Sorting Integers in the RAM Model

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Sorting (Integers)

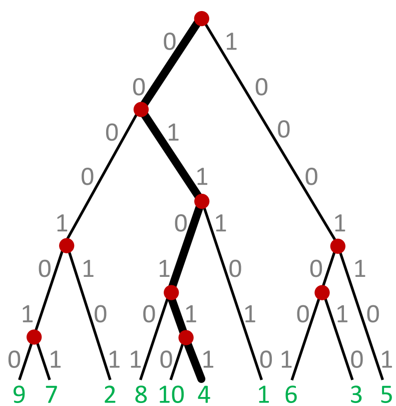
n integers

	1	2	3	4	5	6	7	8	9	10
Input	**58**	**13**	**138**	**55**	**141**	**137**	**11**	**53**	**10**	**54**
	00111010	00001101	10001010	00110111	10001101	10001001	00001010	00110101	00001010	00110110

w bits

Output	**10**	**11**	**13**	**53**	**54**	**55**	**58**	**137**	**138**	**141**
	00001010	00001010	00001101	00110101	00110110	00110111	00111010	10001001	10001010	10001101

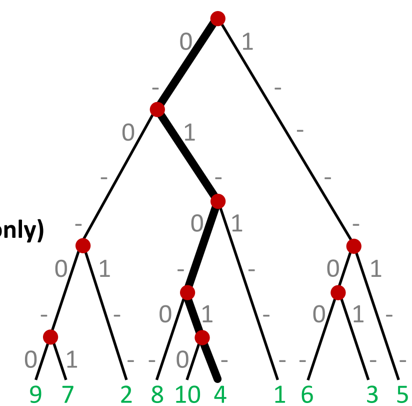
Trie



permutation

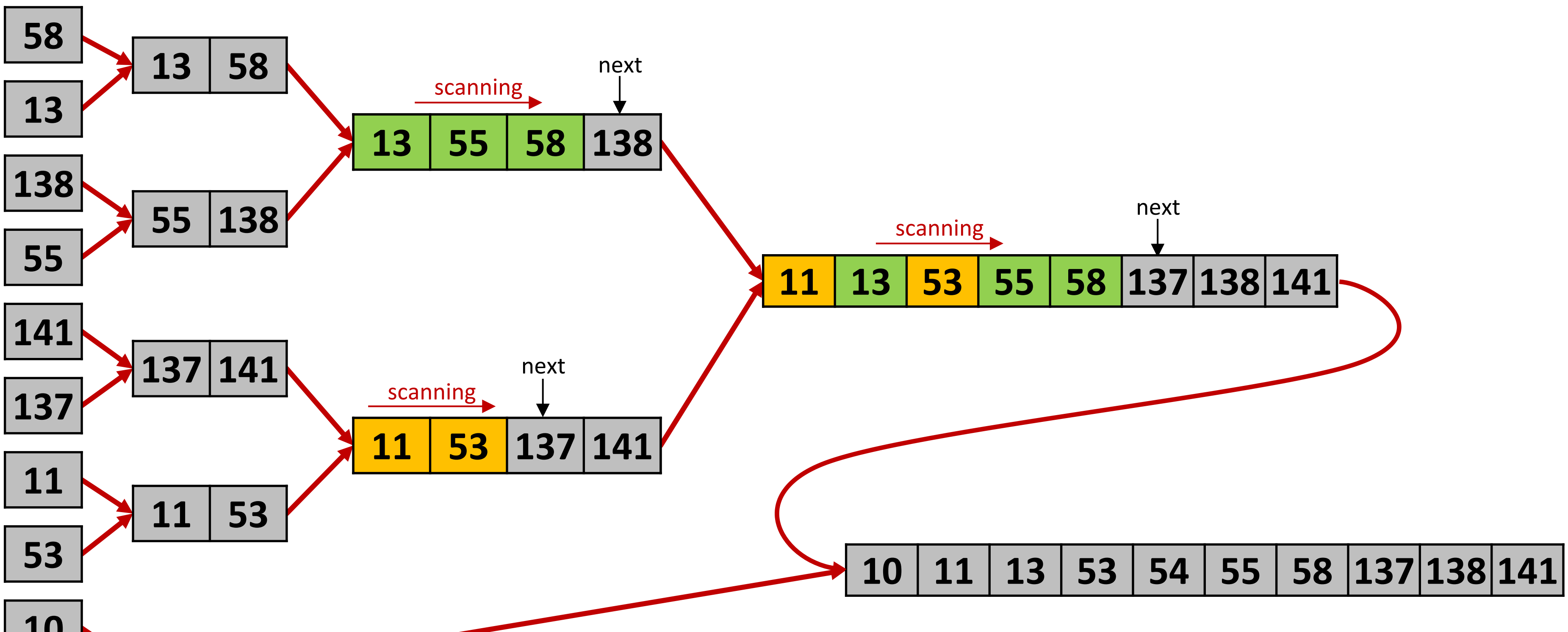
Patricia trie

(branching charcters only)



Comparison Based Sorting

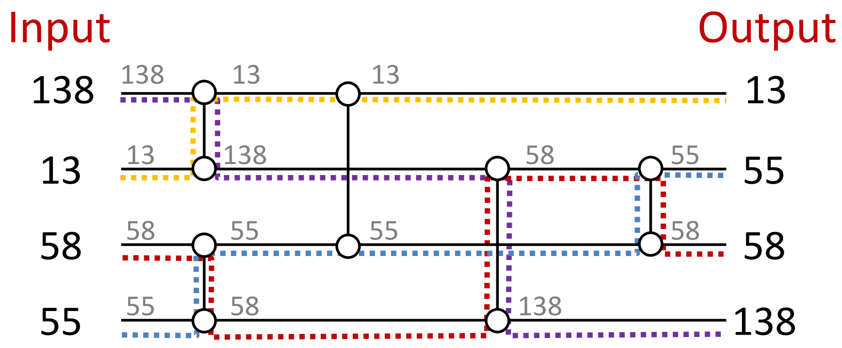
- Merge sort: von Neumann 1945



$\leq n\lceil \log_2 n \rceil$ comparions

- Ford et al. 1959: Lower bound $\geq n \log_2 n - \dfrac{n}{\ln 2}$ comparions

Sorting Networks

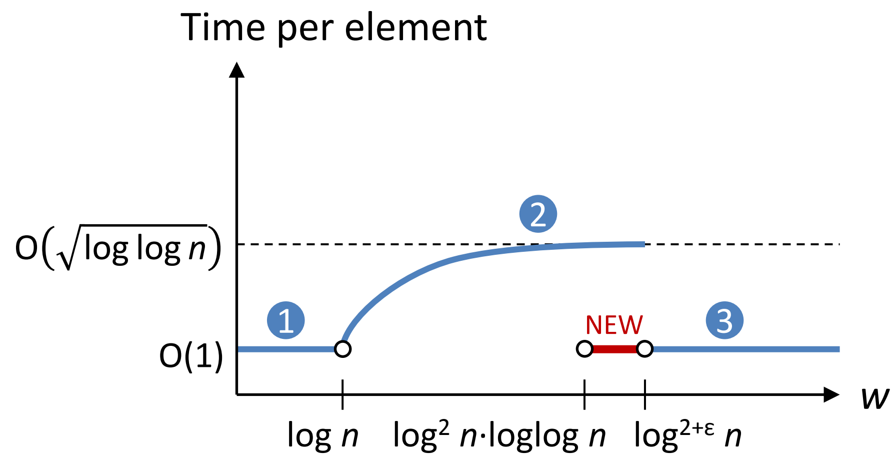


- Original motivated by hardware implementations
- Renewed interest since *data-oblivious*
  - applications in privacy preserving computations
  - allows for parallel computations on bit-level and GPU

$O(n \cdot \log^2 n)$ size : Batcher 1968

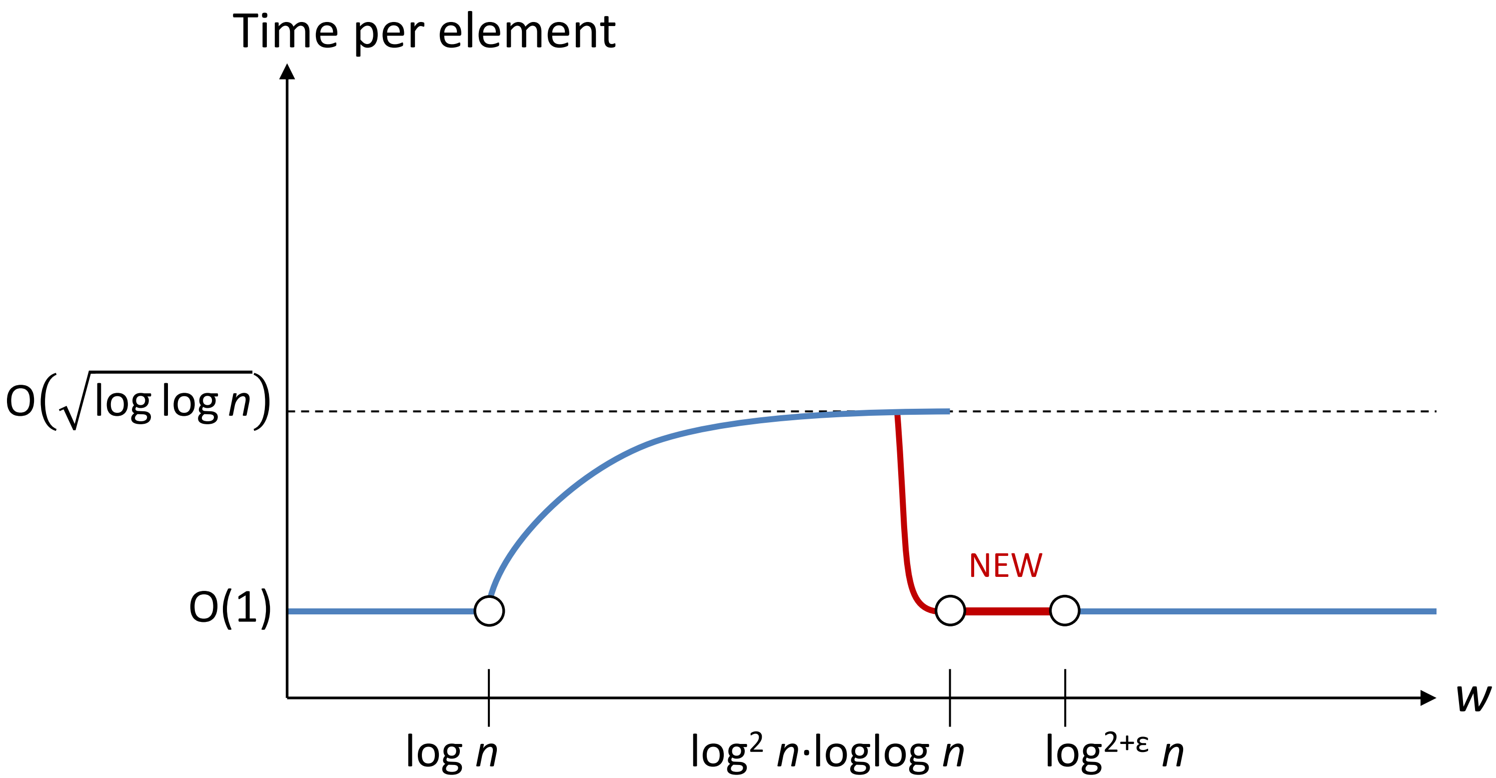
$O(n \cdot \log n)$ size : Ajtai, Komlós, Szemerédi 1983; Paterson 1990; Goodrich 2014

Results



1	Bucket sort	$O(n+2^w)$	
	Radix sort; Hollerith 1887	$O\left(n \frac{w}{\log n}\right)$	
	van Emde Boas 1975 Willard 1983	$O(n \log w)$	superlinear space expected
	Kirkpatrick and Reicsh 1983	$O\left(n \log \frac{w}{\log n}\right)$	
	Merge sort: von Neumann 1945	$O(n \log n)$	comparison based optimal
2	Thorup and Han 2002	$O\left(n\sqrt{\log(w/\log n)}\right)$ $O\left(n\sqrt{\log\log n}\right)$	expected
3	Andersson et al. 1998	$O(n)$	expected, $w \geq \Omega\left(\log^{2+\varepsilon} n\right)$
	NEW	$O(n)$	expected, $w \geq \Omega\left(\log^2 n \cdot \log\log n\right)$

Open Question



Sorting in linear time for all *w*?