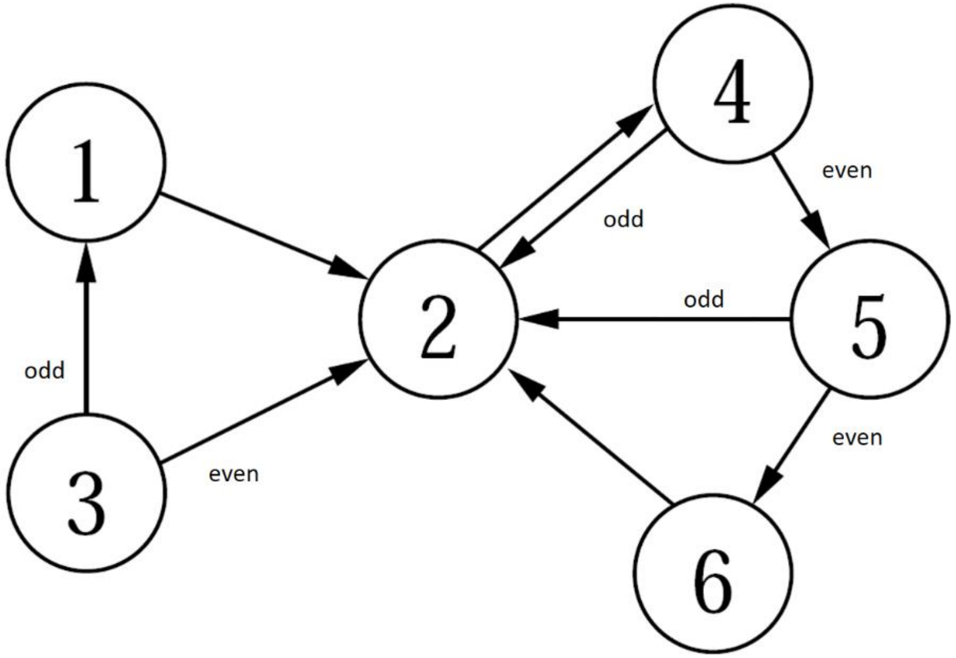
Five different ways to compute PageRank probabilities

1) Simulate random process manually by rolling dices
2) Simulate random process in Python
3) Computing probabilities using matrix multiplication
4) Repeated matrix squaring
5) Eigenvector for $\lambda = 1$



Random surfer model (simplified)

The PageRank of a node (web page) is the fraction of the time one visits a node by performing an *infinite random traversal* of the graph starting at node 1, and in each step



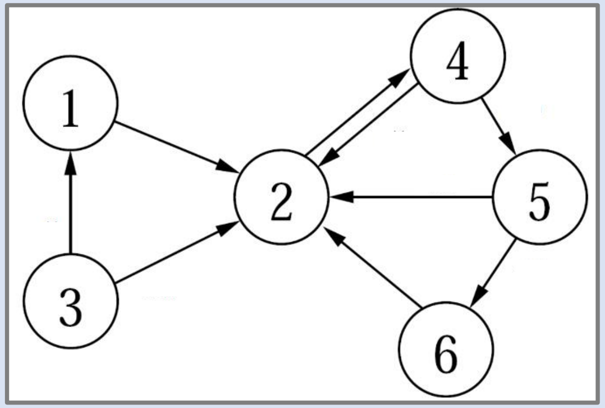
- with probability 1/6 jumps to a random page (probability 1/6 for each node)
- with probability 5/6 follows an outgoing edge to an adjacent node (selected uniformly)

The above can be simulated by using a dice: Roll a *dice*. If it shows 6, jump to a random page by rolling the dice again to figure out which node to jump to. If the dice shows 1-5, follow an outgoing edge - if two outgoing edges roll the dice again and go to the lower number neighbor if it is odd.

Adjacency matrix and degree vector

pagerank.ipynb

```
import numpy as np

# Adjacency matrix of the directed graph in the figure
# (note that the rows/colums are 0-indexed, whereas in the figure the nodes are 1-indexed)

G = np.array([[0, 1, 0, 0, 0, 0],
              [0, 0, 0, 1, 0, 0],
              [1, 1, 0, 0, 0, 0],
              [0, 1, 0, 0, 1, 0],
              [0, 1, 0, 0, 0, 1],
              [0, 1, 0, 0, 0, 0]])

n = G.shape[0]  # number of rows in G
degree = np.sum(G, axis=1, keepdims=True)  # column vector with row sums = out-degrees

# The below code handles sinks, i.e. nodes with outdegree zero (no effect on the graph above)

G = G + (degree == 0)  # add edges from sinks to all nodes
degree = np.sum(G, axis=1, keepdims=True)
```



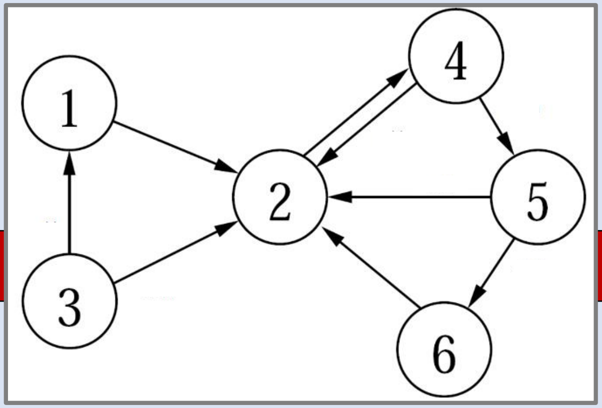
Simulate random walk (random surfer model)

pagerank.ipynb

```
from random import randint
STEPS = 1000000
# adjacency_list[i] is a list of all j where (i, j) is an edge of the graph.
adjacency_list = [[j for j, e in enumerate(row) if e] for row in G]
count = np.zeros(n)          # histogram over number of node visits
state = 0                    # start at node with index 0
for _ in range(STEPS):
    count[state] += 1        # increment count for state
    if randint(1, 6) == 6:   # original paper uses 15% instead of 1/6
        state = randint(0, 5)
    else:
        state = adjacency_list[state][randint(0, degree[state] - 1)]
print(adjacency_list, count / STEPS, sep='\n')
```

Python shell

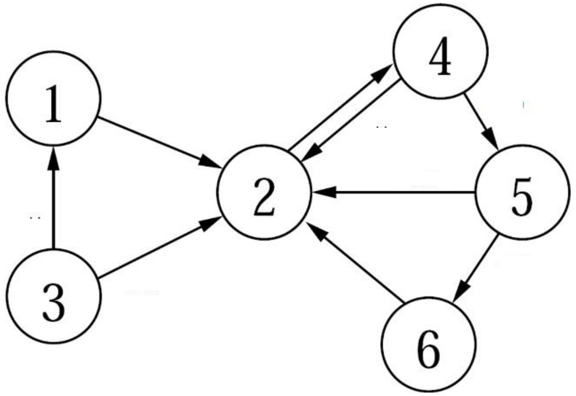
```
| [[1], [3], [0, 1], [1, 4], [1, 5], [1]]
  [0.039365 0.353211 0.02751  0.322593 0.1623   0.095021]
```

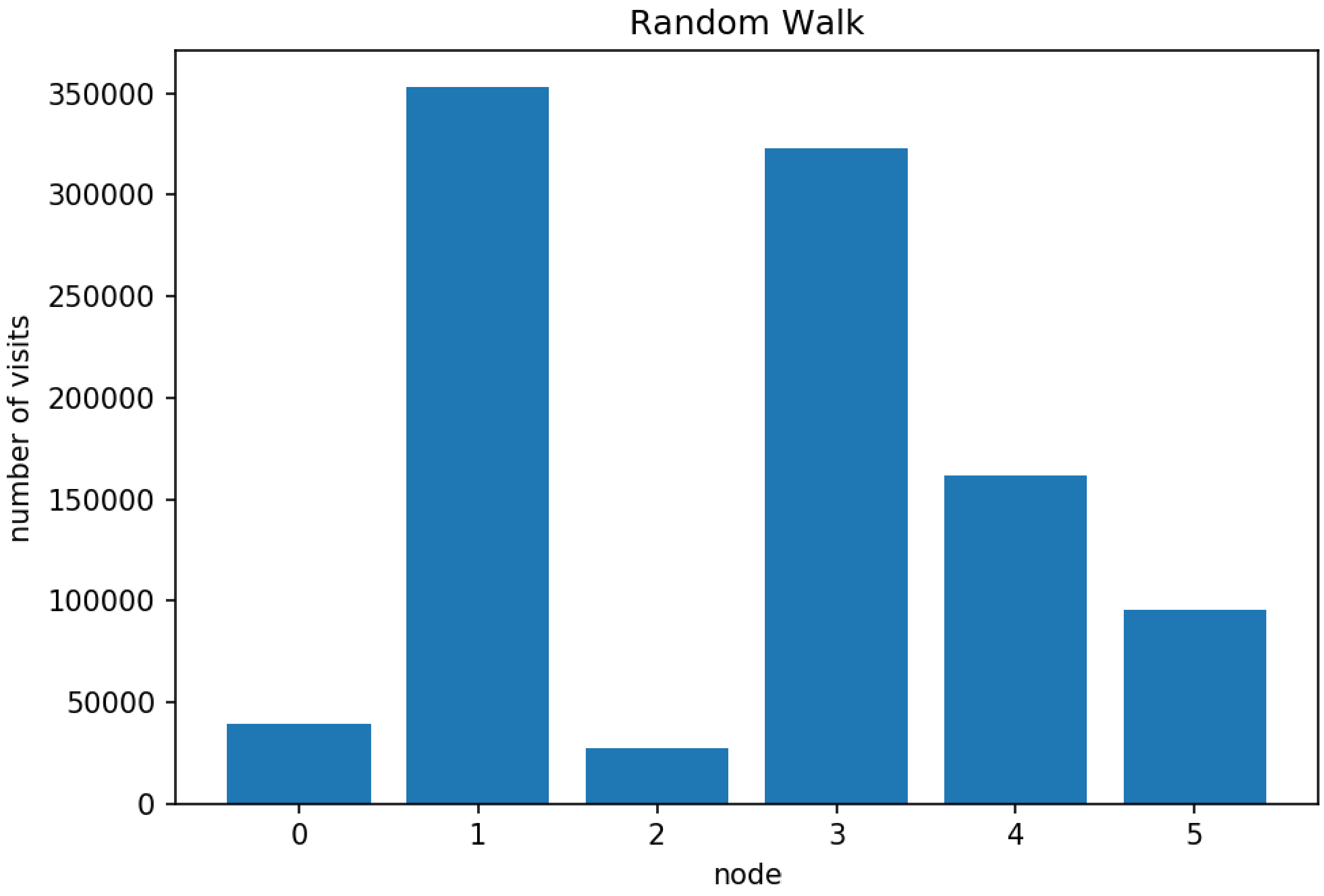


Simulate random walk (random surfer model)

pagerank.ipynb

```
import matplotlib.pyplot as plt
plt.bar(range(6), count)
plt.title("Random Walk")
plt.xlabel("node")
plt.ylabel("number of visits")
plt.show()
```





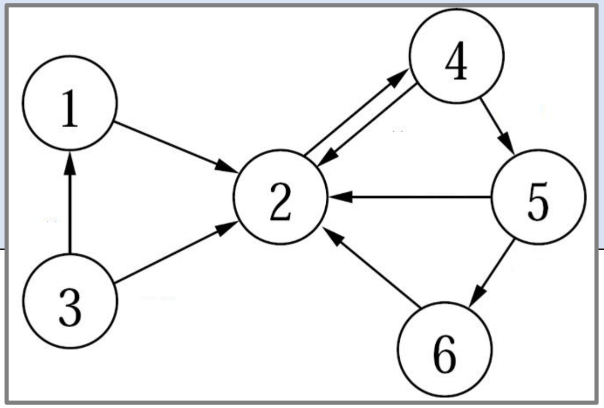
Transition matrix *A*

pagerank.ipynb

```
A = G / degree   # Normalize row sums to one. Note that 'degree'
                 # is an n x 1 matrix, whereas G is an n x n matrix.
                 # The elementwise division is repeated for each column of G
print(A)
```

Python shell

```
[[0.  1.  0.  0.  0.  0. ]
 [0.  0.  0.  1.  0.  0. ]
 [0.5 0.5 0.  0.  0.  0. ]
 [0.  0.5 0.  0.  0.5 0. ]
 [0.  0.5 0.  0.  0.  0.5]
 [0.  1.  0.  0.  0.  0. ]]
```



Repeated matrix multiplication

We now want to compute the probability $p^{(i)}_j$ to be in vertex j after i steps. Let $p^{(i)} = (p^{(i)}_0, \ldots, p^{(i)}_{n-1})$.

Initially we have $p^{(0)} = (1, 0, \ldots, 0)$.

We compute a matrix M, such that $p^{(i)} = M^i \cdot p^{(0)}$ (assuming $p^{(0)}$ is a column vector).

If we let $\mathbf{1}_n$ denote the $n \times n$ matrix with 1 in each entry, then M can be computed as:

$$p_j^{(i+1)} = \frac{1}{6} \cdot \frac{1}{n} + \frac{5}{6} \sum_k p_k^{(i)} \cdot A_{k,j}$$

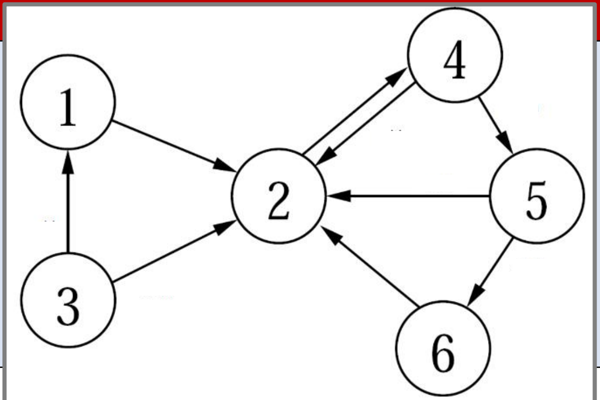
$$p^{(i+1)} = \Big(\underbrace{\frac{1}{6} \cdot \frac{1}{n} \mathbf{1}_n + \frac{5}{6} A^T}_{M}\Big) \cdot p^{(i)}$$

pagerank.ipynb

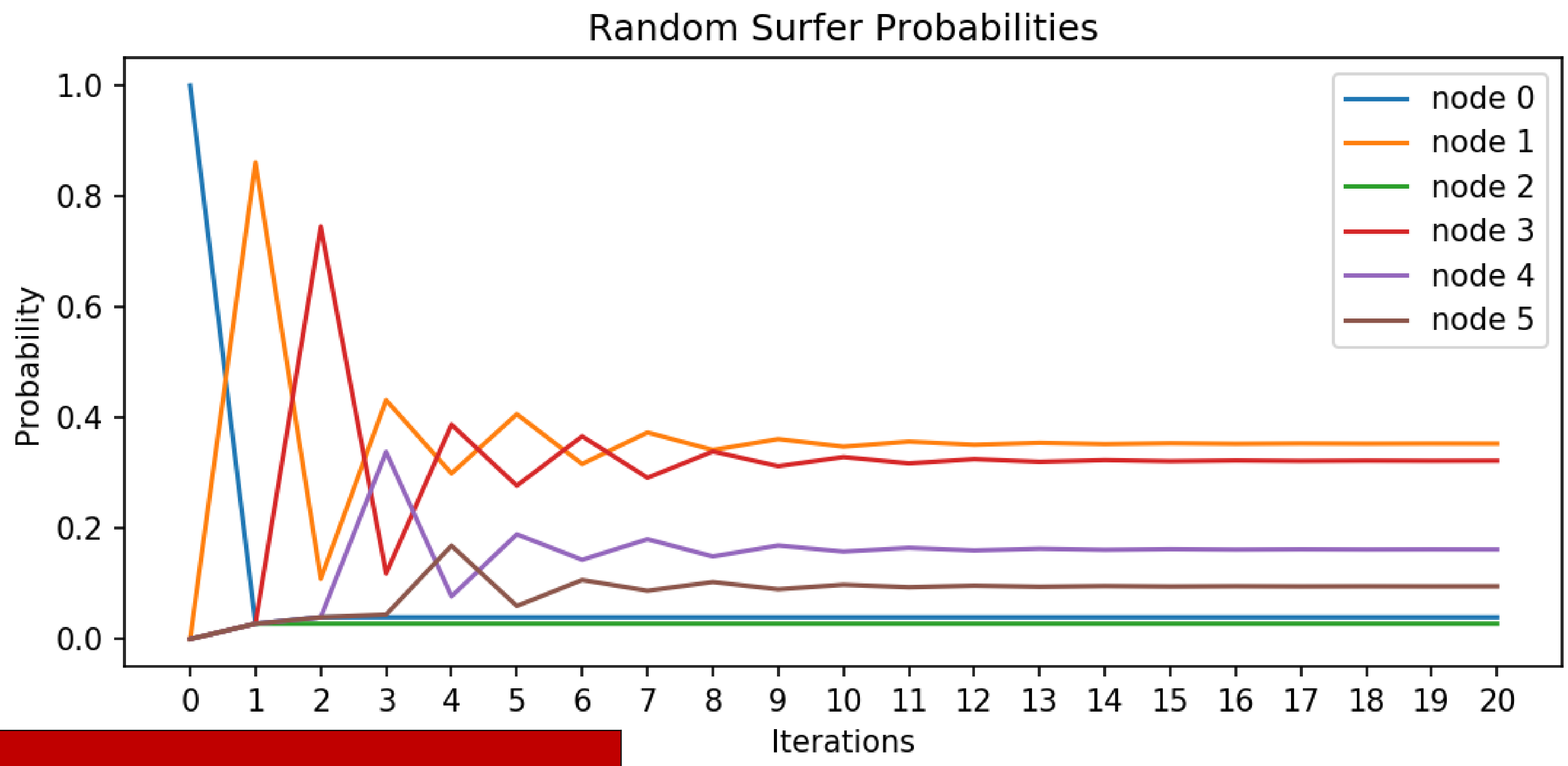
```
ITERATIONS = 20
p_0 = np.zeros((n, 1))
p_0[0, 0] = 1.0
M = 1 / (6 * n) + 5 / 6 * A.T
p = p_0
prob = p  # 'prob' will contain each
          # computed 'p' as a new column
for _ in range(ITERATIONS):
    p = M @ p
    prob = np.append(prob, p, axis=1)
print(p)
```

Python shell

```
| [[0.03935185]
   [0.35326184]
   [0.02777778]
   [0.32230071]
   [0.16198059]
   [0.09532722]]
```

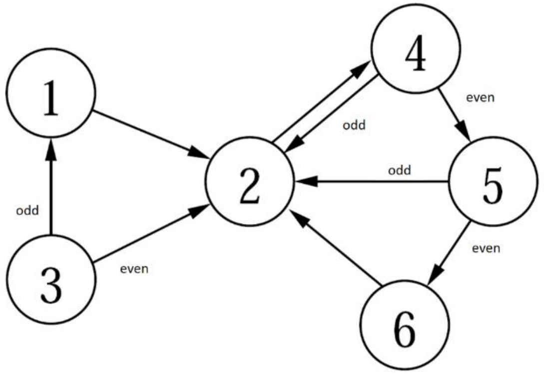


Rate of convergence



pagerank.ipynb

```
x = range(ITERATIONS + 1)
for node in range(n):
    plt.plot(x, prob[node], label="node %s" % node)
plt.xticks(x)
plt.title("Random Surfer Probabilities")
plt.xlabel("Iterations")
plt.ylabel("Probability")
plt.legend()
plt.show()
```



Repeated squaring

$$\underbrace{M\cdot(\cdots(M\cdot(M}_{k\text{ multiplications, }k\text{ power of 2}}\cdot p^{(0)}))\cdots) = M^k\cdot p^{(0)} = M^{2^{\log k}}\cdot p^{(0)} = (\cdots((M\overbrace{^2)^2)^2\cdots)^2}^{\log k}\cdot p^{(0)}$$

pagerank.ipynb

```
from math import log
MP = M
for _ in range(1 + int(log(ITERATIONS, 2))):
    MP = MP @ MP
p = MP @ p_0
print(p)
```

Python shell

```
[[0.03935185]
 [0.35332637]
 [0.02777778]
 [0.32221711]
 [0.16203446]
 [0.09529243]]
```

PageRank : Computing eigenvector for λ = 1

- We want to find a vector *p*, with |*p*| = 1, where *Mp* = *p*, i.e. an *eigenvector* *p* for the eigenvalue λ = 1

pagerank.ipynb

```
eigenvalues, eigenvectors = np.linalg.eig(M)
idx = eigenvalues.argmax()                # find the largest eigenvalue (= 1)
p =  np.real(eigenvectors[:, idx])        # .real returns the real part of complex numbers
p /= p.sum()                              # normalize p to have sum 1
print(p)
```

Python shell

```
| [0.03935185 0.3533267  0.02777778 0.32221669 0.16203473 0.09529225]
```

PageRank : Note on practicality

- In practice an explicit matrix for billions of nodes is infeasible, since the number of entries would be order of 10^{18}
- Instead use sparse matrices (in Python modul `scipy.sparse`) and stay with repeated multiplication

Linear programming

scipy.optimize.linprog

- scipy.optimize.linprog can solve *linear programs* of the following form, where one wants to find an n x 1 vector x satisfying:

Minimize: $c^T \cdot x$

Subject to: $A_{ub} \cdot x \leq b_{ub}$
$A_{eq} \cdot x = b_{eq}$

dimension

$c : n \times 1$

$A_{ub} : m \times n$ $\quad b_{ub} : m \times 1$
$A_{eq} : k \times n$ $\quad b_{eq} : k \times 1$

NB: For industrial strength linear solvers, use solvers like Cplex or Gurobi

Linear programming example

Maximize

$3 \cdot x_1 + 2 \cdot x_2$

Subject to

$2 \cdot x_1 + 1 \cdot x_2 \leq 10$

$5 \cdot x_1 + 6 \cdot x_2 \geq 4$

$-3 \cdot x_1 + 7 \cdot x_2 = 8$

$\Updownarrow$

Minimize

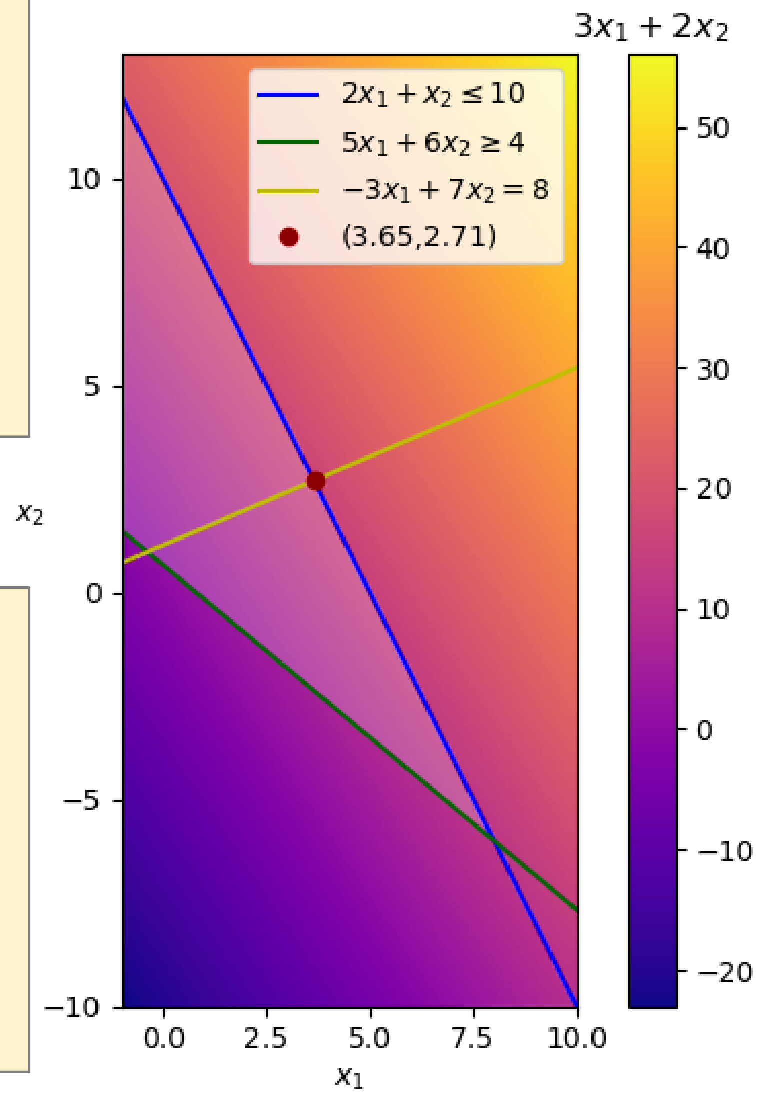
$-(3 \cdot x_1 + 2 \cdot x_2)$

Subject to

$2 \cdot x_1 + 1 \cdot x_2 \leq 10$

$-5 \cdot x_1 + -6 \cdot x_2 \leq -4$

$-3 \cdot x_1 + 7 \cdot x_2 = 8$



linear_programming.py

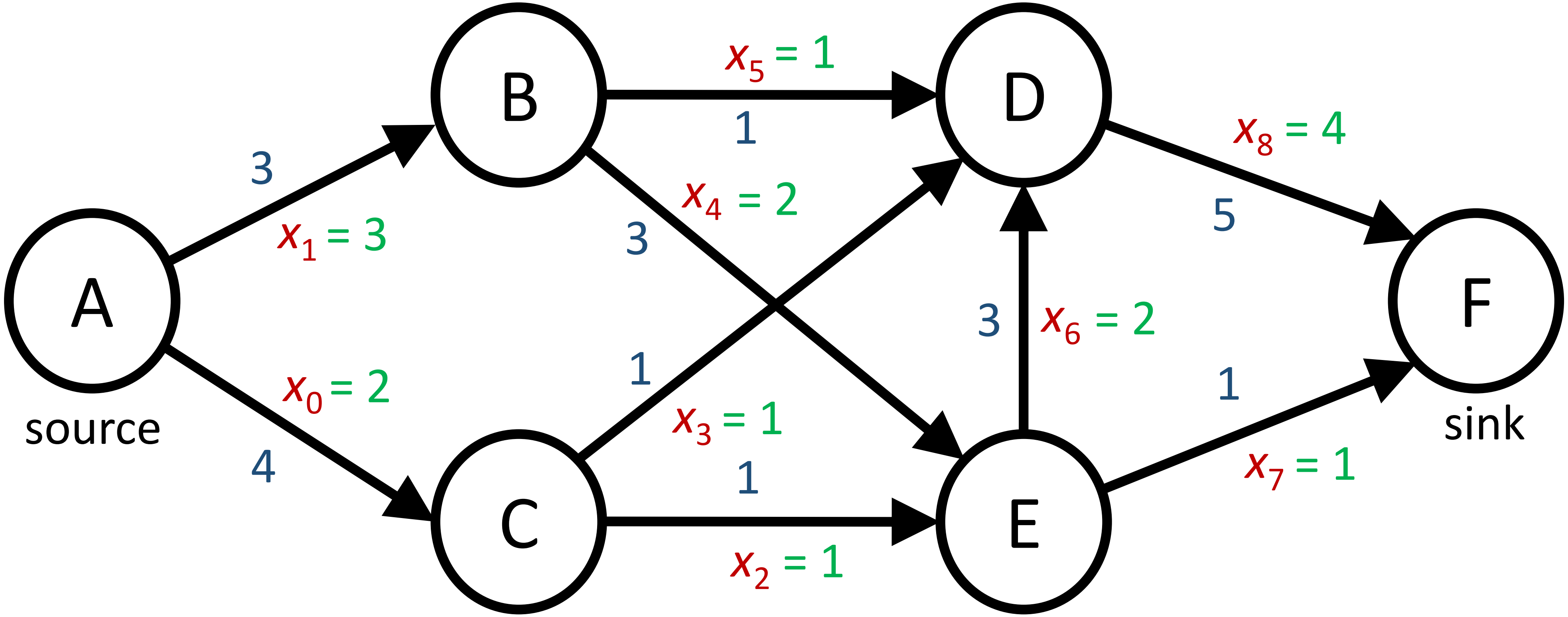
```python
import numpy as np
from scipy.optimize import linprog
c = np.array([3, 2])
A_ub = np.array([[ 2,  1],
                 [-5, -6]]) # multiplied by -1
b_ub = np.array([10, -4])
A_eq = np.array([[-3, 7]])
b_eq = np.array([8])
res = linprog(-c,  # maximize = minimize the negated
              A_ub=A_ub,
              b_ub=b_ub,
              A_eq=A_eq,
              b_eq=b_eq)
print(res)  # res.x is the optimal vector
```

Python shell

```
|    fun: -16.35294117647059
 message: 'Optimization terminated successfully.'
     nit: 3
   slack: array([ 0.        , 30.47058824])
  status: 0
 success: True
       x: array([3.64705882, 2.70588235])
```

Maxmium flow

Solving maximum flow using linear programming



Maximize

$x_7 + x_8$ flow value

Subject to

capacity constraints:

$x_0 \leq 4$
$x_1 \leq 3$
$x_2 \leq 1$
$x_3 \leq 1$
$x_4 \leq 3$
$x_5 \leq 1$
$x_6 \leq 3$
$x_7 \leq 1$
$x_8 \leq 5$

flow conservation:

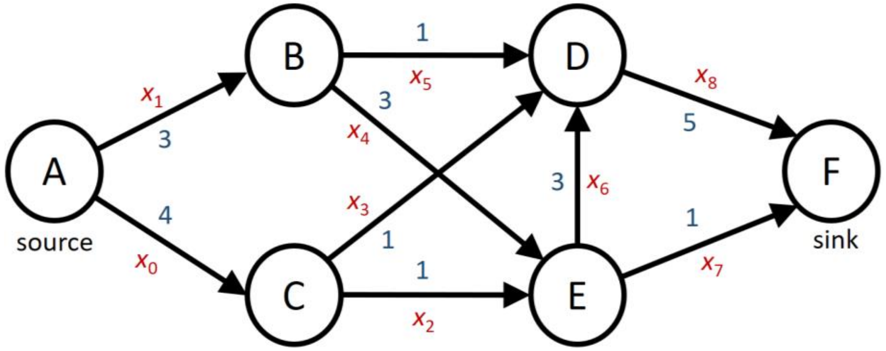
$x_1 = x_4 + x_5$
$x_0 = x_2 + x_3$
$x_3 + x_5 + x_6 = x_8$
$x_2 + x_4 = x_6 + x_7$

Note: solution not unique

We will use the scipy.optimize.linprog function to solve the *maximum flow* problem on the above directed graph. We want to send as much *flow* from node A to node F. Edges are numbered 0..8 and each edge has a maximum *capacity*.

Solving maximum flow using linear programming

- x is a vector describing the flow along each edge
- c is a vector that to add the flow along the edges (7 and 8) to the sink (F), i.e. a function computing *the flow value*
- A_{ub} and b_{ub} is a set of *capacity constraints*, for each edge flow ≤ capacity
- A_{eq} and b_{eq} is a set of *flow conservation* constraints, for each non-source and non-sink node (B, C, D, E), requiring that the flow into equals the flow out of a node



Maximize

$x_7 + x_8$ $\quad c^T \cdot x \quad$ flow value

Subject to

$x_0 \leq 4$
$x_1 \leq 3$
$x_2 \leq 1$
$x_3 \leq 1$
$x_4 \leq 3$
$x_5 \leq 1$
$x_6 \leq 3$
$x_7 \leq 1$
$x_8 \leq 5$

$A_{ub} \cdot x \leq b_{ub}$
⇳
$I \cdot x \leq$ capacity

capacity constraints

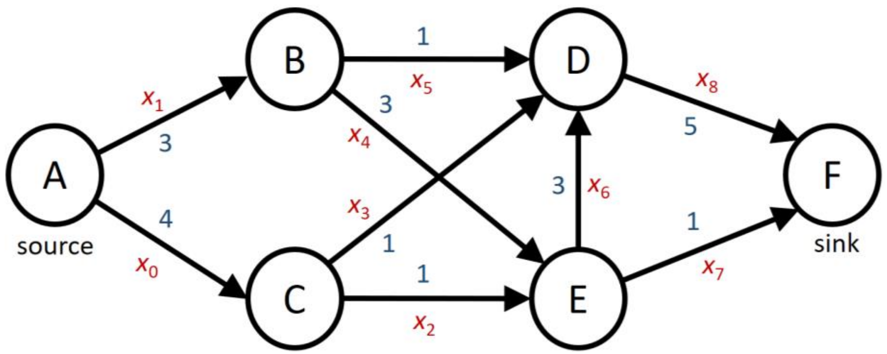
$A_{eq} \cdot x = b_{eq} = 0$

$0 = -x_1 + x_4 + x_5$
$0 = -x_0 + x_2 + x_3$
$0 = -x_3 - x_5 - x_6 + x_8$
$0 = -x_2 - x_4 + x_6 + x_7$

flow conservation

maximum-flow.py

```python
import numpy as np
from scipy.optimize import linprog

#                            0  1  2  3  4  5  6  7  8
conservation = np.array([[ 0,-1, 0, 0, 1, 1, 0, 0, 0],   # B
                         [-1, 0, 1, 1, 0, 0, 0, 0, 0],   # C
                         [ 0, 0, 0,-1, 0,-1,-1, 0, 1],   # D
                         [ 0, 0,-1, 0,-1, 0, 1, 1, 0]])  # E

#                   0  1  2  3  4  5  6  7  8
sinks = np.array([0, 0, 0, 0, 0, 0, 0, 1, 1])

#                      0  1  2  3  4  5  6  7  8
capacity = np.array([4, 3, 1, 1, 3, 1, 3, 1, 5])

res = linprog(-sinks,
              A_eq=conservation,
              b_eq=np.zeros(conservation.shape[0]),
              A_ub=np.eye(capacity.size),
              b_ub=capacity)

print(res)
```



Python shell

```
     fun: -5.0
 message: 'Optimization terminated successfully.'
     nit: 9
   slack: array([2., 0., 0., 0., 1., 0., 1., 0., 1.])
  status: 0
 success: True
       x: array([2., 3., 1., 1., 2., 1., 2., 1., 4.])
```

the solution found varies
with the scipy version

Generators, iterators

- __iter__, __next__
- yield
- generator expression
- measuring memory usage

Iterable & Iterator

Python shell

```
> L = ['a', 'b', 'c']
> type(L)
| <class 'list'>
> it = L.__iter__()
> type(it)
| <class 'list_iterator'>
> it.__next__()
| 'a'
> it.__next__()
| 'b'
> it.__next__()
| 'c'
> it.__next__()
| StopIteration   # Exception
```

Python shell

```
> L = ['a', 'b', 'c']
> it = iter(L)   # calls L.__iter__()
> next(it)       # calls it.__next__()
| 'a'
> next(it)
| 'b'
> next(it)
| 'c'
> next(it)
| StopIteration
```

iterator ≈ pointer into list

['a', 'b', 'c']

- Lists are iterable (must support `__iter__`)
- `iter` returns an iterator (must support `__next__`)

Some iterables in Python: string, list, set, tuple, dict, range, enumerate, zip, map, reversed

Iterator

- next(*iterator_object*) returns the next element from the iterator, by calling the *iterator_object*`.__next__()`. If no more elements to be report raise exception `StopIteration`
- next(*iterator_object*, *default*) returns *default* when no more elements are available (no exception is raised)
- for-loops and list comprehensions require iterable objects
  `for x in range(5):` and `[2**x for x in range(5)]`
- The iterator concept is also central to Java and C++

`for` loop

Python shell

```
> for x in ['a', 'b', 'c']:
      print(x)
| a
| b
| c
```

result of `next` on iterator

iterable object (can call `iter` on it to generate an iterator)

≡

Python shell

```
> L = ['a', 'b', 'c']
> it = iter(L)
> while True:
      try:
          x = next(it)
      except StopIteration:
          break
      print(x)
| a
| b
| c
```

8.3. The `for` statement

The `for` statement is used to iterate over the elements of a sequence (such as a string, tuple or list) or other iterable object:

```
for_stmt ::=  "for" target_list "in" expression_list ":" suite
              ["else" ":" suite]
```

The expression list is evaluated once; it should yield an iterable object. An iterator is created for the result of the `expression_list`. The suite is then executed once for each item provided by the iterator, in the order returned by the iterator. Each item in turn is assigned to the target list using the standard rules for assignments (see Assignment statements), and then the suite is executed. When the items are exhausted (which is immediately when the sequence is empty or an iterator raises a `StopIteration` exception), the suite in the `else` clause, if present, is executed, and the loop terminates.

docs.python.org/3/reference/compound_stmts.html#the-for-statement

`for` loop over changing iterable

⚠ Changing (extending) the list while scanning
The iterator over a list is just an index into the list

Python shell

```
> L = [1, 2]
> for x in L:
      print(x, L)
      L.append(x + 2)
| 1 [1, 2]
| 2 [1, 2, 3]
| 3 [1, 2, 3, 4]
| 4 [1, 2, 3, 4, 5]
| 5 [1, 2, 3, 4, 5, 6]
...
```

Python shell

```
> L = [1, 2]
> for x in L:
      print(x, L)
      L[:0] = [L[0] - 2, L[0] - 1]
| 1 [1,2]
| 0 [-1,0,1,2]
| -1 [-3,-2,-1,0,1,2]
| -2 [-5,-4,-3,-2,-1,0,1,2]
| -3 [-7,-6,-5,-4,-3,-2,-1,0,1,2]
...
```

range

Python shell

```
> r = range(1, 6)   # 1,2,3,4,5
> type(r)
| <class 'range'>
> it = iter(r)
> type(it)
| <class 'range_iterator'>
> next(it)
| 1
> next(it)
| 2
> for x in it:
      print(x)
| 3
| 4
| 5
```

iterable expected
but got iterator ?

Python shell

```
> it
| <range_iterator object at 0x03E7FFC8>
> iter(it)
| <range_iterator object at 0x03E7FFC8>
> it is iter(it)
| True
```

Calling `iter` on a `range_iterator` just returns the iterator itself, i.e. can use the iterator wherever an iterable is expected

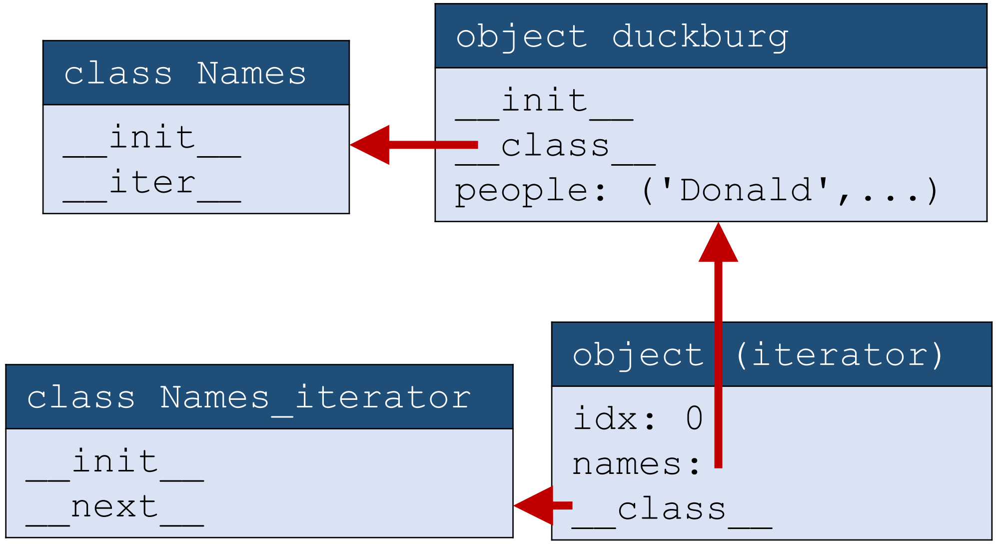
Creating an interable class

names.py

```
class Names:
    def __init__(self, *arg):
        self.people = arg
    def __iter__(self):
        return Names_iterator(self)
class Names_iterator:
    def __init__(self, names):
        self.idx = 0
        self.names = names
    def __next__(self):
        if self.idx >= len(self.names.people):
            raise StopIteration
        self.idx += 1
        return self.names.people[self.idx - 1]
duckburg = Names('Donald', 'Goofy', 'Mickey', 'Minnie')
for name in duckburg:
    print(name)
```

Python shell

```
| Donald
| Goofy
| Mickey
| Minnie
```



An infinite iterable

infinite_range.py

```python
class infinite_range:
    def __init__(self, start=0, step=1):
        self.start = start
        self.step = step
    def __iter__(self):
        return infinite_range_iterator(self)
class infinite_range_iterator:
    def __init__(self, inf_range):
        self.range = inf_range
        self.current = self.range.start
    def __next__(self):
        value = self.current
        self.current += self.range.step
        return value
    def __iter__(self):   # make iterator iterable
        return self
```

Python shell

```
> r = infinite_range(42, -3)
> it = iter(r)
> for idx, value in zip(range(5), it):
      print(idx, value)
| 0 42
| 1 39
| 2 36
| 3 33
| 4 30
> for idx, value in zip(range(5), it):
      print(idx, value)
| 0 27
| 1 24
| 2 21
| 3 18
| 4 15
> print(sum(r))   # don't do this
| (runs forever)
```

`sum` and `zip` take iterables

(`zip` stops when shortest iterable is exhausted)

Creating an iterable class (iterable = iterator)

my_range.py

```python
class my_range:
    def __init__(self, start, end, step):
        self.start = start
        self.end = end
        self.step = step
        self.x = start

    def __iter__(self):
        return self   # self also iterator

    def __next__(self):
        if self.x >= self.end:
            raise StopIteration
        answer = self.x
        self.x += self.step
        return answer

r = my_range(1.5, 2.0, 0.1)
```

Python shell

```
> list(r)
| [1.5, 1.6,
  1.7000000000000002,
  1.8000000000000003,
  1.9000000000000004]
> list(r)
| []
```

- Note that objects act both as an iterable and an iterator
- This e.g. also applies to `zip` objects
- Can only iterate over a `my_range` once

itertools

Function	Description
`count(start, step)`	Inifinite sequence: `start`, `stat` + `step`, …
`cycle(seq)`	Infinite repeats of the elements from `seq`
`repeat(value[, times])`	Infinite repeats of `value` or `times` repeats
`chain(seq0,...,seqk)`	Concatenate sequences
`starmap(func, seq)`	`func(*seq[0]), func(*seq[1]),` …
`permutations(seq)`	Genereate all possible permutations of `seq`
`islice(seq, start, stop, step)`	Create a slice of `seq`
…	…

https://docs.python.org/3/library/itertools.html

Example : Java iterators

vector-iterator.java

```
import java.util.Vector;
import java.util.Iterator;

class IteratorTest {
   public static void main(String[] args) {
       Vector<Integer> a = new Vector<Integer>();
       a.add(7);
       a.add(42);
       // "C" for-loop & get method
       for (int i=0; i<a.size(); i++)
          System.out.println(a.get(i));
       // iterator
       for (Iterator it = a.iterator(); it.hasNext(); )
          System.out.println(it.next());
       // for-each loop - syntax sugar since Java 5
       for (Integer e : a)
          System.out.println(e);
   }
}
```

In Java iteration does not stop using exceptions, but instead the iterator can be tested if it is at the end of the iterable

Example : C++ iterators

vector-iterator.cpp

```
#include <iostream>
#include <vector>
int main() {
  // Vector is part of STL (Standard Template Library)
  std::vector<int> A = {20, 23, 26};
  // "C" indexing - since C++98
  for (int i = 0; i < A.size(); i++)
    std::cout << A[i] << std::endl;
  // iterator - since C++98
  for (std::vector<int>::iterator it = A.begin(); it != A.end(); ++it)
    std::cout << *it << std:: endl;
  // "auto" iterator - since C++11
  for (auto it = A.begin(); it != A.end(); ++it)
    std::cout << *it << std:: endl;
  // Range-based for-loop - since C++11
  for (auto e : A)
    std::cout << e << std:: endl;
}
```

In C++ iterators can be tested if they reach the end of the iterable

move iterator to next element

Generators

Generator expressions

Python shell

```
> [x**2 for x in range(5)]   # list comprehension
| [0, 1, 4, 9, 16]   # list
> (x**2 for x in range(3))   # generator expression
| <generator object <genexpr> at 0x03D9F8A0>
> o = (x**2 for x in range(3))
> next(o)
| 0
> next(o)
| 1
> next(o)
| 4
> next(o)
| StopIteration
```

- A generator expression
  `(... for x in ...)`
  looks like a list comprehension, except square brackets are replaced by parenthesis
- Is an iterable and iterator, that uses less memory than a list comprehension
- computation is done *lazily*, i.e. first when needed

https://docs.python.org/3/reference/expressions.html#generator-expressions

Nested generator expressions

```
Python shell
> squares = (x**2 for x in range(1, 6))   # generator expression
> ratios = (1 / y for y in squares)   # generator expression
> ratios
| <generator object <genexpr> at 0x031FC230>
> next(ratios)
| 1.0
> next(ratios)
| 0.25
> print(list(ratios))
| [0.1111111111111111, 0.0625, 0.04]   # remaining 3
```

- Each fraction is first computed when requested by `next(ratios)` (implicitly called repeatedly in `list(ratios)`)
- The next value of `squares` is first computed when needed by `ratios`

Generator expressions as function arguments

Python shell

```
> squares = (x*2 for x in range(1, 6))
> sum(squares)   # sum takes an iterable
| 30
> sum((x*2 for x in range(1, 6)))
| 30
> sum(x*2 for x in range(1, 6))   # one pair of parenthesis omitted
| 30
```

- Python allows to omit a pair of parenthesis when a generator expression is the only argument to a function

```
f(... for x in ...)   ≡   f((... for x in ...))
```

[PEP 289 – Generator Expressions](#)

Generator functions

two.py

```
def two():
    yield 1
    yield 2
```

Python shell

```
> two()
| <generator object two at 0x03629510>
> t = two()
> next(t)
| 1
> next(t)
| 2
> next(t)
| StopIteration
```

- A *generator function* contains one or more `yield` statements
- Python automatically makes a call to a generator function into an iterable and iterator (provides `__iter__` and `__next__`)
- Calling a generator function returns a *generator object*
- Whenever `next` is called on a generator object, the excuting of the function continues until the next `yield` ***exp*** and the value of *exp* is returned as a result of `next`
- Reaching the end of the function or a return statement, will raise `StopIteration`
- Once consumed, can't be reused

https://docs.python.org/3/reference/expressions.html#yield-expressions

Generator functions (II)

my_generator.py

```python
def my_generator(n):
    yield 'Start'
    for i in range(n):
        yield chr(ord('A') + i)
    yield 'Done'
```

Python shell

```
> g = my_generator(3)
> print(g)
| <generator object my_generator at 0x03E2F6F0>
> print(list(g))
| ['Start', 'A', 'B', 'C', 'Done']
> print(list(g))  # generator object g exhausted
| []
> print(*my_generator(5))  # * takes an iterable (PEP 448)
| Start A B C D E Done
```

Generator functions (III)

my_range_generator.py

```python
def my_range(start, end, step):
    x = start
    while x < end:
        yield x
        x += step
```

Python shell

```
> list(my_range(1.5, 2.0, 0.1))
| [1.5, 1.6, 1.7000000000000002, 1.8000000000000003, 1.9000000000000004]
```

Pipelining generators

Python shell

```
> def squares(seq):   # seq should be an iterable object
      for x in seq:   # use iterator to run through seq
          yield x**2   # generator
> list(squares(range(5)))
| [0, 1, 4, 9, 16]
> list(squares(squares(range(5))))   # pipelining generators
| [0, 1, 16, 81, 256]
> sum(squares(squares(range(100000000))))   # pipelining generators
| 1999999950000000333333333333333330000000
> sum((x**2)**2 for x in range(100000000))   # generator expression
| 1999999950000000333333333333333330000000
> sum([(x**2)**2 for x in range(100000000)])   # list comprehension
| MemoryError
```

yield vs yield from

Python shell

```
> def g():
        yield 1
        yield [2,3,4]
        yield 5
> list(g())
| [1, [2, 3, 4], 5]
```

Python shell

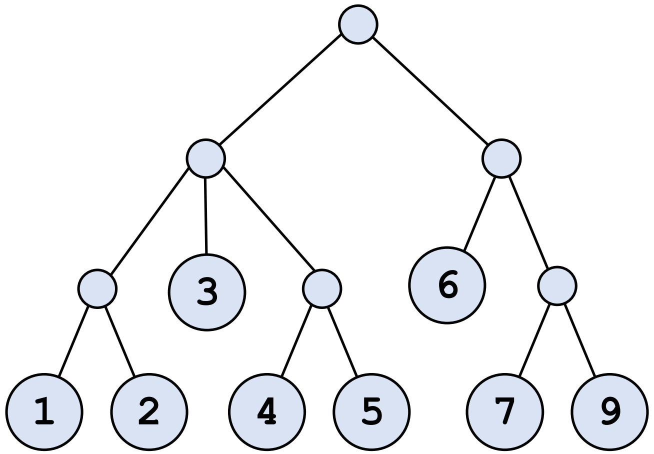
```
> def g():
        yield 1
        yield from [2,3,4]
        yield 5
> list(g())
| [1, 2, 3, 4, 5]
```

- `yield from` available since Python 3.3
- `yield from` ***`exp`*** ≈ `for x in` ***`exp`***`: yield x`

Recursive `yield from`

Python shell

```
> def traverse(T):  # recursive generator
      if isinstance(T, tuple):
          for child in T:
              yield from traverse(child)
      else:
          yield T
> T = (((1,2),3,(4,5)),(6,(7,9)))
> traverse(T)
| <generator object traverse at 0x03279F30>
> list(traverse(T))
| [1, 2, 3, 4, 5, 6, 7, 9]
```



Making objects iterable using `yield`

vector2D.py

```
class vector2D:
    def __init__(self, x_value, y_value):
        self.x = x_value
        self.y = y_value

    def __iter__(self):  # generator
        yield self.x
        yield self.y

    def __iter__(self):  # alternative generator
        yield from (self.x, self.y)

v = vector2D(5, 7)

print(list(v))
print(tuple(v))
print(set(v))
```

Python shell

```
| [5, 7]
| (5, 7)
| {5, 7}
```

Generators vs iterators

- Iterators can often be reused (can copy the current state)
- Generators cannot be reused (only if a new generator is created, starting over again)

- David Beazley's tutorial on "*Generators: The Final Frontier*", PyCon 2014 (3:50:54) Throughout advanced discussion of generators, e.g. how to use `.send` method to implement coroutines https://www.youtube.com/watch?v=D1twn9kLmYg

Measuring memory usage

Measuring memory usage (memory profiling)

- Macro level:

  Task Manager (Windows)
  Activity Monitor (Mac)
  top (Linux)

- Variable level:

  `getsizeof` from `sys` module

- Detailed overview:

  Module `memory_profiler`

  Allows detailed space usage of the code line-by-line (using @profile function decorator) or a plot of total space usage over time

  pip install memory-profiler

Python shell

```
> import sys
> sys.getsizeof(42)
| 14  # size of the integer 42 is 14 bytes
> sys.getsizeof(42**42)
| 44  # the size increases with value
> sys.getsizeof('42')
| 27  # size of a string
> import numpy as np
> sys.getsizeof(np.array(range(100), dtype='int32'))
| 448  # also works on Numpy arrays
> squares = [x**2 for x in range(1000000)]
> sys.getsizeof(squares)
| 4348736
> g = (x**2 for x in range(1000000))
> sys.getsizeof(g)
| 64
```

Module memory-profiler

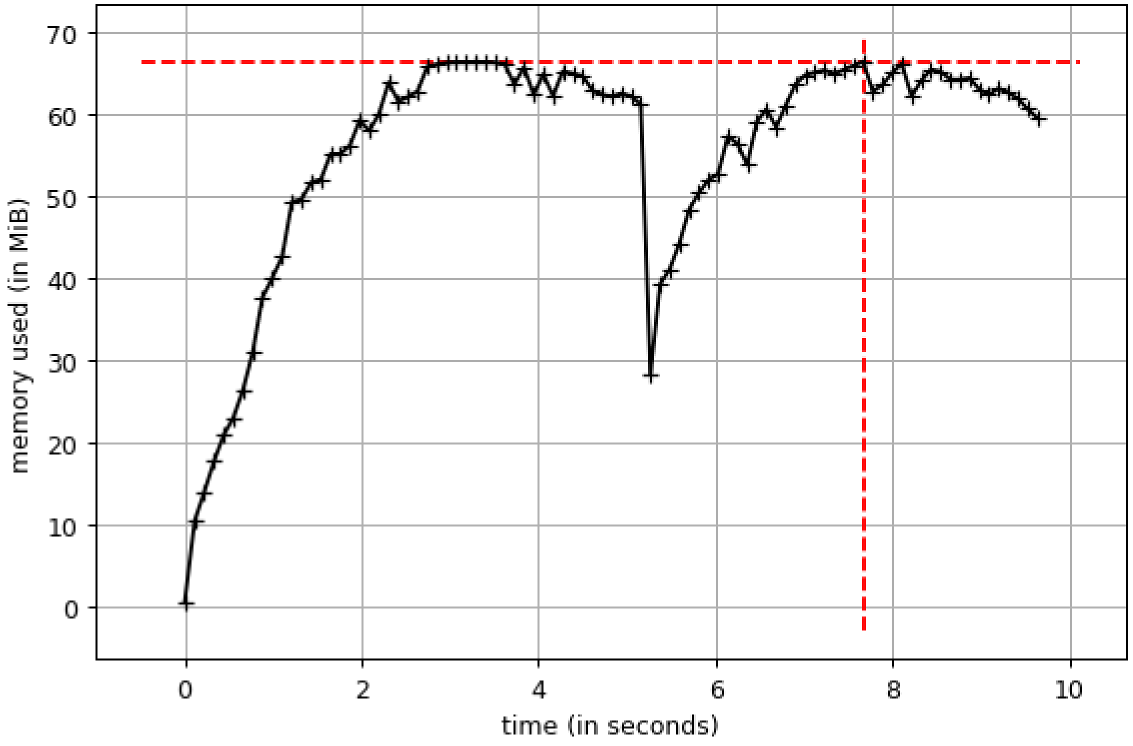
pypi.org/project/memory-profiler/

memory_usage.py

```python
from memory_profiler import profile

@profile  # prints new statistics for each call
def use_memory():
    s = 0
    x = list(range(20_000_000))
    s += sum(x)
    y = list(range(10_000_000))
    s += sum(x)

use_memory()
```

Python Shell

```
| Filename: C:/.../memory_usage.py
|
| Line #  Mem usage  Increment  Line Contents
| ================================================
|      3   32.0 MiB   32.0 MiB  @profile
|      4                         def use_memory():
|      5   32.0 MiB    0.0 MiB      s = 0
|      6  415.9 MiB  383.9 MiB      x = list(range(20_000_000))
|      7  415.9 MiB    0.0 MiB      s += sum(x)
|      8  607.8 MiB  191.9 MiB      y = list(range(10_000_000))
|      9  607.8 MiB    0.0 MiB      s += sum(x)
```

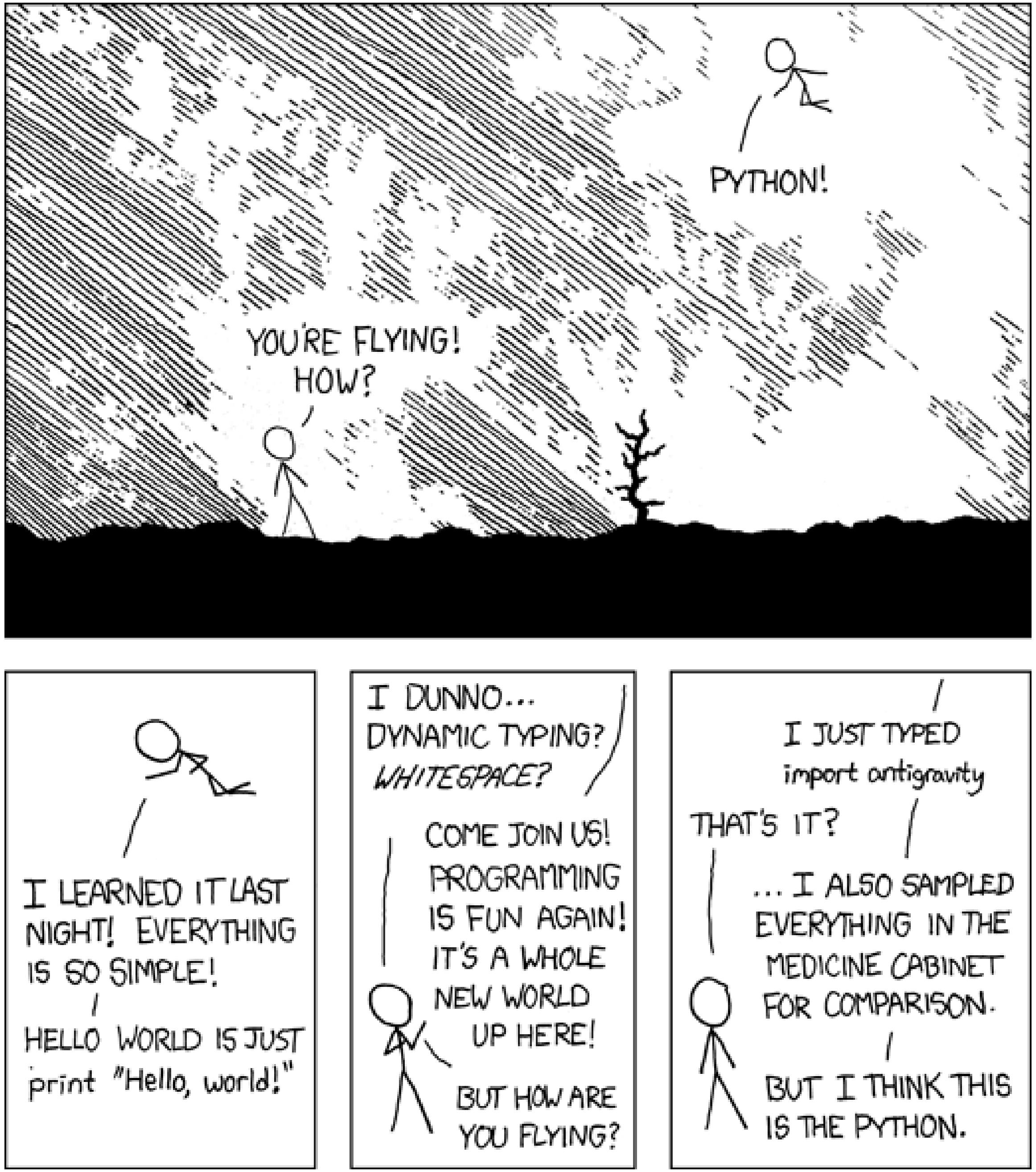


memory_sin_usage.py

```python
from math import sin, pi

for a in range(1000):
    x = list(range(int(1000000 * sin(pi * a / 250))))
```

Windows Shell

```
> pip install memory-profiler
> mprof run memory_sin_usage.py
| mprof: Sampling memory every 0.1s
| running as a Python program...
> mprof plot
```



xkcd.com/353

Modules and packages

- import – from – as
- __name__, "__main__"

docs.python.org/3/tutorial/modules.html

Python modules and packages

- A Python module is a *module_name*.py file containing Python code
- A Python package is a collection of modules

Why do you need modules ?

- A way to structure code into smaller logical units
- Encapsulation of functionality
- Reuse of code in different programs

- Your can write your own modules and packages or use any of the +100.000 existing packages from pypi.org
- The Python Standard Library consists of the modules listed on docs.python.org/3/library

Defining and importing a module

mymodule.py

```
""" This is a 'print something' module """

from random import randint

print("Running my module")

def print_something(n):
    W = ['Eat', 'Sleep', 'Rave', 'Repeat']
    words = (W[randint(0, len(W) - 1)] for _ in range(n))
    print(' '.join(words))

def the_name():
    print('__name__ = "' + __name__ +'"')
```

using_mymodule.py

```
import mymodule
mymodule.the_name()
mymodule.print_something(5)
from mymodule import print_something
print_something(5)
```

Python shell

```
| Running my module
| __name__ = "mymodule"
| Eat Sleep Sleep Sleep Rave
| Eat Sleep Rave Repeat Sleep
```

- A module is only run once when imported several times

Some modules mentioned in the course

Module (example functions)	Description
math (pi sqrt ceil log sin)	*basic math*
random (random randint)	*random number generator*
numpy (array shape)	*multi-dimensional data*
pandas	*data tables*
SQLlite	*SQL database*
scipy scipy.optimize (minimize linprog) scipy.spatial (ConvexHull)	*mathematical optimization*
matplotlib matplotlib.pyplot (plot show style) matplotlib.backends.backend_pdf (PdfPages) mpl_toolkits.mplot3d (Axes3D)	 *plotting data* *print plots to PDF* *3D plot tools*
doctest (testmod) unittest (assertEqual assertTrue)	*testing using doc strings* *unit testing*
time (time) datetime (date.today)	*current time, coversion of time values*
timeit (timeit)	*time execution of simple code*
heapq	*use a list as a heap*

Module (example functions)	Description
functools (lru_cache total_ordering)	*higher order functions and decorators*
itertools (islice permutations)	*Iterator tools*
collections (Counter deque)	*datat structures for collections*
builtins	*module containing the Python builtins*
os (path)	*operating system interface*
sys (argv path)	*system specific functions*
Tkinter PyQt	*graphic user interface*
xml	*xml files (eXtensible Markup Language)*
json	*JSON (JavaScript Object Notation) files*
csv	*comma separated files*
openpyxl	*EXCEL files*
re	*regular expression, string searching*
string (split join lower ascii_letters digits)	*string functions*

Ways of importing modules

import.py

```
# Import a module name in the current namespace
# All definitions in the module are available as <module>.<name>

import math
print(math.sqrt(2))

# Import only one or more specific definitions into current namespace

from math import sqrt, log, ceil
print(ceil(log(sqrt(100), 2)))

# Import specific modules/definitions from a module into current namespace under new names

from math import sqrt as kvadratrod, \  # long import line broken onto multiple lines
                 log as logaritme
import matplotlib.pyplot as plt
print(logaritme(kvadratrod(100)))

# Import all definitions form a module in current namespace
# Deprecated, since unclear what happens to the namespace

from math import *
print(pi)  # where did 'pi' come from?
```

Python shell

```
| 1.4142135623730951
| 4
| 2.302585092994046
| 3.141592653589793
```

Performance of different ways of importing

`from math import sqrt`

appears to be faster than

`math.sqrt`

sqrt_performance.py

```
from time import time

import math
start = time()
x = sum(math.sqrt(x) for x in range(10000000))
end = time()
print("math.sqrt", end - start)

from math import sqrt
start = time()
x = sum(sqrt(x) for x in range(10000000))
end = time()
print("from math import sqrt", end - start)

def test(sqrt=math.sqrt):  # abuse of keyword argument
    start = time()
    x = sum(sqrt(x) for x in range(10000000))
    end = time()
    print("bind sqrt to keyword argument", end - start)

test()
```

Python shell

```
| math.sqrt 4.05187726020813
| from math import sqrt 3.5011463165283203
| bind sqrt to keyword argument 3.261594772338867
```

Listing definitions in a module: dir(*module*)

Python shell

```
> import math
> import matplotlib.pyplot as plt
> dir(math)
| ['__doc__', '__loader__', '__name__', '__package__', '__spec__', 'acos',
  'acosh', 'asin', 'asinh', 'atan', 'atan2', 'atanh', 'ceil', 'copysign',
  'cos', 'cosh', 'degrees', 'e', 'erf', 'erfc', 'exp', 'expm1', 'fabs',
  'factorial', 'floor', 'fmod', 'frexp', 'fsum', 'gamma', 'gcd', 'hypot',
  'inf', 'isclose', 'isfinite', 'isinf', 'isnan', 'ldexp', 'lgamma', 'log',
  'log10', 'log1p', 'log2', 'modf', 'nan', 'pi', 'pow', 'radians', 'sin',
  'sinh', 'sqrt', 'tan', 'tanh', 'tau', 'trunc']
> help(math)
| Help on built-in module math:
| NAME
|     math
| DESCRIPTION
| ...
```

https://docs.python.org/3/library/functions.html#dir

__name__

double.py

```
""" Module double """

def f(x):
    """
    Some doc test code:

    >>> f(21)
    42
    >>> f(7)
    14
    """

    return 2 * x

print('__name__ =', __name__)

if __name__ == "__main__":
    import doctest
    doctest.testmod(verbose=True)
```

Python shell

```
| __name__ = __main__
  ...
  2 passed and 0 failed.
  Test passed.
```

using_double.py

```
import double

print(__name__)
print(double.f(5))
```

Python shell

```
| __name__ = double
  __main__
  10
```

- The variable `__name__` contains the name of the module, or `'__main__'` if the file is run as the main file by the interpreter
- Can e.g. be used to test a module if the module is run independently

module `importlib`

- Implements the `import` statement (Python internal implementation details)
- `importlib.reload(`*module*`)`
  - Reloads a previously imported *module*. Relevant if you have edited the code for the module and want to load the new version in the Python interpreter, without restarting the full program from scratch.

a_constant.py

```python
the_constant = 7
```

Python shell

```
> import a_constant   # import module
> a_constant.the_constant
| 7
> from a_constant import the_constant
> the_constant
| 7
# Update 7 to 42 in a_constant.py
> a_constant.the_constant   # new value not reflected
| 7
> import a_constant   # void, module already loaded
> a_constant.the_constant
| 7  # unchanged
> import importlib
> importlib.reload(a_constant)
| <module 'a_constant' from 'C:\\...\\a_constant.py'>
> a_constant.the_constant
| 42
> the_constant
| 7  # imported attributes are not updated by reload
> from a_constant import the_constant   # force update
> the_constant
| 42  # the new value
```

Packages

- A package is a collection of modules (and subpackages) in a folder = package name
- Only folders having an `__init__.py` file are considered packages
- The `__init__.py` can be empty, or contain code that will be loaded when the package is imported, e.g. importing specific modules

mypackage/__init__.py

```
```

mypackage/a.py

```python
print("Loading mypackage.a")
def f():
    print("mypackage.a.f")
```

using_mypackage.py

```python
import mypackage.a
mypackage.a.f()
```

Python shell

```
| Loading mypackage.a
| mypackage.a.f
```

A package with a subpackage

mypackage/__init__.py

```python
print('loading mypackage')
```

mypackage/a.py

```python
print('Loading mypackage.a')
def f():
    print('mypackage.a.f')
```

mypackage/mysubpackage/__init__.py

```python
print('loading mypackage.mysubpackage')
import mypackage.mysubpackage.b
```

mypackage/mysubpackage/b.py

```python
print('Loading mypackage.mysubpackage.b')
def g():
    print('mypackage.mysubpackage.b.g')
```

using_mysubpackage.py

```python
import mypackage.a
mypackage.a.f()
import mypackage.mysubpackage
mypackage.mysubpackage.b.g()
from mypackage.mysubpackage.b import g
g()
```

Python shell

```
| loading mypackage
| Loading mypackage.a
| mypackage.a.f
| loading mypackage.mysubpackage
| Loading mypackage.mysubpackage.b
| mypackage.mysubpackage.b.g
| mypackage.mysubpackage.b.g
```

`__pycache__` folder

- When Python loads a module the first time it is *compiled* to some intermediate code, and stored as a .pyc file in the `__pycache__` folder.
- If a .pyc file exists for a module, and the .pyc file is newer than the .py file, then `import` loads .pyc – saving time to load the module (but does not make the program itself faster).
- It is safe to delete the `__pycache__` folder – but it will be created again next time a module is loaded.

Path to modules

Python searches the following folders for a module in the following order:

1) The directory containing the input script / current directory
2) *Environment* variable `PYTHONPATH`
3) Installation defaults

The function `path` in the modul `sys` returns a list of the paths

Python 3.6.4 Shell

File Edit Shell Debug Options Window Help

```
Python 3.6.4 (v3.6.4:d48eceb, Dec 19 2017, 06:04:45) [MSC v.1900 32 bit (Intel)] on win32
Type "copyright", "credits" or "license()" for more information.
>>> import sys
>>> sys.path
['', 'C:\\Program Files (x86)\\Python3\\Lib\\idlelib', 'C:\\Program Files (x86)\\Python3\\python36.zip', 'C:\\Program Files (x86)\\Python3\\DLLs', 'C:\\Program Files (x86)\\Python3\\lib', 'C:\\Program Files (x86)\\Python3', 'C:\\Program Files (x86)\\Python3\\lib\\site-packages']
>>> 
```

Ln: 6 Col: 4

Setting PYTHONPATH from windows shell

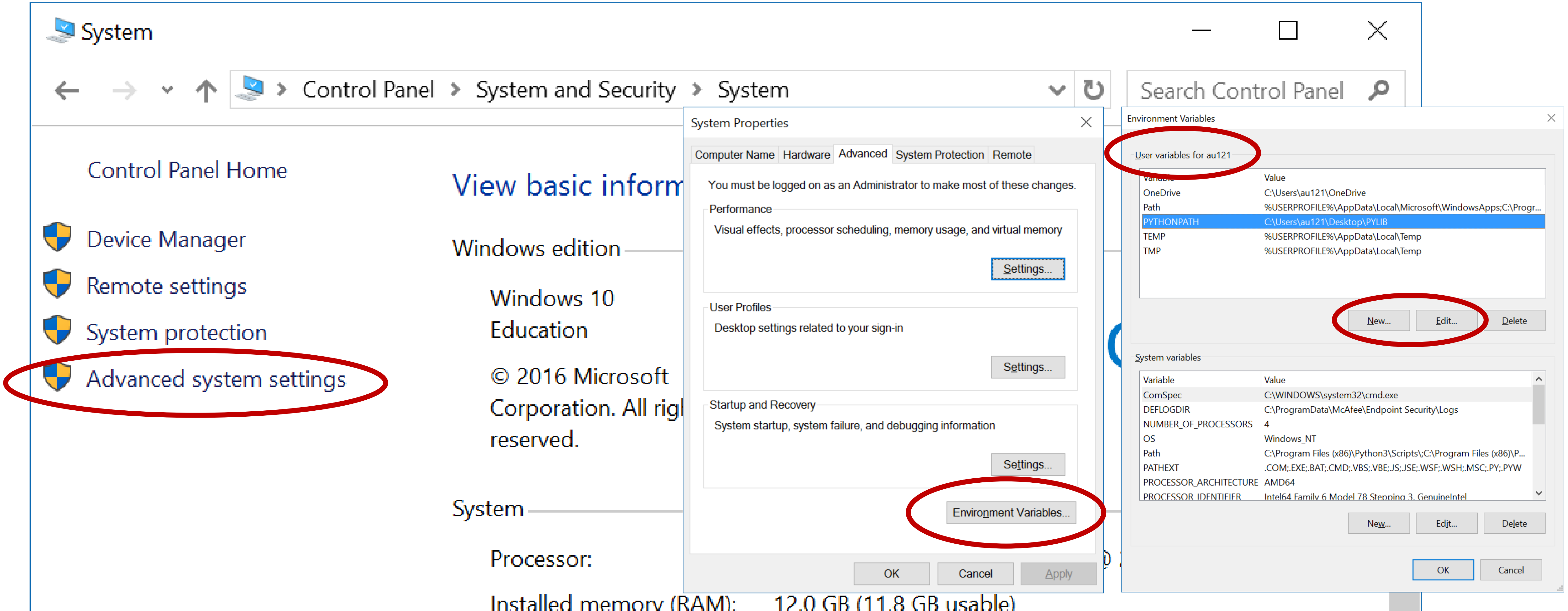
- set PYTHONPATH=*paths separated by semicolon*
  (only valid until shell is closed)

Select Command Prompt - python

```
Microsoft Windows [Version 10.0.14393]
(c) 2016 Microsoft Corporation. All rights reserved.

C:\Users\au121>set PYTHONPATH=C:\Users\au121\Desktop\PYLIB

C:\Users\au121>python
Python 3.6.4 (v3.6.4:d48eceb, Dec 19 2017, 06:04:45) [MSC v.1900 32 bit (Intel)] on win32
Type "help", "copyright", "credits" or "license" for more information.
>>> import sys
>>> sys.path
['', 'C:\\Users\\au121\\Desktop\\PYLIB', 'C:\\Program Files (x86)\\Python3\\python36.zip',
'C:\\Program Files (x86)\\Python3\\DLLs', 'C:\\Program Files (x86)\\Python3\\lib', 'C:\\Pro
gram Files (x86)\\Python3', 'C:\\Program Files (x86)\\Python3\\lib\\site-packages']
>>>
```

Setting PYTHONPATH from control panel

- Control panel > System > Advanced system settings > Environment Variables > User variables > Edit or New PYTHONPATH



Python shell www.python.org/dev/peps/pep-0020/

```
> import this
| The Zen of Python, by Tim Peters
|
| Beautiful is better than ugly.
| Explicit is better than implicit.
| Simple is better than complex.
| Complex is better than complicated.
| Flat is better than nested.
| Sparse is better than dense.
| Readability counts.
| Special cases aren't special enough to break the rules.
| Although practicality beats purity.
| Errors should never pass silently.
| Unless explicitly silenced.
| In the face of ambiguity, refuse the temptation to guess.
| There should be one-- and preferably only one --obvious way to do it.
| Although that way may not be obvious at first unless you're Dutch.
| Now is better than never.
| Although never is often better than *right* now.
| If the implementation is hard to explain, it's a bad idea.
| If the implementation is easy to explain, it may be a good idea.
| Namespaces are one honking great idea -- let's do more of those!
```

module `heapq` (Priority Queue)

- Implements a binary **heap** (Williams 1964).
- Stores a set of elements in a standard list, where arbitrary elements can be inserted efficiently and the smallest element can be extracted efficiently

`heapq.heappush`
`heapq.heappop`

docs.python.org/3/library/heapq.html

J. W. J. Williams. *Algorithm 232: Heapsort*. Communications of the ACM (1964)

heap.py

```python
import heapq
from random import random

H = []  # a heap is just a list

for _ in range(10):
    heapq.heappush(H, random())

while True:
    x = heapq.heappop(H)
    print(x)
    heapq.heappush(H, x + random())
```

Python shell

```
0.20569933892764458
0.27057819339616174
0.31115615362876237
0.4841062272152259
0.5054280956005357
0.509387117524076
0.598647195480462
0.7035150735555027
0.7073929685826221
0.7091224012815325
0.714213496127318
0.727868481291271
0.8051275413759873
0.8279523767282903
0.8626022363202895
0.9376631236263869
```

Valid heap

- A *valid heap* satisfies for all i:

  $L[i] \le L[2 \cdot i + 1]$ and $L[i] \le L[2 \cdot i + 2]$

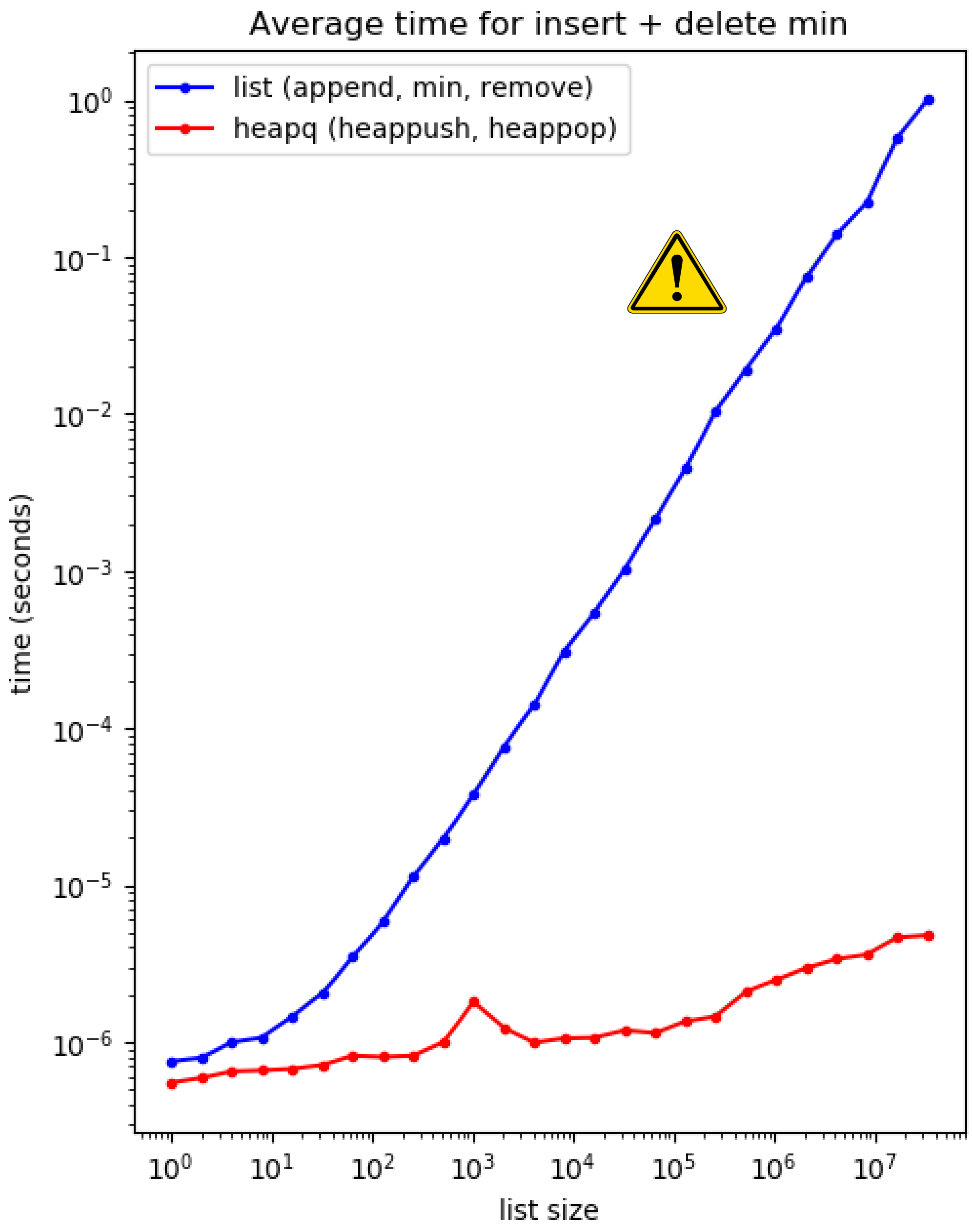
- **heapify(L)** rearranges the elements in a list to make the list a valid heap

Python shell

```
> from random import randint
> L = [randint(1, 20) for _ in range(10)]
> L  # just random numbers
| [18, 1, 15, 17, 4, 14, 11, 3, 4, 9]
> import heapq
> heapq.heapify(L)  # make L a valid heap
> L
| [1, 3, 11, 4, 4, 14, 15, 17, 18, 9]
> print(heapq.heappop(L))
| 1
> L
| [3, 4, 11, 4, 9, 14, 15, 17, 18]
> heapq.heappush(L, 7)
> L
| [3, 4, 11, 4, 7, 14, 15, 17, 18, 9]
```

Why `heapq` ?

- `min` and `remove` on a list take *linear time* (runs through the whole list)
- heapq supports `heappush` and `heappop` in *logarithmic time*
- For lists of length 30.000.000 the performance gain is a factor 200.000



heap_performance.py (generating plot on previous slide)

```
import heapq
from random import random
import matplotlib.pyplot as plt
from time import time
import gc  # garbage collection

size = []
time_heap = []
time_list = []

for i in range(26):
    n = 2 ** i
    size.append(n)

    L = [random() for _ in range(n)]
    R = max(1, 2 ** 23 // n)
(B) gc.collect()
    start = time()
    for _ in range(R):
        L.append(random())
        x = min(L)
        L.remove(x)
    end = time()
    time_list.append((end - start) / R)
```

```
(A) L = None  # avoid MemoryError
    L = [random() for _ in range(n)]
    heapq.heapify(L)  # make L a legal heap
(B) gc.collect()
    start = time()
    for _ in range(100000):
        heapq.heappush(L, random())
        x = heapq.heappop(L)
    end = time()
    time_heap.append((end - start) / 100000)

plt.title("Average time for insert + delete min")
plt.xlabel("list size")
plt.ylabel("time (seconds)")
plt.plot(size, time_list, 'b.-',
         label='list (append, min, remove)')
plt.plot(size, time_heap, 'r.-',
         label='heapq (heappush, heappop)')
plt.xscale('log')
plt.yscale('log')
plt.legend()
plt.show()
```

A Avoid out of memory error for largest experiment, by allowing old **L** to be garbage collected

B Reduce noise in experiments by forcing Python garbage collection before measurement

Working with text

- file formats
- CSV, JSON, XML, Excel
- regular expressions
- module re, finditer

Some file formats

File extension	Content
.html	HyperText Markup Language
.mp3	Audio File
.png .jpeg .jpg	Image files
.svg	Scalable Vector Graphics file
.json	JavaScript Object Notation
.csv	Comma separated values
.xml	eXtensible Markup Language
.xlmx	Micosoft Excel 2010/2007 Workbook

File extension	Description
.exe	Windows executable file
.app	Max OS X Application
.py	Python program
.pyc	Python compiled file
.java	Java program
.cpp	C++ program
.c	C program
.txt	Raw text file

PIL – the Python Imaging Library

- pip install Pillow

rotate_image.py

```
from PIL import Image
img = Image.open("Python-Logo.png")
img_out = img.rotate(45, expand=True)
img_out.save("Python-rotated.png")
```

- For many file types there exist Python packages handling such files, e.g. for images Pillow supports 40+ different file formats
- For more advanced computer vision tasks you should consider [OpenCV]

Python-Logo.png

Python-rotated.png

[python-pillow.org]

CSV files - Comma Separated Values

- Simple 2D tables are stored as rows in a file, with values separated by comma
- Strings stored are quoted if necessary
- Values read are strings
- The deliminator (default comma) can be changed by keyword argument `delimiter`.
  Other typical deliminators are tabs '`\t`', and semicolon '`;`'

docs.python.org/3/library/csv.html

csv-example.py

```
import csv
FILE = 'csv-data.csv'
data = [[1, 2, 3],
        ['a', '"b"'],
        [1.0, ['x',"y"], 'd']]
with open(FILE, 'w', newline="") as outfile:
    csv_out = csv.writer(outfile)
    for row in data:
        csv_out.writerow(row)
with open(FILE, 'r', newline="") as infile:
    for row in csv.reader(infile):
        print(row)
```

Python shell

```
| ['1', '2', '3']
  ['a', '"b"']
  ['1.0', "['x', 'y']", 'd']
```

csv-data.csv

```
1,2,3
a,"""b"""
1.0,"['x', 'y']",d
```

CSV files - Tab Separated Values

csv-tab-separated.py

```python
import csv

FILE = 'tab-separated.csv'

with open(FILE) as infile:
    for row in csv.reader(infile, delimiter='\t'):
        print(row)
```

tab-separated.csv

```
1	2	3
4	5	6
7	8	9
```

Python shell

```
['1', '2', '3']
['4', '5', '6']
['7', '8', '9']
```

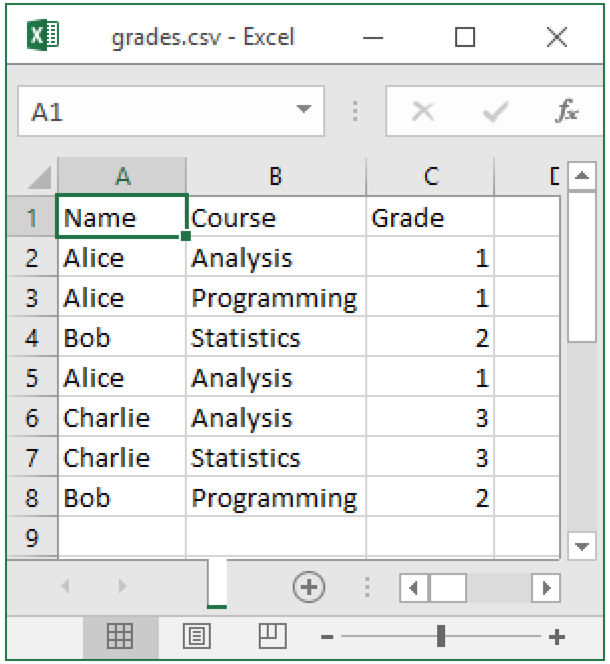
Reading an Excel generated CSV file

average.py

```python
import csv

with open('grades.csv') as file:
    data = csv.reader(file, delimiter=';')  # data = iterator over the rows
    header = next(data)                     # ['Name', 'Course', 'Grade']
    count = {}
    total = {}
    for row in data:                        # iterate over data rows
        course = row[header.index('Course')]
        grade  = int(row[header.index('Grade')])
        count[course] = count.get(course, 0) + 1
        total[course] = total.get(course, 0) + grade
    print('Average grades:')
    width = max(map(len, count))  # maximum course name length
    for course in count:
        print(f'{course:>{width}s} : {total[course] / count[course]:.2f}')
```

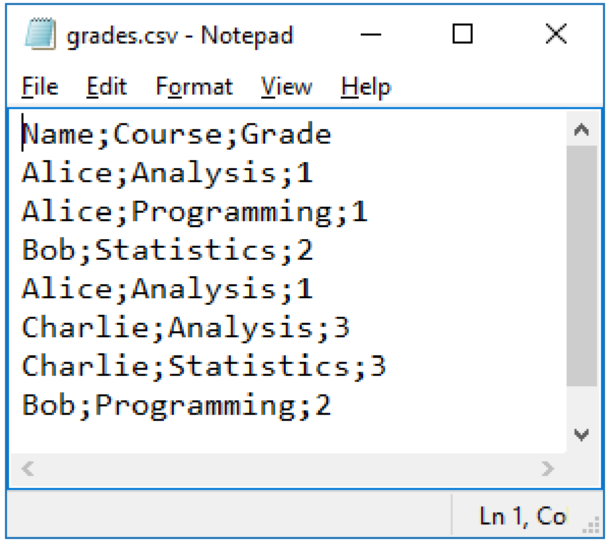
Python shell

```
| Average grades:
|    Analysis : 1.67
| Programming : 1.50
|  Statistics : 2.50
```



	A	B	C
1	Name	Course	Grade
2	Alice	Analysis	1
3	Alice	Programming	1
4	Bob	Statistics	2
5	Alice	Analysis	1
6	Charlie	Analysis	3
7	Charlie	Statistics	3
8	Bob	Programming	2
9			

Saving a file in Excel as
CSV (Comma delimited) (*.csv)
apparently uses ';' as the separator...




```
Name;Course;Grade
Alice;Analysis;1
Alice;Programming;1
Bob;Statistics;2
Alice;Analysis;1
Charlie;Analysis;3
Charlie;Statistics;3
Bob;Programming;2
```

CSV files - Quoting

- The amount of quoting is controlled with keyword argument **`quoting`**
- `csv.QUOTE_MINIMAL` etc. can be used to select the quoting level
- Depending on choice of quoting, numeric values and strings cannot be distinguished in CSV file (`csv.reader` will read all as strings anyway)

csv-quoting.py

```
import csv
import sys

data = [[1, 1.0, '1.0'], ['abc', '"', '\t"', ',']]

quoting_options = [(csv.QUOTE_MINIMAL,    "QUOTE_MINIMAL"),
                   (csv.QUOTE_ALL,        "QUOTE_ALL"),
                   (csv.QUOTE_NONNUMERIC, "QUOTE_NONNUMERIC"),
                   (csv.QUOTE_NONE,       "QUOTE_NONE")]

for quoting, name in quoting_options:
    print(name)
    csv_out = csv.writer(sys.stdout, quoting=quoting, escapechar='\\')
    for row in data:
        csv_out.writerow(row)
```

Python shell

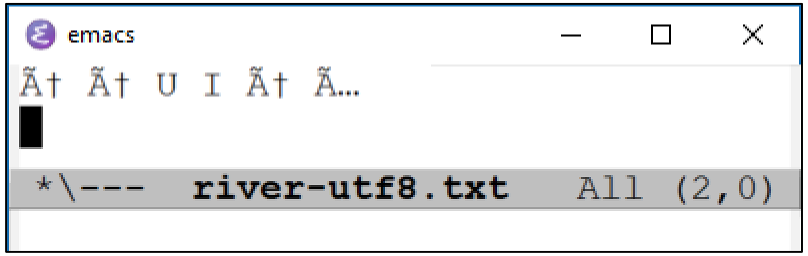
```
QUOTE_MINIMAL  # cannot distinguish 1.0 and "1.0"
1,1.0,1.0
abc,"""","	""",","
QUOTE_ALL  # cannot distinguish 1.0 and "1.0"
"1","1.0","1.0"
"abc","""","	""",","
QUOTE_NONNUMERIC
1,1.0,"1.0"
"abc","""","	""",","
QUOTE_NONE  # cannot distinguish 1.0 and "1.0"
1,1.0,1.0
abc,\",	\",\,
```

File encodings...

```
river-utf8.py   (size 17 bytes, encoding UTF-8)
Æ Æ U I Æ Å
```

```
river-windows1252.py   (size 13 bytes, encoding Windows-1252)
Æ Æ U I Æ Å
```

- Text files can be *encoded* using many different encodings (UTF-8, UTF-16, UTF-32, Windows-1252, ANSI, ASCII, ISO-8859-1, ...)
- Different encodings can result in different file sizes, in particular when containing non-ASCII symbols
- Programs often try to predict the encoding of text files (often with success, but not always)
- Opening files assuming wrong encoding can give strange results....



Opening UTF-8 encoded file but trying to decode using Windows-1252

Opening Windows-1252 encoded file but trying to decode using UTF-8

en.wikipedia.org/wiki/Character_encoding

encoding.py

```
for filename in ['river-utf8.txt', 'river-windows1252.txt']:
    print(filename)
    f = open(filename, 'rb')  # open input in binary mode, default = text mode = 't'
    line = f.readline()       # type(line) = bytes = immutable list of integers in 0..255
    print(line)                              # byte literals look like strings, prefixed 'b'
    print(list(line))                        # print bytes as list of integers
    f = open(filename, 'r', encoding='utf-8')  # try to open file as UTF-8
    line = f.readline()                      # fails if input line is not utf-8
    print(line)
```

river-utf8.py

```
Æ Æ U I Æ Å
```

Python shell

```
| river-utf8.txt
| b'\xc3\x86 \xc3\x86 U I \xc3\x86 \xc3\x85\r\n'  # \x = hexadecimal value follows
| [195, 134, 32, 195, 134, 32, 85, 32, 73, 32, 195, 134, 32, 195, 133, 13, 10]
| Æ Æ U I Æ Å
| river-windows1252.txt
| b'\xc6 \xc6 U I \xc6 \xc5\r\n'
| [198, 32, 198, 32, 85, 32, 73, 32, 198, 32, 197, 13, 10]
| UnicodeDecodeError: 'utf-8' codec can't decode byte 0xc6 in position 0: invalid continuation
  byte
> 'Æ Æ U I Æ Å'.encode('utf8')  # convert string to (an immutable array of) bytes
| b'\xc3\x86 \xc3\x86 U I \xc3\x86 \xc3\x85'
> 'Æ Æ U I Æ Å'.encode('utf8').decode('Windows-1252')  # decode bytes to string
| 'Ã† Ã† U I Ã† Ã…'
```

Reading CSV files with specific encoding

read_shopping.py

```
import csv
with open("shopping.csv", encoding="Windows-1252") as file:
    for article, amount in csv.reader(file):
        print("Buy", amount, article)
```

Python shell

```
| Buy 2 æbler
| Buy 4 pærer
| Buy 3 jordbær
| Buy 10 gulerøder
```

shopping.csv

```
æbler,2
pærer,4
jordbær,3
gulerøder,10
```

CSV file saved with Windows-1252 encoding

JSON

*"**JSON** (**J**ava**S**cript **O**bject **N**otation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is an ideal data-interchange language."*

www.json.org

- Human readable file format
- Easy way to save a Python expression to a file
- Does not support all Python types, e.g. sets are not supported, and tuples are saved (and later loaded) as lists

JSON example

json-example.py

```python
import json
FILE = 'json-data.json'
data = ((None, True), (42.7, (42,)), [3,2,4], (5,6,7),
        {'b':'banana', 'a':'apple', 'c': 'coconut'})

with open(FILE, 'w') as outfile:
    json.dump(data, outfile, indent=2, sort_keys=True)

with open(FILE) as infile:
    indata = json.load(infile)

print(indata)
```

Python shell

```
| [[None, True], [42.7, [42]], [3, 2, 4], [5, 6, 7], {'a':
  'apple', 'b': 'banana', 'c': 'coconut'}]
```

json-data.json


```json
[
  [
    null,
    true
  ],
  [
    42.7,
    [
      42
    ]
  ],
  [
    3,
    2,
    4
  ],
  [
    5,
    6,
    7
  ],
  {
    "a": "apple",
    "b": "banana",
    "c": "coconut"
  }
]
```

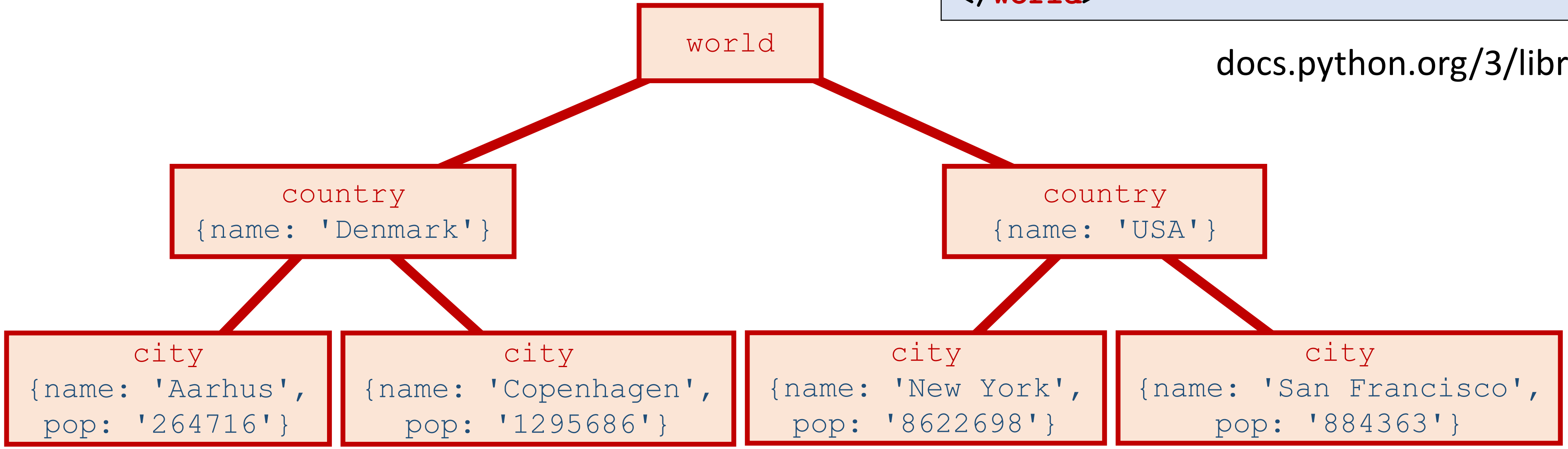
XML - eXtensible Markup Language

- XML is a widespread used data format to store hierarchical data with tags and attributes

cities.xml

```xml
<?xml version="1.0"?>
<world>
  <country name="Denmark">
    <city name="Aarhus" pop="264716"/>
    <city name="Copenhagen" pop="1295686"/>
  </country>
  <country name="USA">
    <city name="New York" pop="8622698"/>
    <city name="San Francisco" pop="884363"/>
  </country>
</world>
```

docs.python.org/3/library/xml.html



xml-example.py

```
import xml.etree.ElementTree as ET
FILE = 'cities.xml'
tree = ET.parse(FILE)  # parse XML file to internal representation
root = tree.getroot()  # get root element
for country in root:
    for city in country:
        print(city.attrib['name'],  # get value of attribute for an element
              'in',
              country.attrib['name'],
              'has a population of',
              city.attrib['pop'])
print(root.tag, root[0][1].attrib)  # the tag & indexing the children of an element
print([city.attrib['name'] for city in root.iter('city')])  # .iter finds elements
```

Python shell

```
| Aarhus in Denmark has a population of 264716
  Copenhagen in Denmark has a population of 1295686
  New York in USA has a population of 8622698
  San Francisco in USA has a population of 884363
  world {'name': 'Copenhagen', 'pop': '1295686'}
  ['Aarhus', 'Copenhagen', 'New York', 'San Francisco']
```

cities.xml

```xml
<?xml version="1.0"?>
<world>
  <country name="Denmark">
    <city name="Aarhus" pop="264716"/>
    <city name="Copenhagen" pop="1295686"/>
  </country>
  <country name="USA">
    <city name="New York" pop="8622698"/>
    <city name="San Francisco" pop="884363"/>
  </country>
</world>
```

XML tags with text

city-descriptions.xml

```
<?xml version="1.0"?>
<world>
  <country name="Denmark">
    <city name="Aarhus" pop="264716">The capital of Jutland</city>
    <city name="Copenhagen" pop="1295686">The capital of Denmark</city>
  </country>
  <country name="USA">
    <city name="New York" pop="8622698">Known as Big Apple</city>
    <city name="San Francisco" pop="884363">Home of the Golden Gate Bridge</city>
  </country>
</world>
```

xml-descriptions.py

```
import xml.etree.ElementTree as ET
FILE = 'city-descriptions.xml'
tree = ET.parse(FILE)
root = tree.getroot()

for city in root.iter('city'):
    print(city.get('name'), "-", city.text)
```

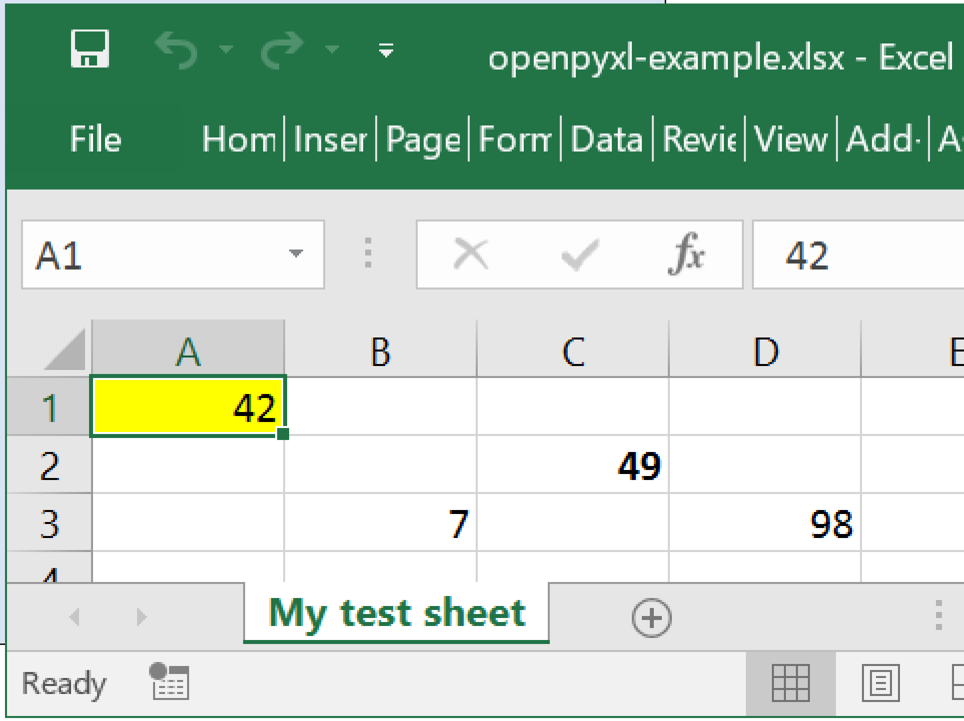
Python shell

```
Aarhus - The capital of Jutland
Copenhagen - The capital of Denmark
New York - Known as Big Apple
San Francisco - Home of the Golden Gate Bridge
```

Openpyxl - Microsoft Excel 2010 manipulation

openpyxl-example.py

```
from openpyxl import Workbook
from openpyxl.styles import Font, PatternFill

wb = Workbook()   # create workbook
ws = wb.active    # active worksheet

ws['A1'] = 42
ws['B3'] = 7
ws['C2'] = ws['A1'].value + ws['B3'].value
ws['D3'] = '=A1+B3+C2'

ws.title = 'My test sheet'

ws['A1'].fill = PatternFill('solid', fgColor='ffff00')
ws['C2'].font = Font(bold=True)

wb.save("openpyxl-example.xlsx")
```



openpyxl.readthedocs.io/en/stable/tutorial.html

String searching using `find`

- Search for first occurence of *substring* in `str[start, end]`

  `str.find(substring[, start[, end]])`

- Returns -1 if no occurence found.
- `.index` similar as `.find`, except raises ValueError exception if substring not found

string-search.py

```
text = 'this is a string - a list of characters'
pattern = 'is'
idx = text.find(pattern)
while idx >= 0:
    print(idx, end=" ")
    idx = text.find(pattern, idx + 1)
```

Python shell

```
| 2 5 22
```

docs.python.org/3/library/stdtypes.html#textseq

Regular expression
– A powerful language to describe sets of strings

- **Examples**
  - `abc` denotes a string of letters
  - `ab*c` any string starting with `a`, followed by an arbitrary number of `b`s and terminated by `c`, i.e. {`ac`, `abc`, `abbc`, `abbbc`, `abbbbc`, ...}
  - `ab+c` equivalent to `abb*c`, i.e. there must be at least one `b`
  - `a\wc` any three letter string, starting with `a` and ending with `c`, where second character is any character in `[a-zA-Z0-9_]`
  - `a[xyz]c` any three letter string, starting with `a` and ending with `c`, where second character is either `x`, `y` or `z`
  - `a[^xyz]c` any three letter string, starting with `a` and ending with `c`, where second character is *none* of `x`, `y` or `z`
  - `^xyz` match at start of string (prefix)
  - `xyz$` match at end of string (suffix)
  - ...
- See docs.python.org/3/library/re.html for more

String searching using regular expressions

- `re.search(`*`pattern`*`, `*`text`*`)`
  – find the first occurence of *`pattern`* in *`text`* – returns `None` or a *match object*
- `re.findall(`*`pattern`*`, `*`text`*`)`
  – returns a list of non-overlapping occurence of *`pattern`* in *`text`* – returns a list of substrings
- `re.finditer(`*`pattern`*`, `*`text`*`)`
  – iterator returning a match object for each non-overlapping occurence of *`pattern`* in *`text`*

Python shell

```
> import re
> text = 'this is a string - a list of characters'
> re.findall(r'i\w*', text)  # prefix with 'r' for raw string literal
| ['is', 'is', 'ing', 'ist']
> for m in re.finditer(r'a[^at]*t', text):
      print('text[%s, %s] = %s' % (m.start(), m.end(), m.group()))
| text[8, 12] = a st
  text[19, 25] = a list
  text[33, 36] = act
```

docs.python.org/3/library/re.html

Substitution and splitting using regular expressions

- `re.sub(`*`pattern, replacement, text`*`)`
  - replace any occurence of the *`pattern`* in *`text`* by *`replacement`*
- `re.split(`*`pattern, text`*`)`
  - split *`text`* at all occurences of *`patern`*

Python shell

```
> text = 'this is a string - a list of characters'
> re.sub(r'\w*i\w*', 'X', text)   # all words containing i
| 'X X a X - a X of characters'
> re.split(r'[^\w]+a[^\w]+', text)   # split around word 'a'
| ['this is', 'string', 'list of characters']
```

docs.python.org/3/library/re.html

Regular expression substitution: \b \w \1 \2 ...

- Assume we want to replace ”a” with ”an” in front of words starting with the vowels a, e, i, o and u.

Python shell

```
> txt = 'A elephant, a zebra and a ape'   # two places to correct
> re.sub('a', 'an', txt)
| 'A elephannt, an zebran annd an anpe'   # replaces all letters 'a' with 'an'
> re.sub(r'\ba\b', 'an', txt)             # raw string + \b boundary of word
| 'A elephant, an zebra and an ape'       # all lower 'a' replaced
> re.sub(r'\b[aA]\b', 'an', txt)
| 'an elephant, an zebra and an ape'      # both 'a' and 'A' replaced by 'an'
> re.sub(r'\b([aA])\b', r'\1n', txt)      # use () and \1 to reinsert match
| 'An elephant, an zebra and an ape'      # kept 'a' and 'A'
> re.sub(r'\b([aA])\s+[aeiou]', r'\1n', txt)        # \s+ = one or more whitespace
| 'Anlephant, a zebra and anpe'           # missing original whitespace + vowel
> re.sub(r'\b([aA])(\s+[aeiou])', r'\1n\2', txt)  # reinsert both () using \1 \2
| 'An elephant, a zebra and an ape'
```

Fun with strings: Lindenmayer systems (L-systems)

Axiom	A
(1)	AB
(2)	ABA
(3)	ABAAB
(4)	ABAABABA

Rules
A → AB
B → A

First four iterations of parallel rewriting

"L-systems were introduced and developed in 1968 by Aristid Lindenmayer, a Hungarian theoretical biologist and botanist at the University of Utrecht. Lindenmayer used L-systems to describe the behaviour of plant cells and to model the growth processes of plant development."

- Wikipedia

L_system.py

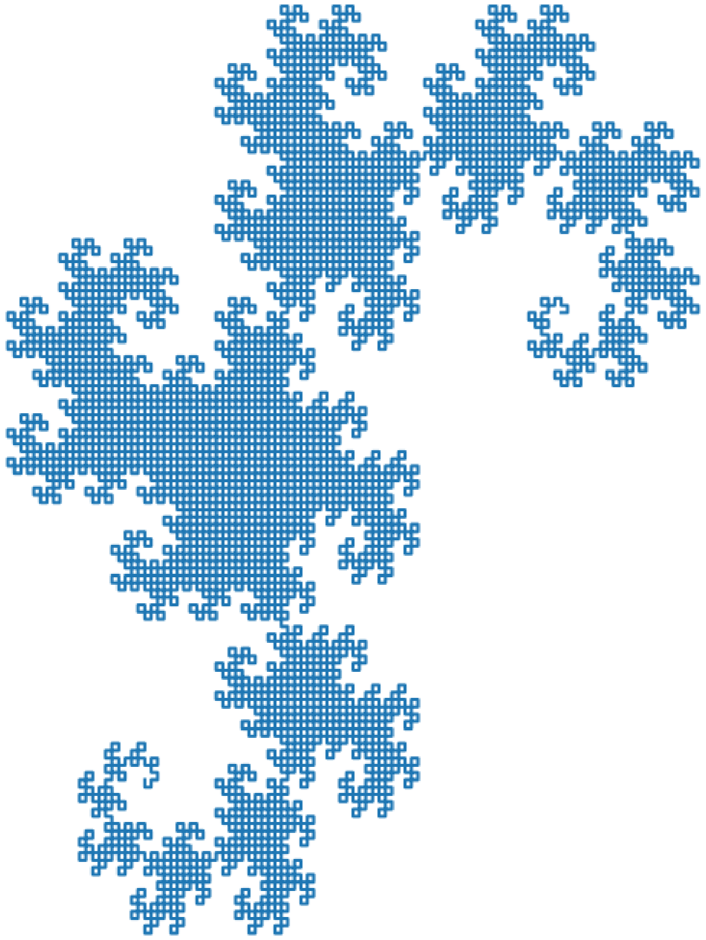
```python
S = 'A'  # axiom
rules = {'A': 'AB', 'B': 'A'}
for i in range(8):
    S = ''.join(rules.get(c, c) for c in S)
    print(S)
```

Python shell

```
| AB
| ABA
| ABAAB
| ABAABABA
| ABAABABAABAAB
| ABAABABAABAABABAABABA
| ABAABABAABAABABAABABAABAABABAABAAB
```

en.wikipedia.org/wiki/L-system

Heighway Dragon



dragon.py

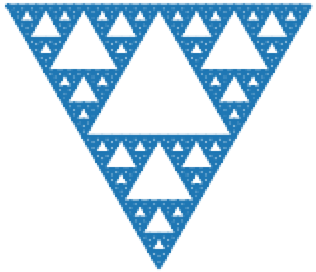
```
import matplotlib.pyplot as plt
from math import sin, cos, radians

axiom = 'FX'
rules = {'X': 'X+YF+', 'Y': '-FX-Y'}

def apply_rules(axiom, rules, repeat):
    for _ in range(repeat):
        axiom = ''.join(rules.get(symbol, symbol) for symbol in axiom)
    return axiom

def walk(commands, position=(0, 0), angle=0, turn=90):
    path = [position]
    for move in commands:
        if move == 'F':
            position = (position[0] + cos(radians(angle)),
                        position[1] + sin(radians(angle)))
            path.append(position)
        elif move == '-': angle -= turn
        elif move == '+': angle += turn
    return path

path = walk(apply_rules(axiom, rules, 13))
plt.plot(*zip(*path), '-')
plt.title('Heighway dragon')
plt.show()
```

Interprete the symbols of the resulting string as a walk where 'F' = draw line forward, and '+' and '-' are turn left and right 90° (X and Y are skipped)

FX+YF++-FX-YF++-FX+YF+--FX-YF++-FX+YF++-FX-YF+--FX+YF+--FX-YF++-FX+YF++-FX-YF++-FX+YF+--FX-YF+--FX+YF++-FX-YF+--FX+YF+--FX-YF++-FX+YF++-FX-YF++-FX+YF+--FX-YF++-FX+YF++-FX-YF+--FX+Y

More space filling curves...

Sierpinski triangle



Axiom F-G-G
F → F-G+F+G-F
G→ GG

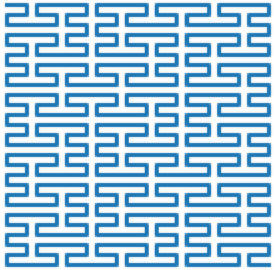
Forward F and G
Turns 120°

Sierpinski arrowhead curve

Axiom A
A → B-A-B
B → A+B+A

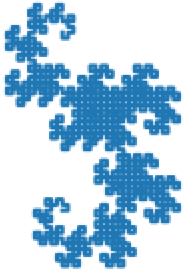
Forward A and B
Turns 60°

Peano curve



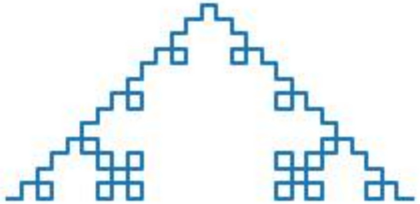
Axiom L
L → LFRFL-F-RFLFR+F+LFRFL
R→ RFLFR+F+LFRFL-F-RFLFR

Heighway dragon



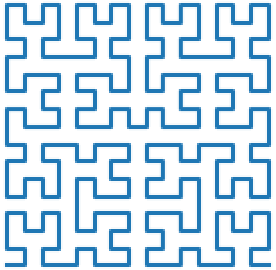
Axiom FX
X → X+YF+
Y → -FX-Y

Koch curve



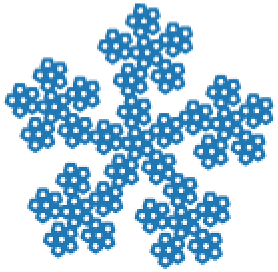
Axiom F
F → F+F-F-F+F

Hilbert curve



Axiom L
L → +RF-LFL-FR+
R → -LF+RFR+FL-

McWorter Pentigree curve



Axiom F-F-F-F-F
F → F-F-F++F+F-F

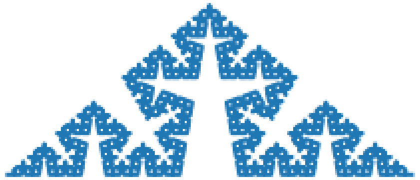
Turns 72°

Tree

Axiom F
F → F[+FF][-FF]F[-F][+F]F

Turns 36°
[and] return to start point when done

Cesero fractal



Axiom F
F → F+F--F+F

Turns 80°

More space filling curves... (source code)

space-filling-L_systems.py

```
import matplotlib.pyplot as plt
from math import sin, cos, radians

def walk(commands,
         pos=(0, 0),
         forward=frozenset('F'),
         angle=0,
         turn=90):
  paths = [[pos]]
  stack = []
  for move in commands:
    if move in forward:
      pos = (pos[0]+cos(radians(angle)),
             pos[1]+sin(radians(angle)))
      paths[-1].append(pos)
    elif move == '-': angle -= turn
    elif move == '+': angle += turn
    elif move == '[':
      stack.append((pos, angle))
    elif move == ']':
      pos, angle = stack.pop()
      paths.append([pos])
  return paths
```

```
def apply_rules(axiom, rules, repeat=1):
    for _ in range(repeat):
        axiom = ''.join(rules.get(symbol, symbol) for symbol in axiom)
    return axiom

curves = [  # Lindenmayer systems (L-systems)
  ('Sierpinski triangle', 'F-G-G', {'F': 'F-G+F+G-F', 'G': 'GG'}, 5, {'turn': 120, 'forward': {'F','G'}}),
  ('Sierpinski arrowhead curve', 'A', {'A': 'B-A-B', 'B': 'A+B+A'}, 5, {'turn': 60, 'forward': {'A','B'}}),
  ('Peano curve', 'L', {'L': 'LFRFL-F-RFLFR+F+LFRFL', 'R': 'RFLFR+F+LFRFL-F-RFLFR'}, 3, {}),
  ('Heighway dragon','FX', {'X': 'X+YF+', 'Y': '-FX-Y'}, 10, {}),
  ('Koch curve', 'F', {'F': 'F+F-F-F+F'}, 3, {}),
  ('Hilbert curve', 'L', {'L': '+RF-LFL-FR+', 'R': '-LF+RFR+FL-'}, 4, {}),
  ('McWorter Pentigree curve', 'F-F-F-F-F', {'F': 'F-F-F++F+F-F'}, 3, {'turn': 72}),
  ('Tree', 'F', {'F': 'F[+FF][-FF]F[-F][+F]F'}, 3, {'turn': 36}),
  ('Cesero fractal', 'F', {'F': 'F+F--F+F'}, 5, {'turn': 80})
]

for idx, (title, axiom, rules, repeat, walk_arg) in enumerate(curves, start=1):
    paths = walk(apply_rules(axiom, rules, repeat), **walk_arg)
    ax = plt.subplot(3, 3, idx, aspect='equal')
    ax.set_title(title)
    for path in paths:
        plt.plot(*zip(*path), '-')
plt.axis('off')
plt.show()
```

Relational data

- SQLite
- pandas

Two tables

Table: country			
name	population	area	capital
'Denmark'	5748769	42931	'Copenhagen'
'Germany'	82800000	357168	'Berlin'
'USA'	325719178	9833520	'Washington, D.C.'
'Iceland'	334252	102775	'Reykjavik'

Table: city			
name	country	population	established
'Copenhagen'	'Denmark'	775033	800
'Aarhus'	'Denmark'	273077	750
'Berlin'	'Germany'	3711930	1237
'Munich'	'Germany'	1464301	1158
'Reykjavik'	'Iceland'	126100	874
'Washington D.C.'	'USA'	693972	1790
'New Orleans'	'USA'	343829	1718
'San Francisco'	'USA'	884363	1776

SQL

pronounced ˌɛsˌkjuːˈɛl or ˈsiːkwəl

Table: country			
name	population	area	capital
'Denmark'	5748769	42931	'Copenhagen'
'Germany'	82800000	357168	'Berlin'
'USA'	325719178	9833520	'Washington, D.C.'
'Iceland'	334252	102775	'Reykjavik'

- SQL = Structured Query **Language**
- **Database = collection of tables** stored persistently on disk
- ANSI and ISO standards since 1986 and 1987, respectively; origin early 70s
- Widespread used SQL databases (can handle many tables/rows/users): Oracle, MySQL, Microsoft SQL Server, PostgreSQL and IBM DB2
- **SQLite** is a very lightweight version storing a database in a single file, without a separate database server
- SQLite is included in both iOS and Android mobil phones

The Course “Database Systems” gives a more in-depth introduction to SQL (MySQL)

SQL examples

Table: country			
name	population	area	capital
'Denmark'	5748769	42931	'Copenhagen'
'Germany'	82800000	357168	'Berlin'
'USA'	325719178	9833520	'Washington, D.C.'
'Iceland'	334252	102775	'Reykjavik'

- CREATE TABLE country (name, population, area, capital)
- INSERT INTO country VALUES ('Denmark', 5748769, 42931, 'Copenhagen')
- UPDATE country SET population=5748770 WHERE name='Denmark'
- SELECT name, capital FROM country WHERE population >= 1000000

  > [('Denmark', 'Copenhagen'), ('Germany', 'Berlin'), ('USA', 'Washington, D.C.')]
- SELECT * FROM country WHERE capital = 'Berlin'

  > [('Germany', 82800000, 357168, 'Berlin')]
- SELECT country.name, city.name, city.established FROM city, country WHERE city.name=country.capital AND city.population < 500000

  > [('Iceland', 'Reykjavik', 874), ('USA', 'Washington, D.C.', 1790)]
- DELETE FROM country WHERE name = 'Germany'
- DROP TABLE country

www.w3schools.com/sql/

sqlite-example.py

```python
import sqlite3
connection = sqlite3.connect('example.sqlite')  # creates file if necessary
c = connection.cursor()
c.executescript('''DROP TABLE IF EXISTS country;  -- multiple SQL statements
                   DROP TABLE IF EXISTS city''')
countries = [('Denmark', 5748769, 42931, 'Copenhagen'),
             ('Germany', 82800000, 357168, 'Berlin'),
             ('USA', 325719178, 9833520, 'Washington, D.C.'),
             ('Iceland', 334252, 102775, 'Reykjavik')]
cities = [('Copenhagen', 'Denmark', 775033, 800),
          ('Aarhus', 'Denmark', 273077, 750),
          ('Berlin', 'Germany', 3711930, 1237),
          ('Munich', 'Germany', 1464301, 1158),
          ('Reykjavik', 'Iceland', 126100, 874),
          ('Washington, D.C.', 'USA', 693972, 1790),
          ('New Orleans', 'USA', 343829, 1718),
          ('San Francisco', 'USA', 884363, 1776)]
c.execute('CREATE TABLE country (name, population, area, capital)')
c.execute('CREATE TABLE city (name, population, established)')
c.executemany('INSERT INTO country VALUES (?,?,?,?)', countries)
c.executemany('INSERT INTO city VALUES (?,?,?,?)', cities)
connection.commit()  # save data to database before closing
connection.close()
```

SQLite

docs.python.org
/3/library/sqlite3.html

SQLite query examples

try to avoid using the asterisk (*) as a good habit
www.sqlitetutorial.net/sqlite-select

sqlite-example.py

```
for row in c.execute('SELECT * FROM country'):   # * = all columns, execute returns iterator
    print(row)                                   # row is by default a Python tuple

for row in c.execute('''SELECT * FROM city, country    -- all pairs of rows from city × country
                        WHERE city.name = country.capital AND city.population < 700000'''):
    print(row)

print(*c.execute('''SELECT country.name,
                           COUNT(city.name) AS cities,
                           100 * SUM(city.population) / country.population
                    FROM city JOIN country ON city.country = country.name  -- SQL join 2 tables
                    WHERE city.population > 500000                       -- only consider big cities
                    GROUP BY city.country                          -- output has one row per group of rows
                    ORDER BY cities DESC, SUM(city.population) DESC'''))   # ordering of output
```

Python shell

```
| ('Denmark', 5748769, 42931, 'Copenhagen')
| ('Germany', 82800000, 357168, 'Berlin')
| ('USA', 325719178, 9833520, 'Washington, D.C.')
| ('Iceland', 334252, 102775, 'Reykjavik')

| ('Reykjavik', 'Iceland', 126100, 874, 'Iceland', 334252, 102775, 'Reykjavik')
| ('Washington, D.C.', 'USA', 693972, 1790, 'USA', 325719178, 9833520, 'Washington, D.C.')

| ('Germany', 2, 6) ('USA', 2, 0) ('Denmark', 1, 13)
```

SQL injection

Right way

```
c.execute('INSERT INTO users VALUES (?)', (user,))
```

unsafe-example.py

```
import sqlite3
connection = sqlite3.connect('users.sqlite')
c = connection.cursor()
c.execute('CREATE TABLE users (name)')
while True:
    user = input('New user: ')
    c.executescript('INSERT INTO users VALUES ("%s")' % user)
    connection.commit()
    print(list(c.execute('SELECT * FROM users')))
```

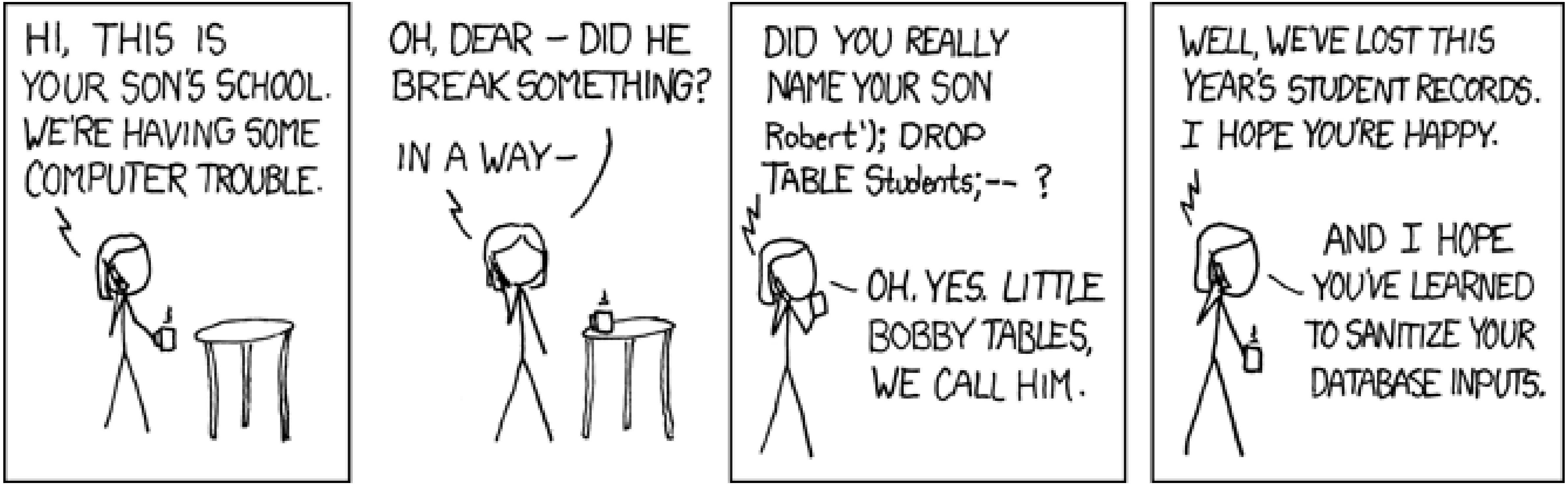
can execute a string containing several SQL statements

Insecure: NEVER use % on user input

Python shell

```
> New user: gerth
| [('gerth',)]
> New user: guido
| [('gerth',), ('guido',)]
> New user: evil"); DROP TABLE users; --
| sqlite3.OperationalError: no such table: users
```

```
INSERT INTO users VALUES ("evil"); DROP TABLE users; --")
```


HI, THIS IS YOUR SON'S SCHOOL. WE'RE HAVING SOME COMPUTER TROUBLE.
OH, DEAR – DID HE BREAK SOMETHING?
IN A WAY–
DID YOU REALLY NAME YOUR SON Robert'); DROP TABLE Students;-- ?
OH. YES. LITTLE BOBBY TABLES, WE CALL HIM.
WELL, WE'VE LOST THIS YEAR'S STUDENT RECORDS. I HOPE YOU'RE HAPPY.
AND I HOPE YOU'VE LEARNED TO SANITIZE YOUR DATABASE INPUTS.

xkcd.com/327/

Pandas

- Comprehensive Python library for data manipulation and analysis, in particular tables and time series
- Pandas **data frames** = tables
- Supports interaction with SQL, CSV, JSON, ...
- Integrates with Jupyter, numpy, matplotlib, ...

pandas.pydata.org

Pandas integration with Jupyter

- Tables (Pandas data frames) are rendered nicely in Jupyter

In [1]:

```python
import pandas as pd
students = pd.read_csv('students.csv')
students
```

Out[1]:

	Name	City
0	Donald Duck	Copenhagen
1	Goofy	Aarhus
2	Mickey Mouse	Aarhus

students.csv

```
Name,City
"Donald Duck","Copenhagen"
"Goofy","Aarhus"
"Mickey Mouse","Aarhus”
```

Reading tables (data frames)

- Pandas provide functions for reading different data formats, e.g. SQLite and .csv files, into pandas.DataFrames

pandas-example.py

```
import pandas as pd

import sqlite3
connection = sqlite3.connect('example.sqlite')
countries = pd.read_sql_query('SELECT * FROM country', connection)
cities = pd.read_sql_query('SELECT * FROM city', connection)

students.to_sql('students', connection, if_exists='replace')
print(students)
```

Python shell

```
|              Name        City
| 0    Donald Duck  Copenhagen
| 1          Goofy      Aarhus
| 2   Mickey Mouse      Aarhus
```

Selecting columns and rows

Table: country			
name	population	area	capital
'Denmark'	5748769	42931	'Copenhagen'
'Germany'	82800000	357168	'Berlin'
'USA'	325719178	9833520	'Washington, D.C.'
'Iceland'	334252	102775	'Reykjavik'

Python shell

```
> countries['name']                # select column
> countries.name                   # same as above
> countries[['name', 'capital']]   # select multiple columns, note double-[]
> countries.head(2)                # first 2 rows
> countries[1:3]                   # slicing rows, rows 1 and 2
> countries[::2]                   # slicing rows, rows 0 and 2
> countries.at[1, 'area']          # indexing cell by (row label, column name)
> cities[(cities['name'] == 'Berlin') | (cities['name'] == 'Munich')]  # select rows
|      name   country  population  established
| 2  Berlin   Germany     3711930         1237  # note original row labels
| 3  Munich   Germany     1464301         1158
> pd.DataFrame([[1,2], [3, 4], [5,6]], columns=['x', 'y'])   # create DF from list
> pd.DataFrame(np.random.random((3,2)), columns=['x', 'y'])  # from numpy
```

pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html

Row labels

Python shell

```
> df = pd.DataFrame(np.arange(1, 13).reshape(3, 4),
                    index=['q', 'w', 'e'],         # row labels
                    columns=['c', 'a', 'd', 'e'])  # column names
> df
|     c   a   d   e
| q   1   2   3   4  # row labels can be strings
| w   5   6   7   8
| e   9  10  11  12
> df.loc['w':'e', ['e', 'a']]  # slice of labeled rows
|      e   a
| w    8   6
| e   12  10
> df.loc['w']  # single row
| c    5
| a    6
| d    7
| e    8
| Name: w, dtype: int32
> df.iloc[:2,:2]  # use iloc to work with integer indexes
|     c  a
| q   1  2
| w   5  6
```

Merging tables and creating a new column

pandas-example.py

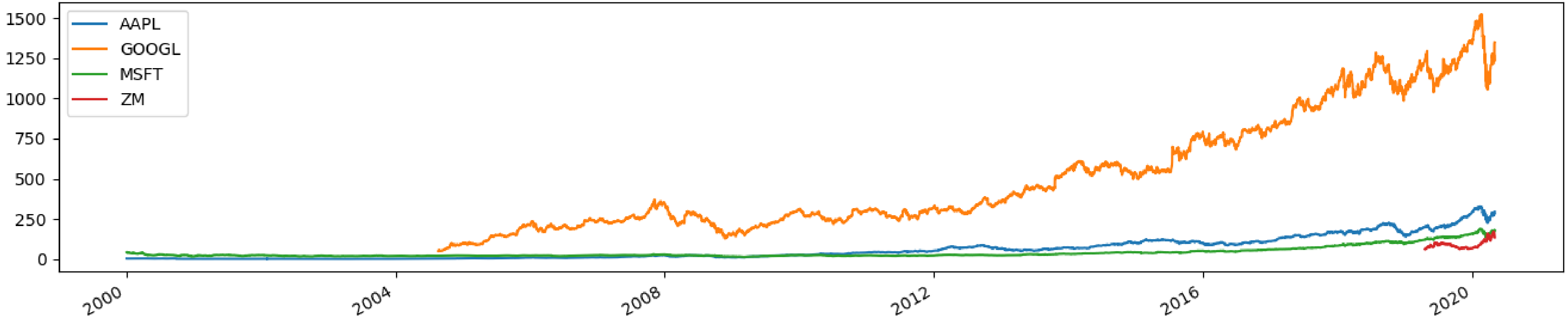
```
M = pd.merge(countries, cities, left_on='capital', right_on='name')
# both data frames had a 'name' and 'population' column
M1 = M.rename(columns={
    'population_x': 'country_population',
    'population_y': 'capital_population'
})
M2 = M1.drop(columns=['name_x', 'name_y'])
M2['%pop in capital'] = M2.capital_population / M2.country_population
M2.sort_values('%pop in capital', ascending=False, inplace=True)
print(M2[['country', '%pop in capital']])
```

Python shell

```
|      country   %pop in capital
| 3   Iceland          0.377260   # note row labels arepermuted
| 0   Denmark          0.134817
| 1   Germany          0.044830
| 2       USA          0.002131
```

Pandas datareader and Matplotlib

- pandas_datareader provides access to many data sources
- dataframes have a .plot method (using matplotlib.pyplot)



pandas-datareader.py

```
import matplotlib.pyplot as plt
import pandas_datareader
#df = pandas_datareader.data.DataReader(['AAPL', 'GOOGL', 'MSFT', 'ZM'], 'stooq')  # ignores start=...
df = pandas_datareader.stooq.StooqDailyReader(['AAPL', 'GOOGL', 'MSFT', 'ZM'], start='2000-01-01').read()
df['Close'].plot()
plt.legend()
plt.show()
```

pandas-datareader.readthedocs.io
pandas-datareader.readthedocs.io/en/latest/readers/stooq.html

Hierarchical / Multi-level indexing (MultiIndex)

Python shell

```
> df.tail(2)
| Attributes   Close                         ...      Volume
| Symbols        AAPL     GOOGL     MSFT     ...       GOOGL         MSFT           ZM
| Date                                       ...
| 2020-04-29   287.73   1342.18   177.43     ...   5417888.0   51286559.0   22033320.0
| 2020-04-30   293.80   1346.70   179.21     ...   2788644.0   53627543.0   16648922.0
> df['Close'].tail(2)
| Symbols        AAPL     GOOGL     MSFT       ZM
| Date
| 2020-04-29   287.73   1342.18   177.43   146.48
| 2020-04-30   293.80   1346.70   179.21   135.17
> df['Close']['GOOGL'].tail(2)
| Date
| 2020-04-29    1342.18
| 2020-04-30    1346.70
| Name: GOOGL, dtype: float64
> df.loc[:, pd.IndexSlice[:,'GOOGL']].tail(2)
| Attributes     Close      High       Low      Open      Volume
| Symbols        GOOGL     GOOGL     GOOGL     GOOGL       GOOGL
| Date
| 2020-04-29   1342.18   1360.15   1326.73   1345.00   5417888.0
| 2020-04-30   1346.70   1350.00   1321.50   1331.36   2788644.0
```

pandas.pydata.org/pandas-docs/stable/user_guide/advanced.html

Both rows and columns can have multi-level indexing

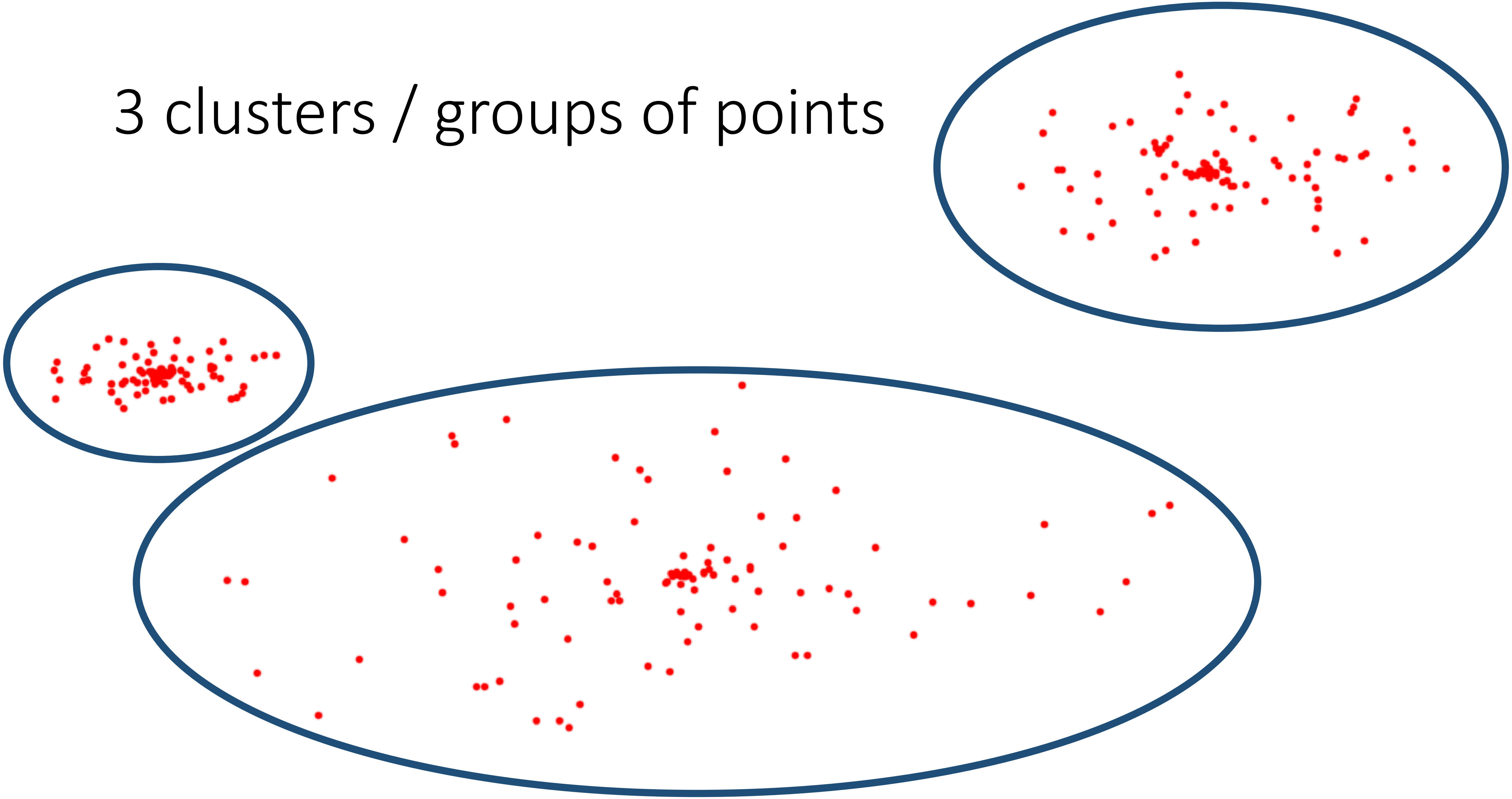
Python shell

```
> df.columns
| MultiIndex([( 'Close',  'AAPL'),
|             ( 'Close', 'GOOGL'),
|             ( 'Close',  'MSFT'),
|             ( 'Close',    'ZM'),
|             (  'High',  'AAPL'),
|             (  'High', 'GOOGL'),
|             (  'High',  'MSFT'),
|             (  'High',    'ZM'),
|             (   'Low',  'AAPL'),
|             (   'Low', 'GOOGL'),
|             (   'Low',  'MSFT'),
|             (   'Low',    'ZM'),
|             (  'Open',  'AAPL'),
|             (  'Open', 'GOOGL'),
|             (  'Open',  'MSFT'),
|             (  'Open',    'ZM'),
|             ('Volume',  'AAPL'),
|             ('Volume', 'GOOGL'),
|             ('Volume',  'MSFT'),
|             ('Volume',    'ZM')],
|  names=['Attributes', 'Symbols'])
```

Clustering

- k-means
- scipy.cluster.vq.kmeans
- neural networks

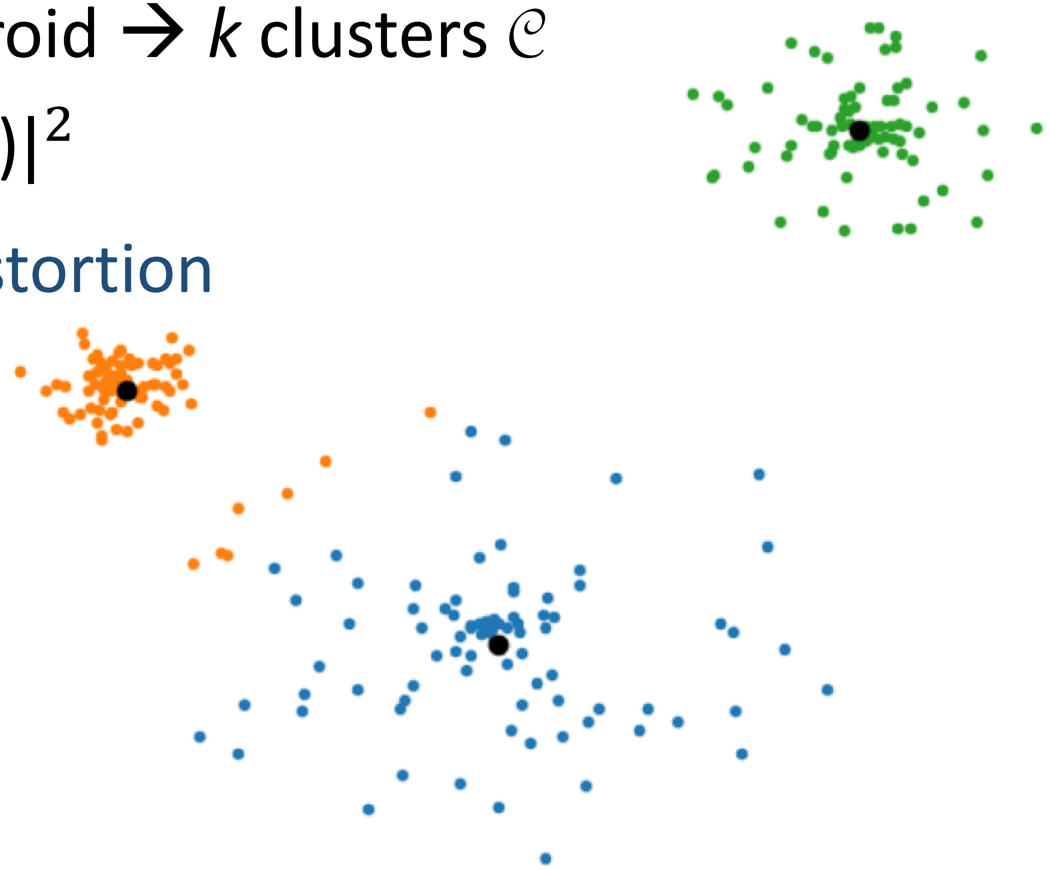
3 clusters / groups of points



Clustering = Optimization problem

Example: k-means

- Find k points *centroids*
- Assign each input point to nearest centroid → k clusters $\mathcal{C}$
- distortion = $\sum_{C \in \mathcal{C}} \sum_{p \in C} |p - \text{centroid}(C)|^2$
- Goal : Find k centroids that minimize distortion

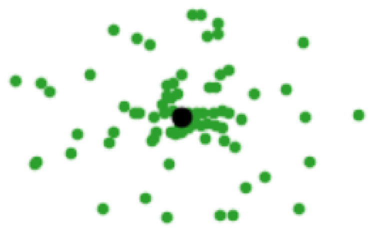


k-means for $k = 1$

- Let the centroid point c for a point set C be the point minimizing the

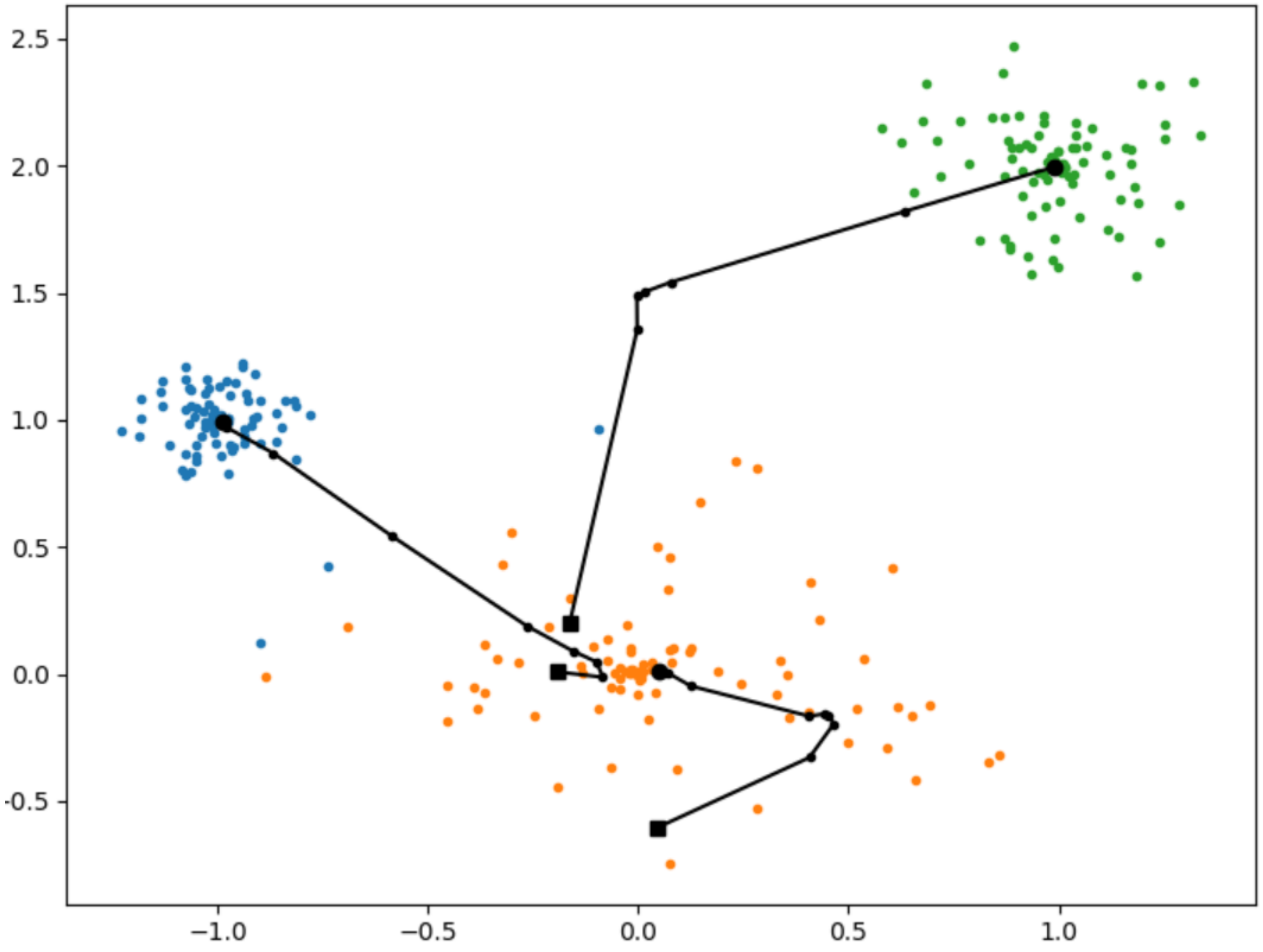
$$\text{distortion} = \sum_{p \in C} |p - c|^2$$

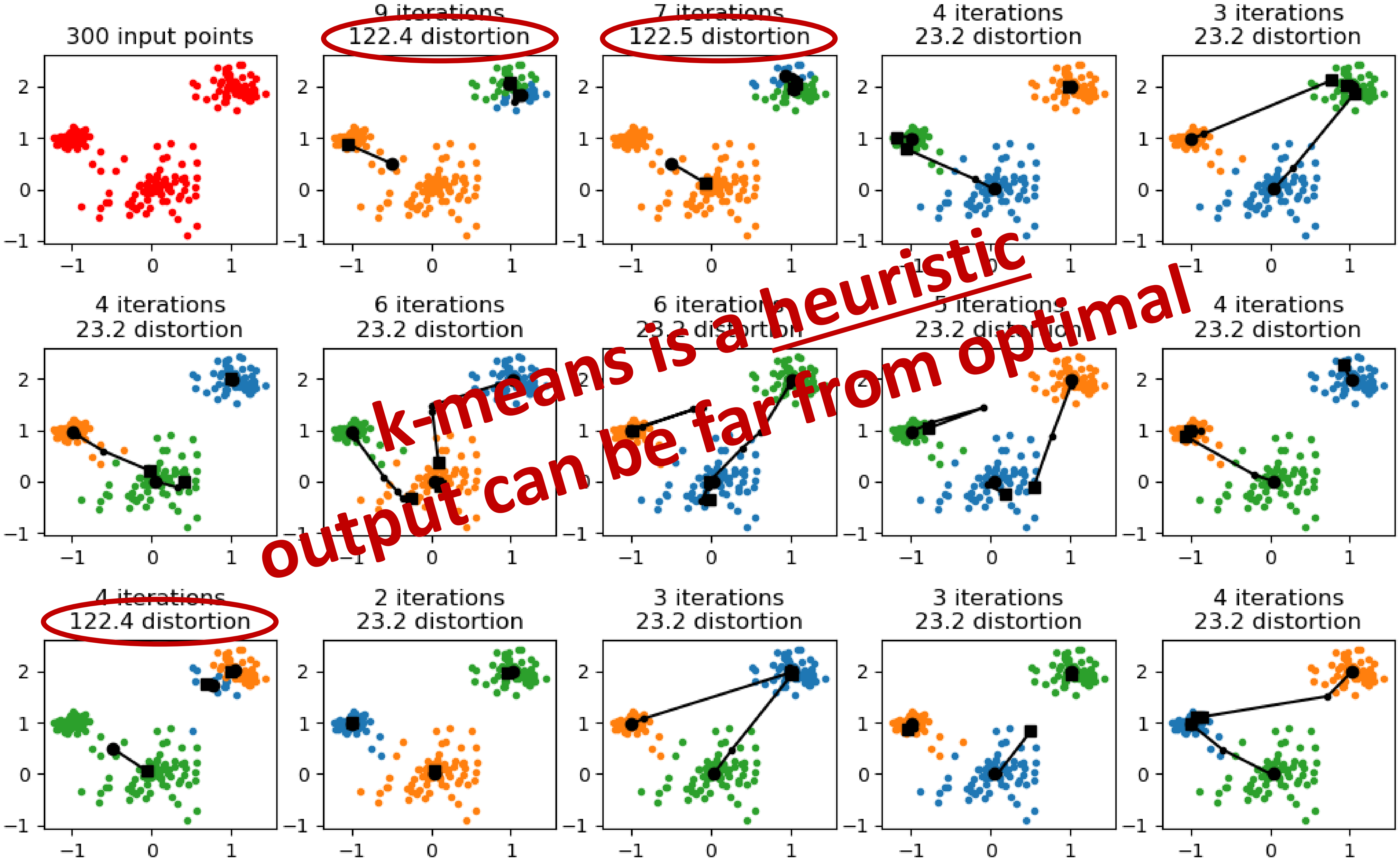
- Theorem $c = \text{average}(C)$



k-means - Lloyd's method (pseudo code)

```
centroids = k distinct random input points
while centroids change:
    create clusters C by assigning points to the nearest centroid
    centroids = average of each cluster
```

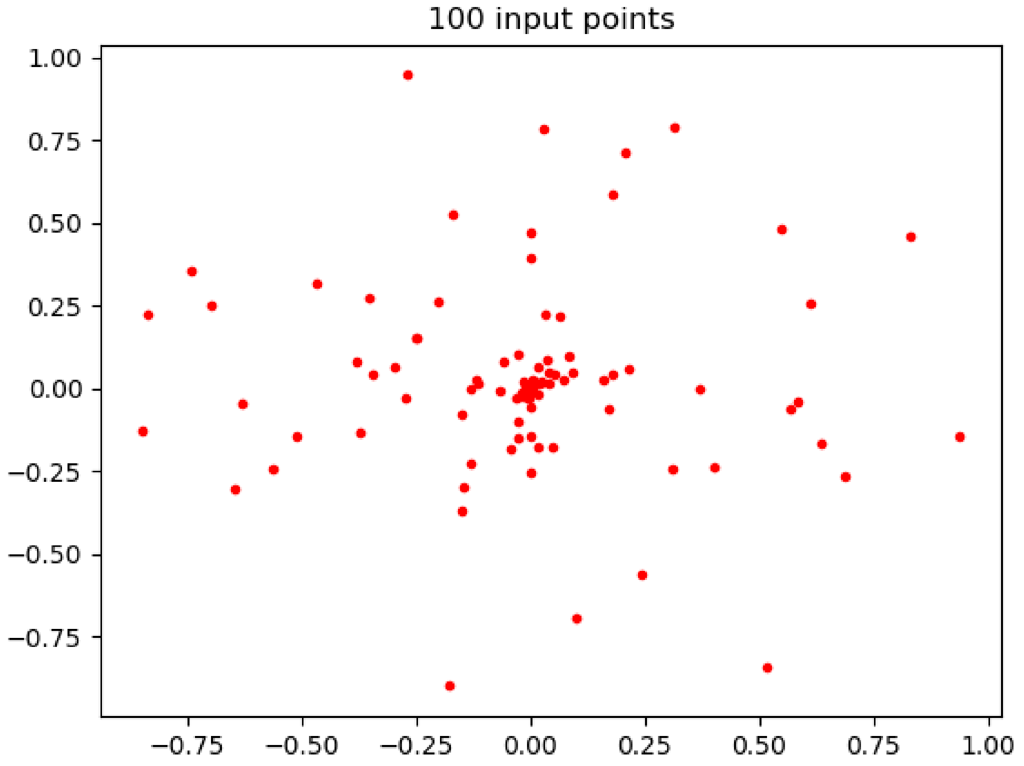



300 input points
9 iterations
122.4 distortion
7 iterations
122.5 distortion
4 iterations
23.2 distortion
3 iterations
23.2 distortion
4 iterations
23.2 distortion
6 iterations
23.2 distortion
6 iterations
23.2 distortion
5 iterations
23.2 distortion
4 iterations
23.2 distortion
4 iterations
122.4 distortion
2 iterations
23.2 distortion
3 iterations
23.2 distortion
3 iterations
23.2 distortion
4 iterations
23.2 distortion
k-means is a heuristic
output can be far from optimal

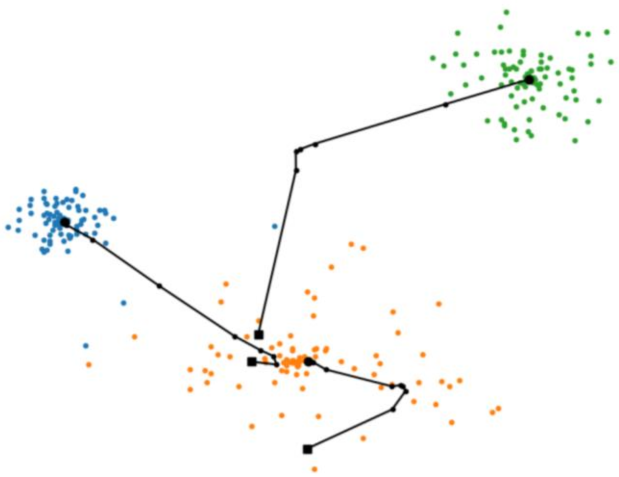
Generating random points
(just one random approach)

k_means.py

```
from random import random
from math import pi, cos, sin

def random_point(x, y, radius):
    angle = 2 * pi * random()
    r = radius * random() ** 2
    return x + r * cos(angle), y + r * sin(angle)

def random_points(n, x, y, radius):
    for _ in range(n):
        yield random_point(x, y, radius)
```



k-means



k_means.py

```
from random import sample
from numpy import argmin, mean

def k_means(points, k):
    centroid = sample(points, k)
    centroids = [ centroid ]  # history for visualization
    while True:
        clusters = [[] for _ in centroid]
        for p in points:
            i = argmin([dist(p, c) for c in centroid])
            clusters[i].append(p)
        centroid = [tuple(map(mean, zip(*c))) for c in clusters]
        if centroid == centroids[-1]:
            break
        centroids.append(centroid)
        if min(len(c) for c in clusters) == 0:
            print("Not good - empty cluster")
            break
    return clusters
```

k-mean limitations

- Can easily converge to a solution far from a global minimum
  - Solution – try several times and take the best
    (possibly since we can measure the quality (= distortion) of a solution)
- Clusters can become empty
  - Solution – discard and restart / take a random point out as a new centroid / take point furthest away from existing centroids / ....
- Sensitive to the scales of the different dimensions
  - Solution – apply some kind of initial normalization of coordinates

k-means - better bounds

- The k-means++ algorithm achieves an expected guarantee to be at most a factor 8(ln *k* +2) from the optimal [Vassilvitskii & Arthur]
- There exist polynomial time approximation schemes that find a solution that is guaranteed 1 + ε of the optimal (but running time exponential in *k* and dimension of points) [Har Peled et al.]
- **In practice: A heuristic is most often the algorithm of choice**

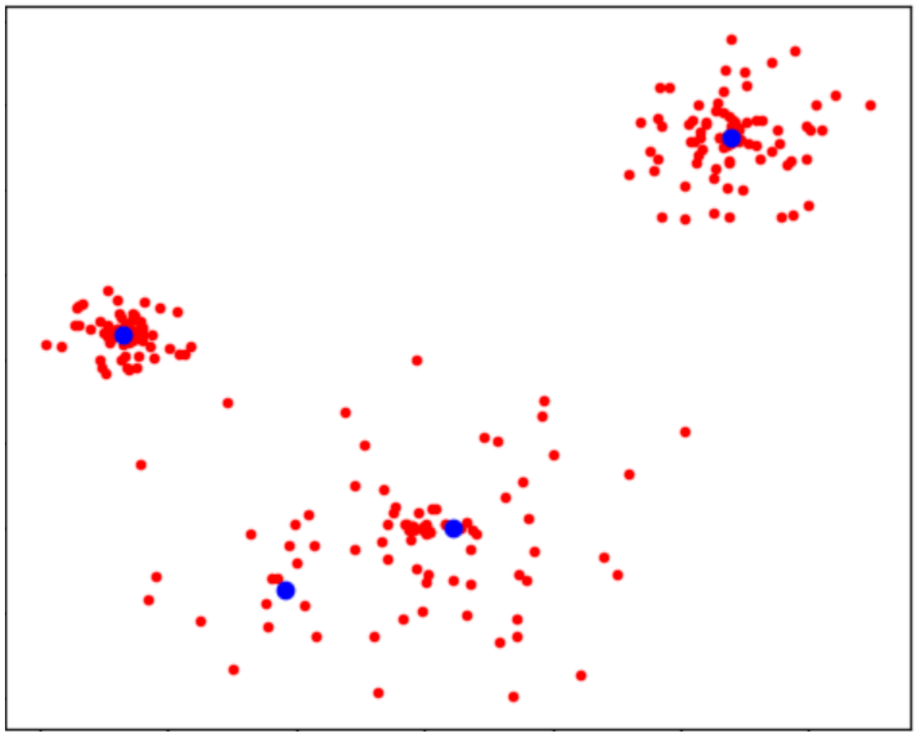
scipy.cluster.vq.kmeans

k_means.py

```
from scipy.cluster.vq import kmeans, whiten
import matplotlib.pyplot as plt

points = whiten(points)  # normalize variance of points

plt.plot(*zip(*points), 'r.')
plt.plot(*zip(*kmeans(points, K)[0]), "bo")
plt.title("scipy.cluster.vq.kmeans")
```

Note: According to the documentation ”`whiten` must be called prior to passing an observation matrix to `kmeans`”

kmeans returns tuple (centroids, distortion)



docs.scipy.org/doc/scipy/reference/generated/scipy.cluster.vq.kmeans.html

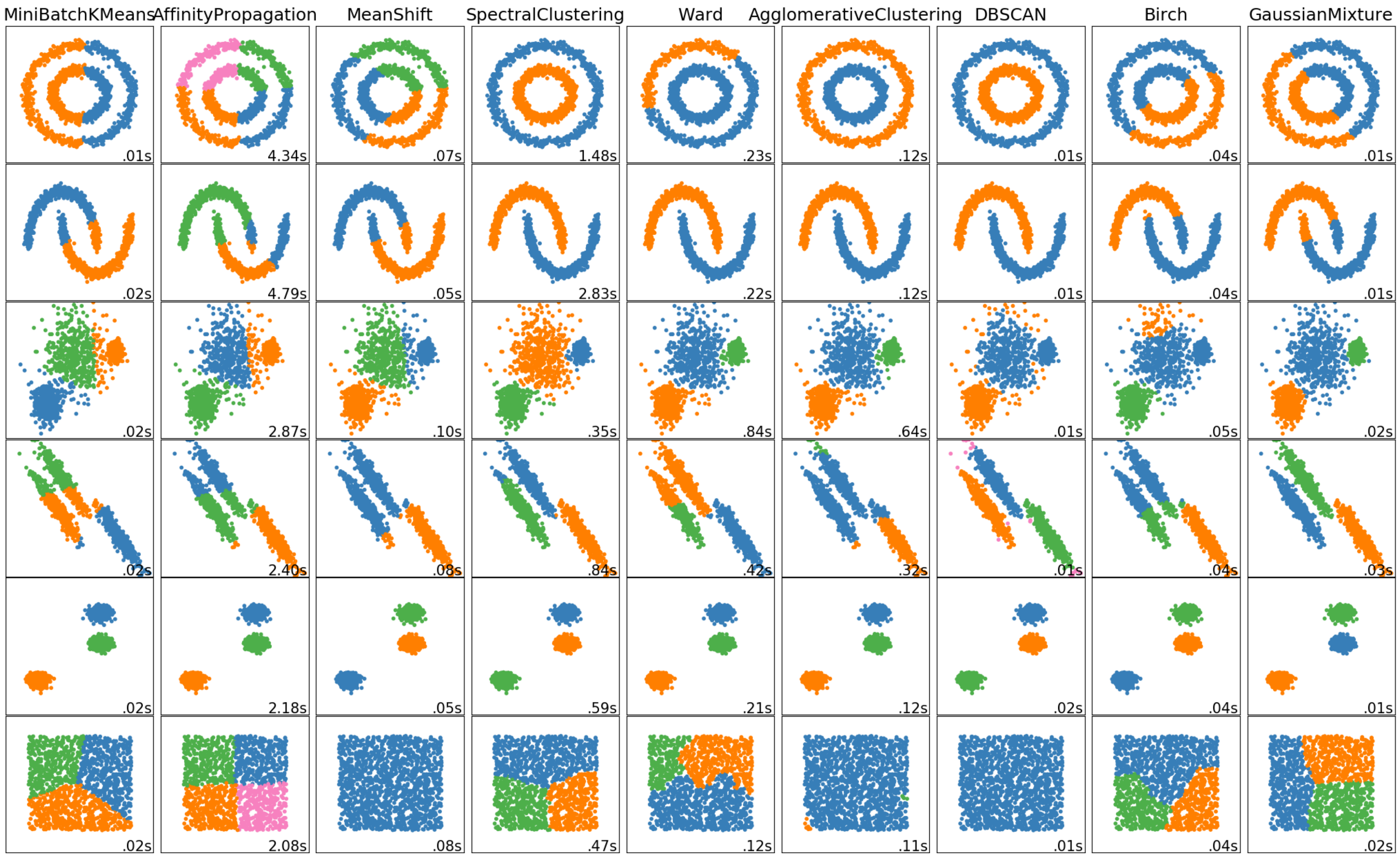
scipy.cluster.vq.whiten

- Normalizes / scales each dimension to have unit variance 1.0

$$\mathrm{Var}(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 \qquad \mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

https://docs.scipy.org/doc/scipy/reference/generated/scipy.cluster.vq.kmeans.html

Other Python clustering methods - `sklearn.cluster`



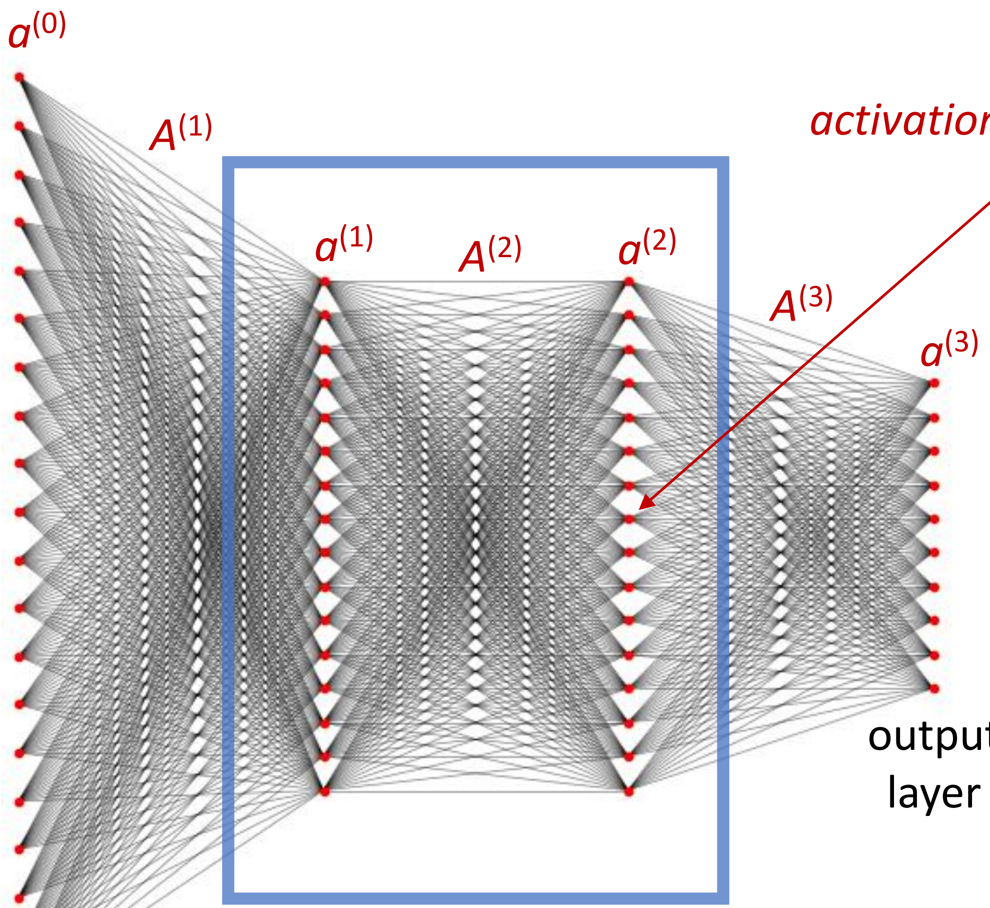
http://scikit-learn.org/stable/modules/clustering.html

Data Mining Algorithms

- k-means, and more generally clustering, is just one field in the area of *Data Mining*
- For more information see the webpage **[Top 10 Data Mining Algorithms, Explained]** a follow up to the below paper
- X. Wu et al., *Top 10 algorithms in data mining*, Knowledge and Information Systems, 14(1):1–37, 2008. DOI 10.1007/s10115-007-0114-2

www.kdnuggets.com/2015/05/top-10-data-mining-algorithms-explained.html

Neural networks (one slide introduction)



output layer

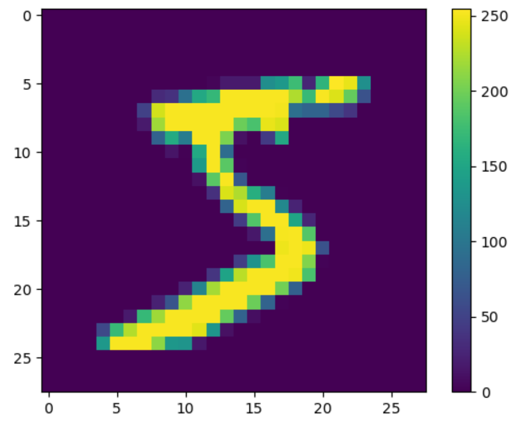
input layer

zero or more hidden layers (zero = linear classifier)

MNIST, 784 input pixels scaled to [0.0, 1.0]

activation $a_i^{(l)} = f^{(l)}\left(\sum_j a_j^{(l-1)} \cdot A_{ji}^{(l)} + b_i^{(l)} \right)$

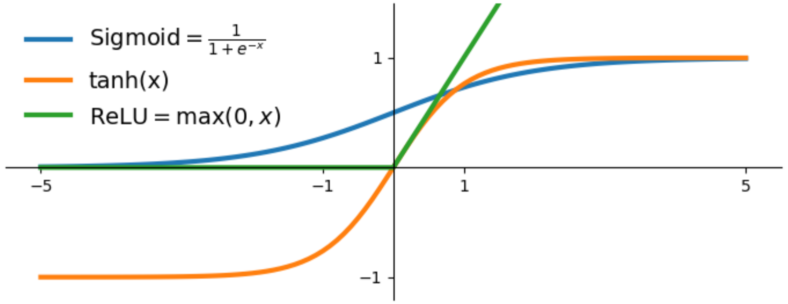
activation function — weight — bias



MNIST : 28 x 28 pixel values from [0, 255]

Classification, like MNIST, prediction = index of node with maximum output

Common activation functions



e.g. mean squared error

$$\frac{1}{n}\sum_{(x,y)} |\mathrm{out}(x) - y|^2$$

Learning
Find *A*s and *b*s performing well (minimize a cost function) on a set of *n* training inputs *x* with known output *y* using *backpropagation / stochastic gradient descend*

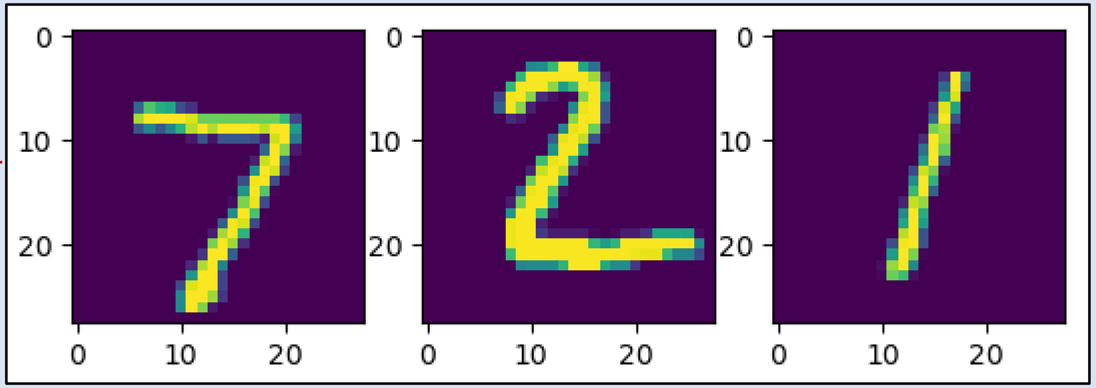
Applying a linear classifier using Numpy: $x \cdot A + b$

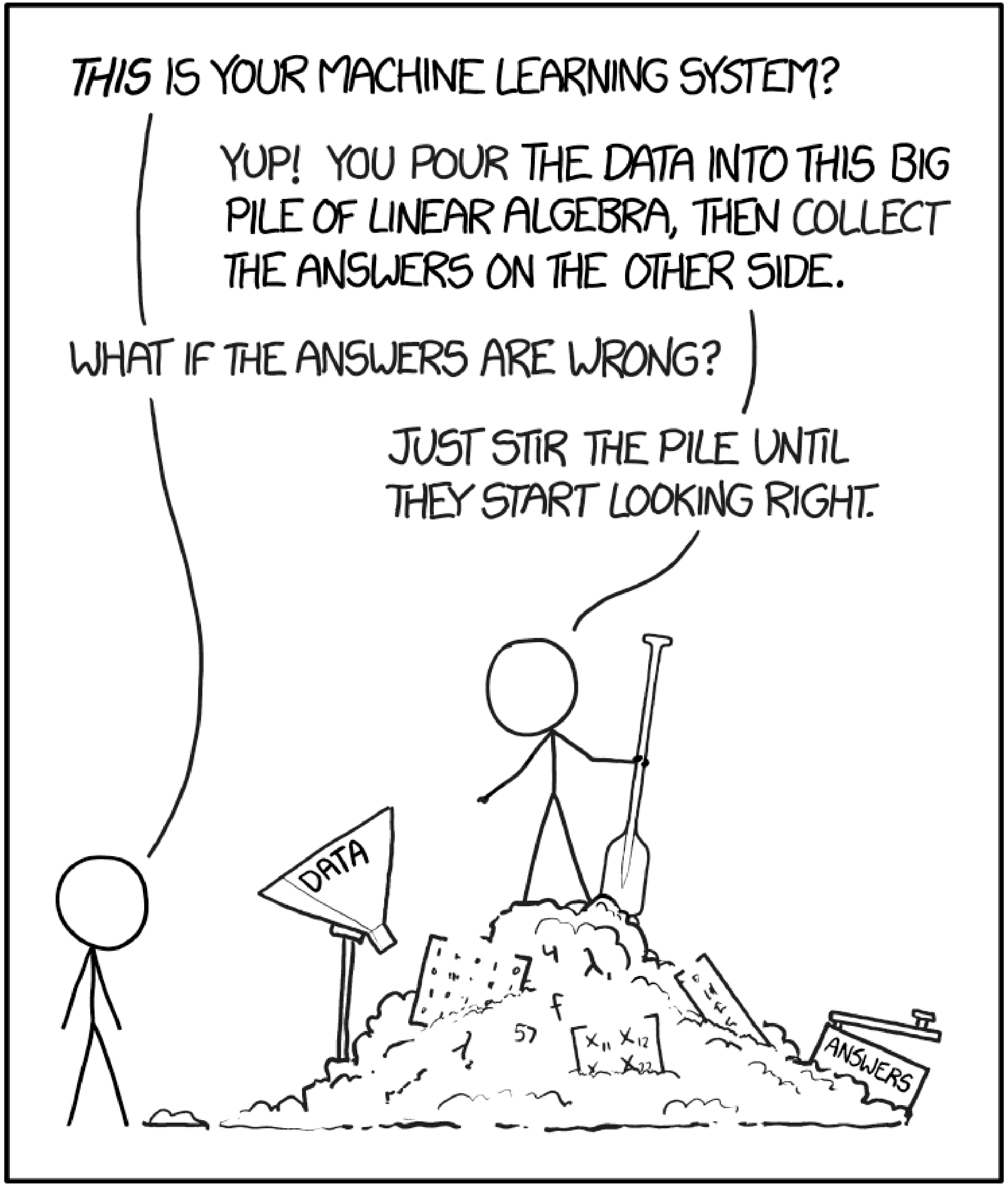
Python shell

```
| import matplotlib.pyplot as plt
| import numpy as np
| import keras
| (train_images, train_labels), (test_images, test_labels) = keras.datasets.mnist.load_data()
| type(test_images)
> <class 'numpy.ndarray'>
| test_images.shape
> (10000, 28, 28)   # 10_000 images 28 x 28
| test_labels.shape
> (10000,)          # 10_000 labels
| test_labels[:3]
> array([7, 2, 1], dtype=uint8)
| for i, image in zip(range(3), test_images):
|       plt.subplot(1, 3, i + 1)
|       plt.imshow(image)
| plt.show()

| A, b = map(np.array, eval(open('mnist_linear.weights').read()))   # read A and b from file
| print(A.shape, A.dtype, b.shape, b.dtype)
> (784, 10) float64 (10,) float64
| print([np.argmax(image.reshape(28 * 28) @ A + b) for image in test_images[:3]])
> [7, 2, 1]   # correct on 9_142 of the 10_000 images for the above file, ie accuracy 91%
```

manually generated labels

network prediction





xkcd.com/1838/

Graphical user interfaces (GUI)

- Tkinter

primitive_calculator.py

```python
accumulator = 0

while True:
    print("Accumulator:", accumulator)
    print("Select:")
    print("   1: clear")
    print("   2: add")
    print("   3: subtract")
    print("   4: multiply")
    print("   5: quit")

    choice = int(input("Choice: "))

    if choice == 1: accumulator = 0
    if choice == 2: accumulator += int(input("add: "))
    if choice == 3: accumulator -= int(input("subtract: "))
    if choice == 4: accumulator *= int(input("multiply by: "))
    if choice == 5: break
```

Python shell

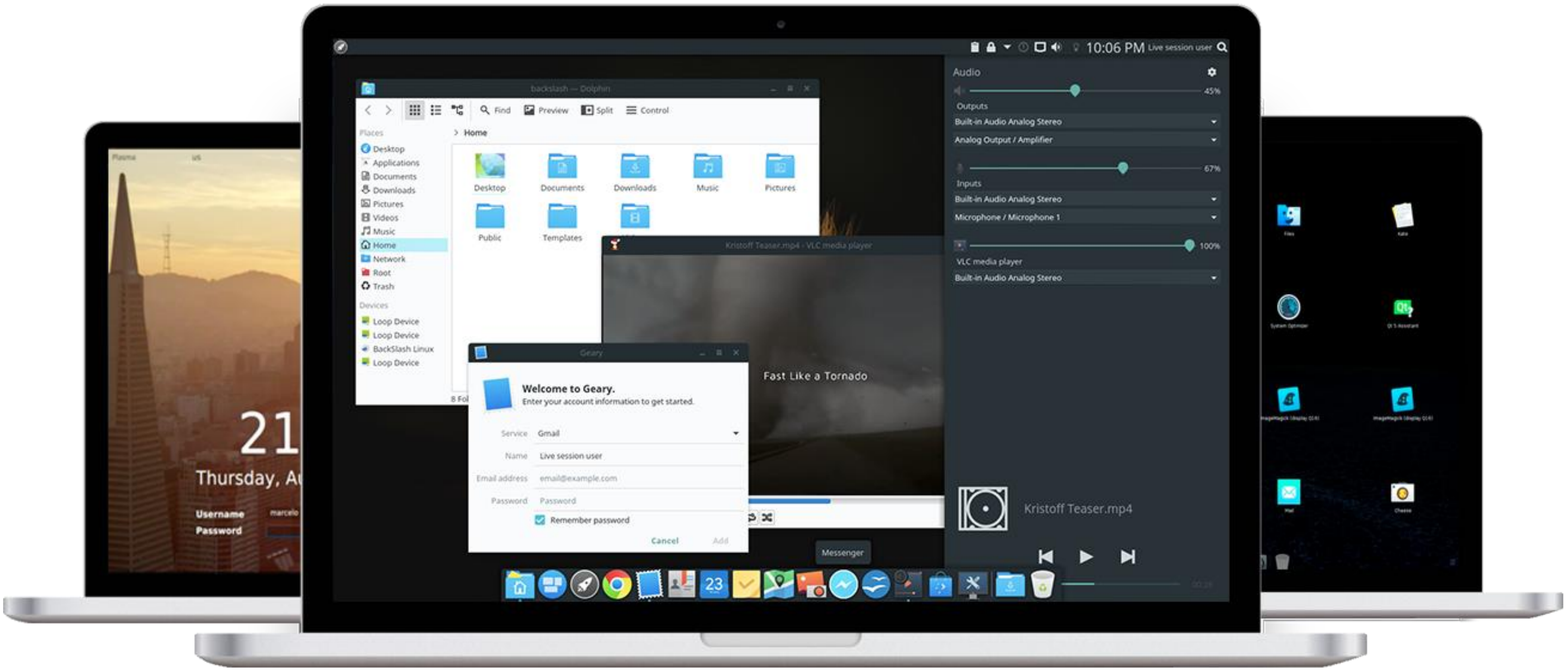
```
Accumulator: 0
Select:
  1: clear
  2: add
  3: subtract
  4: multiply
  5: quit
Choice: 2
add: 10
Accumulator: 10
Select:
  1: clear
  2: add
  3: subtract
  4: multiply
  5: quit
Choice: 2
add: 15
Accumulator: 25
Select:
...
```

Python GUI’s (Graphical Users Interfaces)

- There is a long list of GUI frameworks and toolkits, designer tools
  - we will only briefly look at Tkinter
- GUI are, opposed to a text terminal, easier to use, more intuitive and flexible
- Windows, icons, menus, buttons, scrollbars mouse / touch / keyboard interaction etc.
- Operating system (e.g. Windows, maxOS, iOS, Linux, Android) provides basic functionality in particular a windows manager
- Writing GUI applications from scratch can be painful – frameworks try to provide all standard functionality

en.wikipedia.org/wiki/Colossal_Cave_Adventure

wiki.python.org/moin/GuiProgramming



BackSlash Linux GUI
www.backslashlinux.com

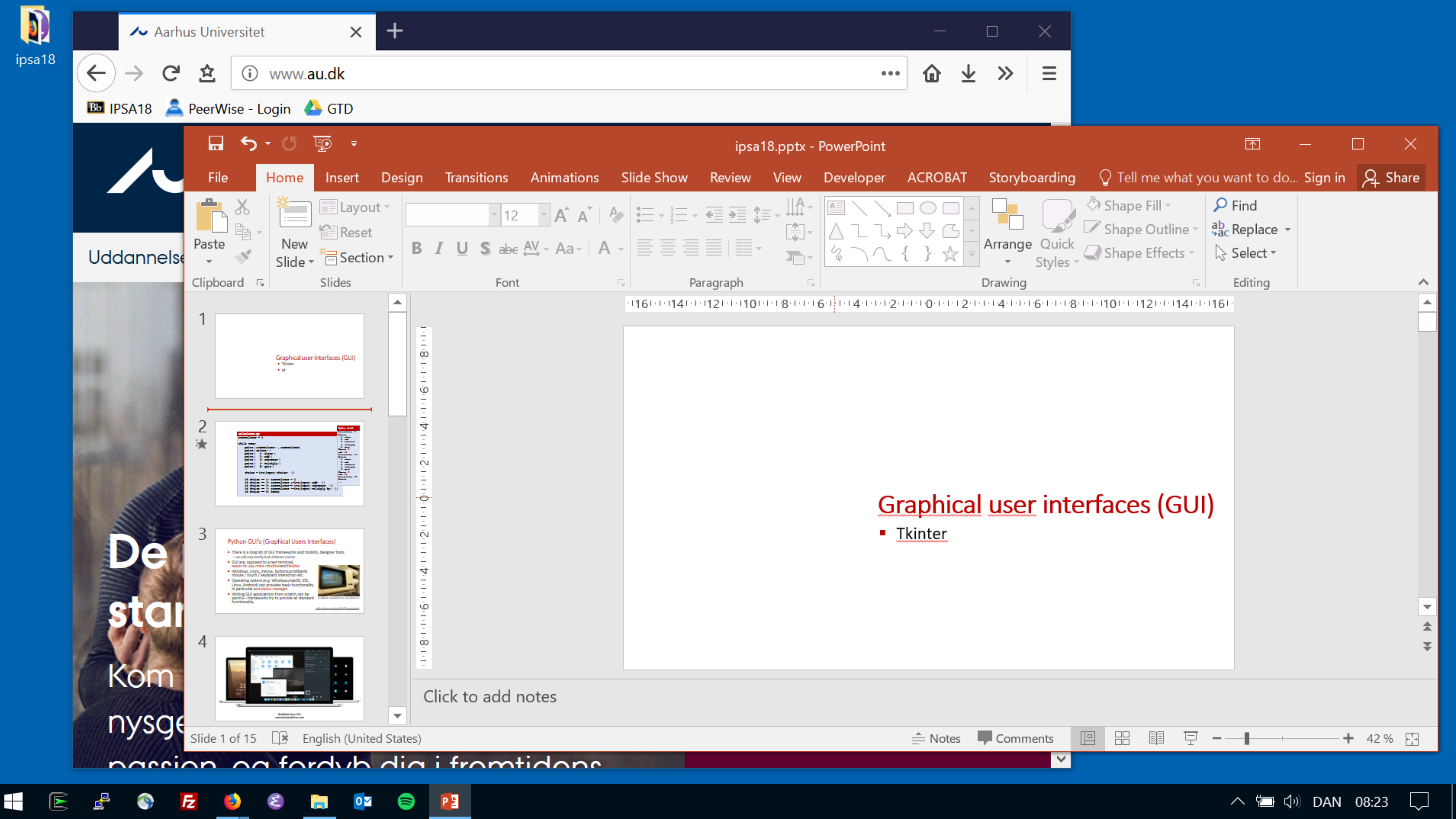
Tkinter

- "Tkinter is Python's de-facto standard GUI (Graphical User Interface) package. It is a thin object-oriented layer on top of Tcl/Tk."
- "Tcl is a high-level, general-purpose, interpreted, dynamic programming language."
- "Tk is a free and open-source, cross-platform widget toolkit that provides a library of basic elements of GUI widgets for building a graphical user interface (GUI) in many programming languages."
- "The popular combination of Tcl with the Tk extension is referred to as Tcl/Tk, and enables building a graphical user interface (GUI) natively in Tcl. Tcl/Tk is included in the standard Python installation in the form of Tkinter."

en.wikipedia.org

Terminology

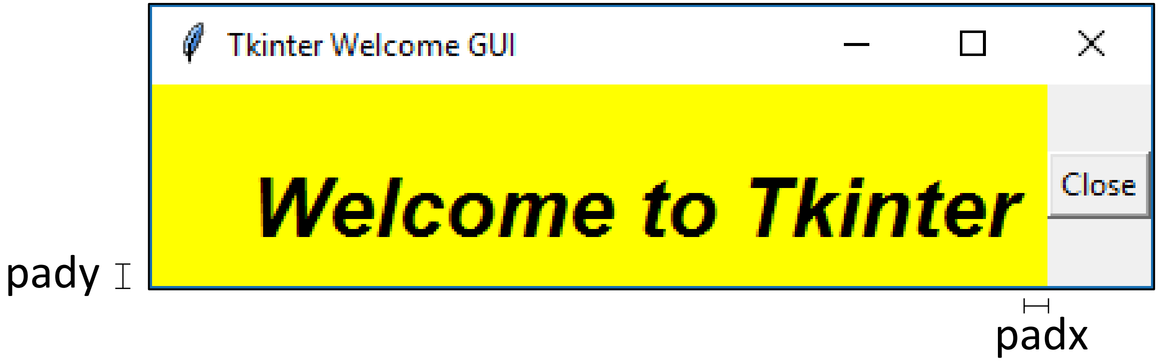
- widgets (e.g. buttons, editable text fields, labels, scrollbars, menus, radio buttons, check buttons, canvas for drawing, frames...)
- events (e.g. mouse click, mouse entering/leaving, resizing windows, redraw requests, ...)
- listening (application waits for events to be fired)
- event handler (a function whose purpose is to handle an event, many triggered by user or OS/Window manager)
- geometry managers (how to organize widgets in a window: Tkinter *pack*, *grid*, *place*)


ipsa18
Aarhus Universitet
www.au.dk
IPSA18
PeerWise - Login
GTD
Uddannelse
ipsa18.pptx - PowerPoint
File
Home
Insert
Design
Transitions
Animations
Slide Show
Review
View
Developer
ACROBAT
Storyboarding
Tell me what you want to do...
Sign in
Share
Paste
New Slide
Layout
Reset
Section
Clipboard
Slides
Font
Paragraph
Arrange
Quick Styles
Shape Fill
Shape Outline
Shape Effects
Drawing
Find
Replace
Select
Editing
Graphical user interfaces (GUI)
Tkinter
Python GUI's (Graphical Users Interfaces)
Click to add notes
Slide 1 of 15
English (United States)
Notes
Comments
42 %
DAN
08:23

docs.python.org/3/library/tk.html

⚠ *“tkinter is also famous for having an outdated look and feel”*

Welcome example



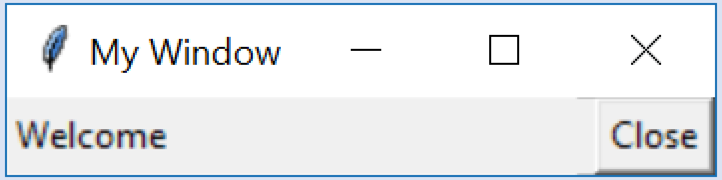
welcome.py

```
import tkinter
root = tkinter.Tk()   # root window
def do_quit():   # event handler for "Close" button
    root.destroy()
root.title("Tkinter Welcome GUI")
label = tkinter.Label(root, text="Welcome to Tkinter", background="yellow",
                      anchor=tkinter.SE, font=("Helvetica", "24", "bold italic"),
                      padx=10, pady=10)
label.pack(side=tkinter.LEFT, fill=tkinter.BOTH, expand=True)
                                         # parent window
close_button = tkinter.Button(root, text="Close", command=do_quit)
close_button.pack(side=tkinter.RIGHT)
tkinter.mainloop()   # loop until all windows are closed/destroyed
```

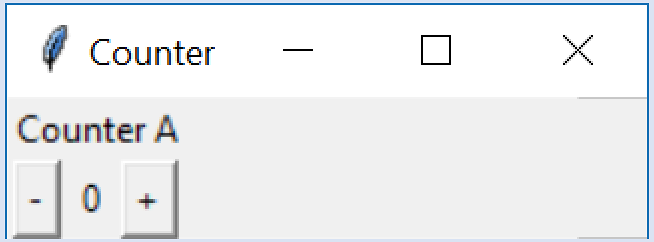
Welcome example (class)

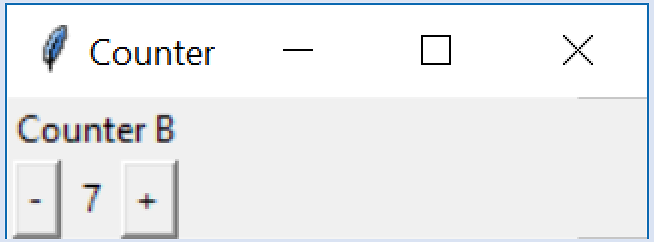
welcome_class.py

```
import tkinter

class Welcome:
    def do_quit(self):  # event handler for "Close"
        self.root.destroy()

    def __init__(self, window_title):
        self.root = tkinter.Tk()
        self.root.title(window_title)

        self.label = tkinter.Label(self.root, text="Welcome")
        self.label.pack(side=tkinter.LEFT)

        self.close_button = tkinter.Button(self.root, text="Close", command=self.do_quit)
        self.close_button.pack(side=tkinter.RIGHT)

Welcome("My Window")

tkinter.mainloop()
```



increment.py (part I)

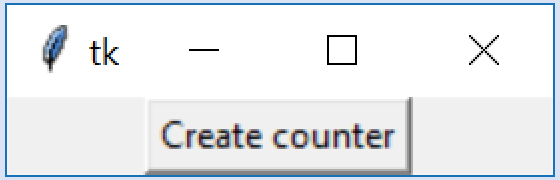





```
import tkinter

class Counter:
    def do_quit(self):
        self.root.destroy()

    def add(self, x):
        self.counter += x
        self.count.set(self.counter)

    def __init__(self, message):
        self.counter = 0

        self.root = tkinter.Toplevel()   # new window
        self.root.title("Counter")

        self.label = tkinter.Label(self.root, text=message)
        self.label.grid(row=0, columnspan=3)

        self.minus_button = tkinter.Button(self.root, text="-", command=lambda: self.add(-1))
        self.minus_button.grid(row=1, column=0)

        self.count = tkinter.IntVar()
        self.count_label = tkinter.Label(self.root, textvariable=self.count)
        self.count_label.grid(row=1, column=1)

        self.plus_button = tkinter.Button(self.root, text="+", command=lambda: self.add(+1))
        self.plus_button.grid(row=1, column=2)
```

```
increment.py (part II)
```

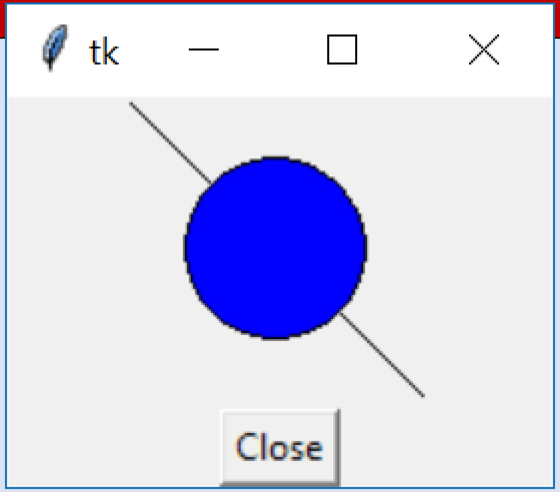
```python
class Counter_app:
    def __init__(self):
        self.counters = 0

        self.root = tkinter.Tk()

        self.create = tkinter.Button(self.root, text="Create counter", command=self.new_counter)
        self.create.pack()

    def new_counter(self):
        Counter("Counter " + chr(ord('A') + self.counters))
        self.counters += 1

Counter_app()

tkinter.mainloop()
```

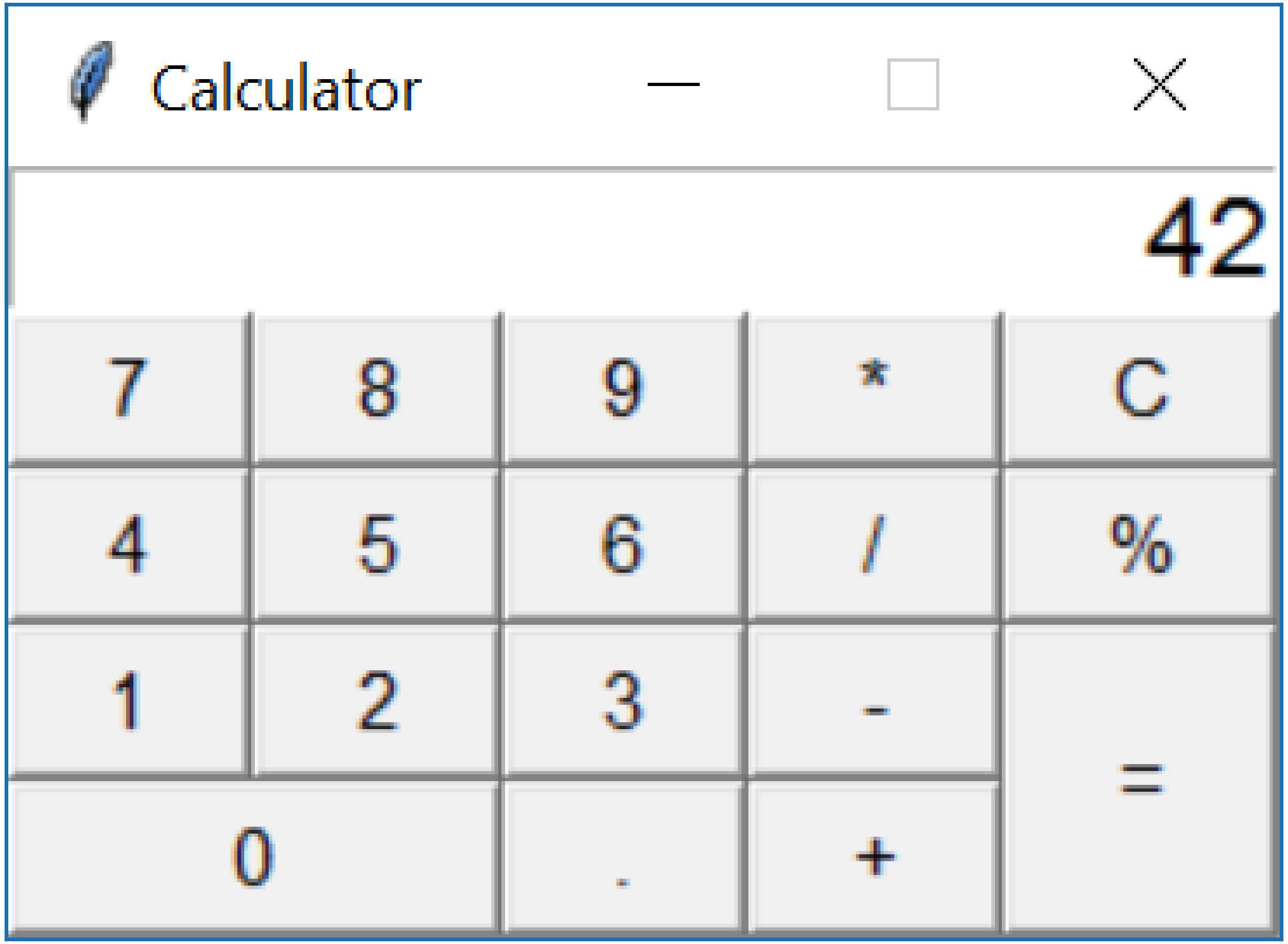


Canvas

canvas.py

```
import tkinter

root = tkinter.Tk()

canvas = tkinter.Canvas(root, width=100, height=100)
canvas.pack()
canvas.create_line(0, 0, 100, 100)
canvas.create_oval(20, 20, 80, 80, fill="blue")

close = tkinter.Button(root, text="Close", command=root.destroy)
close.pack()

tkinter.mainloop()
```




Calculator
42
7
8
9
*
C
4
5
6
/
%
1
2
3
-
=
0
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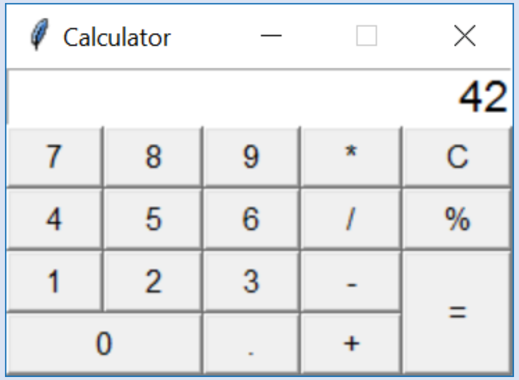
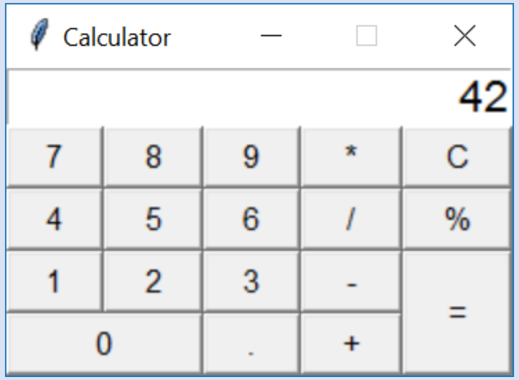
calculator.py (Part I)

```
import tkinter
from tkinter import messagebox

class Calculator:
    def __init__(self, root):
        self.root = root

        self.display = tkinter.Entry(self.root, font=("Helvetica", 16), justify=tkinter.RIGHT)
        self.display.insert(0, "0")
        self.display.grid(row=0, column=0, columnspan=5)   # grid = geometry manager

        self.button(1, 0, '7')
        self.button(1, 1, '8')
        self.button(1, 2, '9')
        self.button(1, 3, '*')
        self.button(1, 4, 'C', command=self.clearText)   # 'C' button
        self.button(2, 0, '4')
        self.button(2, 1, '5')
        self.button(2, 2, '6')
        self.button(2, 3, '/')
        self.button(2, 4, '%')
        self.button(3, 0, '1')
        self.button(3, 1, '2')
        self.button(3, 2, '3')
        self.button(3, 3, '-')
        self.button(3, 4, '=', rowspan=2, command=self.calculateExpression)   # '=' button
        self.button(4, 0, '0', columnspan=2)
        self.button(4, 2, '.')
        self.button(4, 3, '+')
```



calculator.py (Part II)

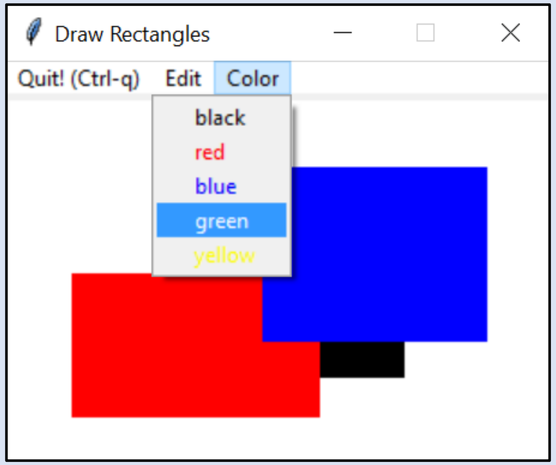
```
    def button(self, row, column, text, command=None, columnspan=1, rowspan=1):
        if command == None:
            command = lambda: self.appendToDisplay(text)
        B = tkinter.Button(self.root, font=("Helvetica", 11), text=text, command=command)
        B.grid(row=row, column=column, rowspan=rowspan, columnspan=columnspan, sticky="NWNESWSE")

    def clearText(self):
        self.replaceText("0")

    def replaceText(self, text):
        self.display.delete(0, tkinter.END)
        self.display.insert(0, text)

    def appendToDisplay(self, text):
        if self.display.get() == "0":
            self.replaceText(text)
        else:
            self.display.insert(tkinter.END, text)

    def calculateExpression(self):
        expression = self.display.get().replace("%", "/ 100")
        try:
            result = eval(expression)  # DON'T DO THIS !!!
            self.replaceText(result)
        except:
            messagebox.showinfo("Message", "Invalid expression", icon="warning")

root = tkinter.Tk()
root.title("Calculator")
root.resizable(0, 0)

Calculator(root)

tkinter.mainloop()
```



Creating a menu

rectangles.py

```
class Rectangles:
    Colors = ['black', 'red', 'blue', 'green', 'yellow']

    def create_menu(self):
        menubar = tkinter.Menu(self.root)
        menubar.add_command(label="Quit! (Ctrl-q)", command=self.do_quit)

        editmenu = tkinter.Menu(menubar, tearoff=0)
        editmenu.add_command(label="Clear", command=self.clear_all)
        editmenu.add_command(label="Delete last (Ctrl-z)", command=self.delete_last_rectangle)

        colormenu = tkinter.Menu(menubar, tearoff=0)
        for color in self.Colors:  # list of color names
            colormenu.add_command(label=color,
                                  foreground=color,
                                  command=self.get_color_handler(color))

        menubar.add_cascade(label="Edit", menu=editmenu)
        menubar.add_cascade(label="Color", menu=colormenu)
        self.root.config(menu=menubar)   # Show menubar

    def get_color_handler(self, color):
        return lambda : self.set_color(color)

    def set_color(self, color):
        self.current_color = color
...
```

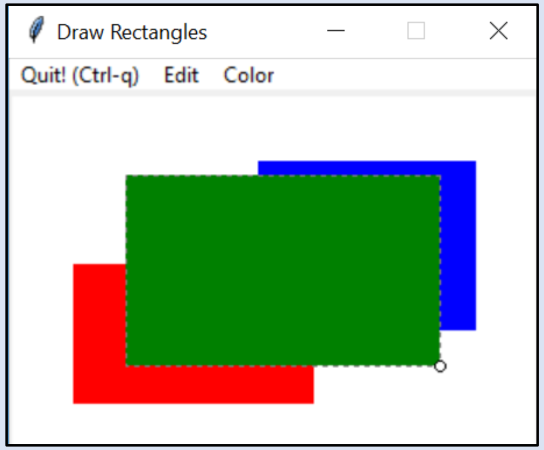


Binding key and mouse events

- Whenever a key is pressed, mouse button is pressed/released, mouse is moved, mouse enters/leaves objects etc. events are triggered that can be bound to call a user defined event handler

rectangles.py (continued)

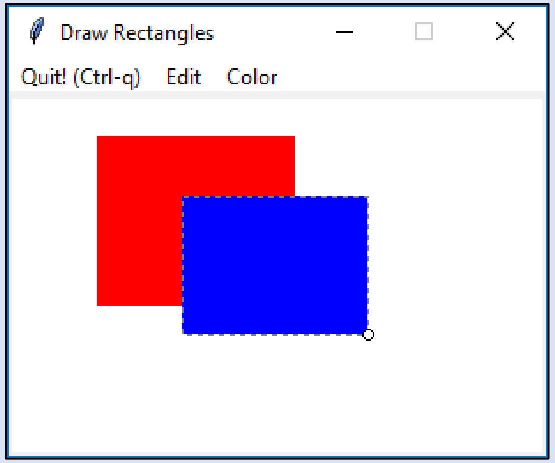
```
...
  self.root = tkinter.Tk()
  self.root.bind('<Control-q>', self.do_quit)
  self.root.bind('<Control-z>', self.delete_last_rectangle)
...
  self.canvas = tkinter.Canvas(self.root, width=300, height=200,
                               background='white')
  self.canvas.bind('<Button-1>', self.create_rectangle_start)
  self.canvas.bind('<B1-Motion>', self.create_rectangle_mouse_move)
  self.canvas.bind('<ButtonRelease-1>', self.create_rectangle_end)
...
```



Handling mouse events

rectangles.py (continued)

```
def create_rectangle_start(self, event):
    radius = 3
    x, y = event.x, event.y
    self.top_pos = (x, y)
    self.bottom_pos = (x, y)
    self.rectangle = self.canvas.create_rectangle(x, y, x, y,  # top-left = bottom-right
                                     fill=self.current_color, width=1, outline='grey', dash=(3, 5))
    self.corner = self.canvas.create_oval(x - radius, y - radius, x + radius, y + radius, fill='white')
def create_rectangle_mouse_move(self, event):
    if self.corner is not None:
        x, y = event.x, event.y
        x_, y_ = self.bottom_pos
        self.bottom_pos = (x, y)
        self.canvas.coords(self.rectangle, *self.top_pos, *self.bottom_pos)
        self.canvas.move(self.corner, x - x_, y - y_)
def create_rectangle_end(self, event):
    if self.corner is not None:
        self.canvas.delete(self.corner)
        self.corner = None
        if self.bottom_pos != self.top_pos:
            self.rectangles.append(self.rectangle)
            self.canvas.itemconfig(self.rectangle, width=0)
        else:  # empty rectangle, skip
            self.canvas.delete(self.rectangle)
        self.rectangle = None
```



Exercise 25.1 (convex hull GUI)