Cache-Oblivious Dynamic Dictionaries with Optimal Update/Query Tradeoff

Indeksstrukturer til Ekstern Hukommelse

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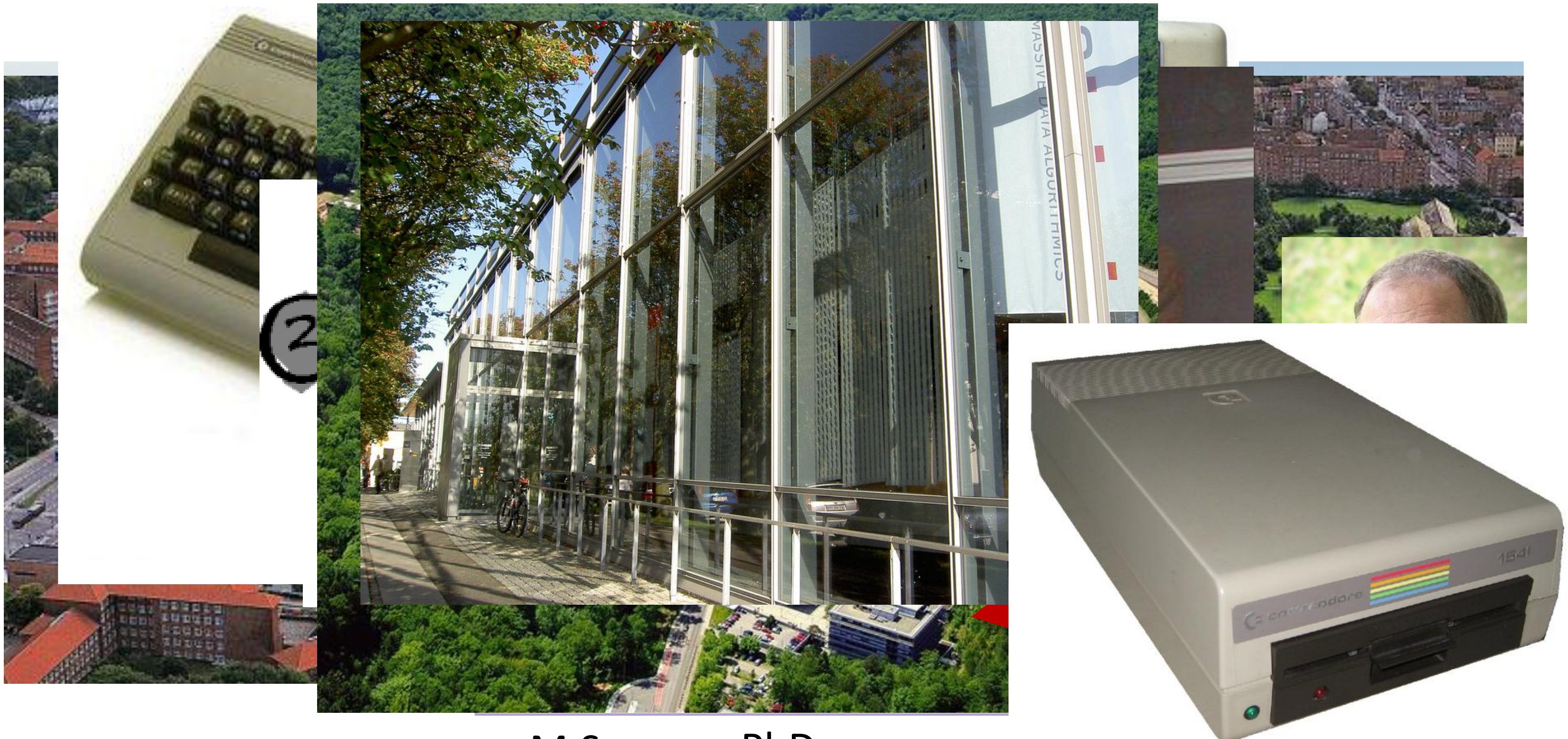
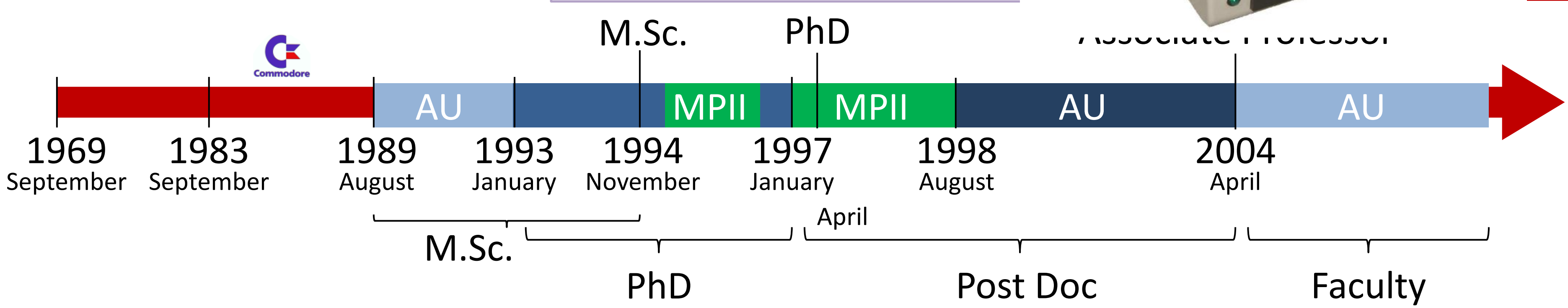
Dansk Selskab for Datalogi, Frederiksbjerg, October 13, 2010

Gerth Stølting Brodal





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- Center of Danmarks Grundforskningsfond Danish National Research Foundation
- **Lars Arge**, Professor, Centerleader
- Gerth S. Brodal, Associate Professor
- 5 Post Docs, 10 PhD students, 4 TAP
- Total budget for 5 years ca. 60 million DKR

AU

Arge

Brodal

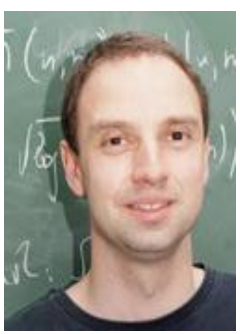
MIT

Demaine

Indyk

MPII

Mehlhorn

Frankfurt

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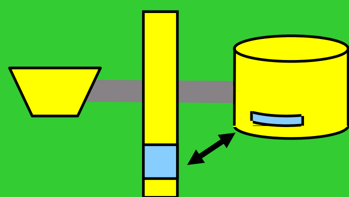
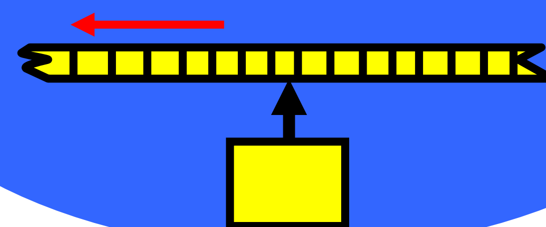
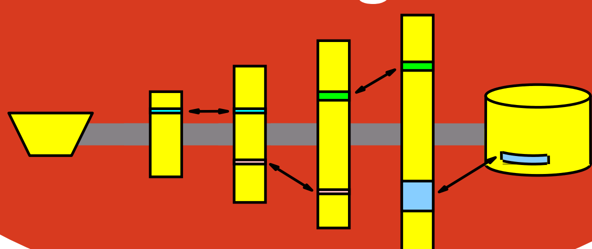
I/O Efficient Algorithms



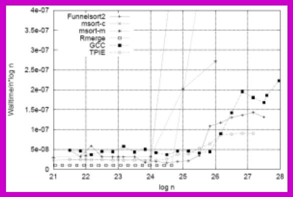
Streaming Algorithms

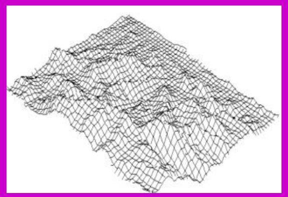


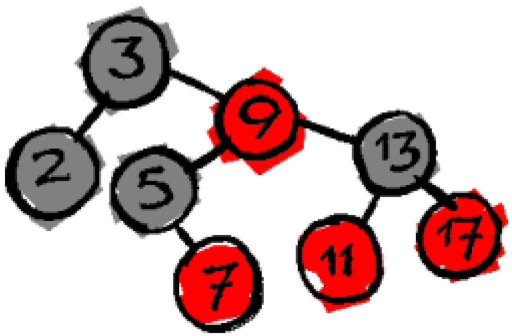
Cache Oblivious Algorithms



Algorithm Engineering



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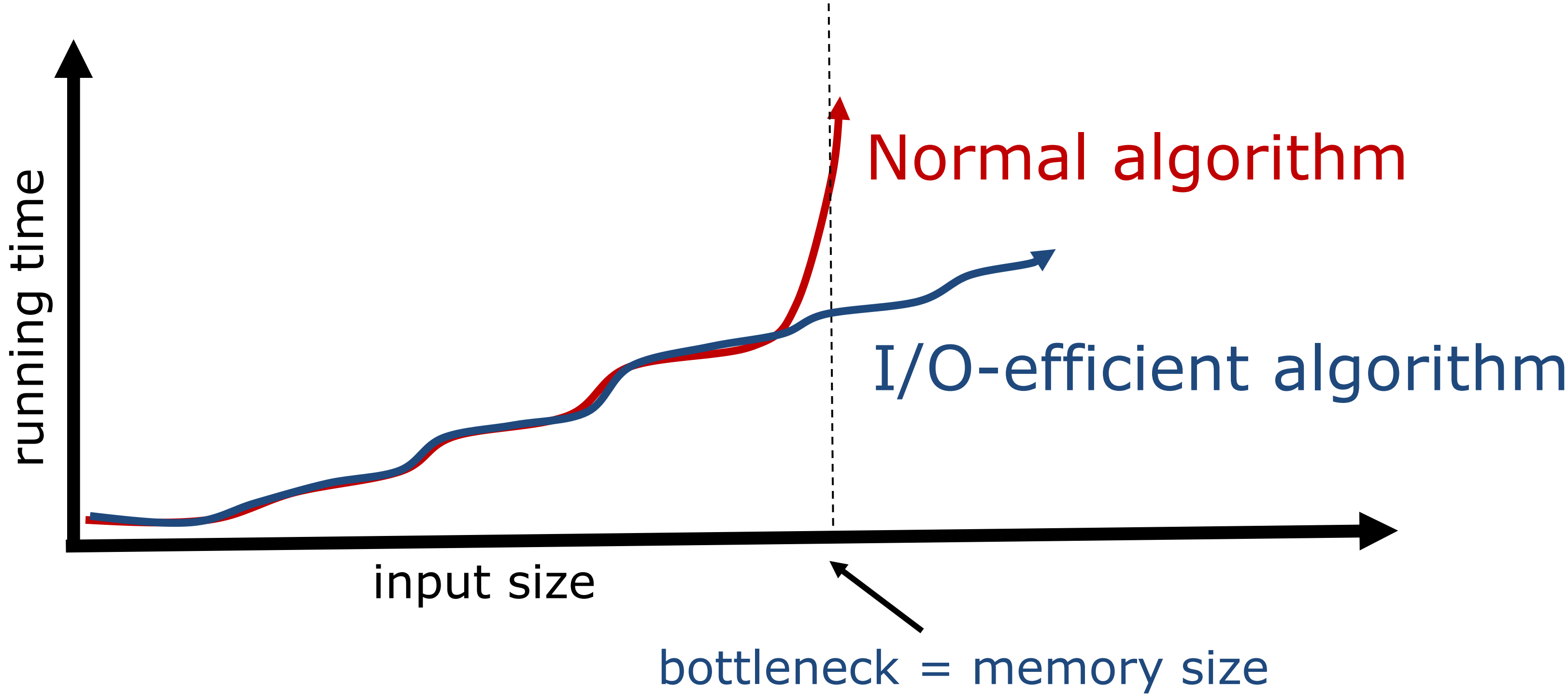
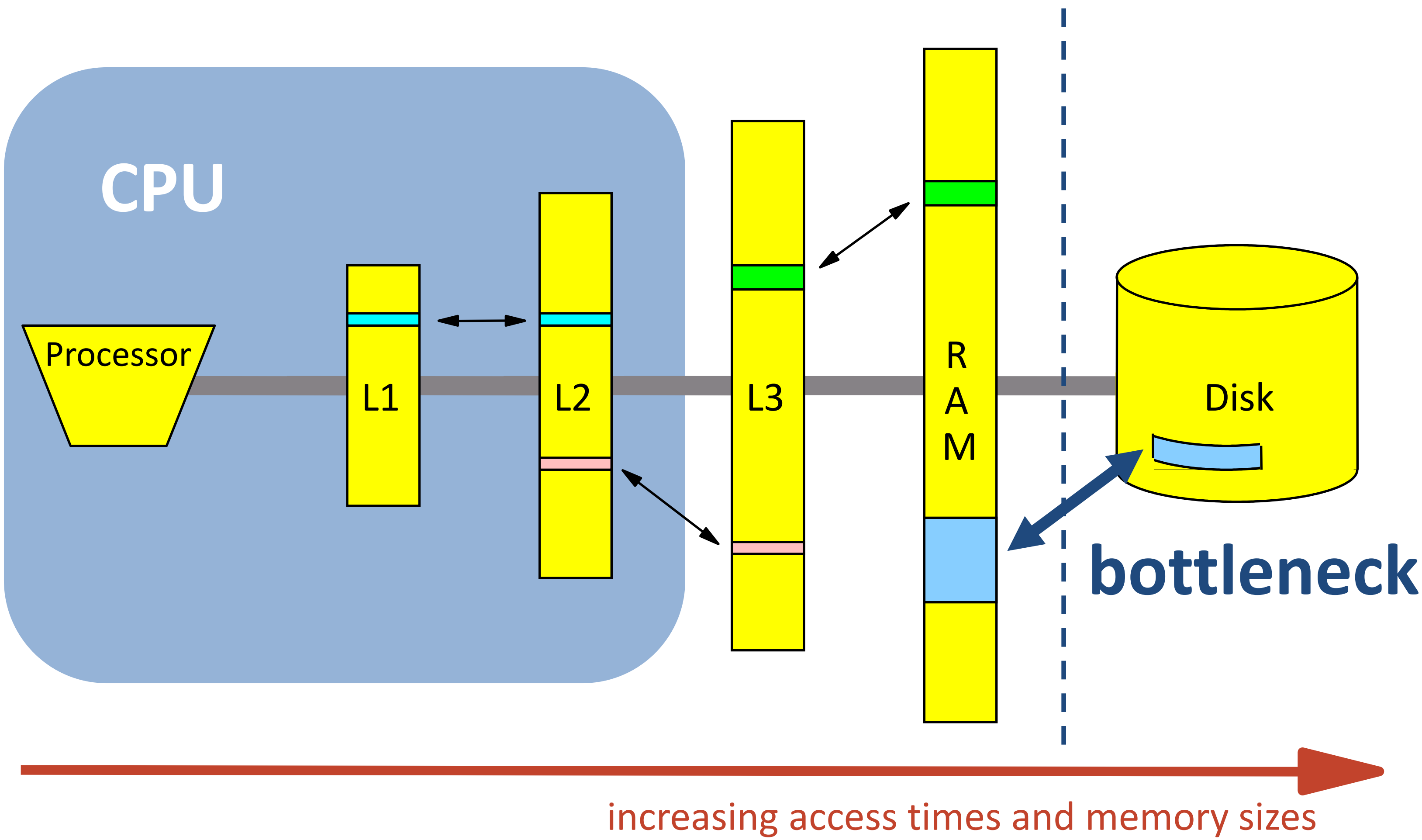
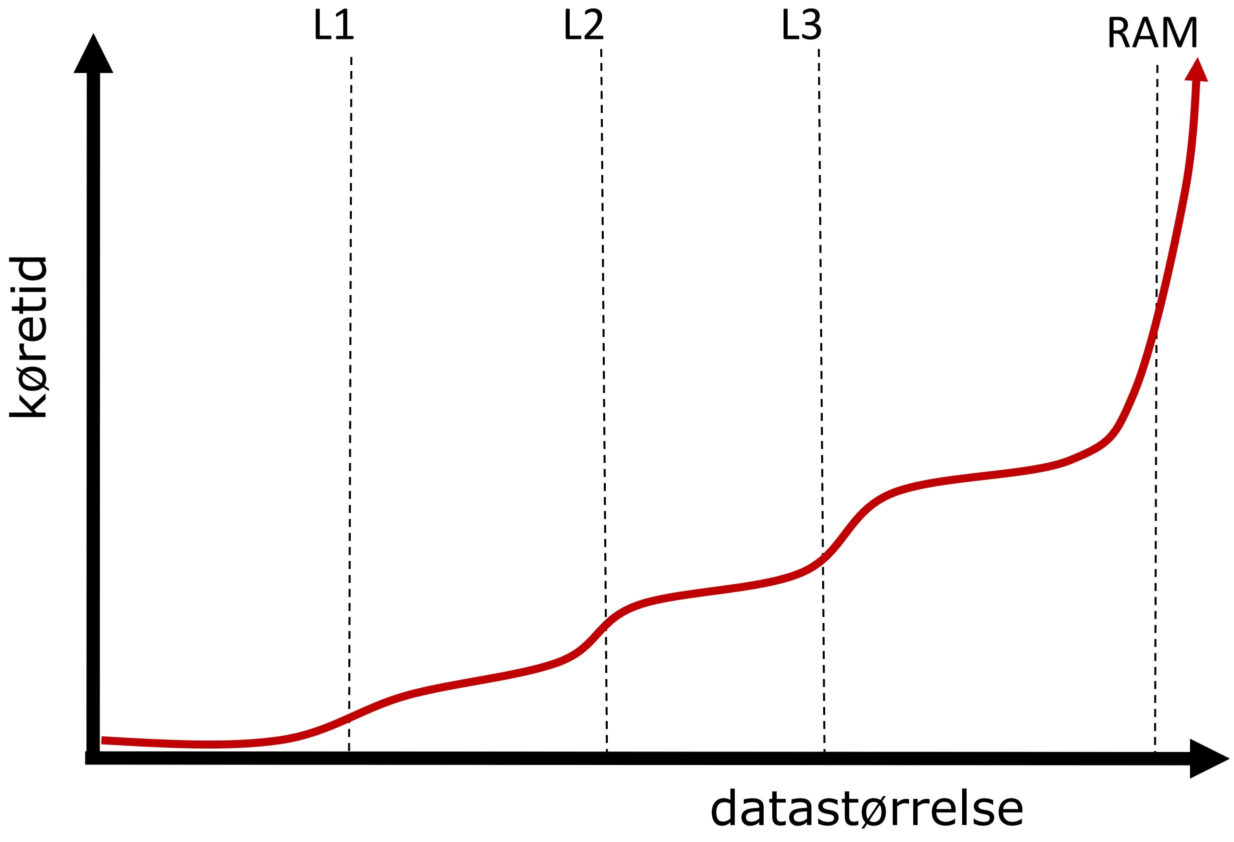
The problem...


Normal algorithm
I/O-efficient algorithm
running time
input size
bottleneck = memory size

Memory Hierarchies



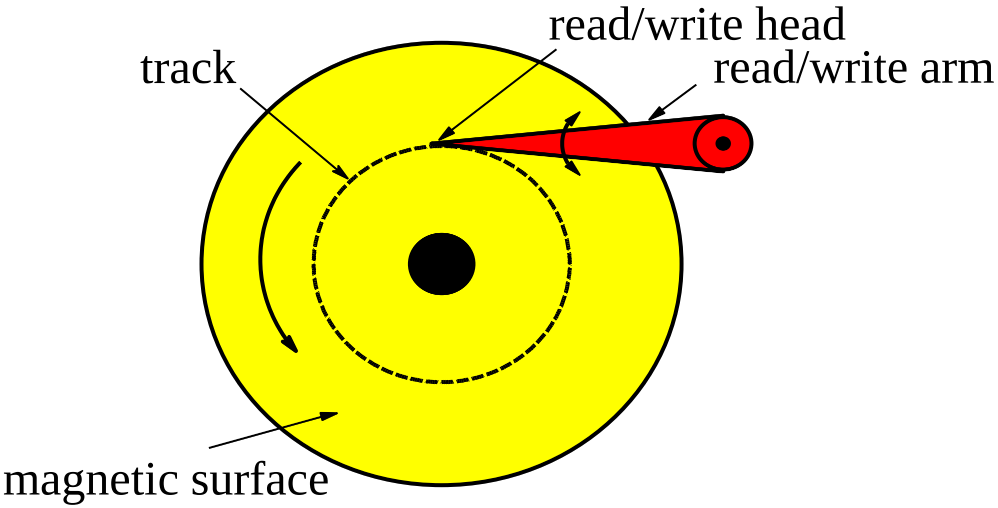
Memory Hierarkies
vs.
Running Time



Memory Access Times

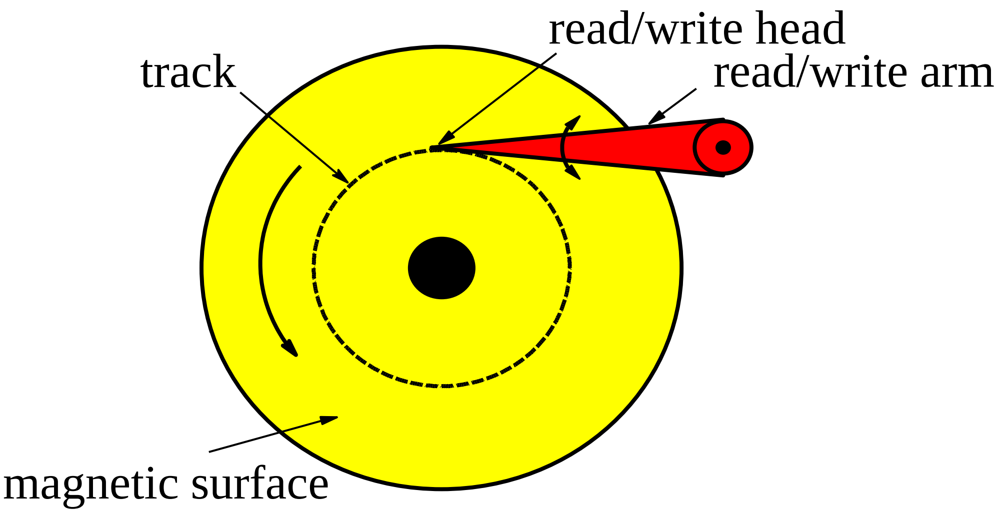
	Latency	Relative to CPU
Register	0.5 ns	1
L1 cache	0.5 ns	1-2
L2 cache	3 ns	2-7
DRAM	150 ns	80-200
TLB	500+ ns	200-2000
Disk	10 ms	10^7

Disk Mechanics



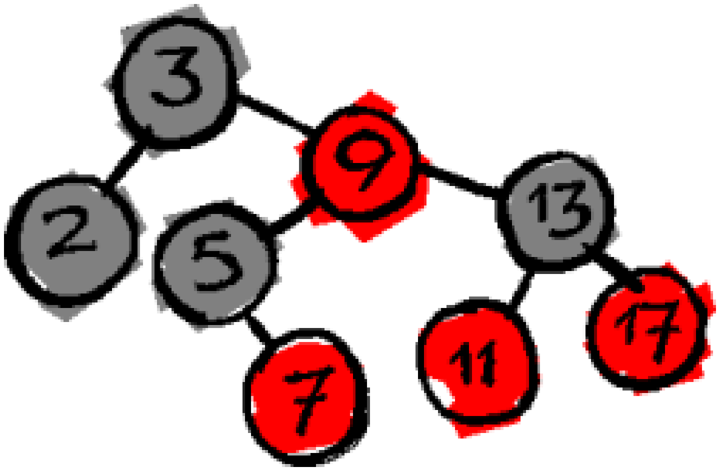
"The difference in speed between modern CPU and disk technologies is analogous to the difference in speed in sharpening a pencil using a sharpener on one's desk or by taking an airplane to the other side of the world and using a sharpener on someone else's desk." (D. Comer)

Disk Mechanics



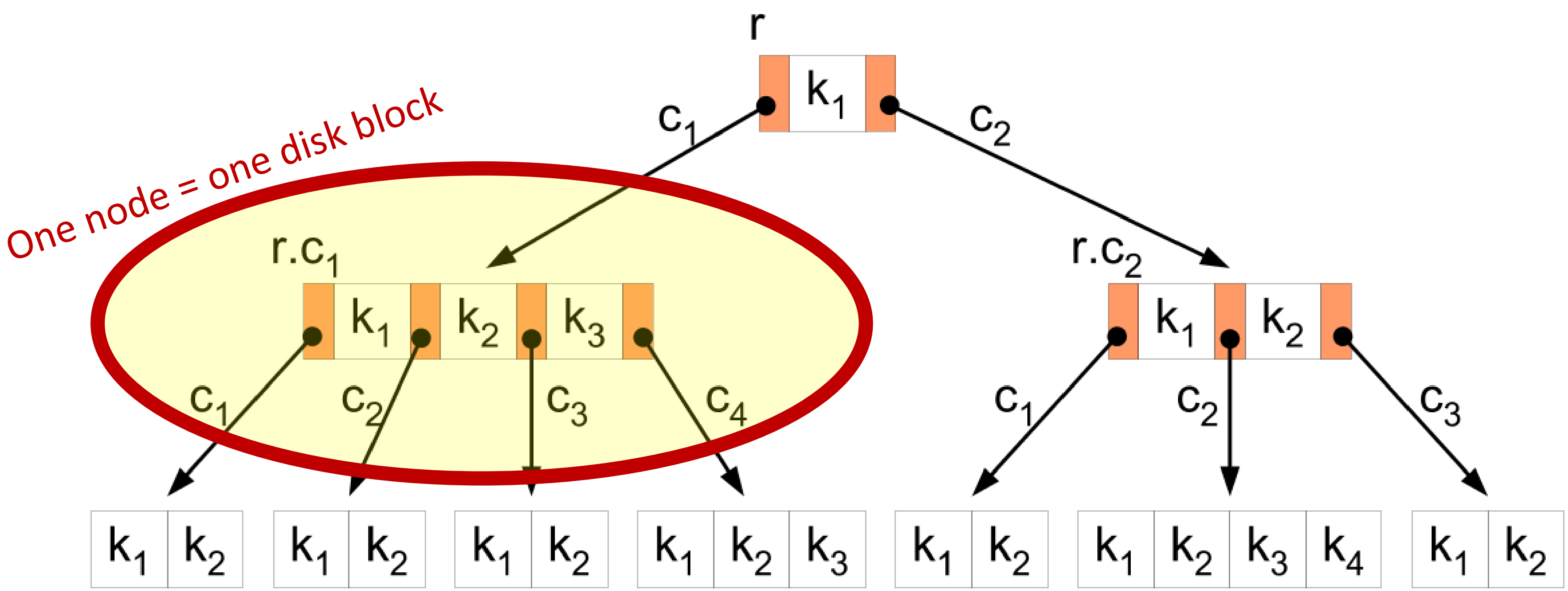
- I/O is often bottleneck when handling massive datasets
- Disk access is 10^7 times slower than main memory access!
- Disk systems try to amortize large access time transferring large contiguous blocks of data
- Need to store and access data to take advantage of blocks !

Internal Memory Index



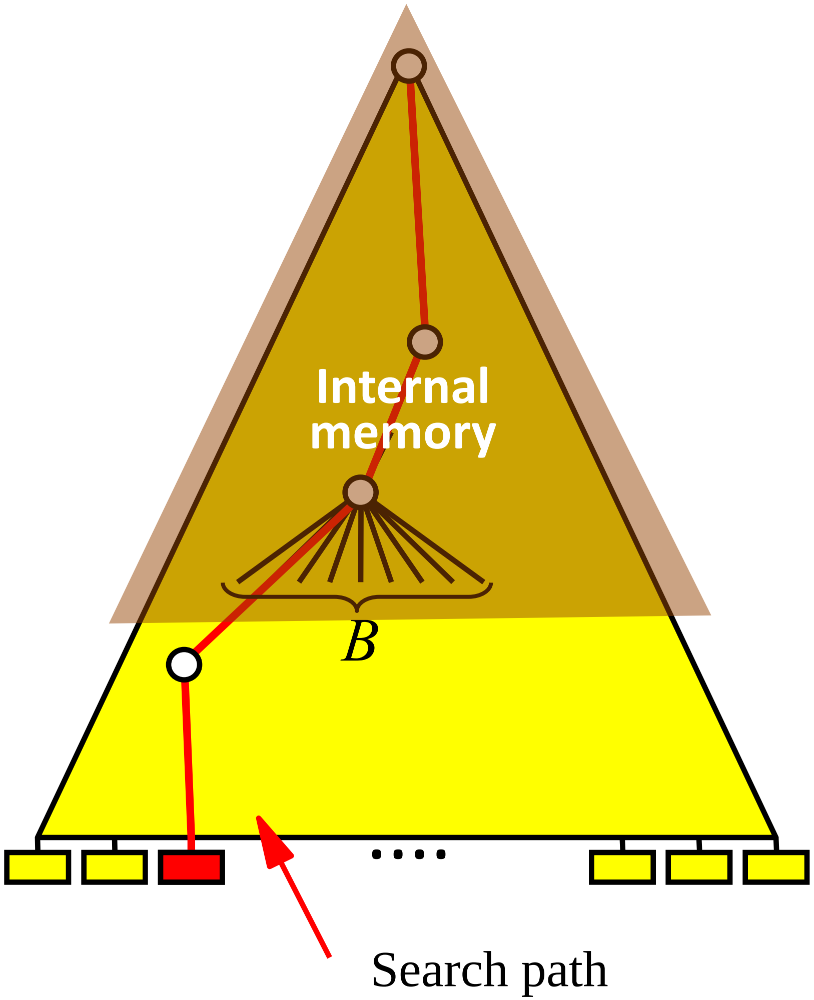
Bayer og McCreight (1971)

B(oing)-Trees



Height $O(\log_B N)$

B-trees -
The Basic Searching Structure



- **Searches**
  Practice: 4-5 I/Os

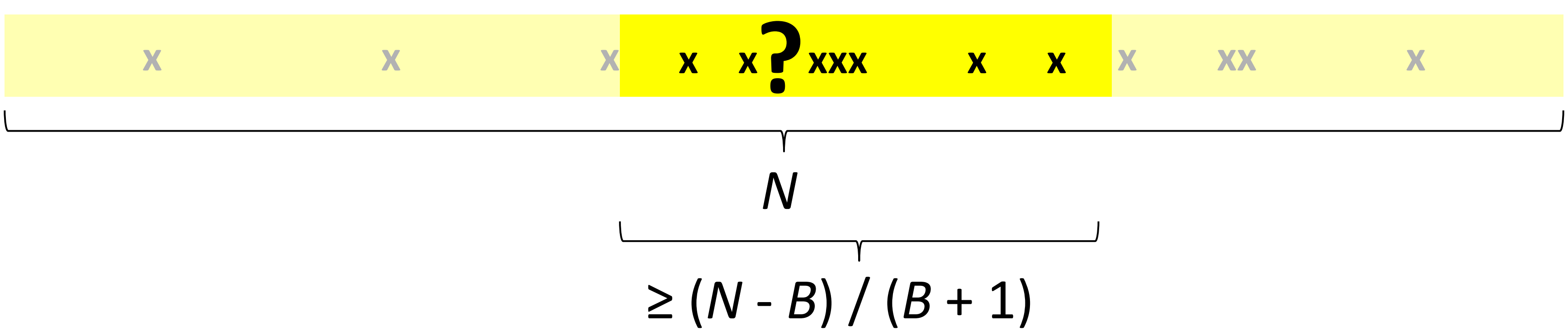
- **Repeated searching**
  Practice: 1-2 I/Os

The Bad News...

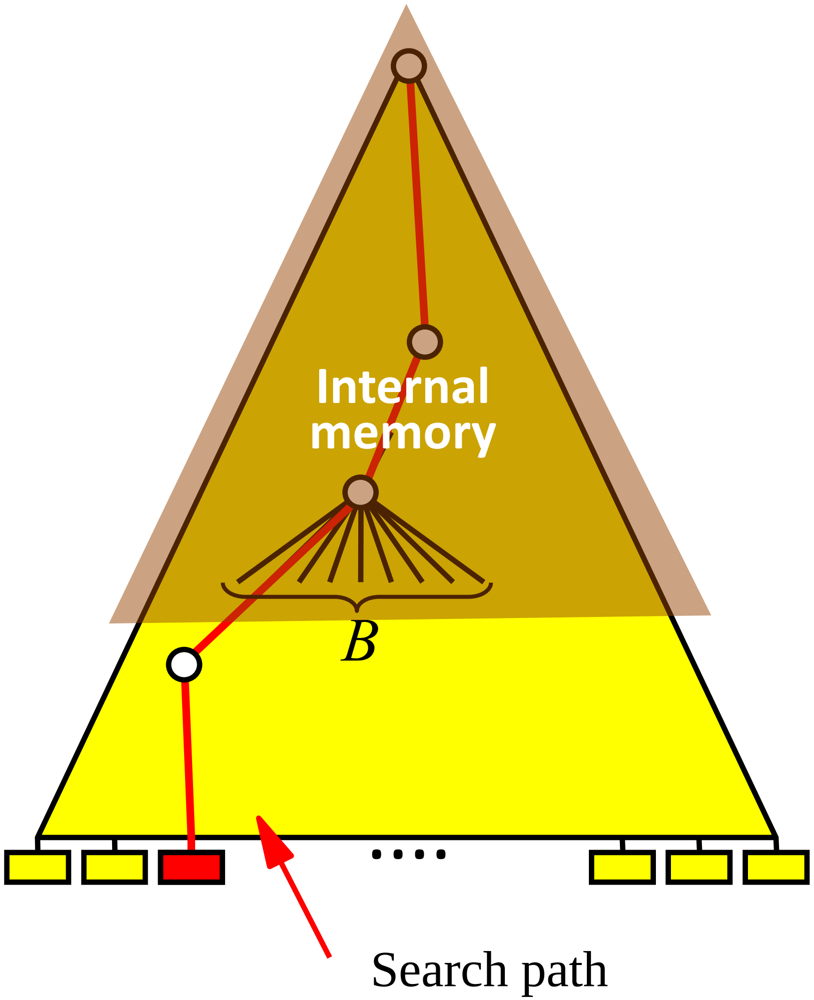
Searching any external memory dictionary (incl. B-trees) requires worst-case

$$\Omega(\log_B N) \quad \text{I/Os}$$

Proof idea:



B-trees

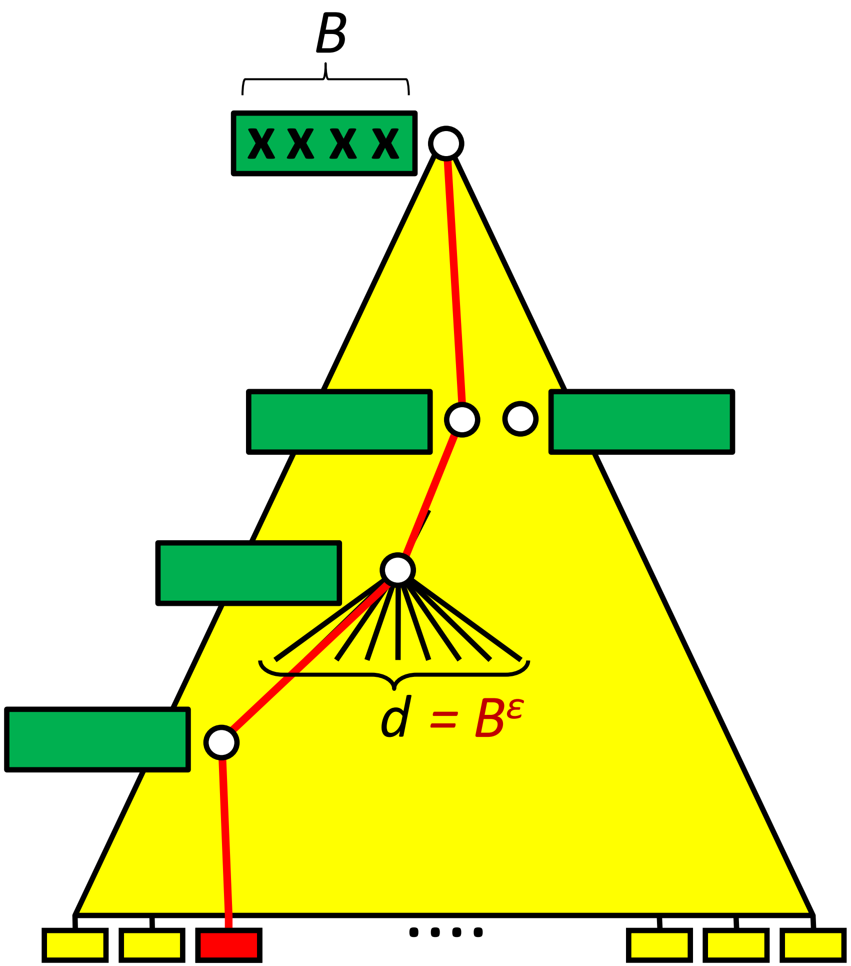


Best possible

- **Searches** $O(\log_B N)$ I/Os
- **Updates** $O(\log_B N)$ I/Os

?

B-trees with Buffered Updates



- **Searches cost**

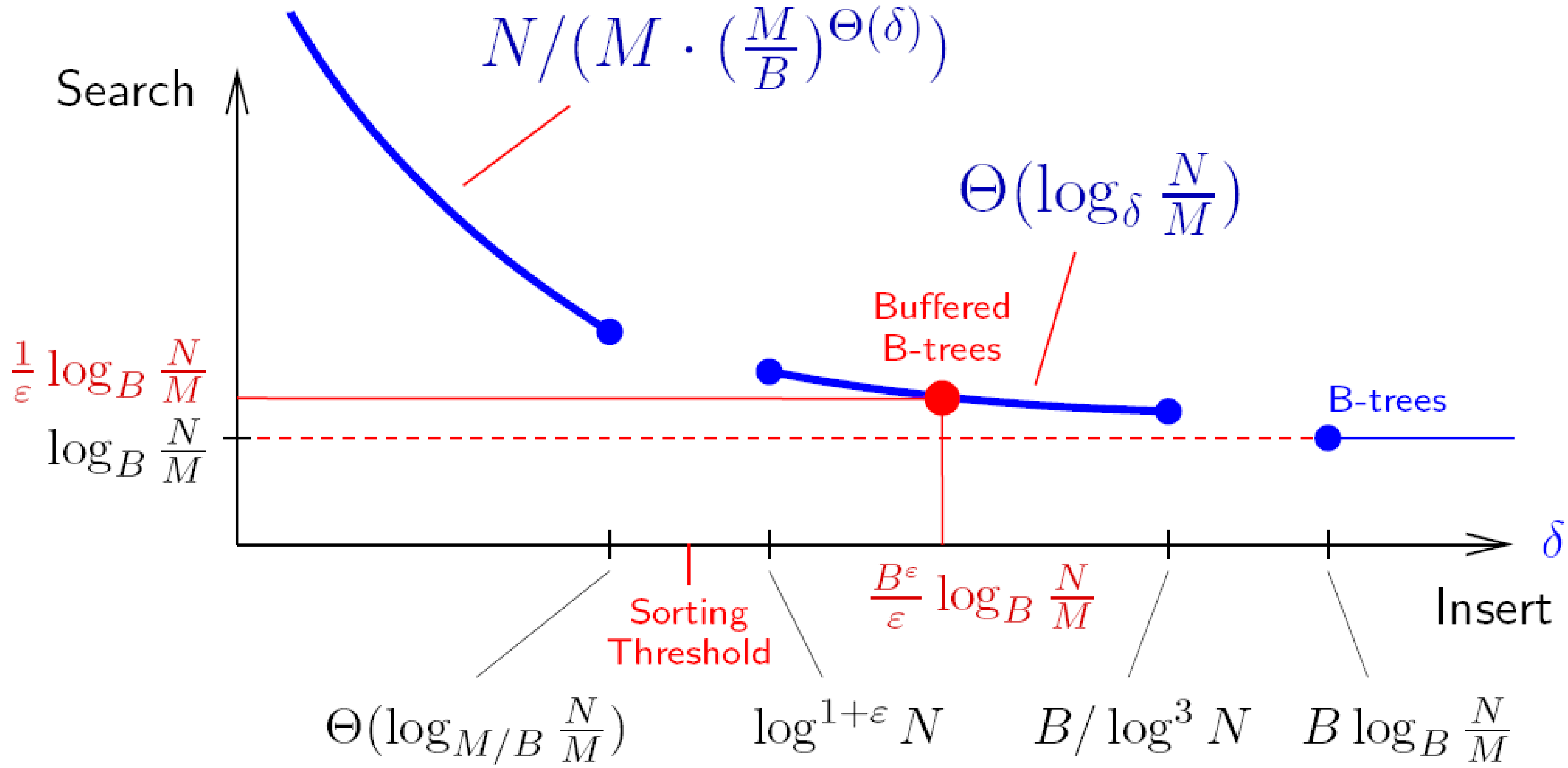
$$O(\log_d N) \quad \text{I/Os}$$
$$= O(\log_B N \cdot \underbrace{1/\varepsilon}_{2}) \quad \text{I/Os}$$

- ***N* updates cost**

$$O(N \cdot \log_d N \cdot d / B) \quad \text{I/Os}$$
$$= O(N \cdot \log_B N \cdot \underbrace{1/\varepsilon B^{1-\varepsilon}}_{2/\sqrt{B}}) \quad \text{I/Os}$$

Trade-off between search and update times – optimal *!*

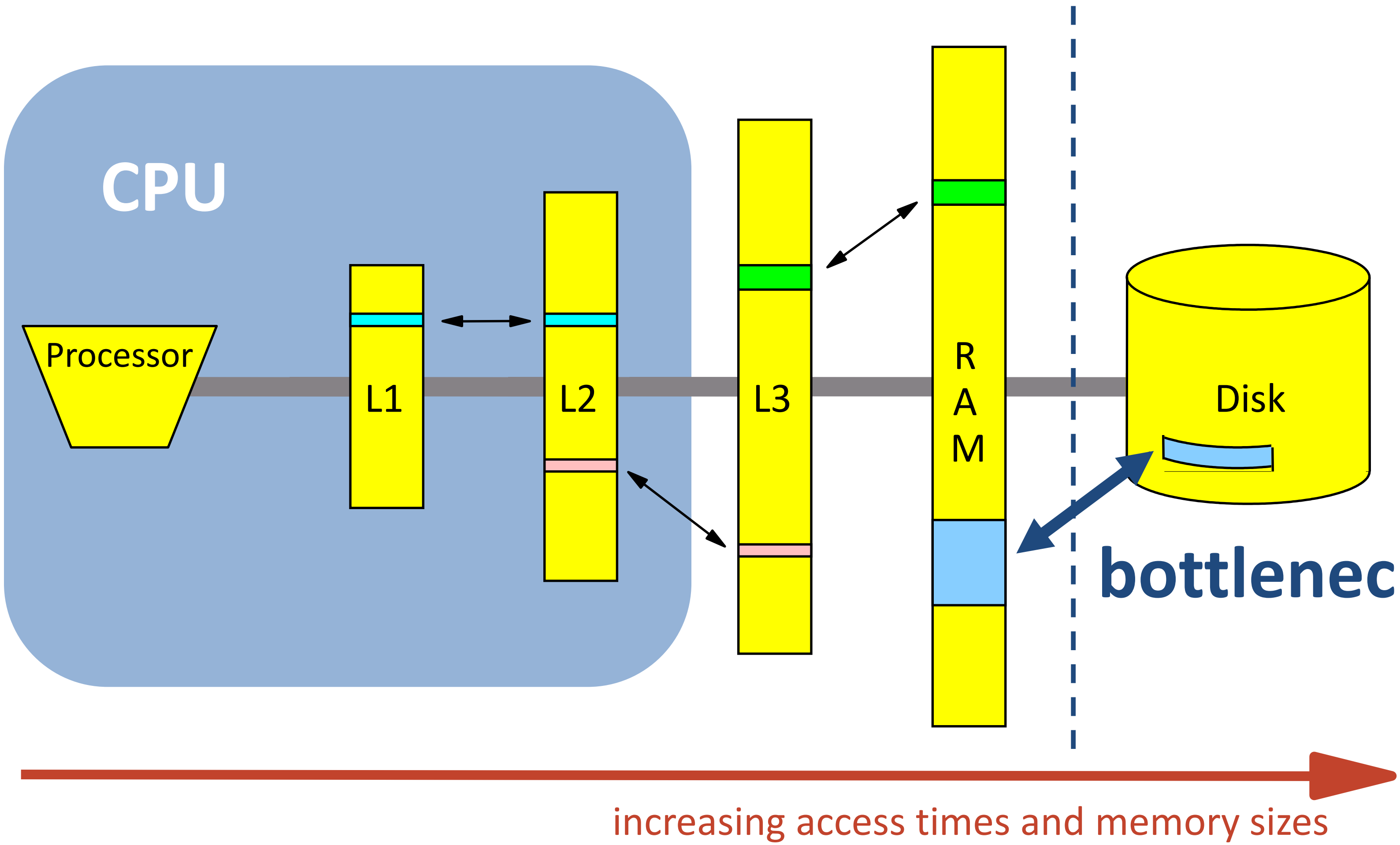
B-trees with Buffered Updates



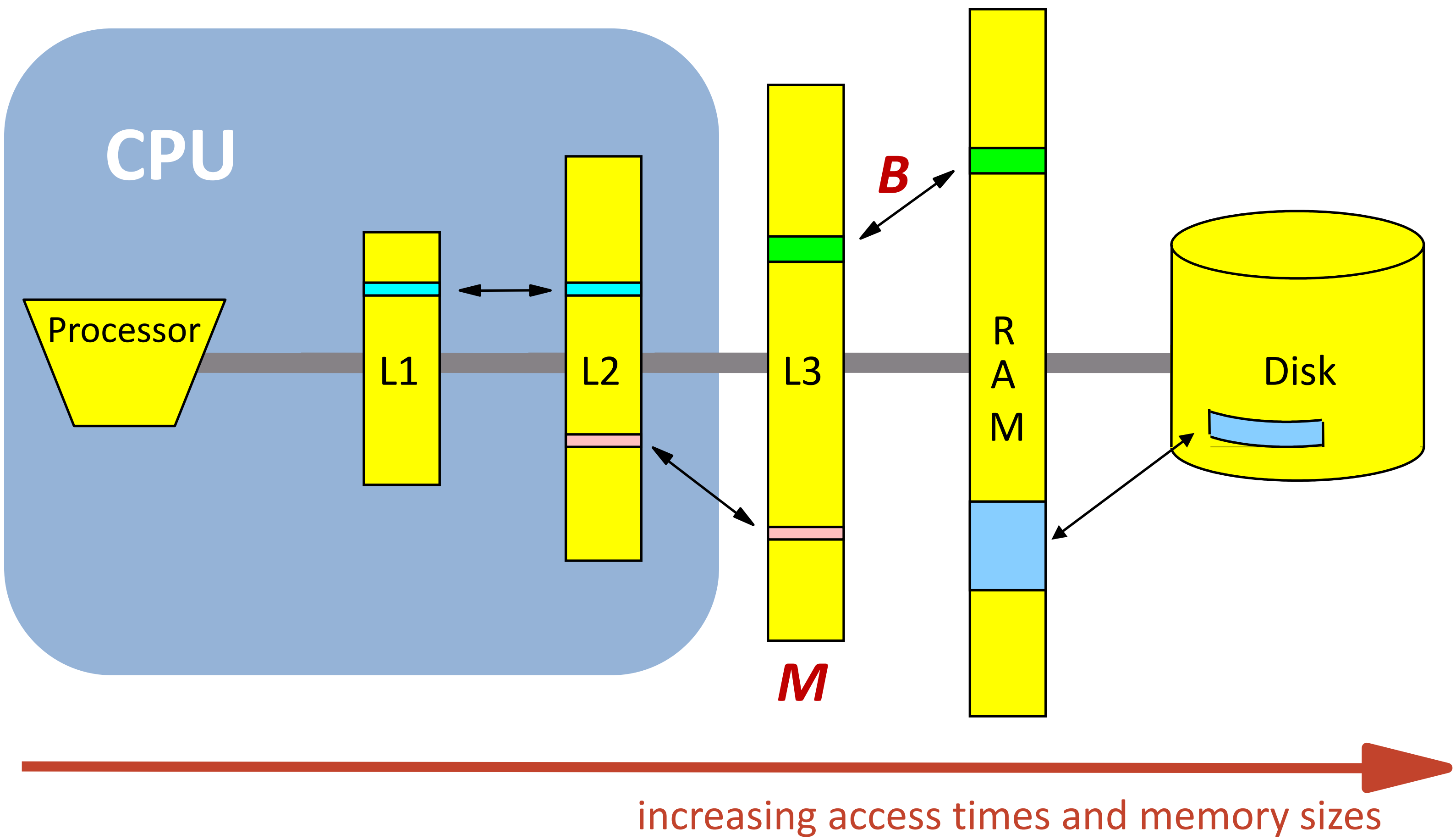
B-trees with Buffered Updates Experimental Study

- 100.000.000 elements
- Search time basically unchanged with buffers
- 100 times faster updates

Assumptions until now : *B* is known + one level is the bottleneck



Cache-Obliviousnes Model

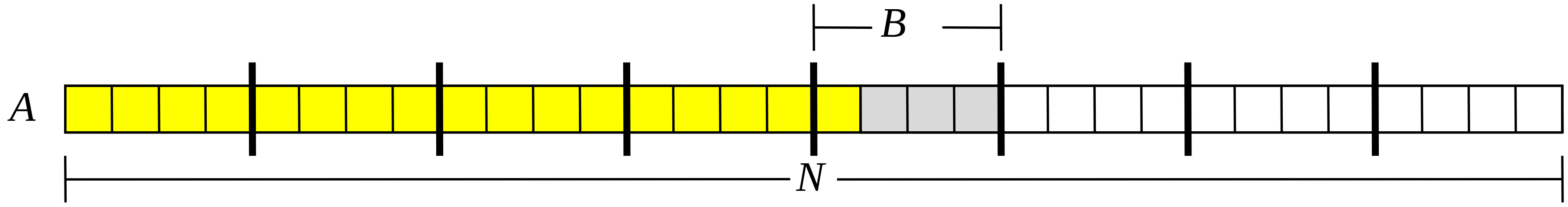


Algorithm **does not know** *B* and *M*

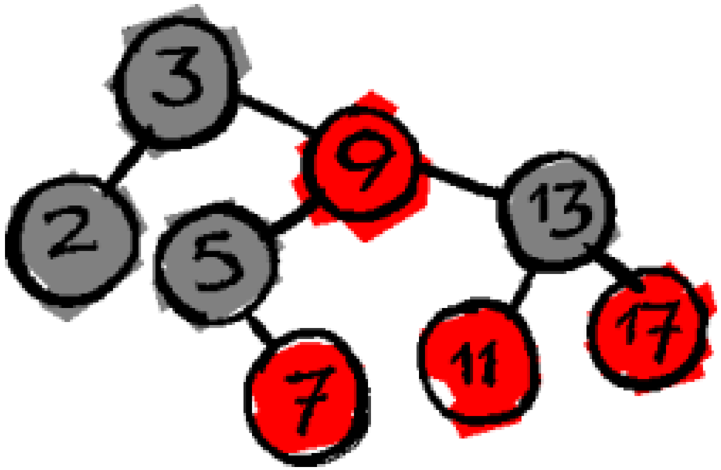
(assume optimal offline cache-replacement strategy)

I/O Efficient Scanning

```
sum = 0
for i = 1 to N do sum = sum + A[i]
```



$O(N/B)$ I/Os



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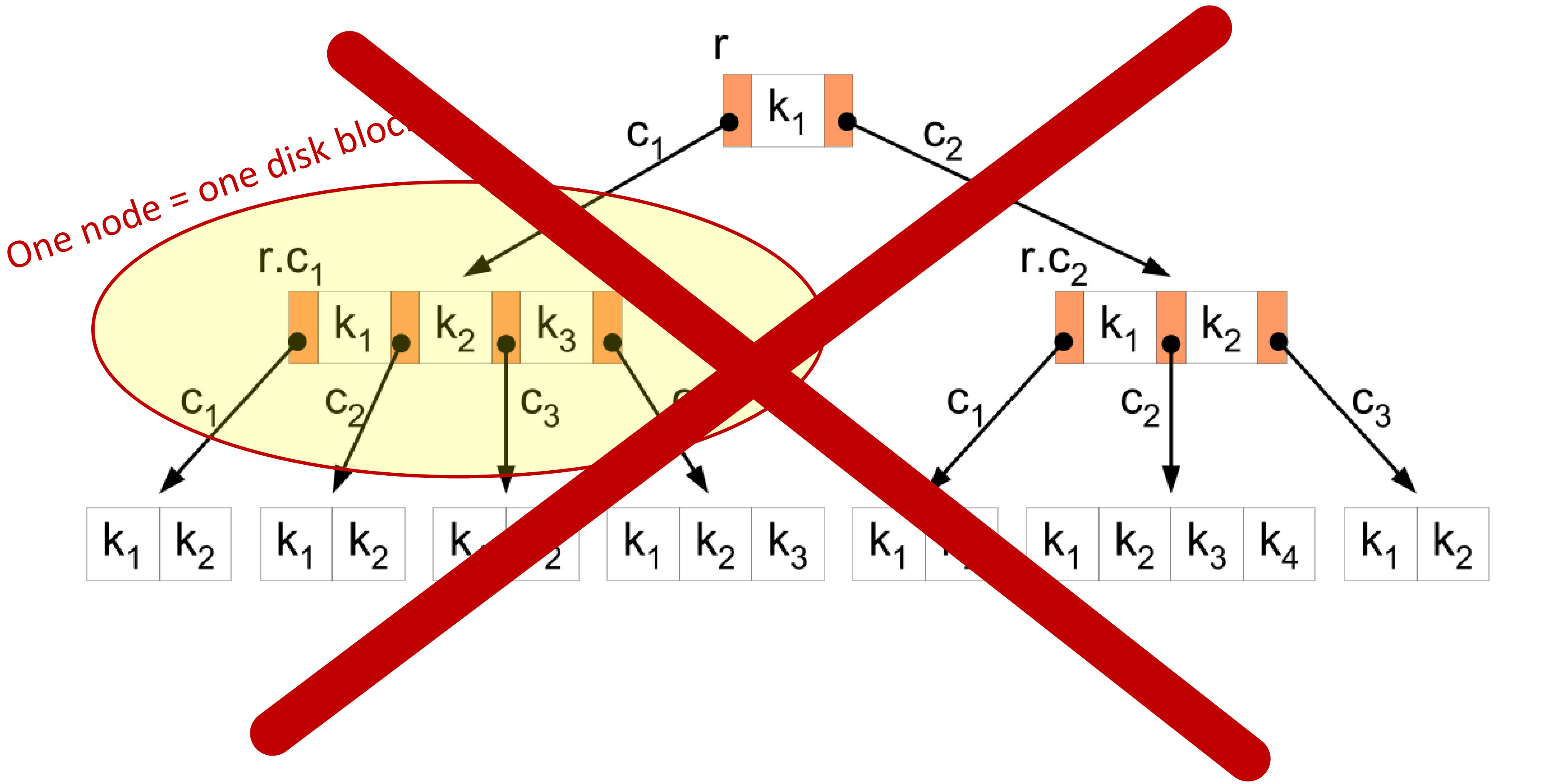
Bayer og McCreight (1971)

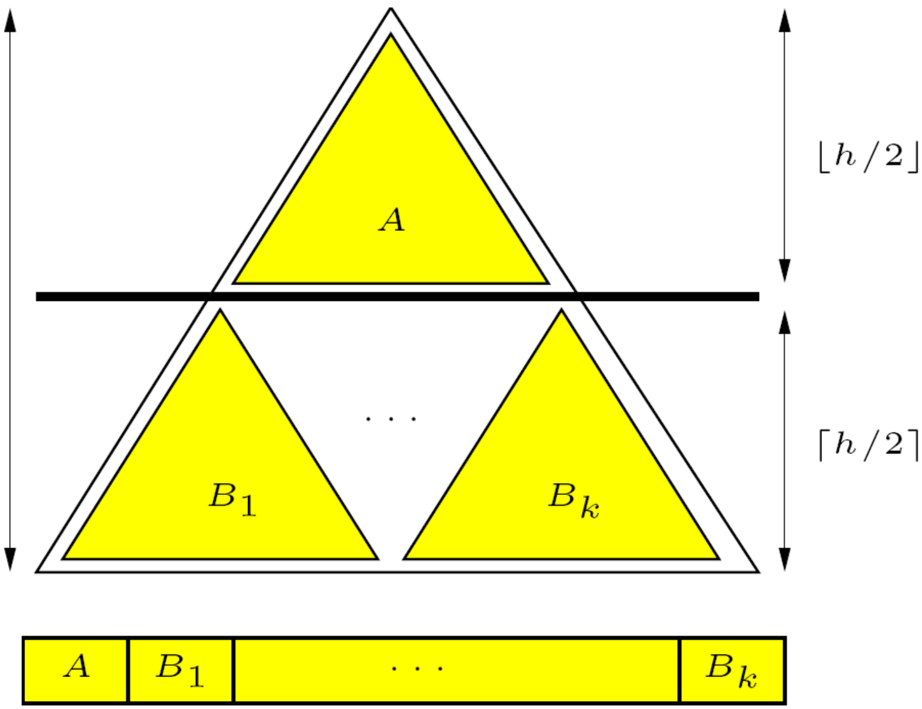
B(oing)-Trees

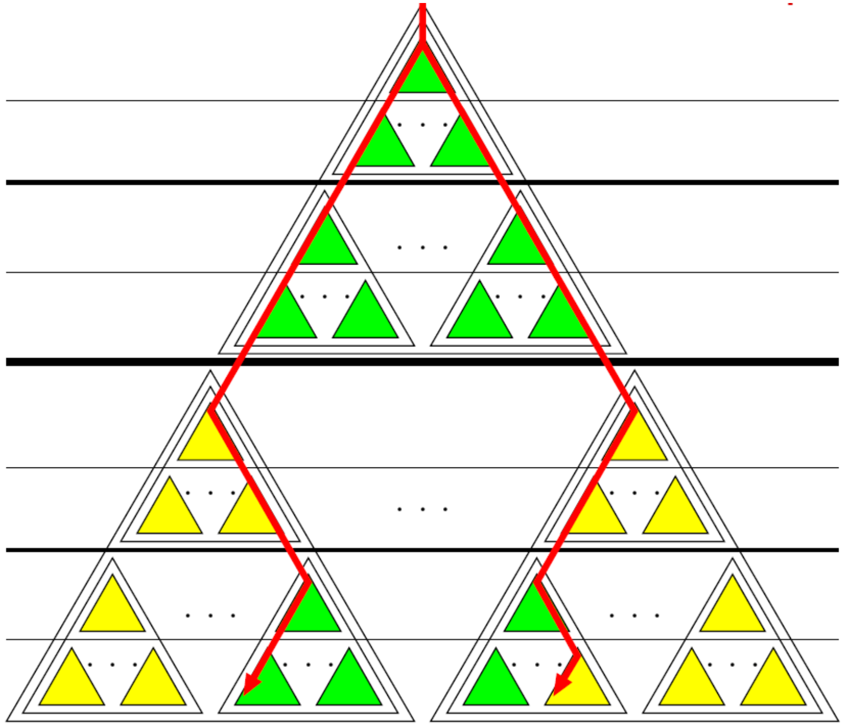


Height $O(\log_B N)$

Frigo (1999)

Recursive Search Tree Layout (Cache-Obliviousness)

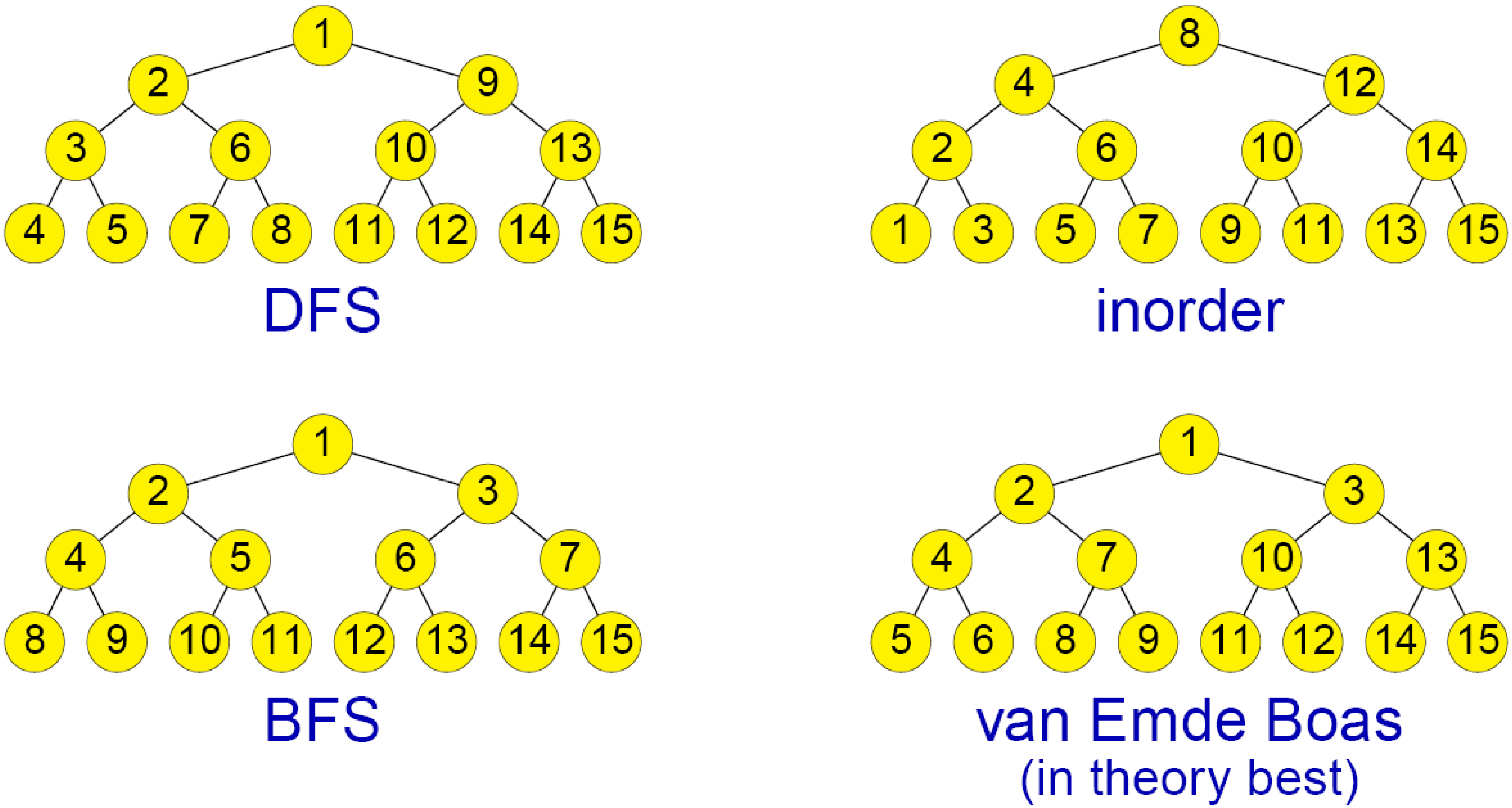




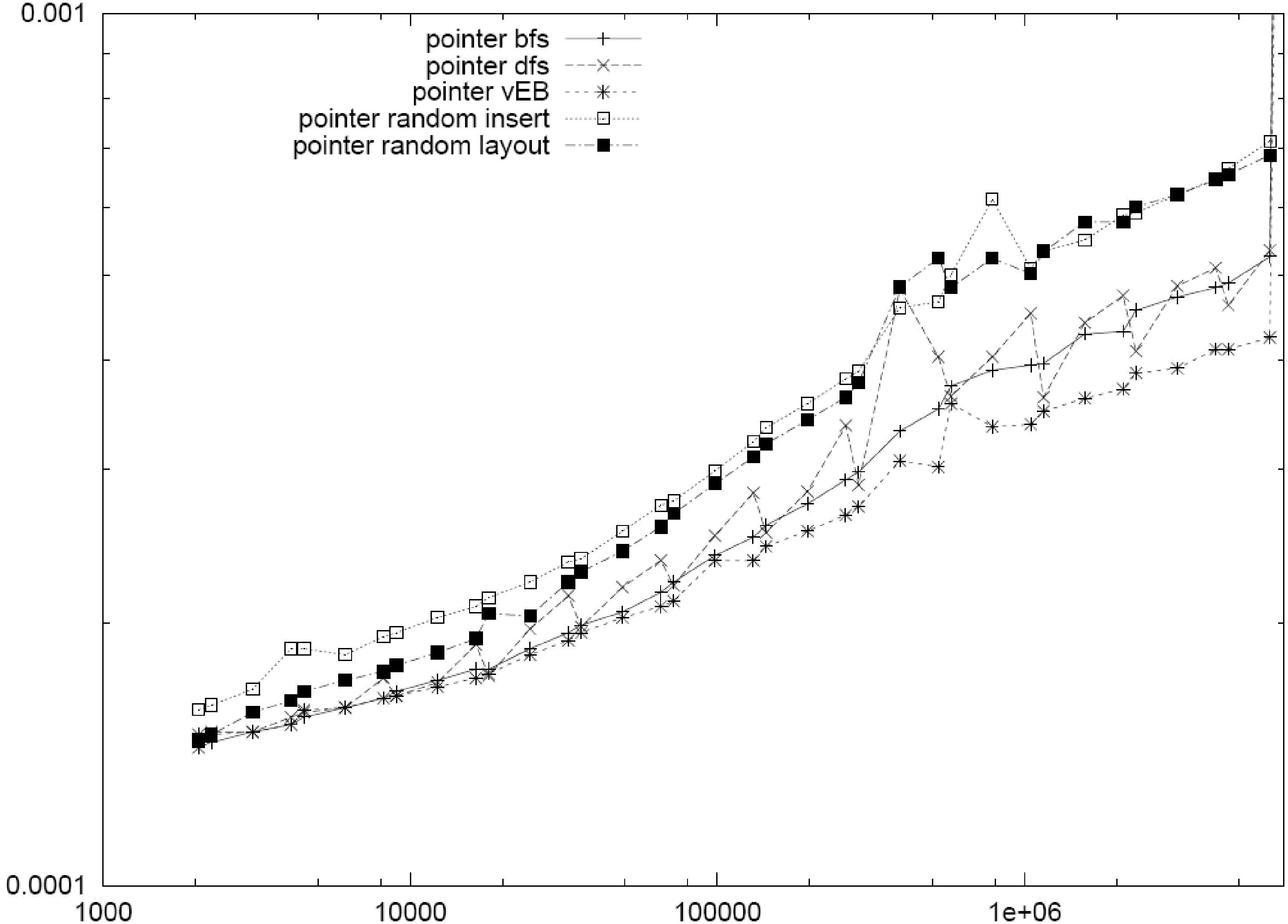
Searches require $O(\log_B N)$ I/Os

(small subtrees have $\sqrt{B} \leq$ size $\leq B$, a path traverses at most $2 \cdot \log_B N$ trees)

Experiments : Binary Tree Layout

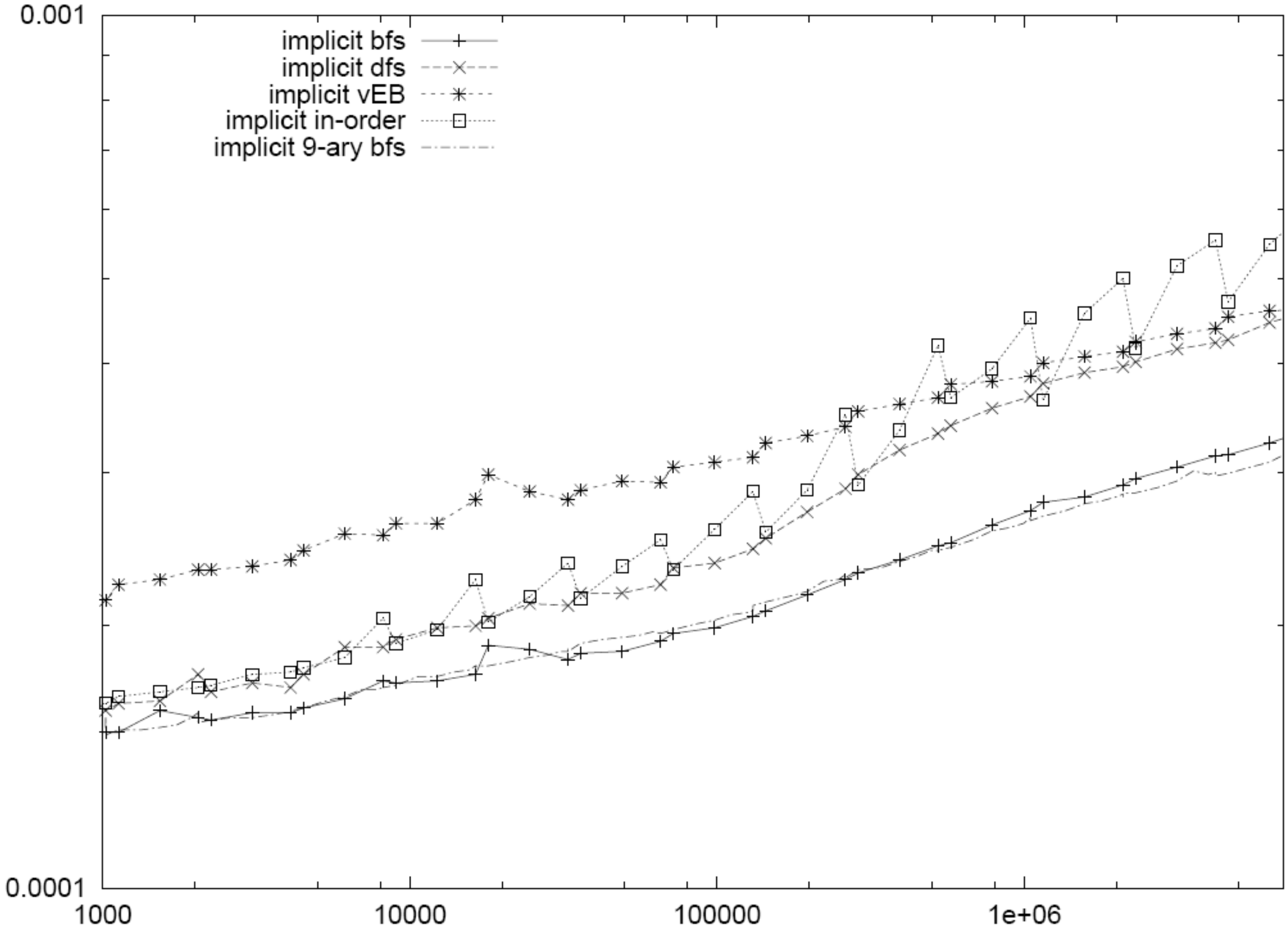


Searches with Pointer Layout



- van Emde Boas layout wins, followed by the BFS layout   van Emde Boas layout

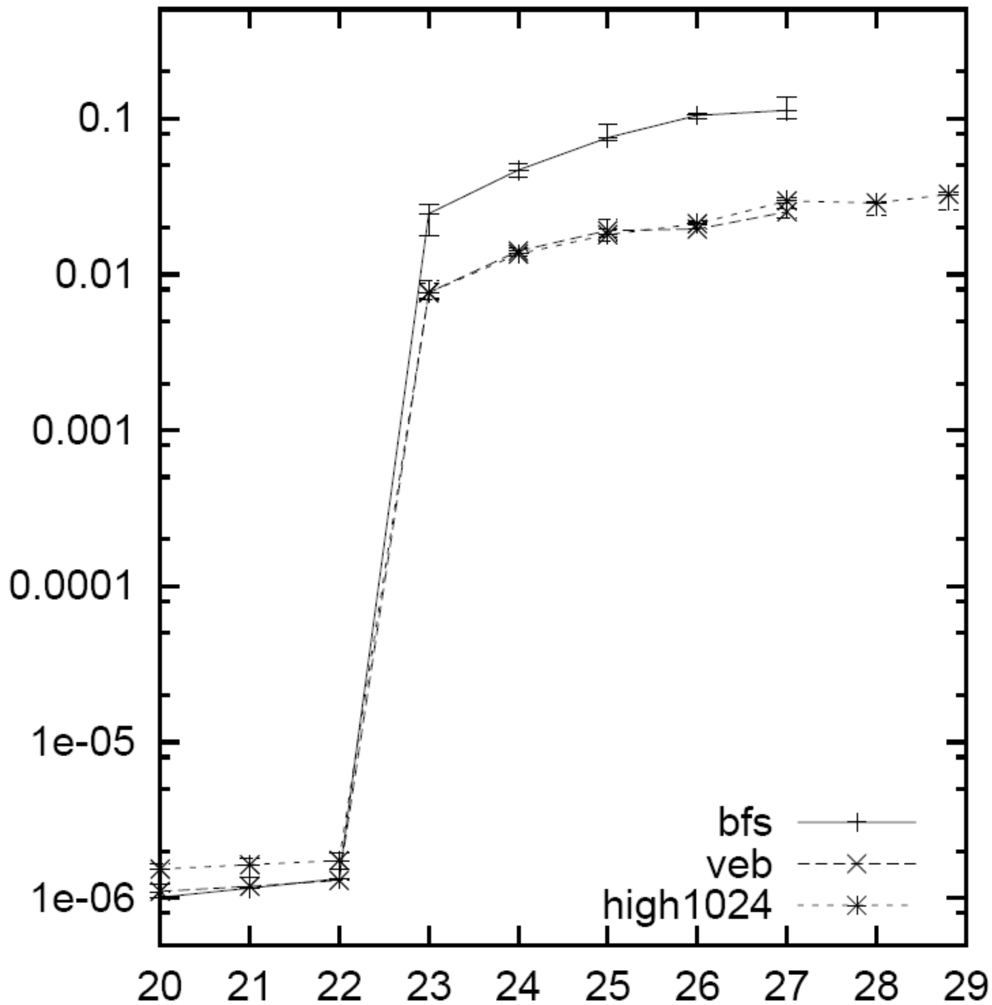
Searches with Implicit Layout



- BFS layout wins due to simplicity and caching of topmost levels
- van Emde Boas layout requires quite complex index computations

Brodal, Fagerberg, Jacob (2002)

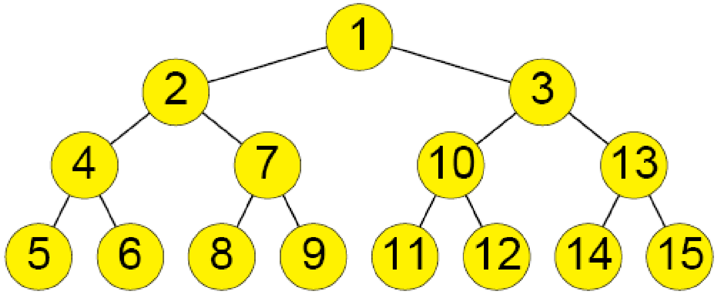
Searches with Pointer Layout - Beyond Main Memory



- van Emde Boas layout wins, followed by the BFS layout

Optimal Cache-Oblivious Static Index

= van Emde Boas Layout of Complete Binary Tree

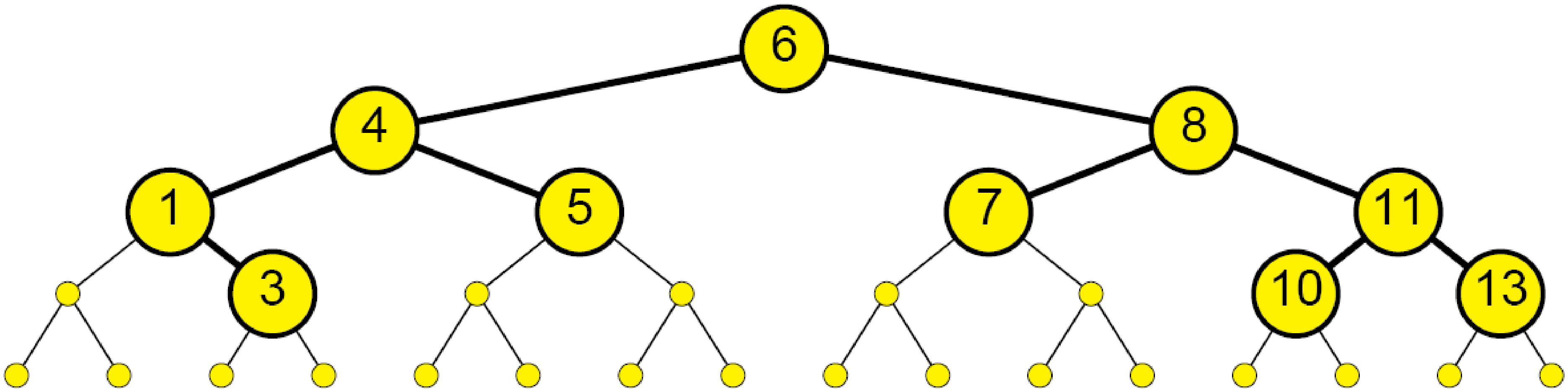


van Emde Boas

Cache-Oblivious <u>Dynamic</u> Index ?

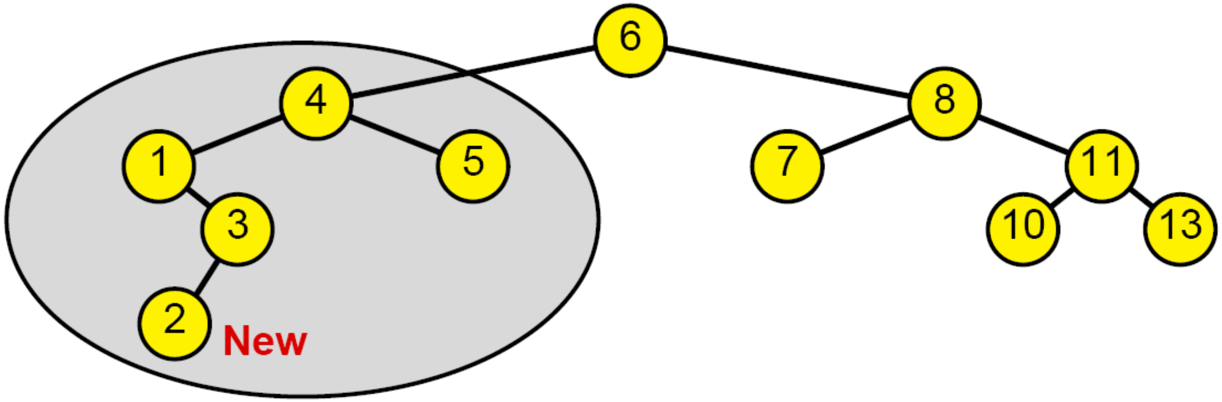
Brodal, Fagerberg, Jacob (2002)

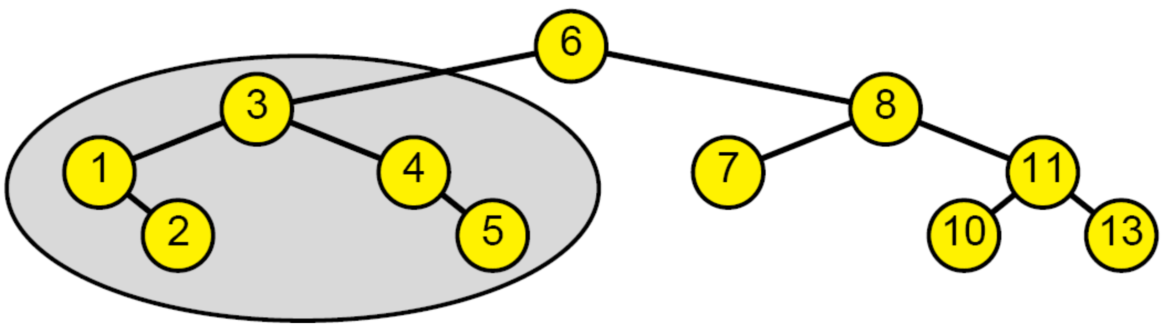
Binary Search Trees
Dynamic



- **Embed** a dynamic tree of small height into a complete tree
- Static van Emde Boas layout
- Rebuild data structure whenever *N* doubles or halves

Rebalancing Subtree during Insertion

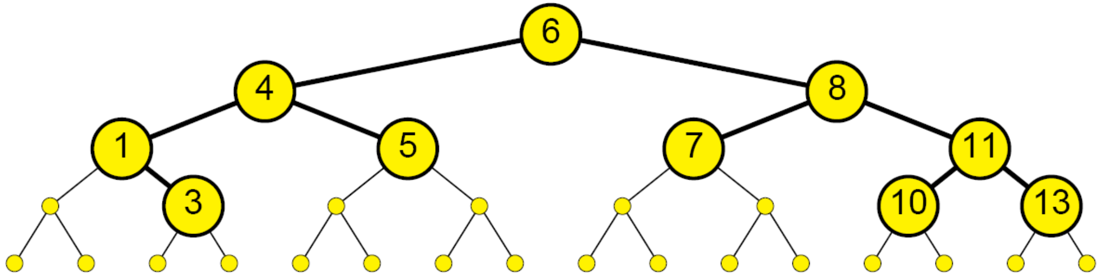




- If an insertion causes non-small height then **rebuild** subtree at nearest ancestor with sufficient few descendants
- Insertions require amortized time $O(\log^2 N)$

Optimal Cache-Oblivious Dynamic Index

- Search $O(\log_B N)$ I/Os (optimal)
- Updates $O(\log_B N + (\log^2 N) / B)$ I/Os



Cache-Oblivious Index
with Query-Update Trade-off ?

Brodal, Demaine, Fineman, Iacono, Langerman, Munro (2010)

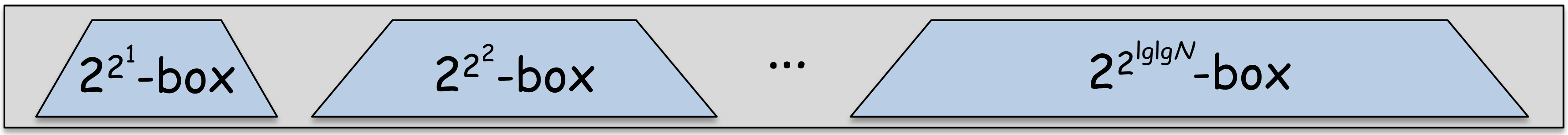
Cache-Oblivious Dynamic Dictionaries with Query-Update Trade-off

- Solution matching the I/O complexity of Buffered B-trees

- Searches $O(\log_B N \cdot 1/\varepsilon)$ I/Os

- N updates $O(N \cdot \log_B N \cdot 1/\varepsilon B^{1-\varepsilon})$ I/Os

xDict



Insert

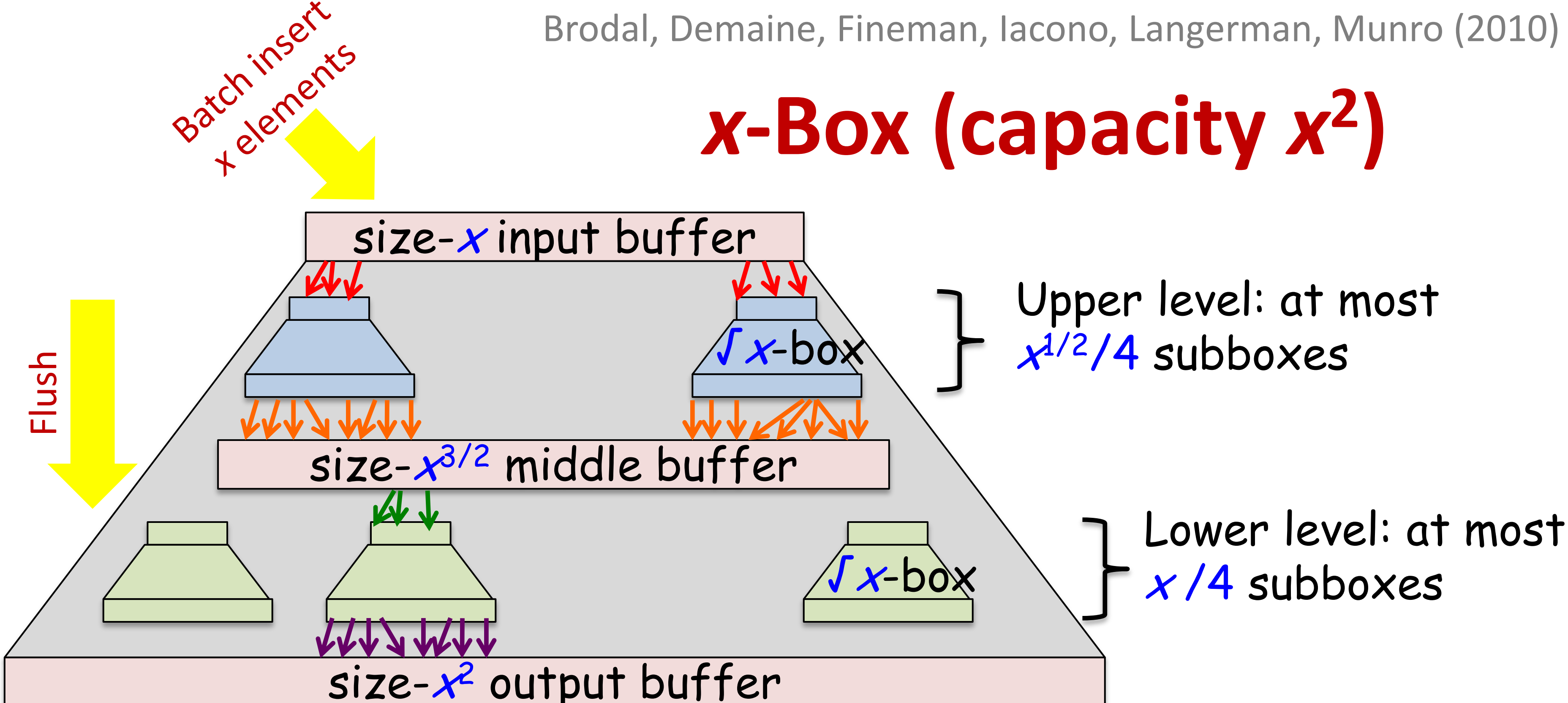
- Insert into smallest box
- When a box reaches capacity, **Flush** it and **Batch-Insert** into the next box

Search

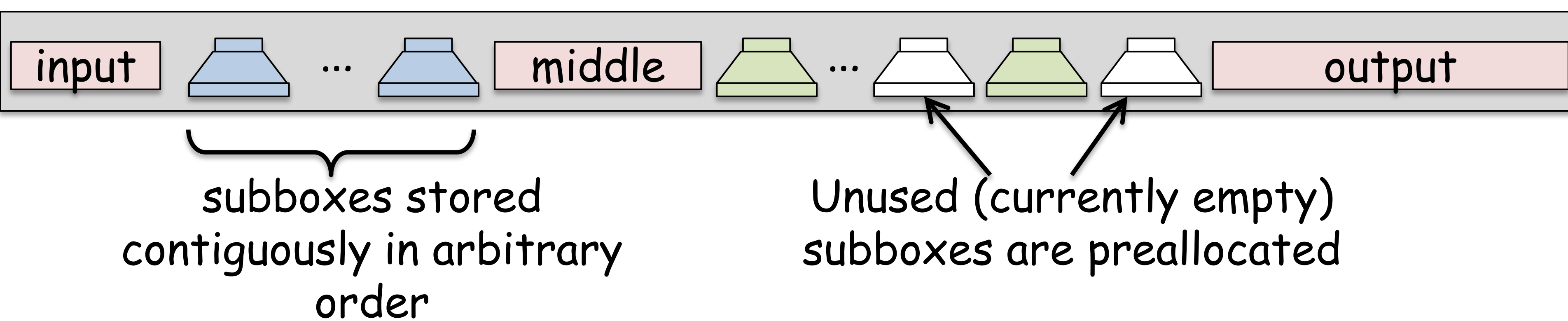
- Search in each x-box
- $O(\log_B x)$ cost is dominated by largest box $O(\log_B N)$

Brodal, Demaine, Fineman, Iacono, Langerman, Munro (2010)

x-Box (capacity x^2)



Memory Layout



External Memory Index

- Searches $O(\log_B N \cdot 1/\varepsilon)$ I/Os
- N updates $O(N \cdot \log_B N \cdot 1/\varepsilon B^{1-\varepsilon})$ I/Os