

Closing a Classical Data Structure Problem: Strict Fibonacci Heaps

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joint work with

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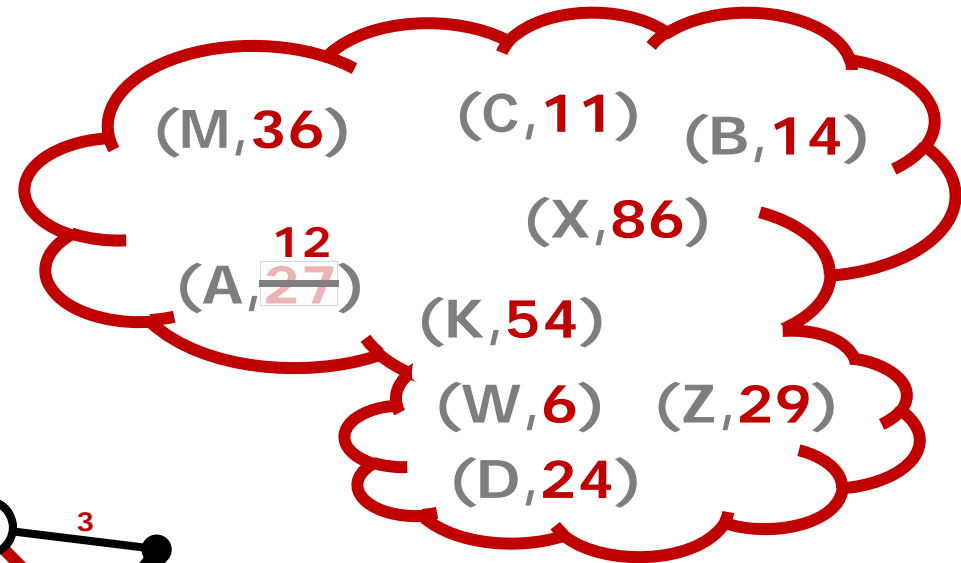
Robert Endre Tarjan
Princeton & HP

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The Problem

- INSERT(value, key)
- FINDMIN
- DELETE / DELETEMIN
- MELD(Q_1, Q_2)
- DECREASEKEY(value, Δ)

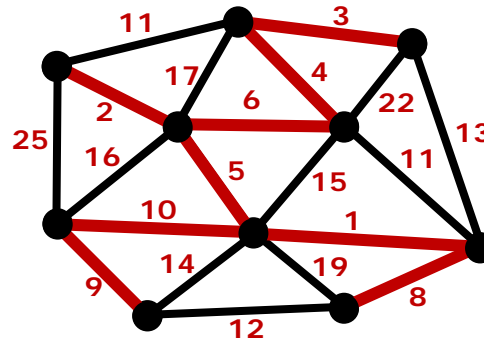
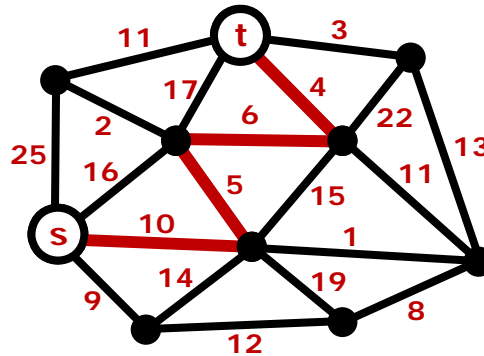
Priority queue



Applications

- Shortest Path Problem
- Dijkstra (1956)
- Minimum Spanning Tree
- Borůvka (1926)
- Jarník (1930)

(n node, m edges)



INSERT/DELETEMIN
 $(m+n) \cdot \log n$
 ↓ Fredman, Tarjan 1984
 + DECREASEKEY
 $m+n \cdot \log n$
 ↓ MST only Fredman, Tarjan 1984
 + DECREASEKEY
 $m \cdot \beta(m, n)$

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History

Binary heaps Binomial queues Fibonacci heaps Run-relaxed heaps Strict Fibonacci heaps

	Williams 1964	Vuillemin 1978	Fredman Tarjan 1984		Tarjan et al. 1988	Brodal 1995	Brodal 1996	Brodal Lagogianis Tarjan 2012
Insert	log n	log n	1	1	1	1	1	1
FindMin	1	1	1	1	1	1	1	1
Delete	log n	log n	log n	n	log n	log n	log n	log n
Meld	-	log n	1	1	log n	1	1	1
DecreaseKey	log n	log n	log n	n	1	log n	1	1

Amortized complexity (Tarjan 1983)

Arrays
Complicated

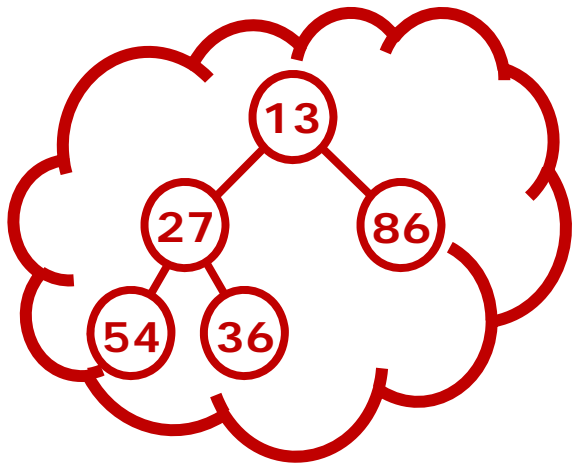
CONFERENCE REVIEWS

- This paper closes one of the few open problems in the area of comparison-based priority queues
- Other PQs from the 1990s ... were extremely complicated
- The data structure is genuinely simple

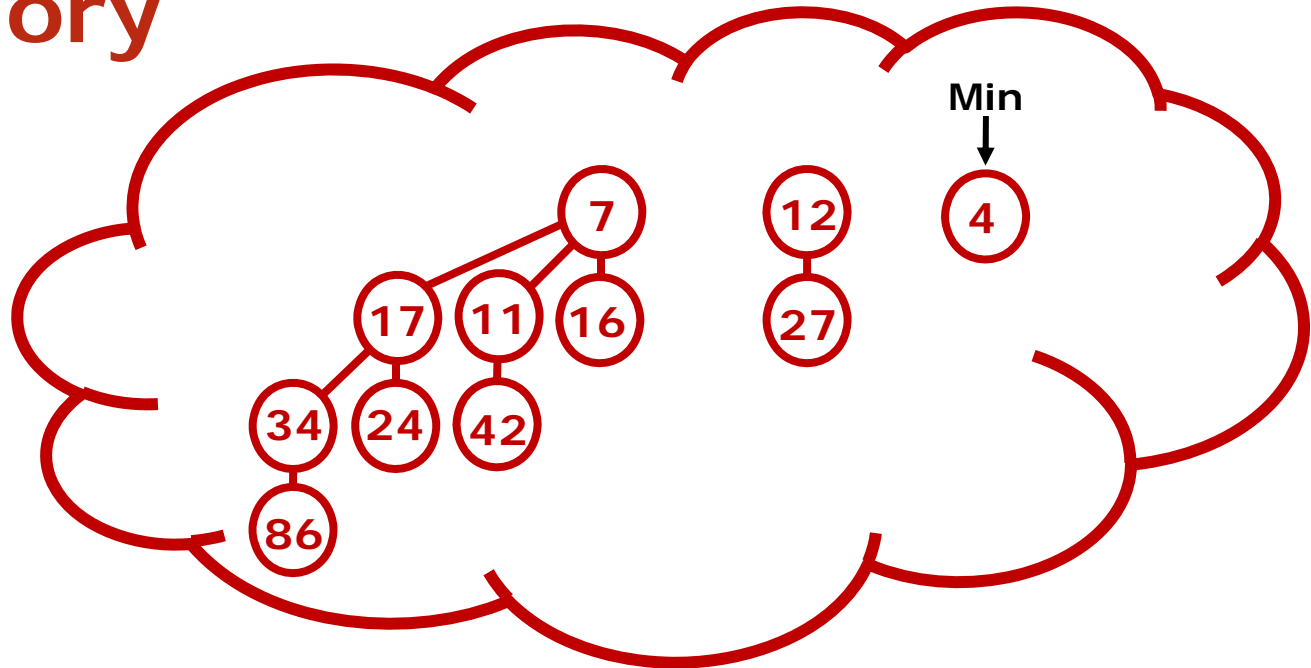
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Technical History



Binary heaps



Binomial queues

Binary heaps 1964	Binomial queues 1978	Fibonacci heaps 1984	Run-relaxed heaps 1988	Brodal 1995	Brodal 1996	Strict Fibonacci heaps 2012
Heap-order Rigid structure	Forest Linking	Subtrees cut Cascades ⇒ Amortized DECREASEKEY	Global control Redundant counters	Local control Redundant counters	Local redundant counters Heap order violations	Global partial control Pigeonhole principle

single tree

don't give up

— Peter Gabriel (1985)

(((I worked on the heap problem 1994-2012)))

Thank You