

Dynamic Planar Range Maxima Queries

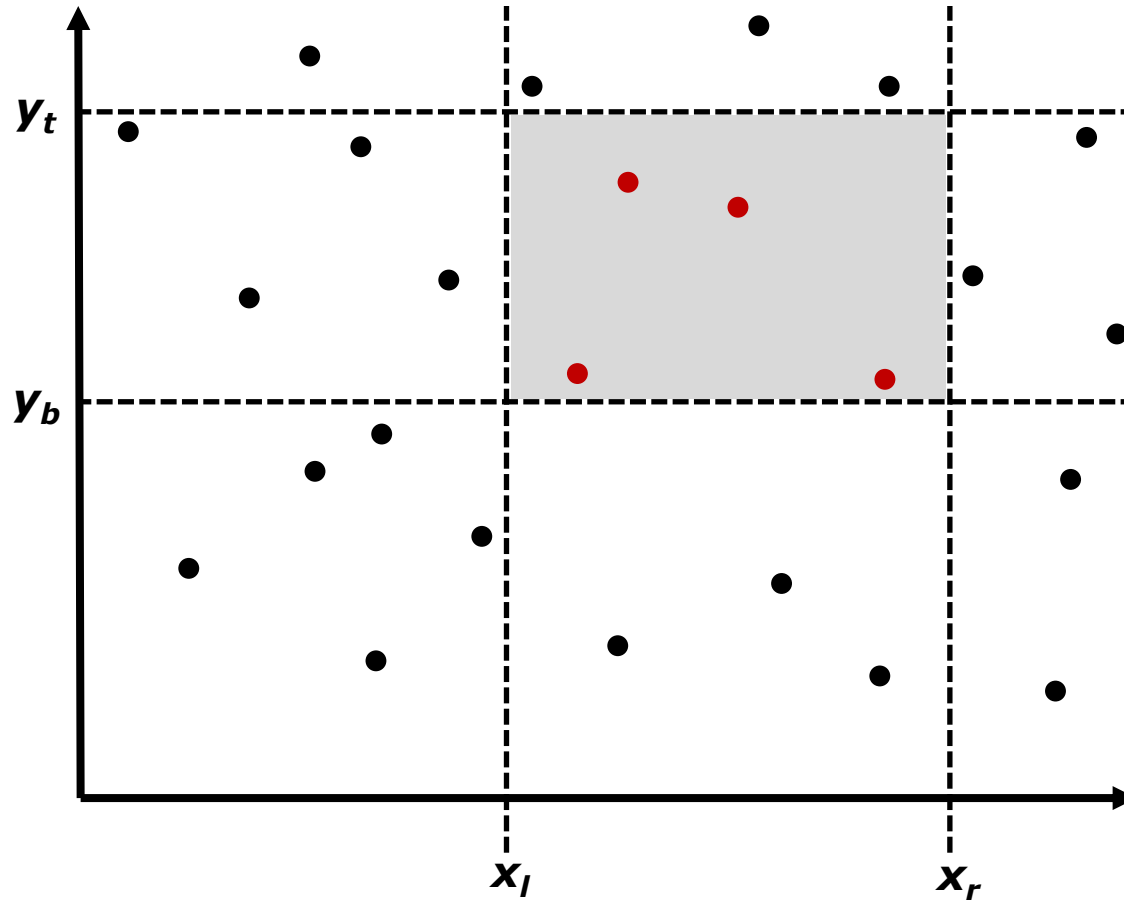
(presented at ICALP 2011)

Gerth Stølting Brodal
Aarhus University

maDALGO 
CENTER FOR MASSIVE DATA ALGORITHMICS

Kostas Tsakalidis

Orthogonal Range Queries



Priority Search Tree [McCreight'75]

Space: $O(n)$

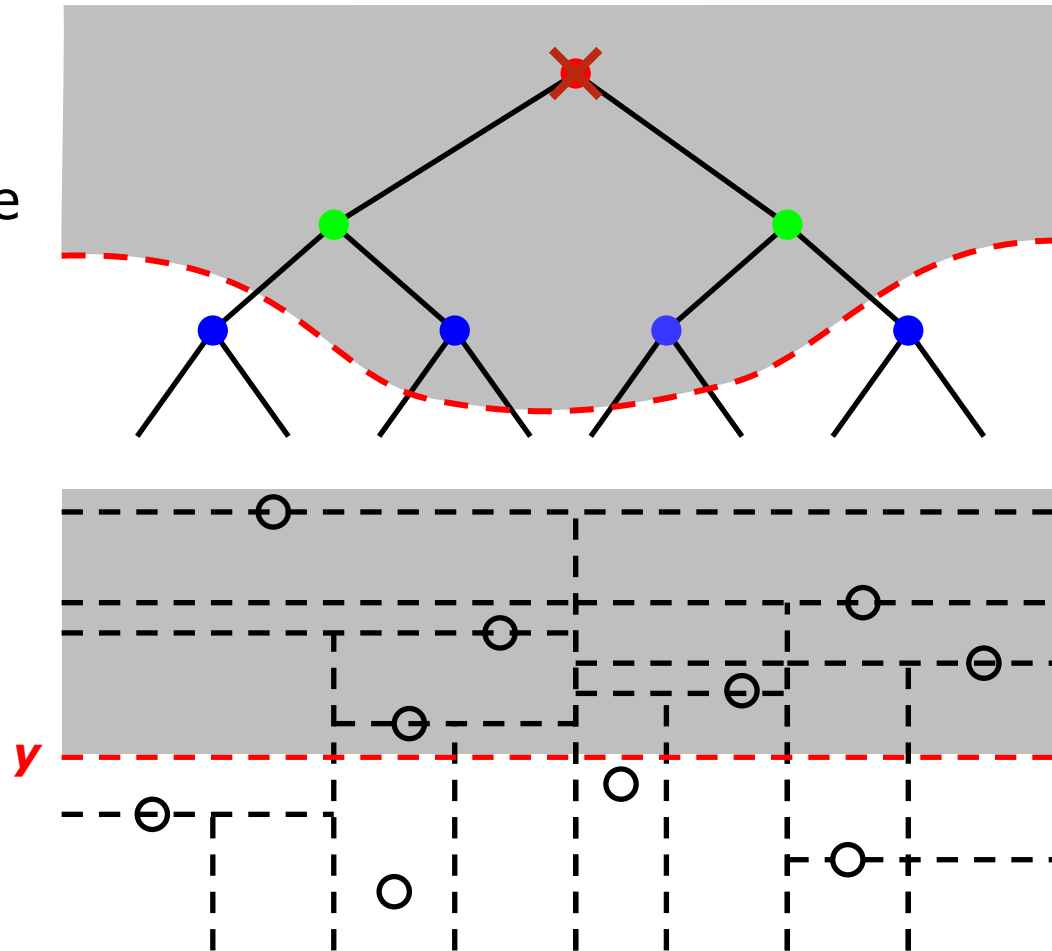
Update:

Recursively move
up maximum y

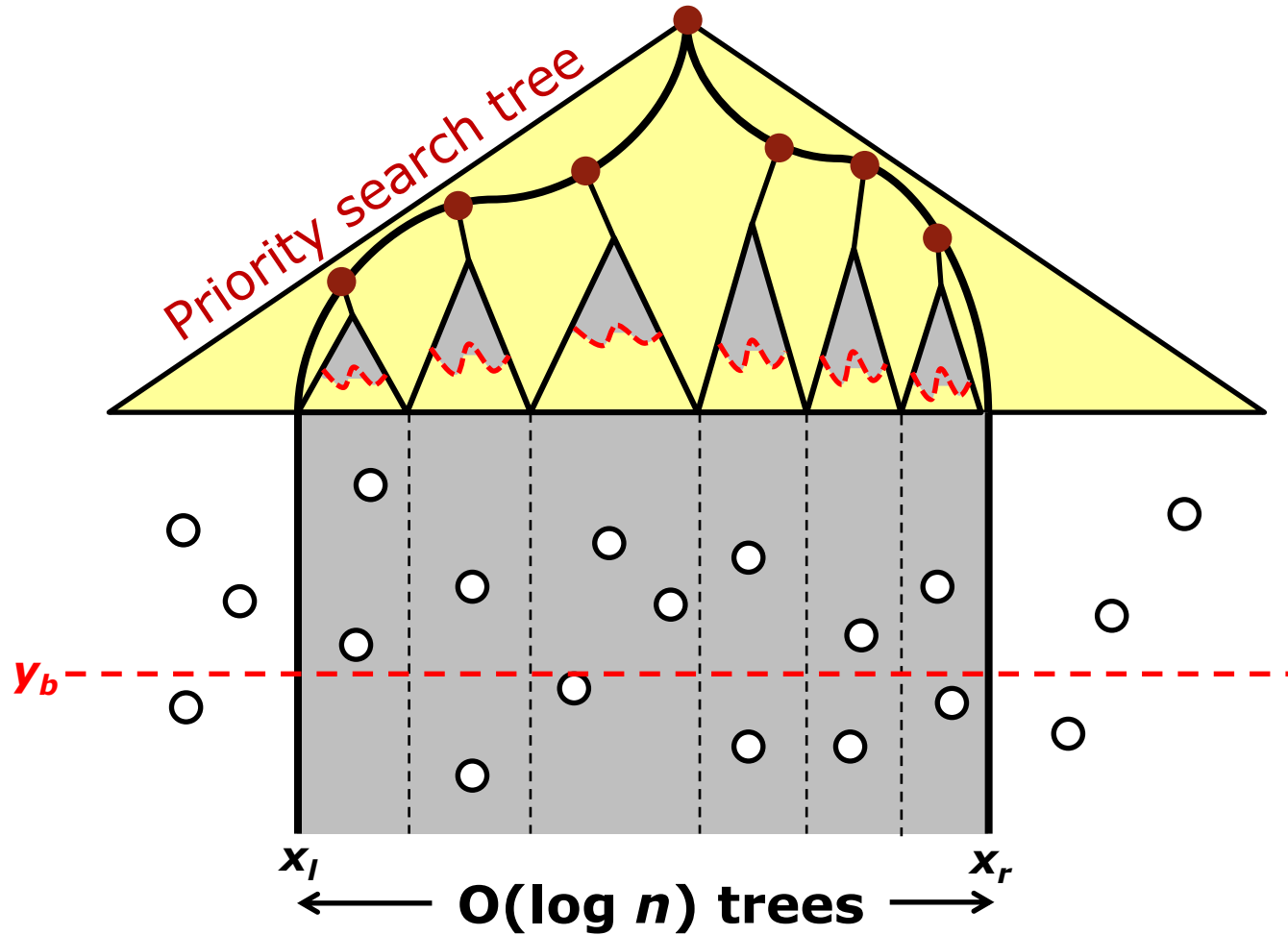
$O(\log n)$

**1-Sided
reporting:**

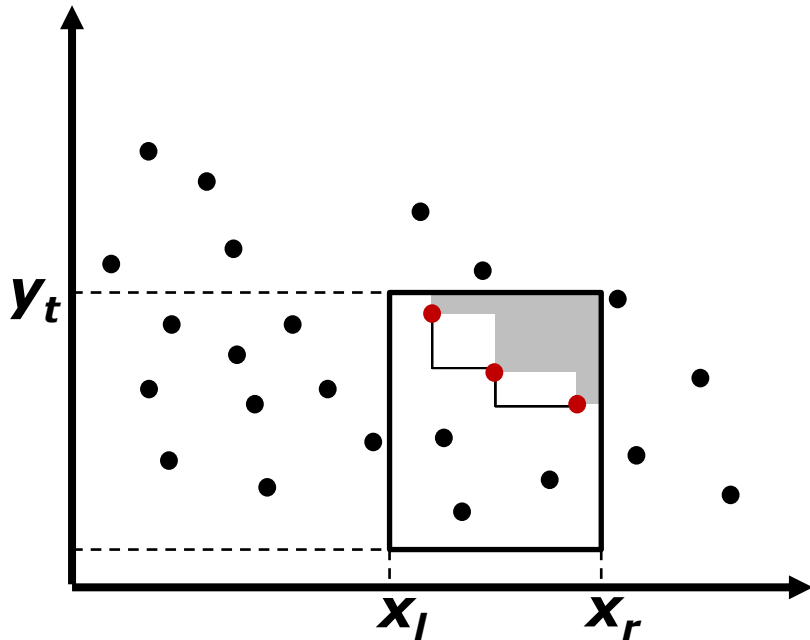
$O(1+t)$





3-Sided Reporting Queries $O(\log n + t)$





Orthogonal Range MAXIMA Reporting alias “Generalized Planar SKYLINE Operator”



 **Dominance Maxima Queries**
Report **all maximal points** among points with x in $[x_l, +\infty)$ and y in $[y_b, +\infty)$



 **Contour Maxima Queries**
Report **all maximal points** among points with x in $(-\infty, x_r]$

 **3-Sided Maxima Queries**
Report **all maximal points** among points with x in $[x_l, x_r]$ and y in $[y_b, +\infty)$

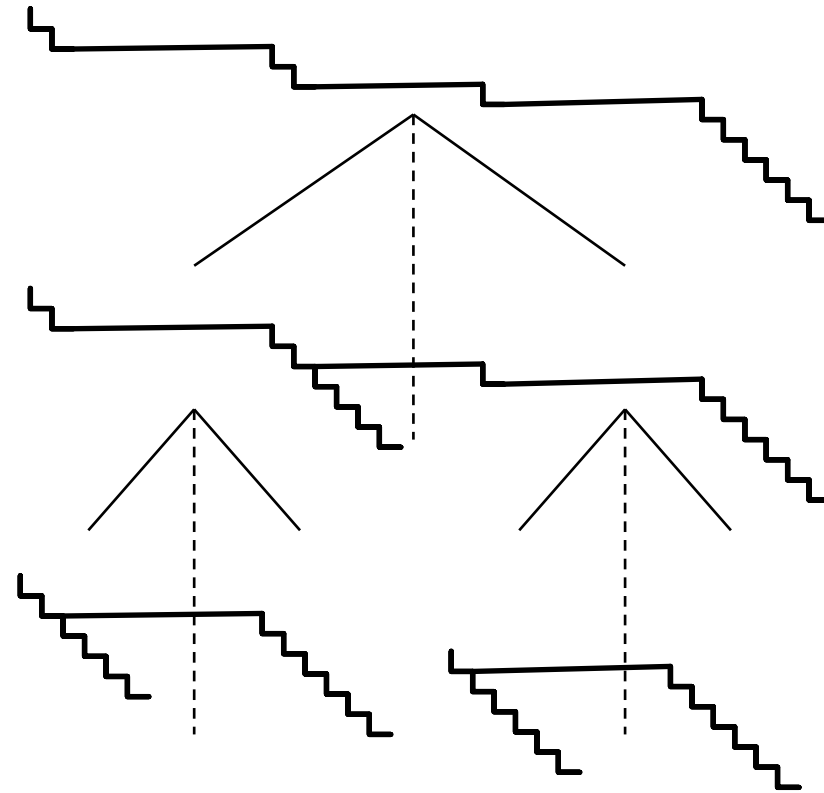
 **4-Sided Maxima Queries**
Report **all maximal points** among points with x in $[x_l, x_r]$ and y in $[y_b, y_t]$

Static maximal points in $O(n \cdot \log n)$ time [Kung, Luccio, Preparata, J.ACM'75]

Dynamic Range Maxima Reporting

	Space			Insert	Delete
Overmars, van Leeuwen '81	n	$\log n + t$	$\log^2 n + t$	$\log^2 n$	
Frederickson, Rodger '90	n	$\log n + t$	$\log^2 n + t$ $(1+t)\log n$	$\log n$	$\log^2 n$
Janardan '91	n	$\log n + t$		$\log n$	$\log^2 n$
Kapoor '00	n	$\log n + t$ amo.	-	$\log n$	

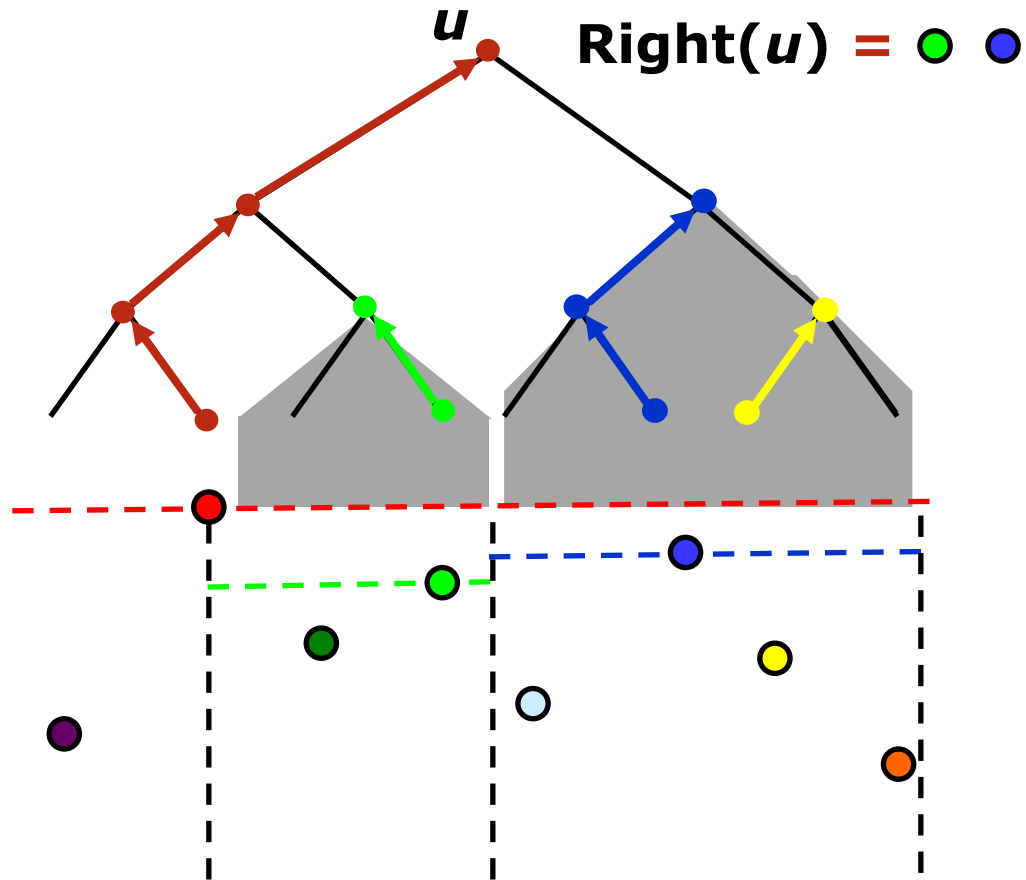
Overmars, van Leeuwen [JCSS '81]



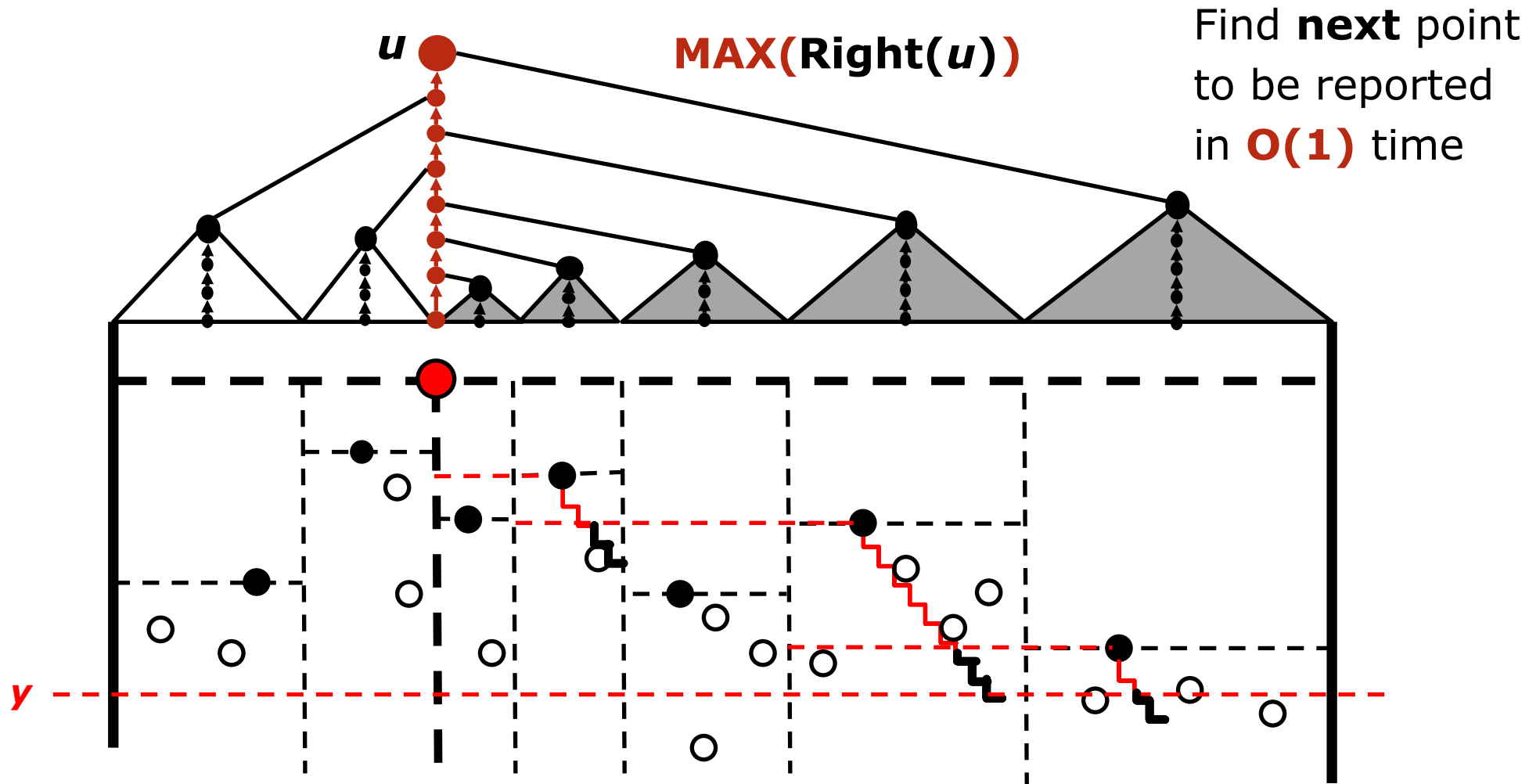
 $O(\log n + t)$

Updates: $O(\log^2 n)$

Our Structure - Tournament Tree



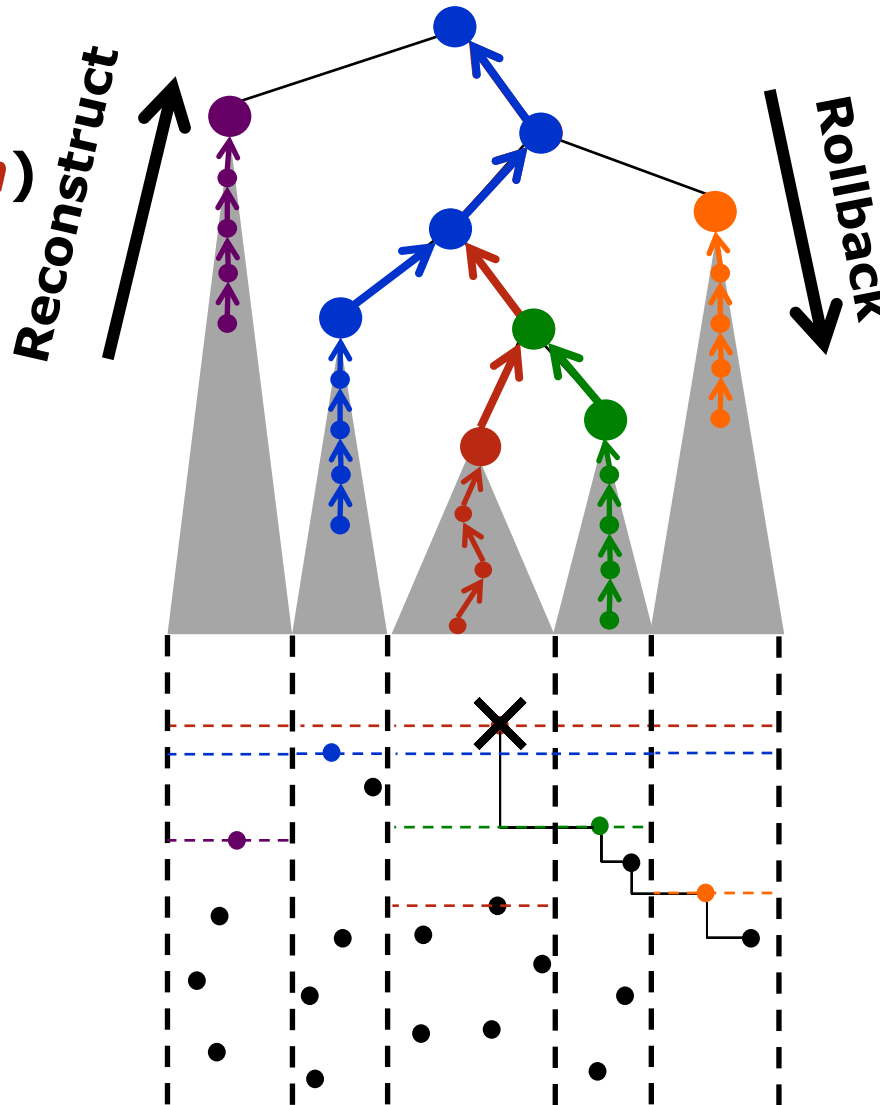
Tournament Tree



Update Operation

Space: $O(n)$

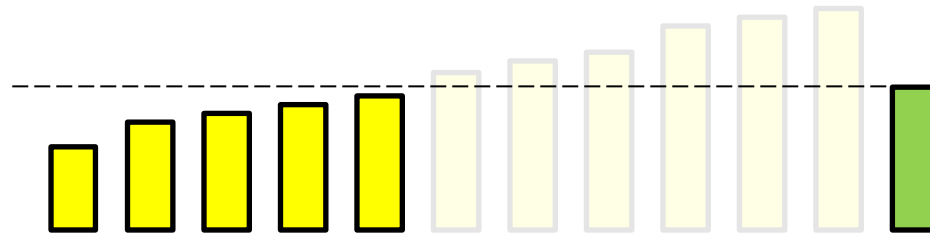
Update: $O(\log n)$



Priority Queue
with **A**ttention
with Rollback

Priority Queues with Attrition [Sundar, IPL '89]

- Deletemin()
- InsertAndAttrite(element)



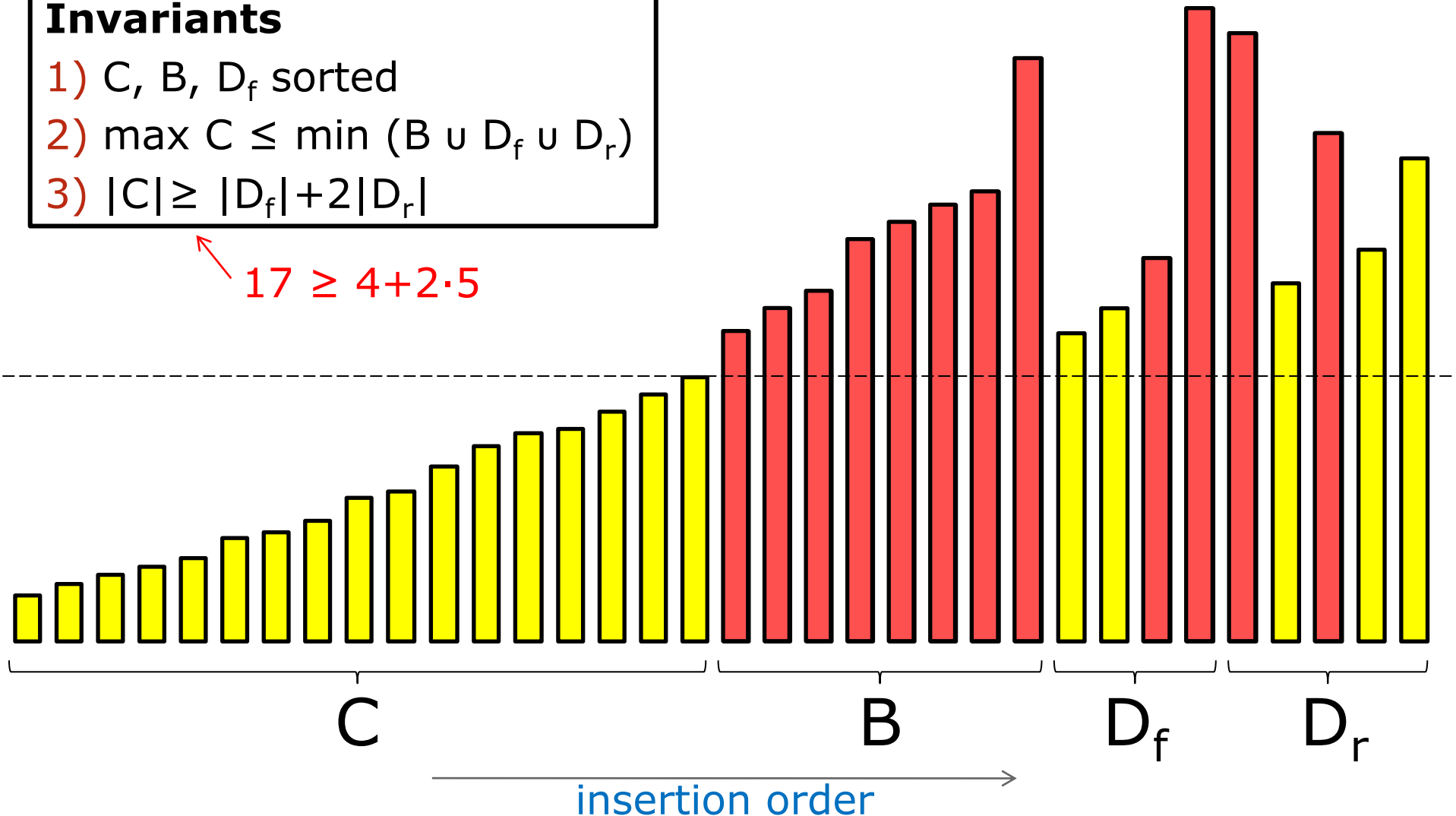
$O(1)$ worst case time

Priority Queues with Attrition [Sundar, IPL '89]

Invariants

- 1) C, B, D_f sorted
- 2) $\max C \leq \min (B \cup D_f \cup D_r)$
- 3) $|C| \geq |D_f| + 2|D_r|$

$17 \geq 4 + 2 \cdot 5$



Invariants

- 1) C, B, D_f sorted
- 2) max C ≤ min (B ∪ D_f ∪ D_r)
- 3) |C| ≥ |D_f| + 2|D_r|

BIAS ≡

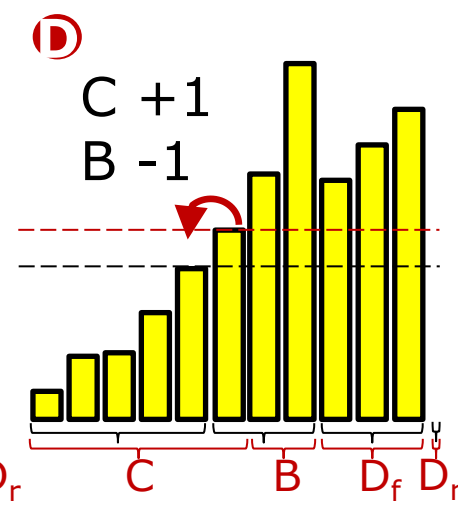
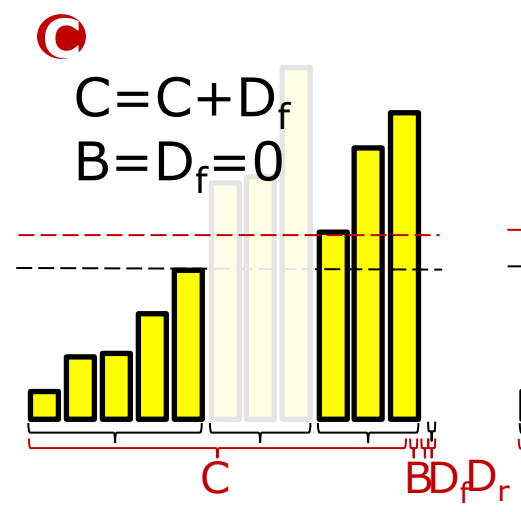
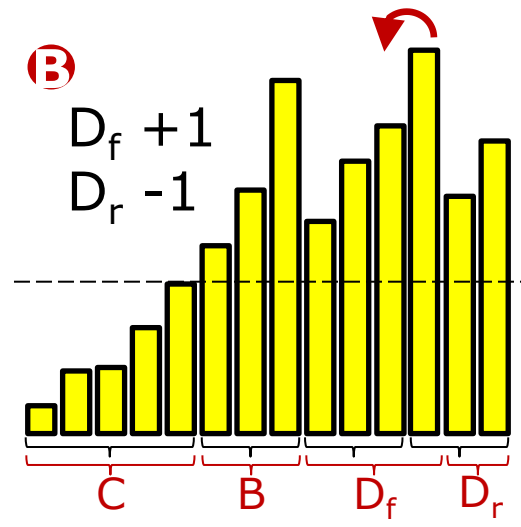
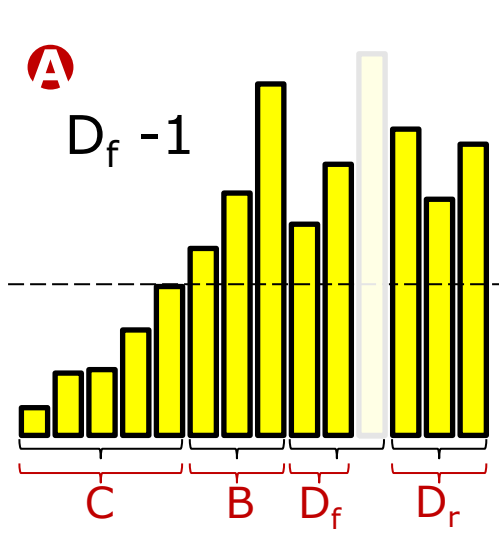
if D_r ≠ () then

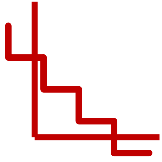
{Clean-up step}

if D_f ≠ () and last(D_f) ≥ first(D_r) then

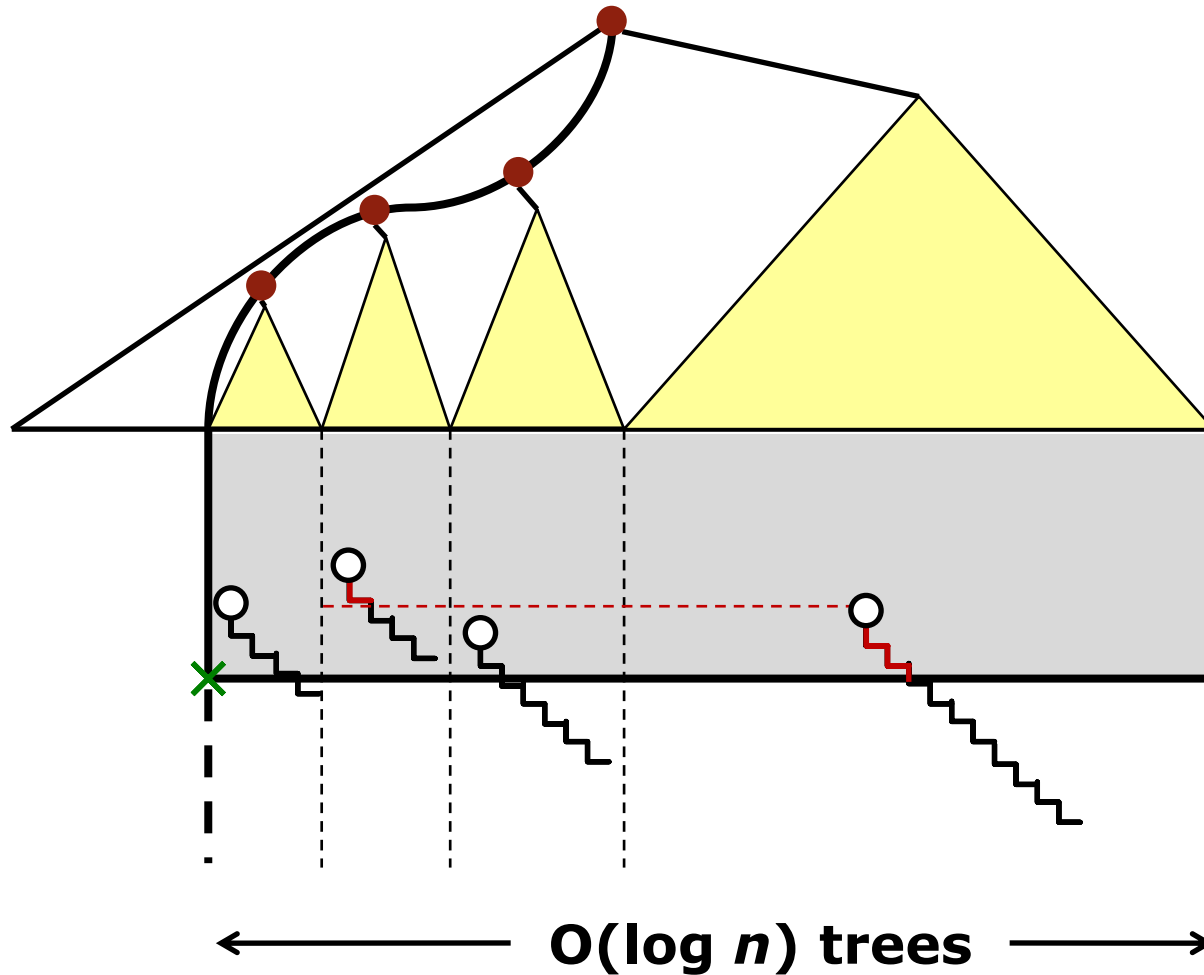
↖ **BIAS** ≥ "+1"

- A** DELETEDLAST(D_f) {decrease |D_f|}
- B** else PASS(D_f, D_r) {decrease |D_r|; increase |D_f|}
- else if D_f ≠ () and (B = () or first(B) ≥ first(D_f)) then
- C** D_f, B, C := (), (), C ∥ D_f {decrease |D_f|; increase |C|}
- D** else if B ≠ () then PASS(B, C) {increase |C|}
- {else B = D_f = D_r = ()}

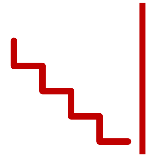




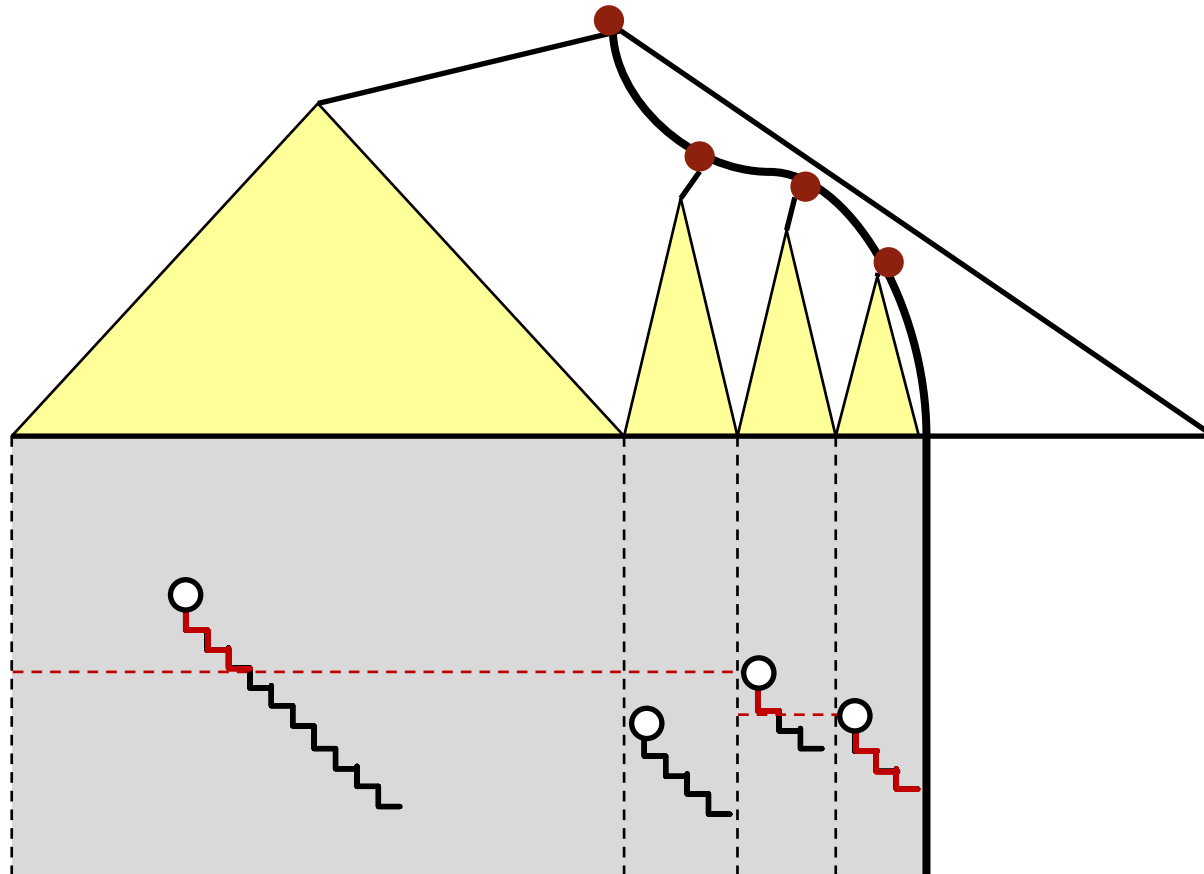
Dominance Range Maxima Queries



Query time $O(\log n + t)$

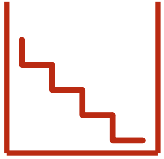


Contour Range Maxima Queries

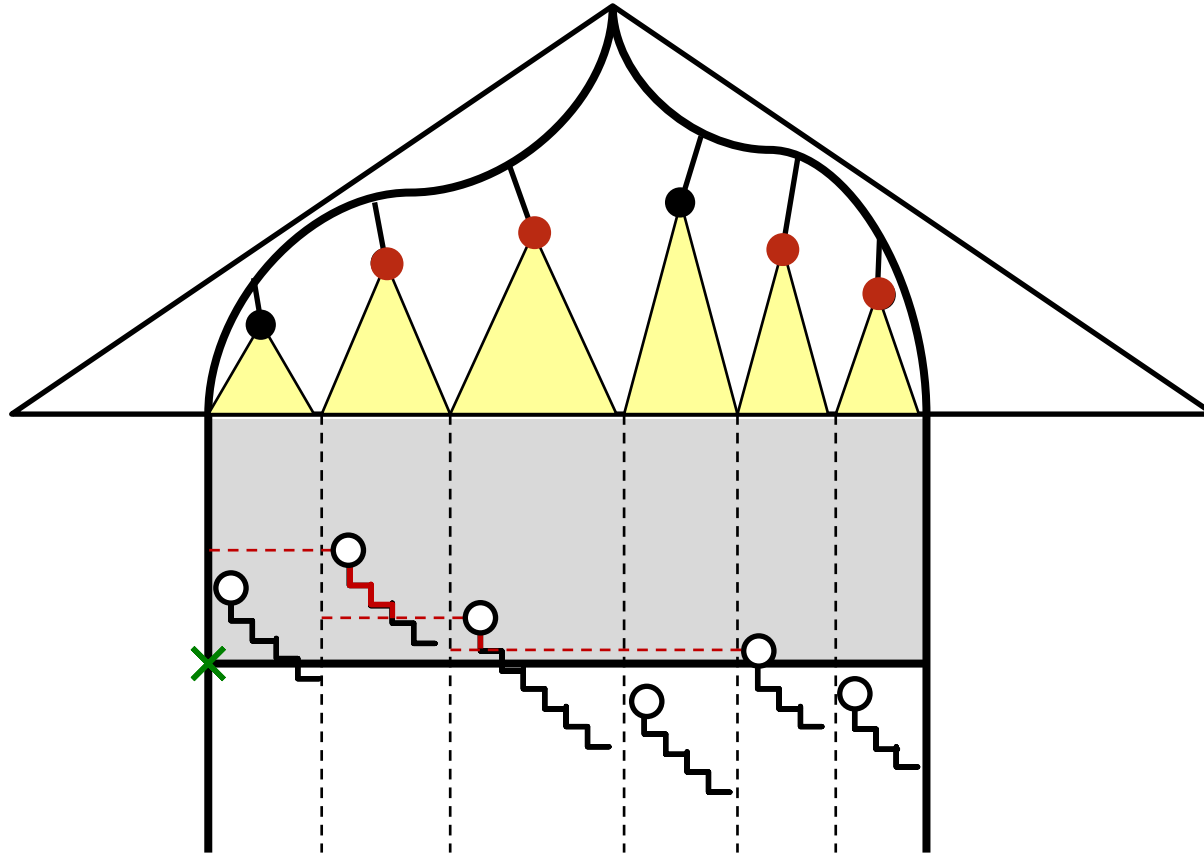


← $O(\log n)$ trees →

Query time $O(\log n + t)$



3-Sided Range Maxima Queries

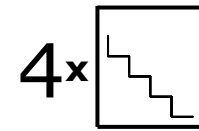
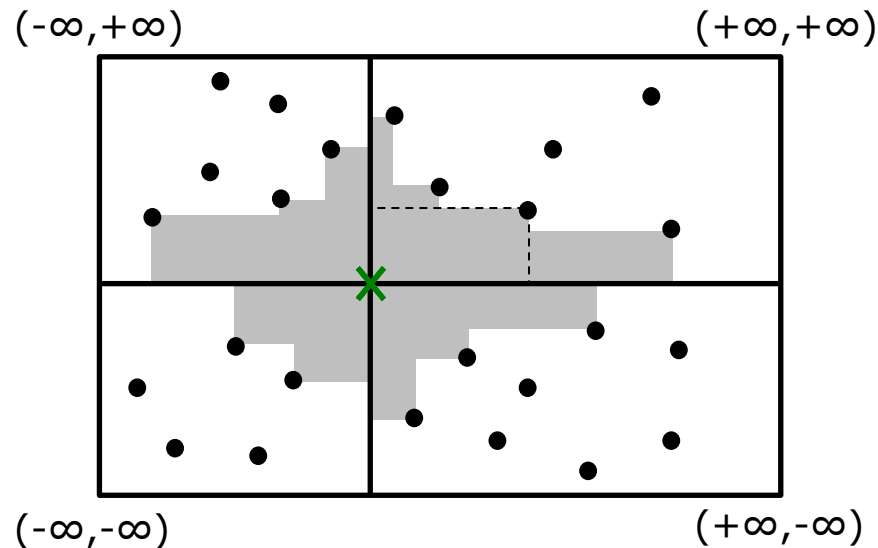


← $O(\log n)$ trees →


Query time $O(\log n + t)$

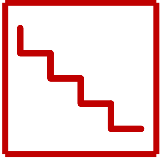
4-Sided Range MAXIMA Reporting and Rectangular Visibility Queries

Proximity Queries/Similarity Search

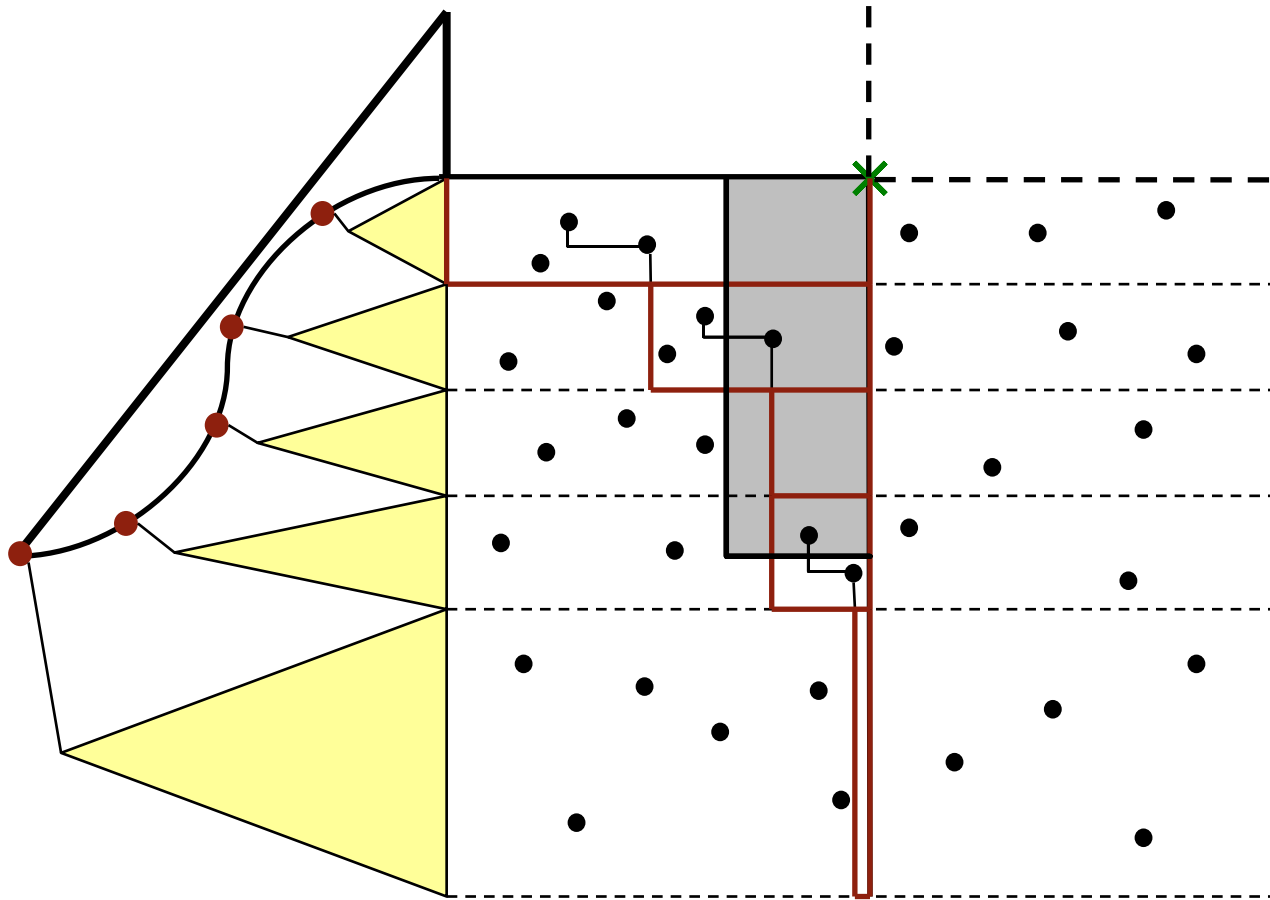


4-Sided Range Maxima Queries

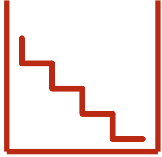
	Space		Insert	Delete
Overmars, Wood '88	$n \cdot \log n$	$\log^2 n + t$ $\log^2 n + t \cdot \log n$	$\log^2 n$	$\log^3 n$ $\log^2 n$
[ICALP '11]	$n \cdot \log n$	$\log^2 n + t$	$\log^2 n$	$\log^2 n$



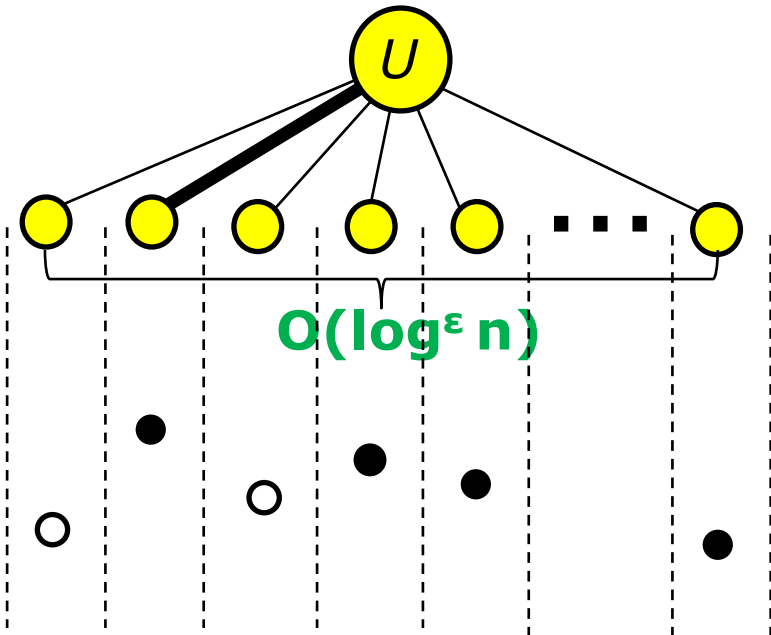
4-sided Range Maxima Queries



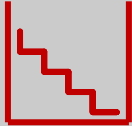
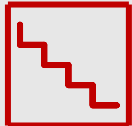
Query time $O(\log^2 n + t)$, space $O(n \cdot \log n)$



RAM - $O(\log n / \log \log n + t)$



- Height $O(\log n / \log \log n)$
- $\text{MAX}(\text{Right}(\underline{u}))$ maintained using **Q-heaps**
[Fredman, Willard, JCSS '94]

	Space	Query	Insert/Delete	
	$O(n)$	$O(\log n / \log \log n + t)$	$O(\log n / \log \log n)$	RAM
	$O(n \cdot \log n)$	$O(\log^2 n + t)$	$O(\log^2 n / \log \log n)$	

Thank You

Gerth Stølting Brodal
Aarhus University