

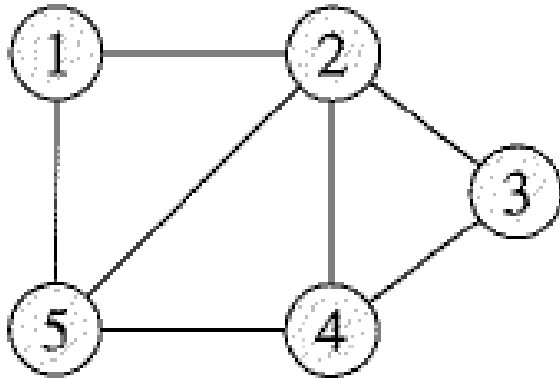
Algoritmer og Datastrukturer 2

Gerth Stølting Brodal

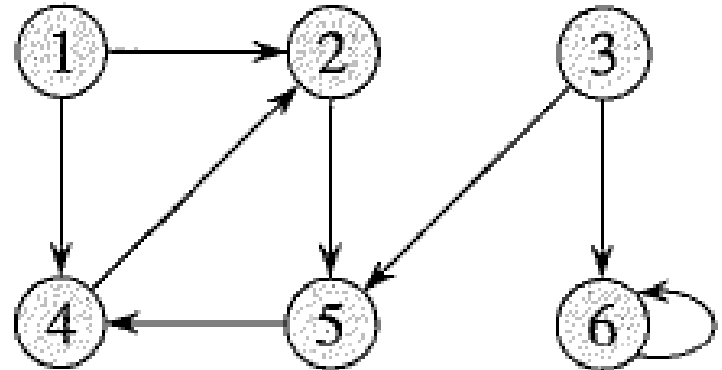
Graf repræsentationer, BFS og DFS
[CLRS, kapitel 22.1-22.3]



Grafer



Uorienterede grafer



Orienterede grafer

$G = (V, E)$ graf med knuder V og kanter E

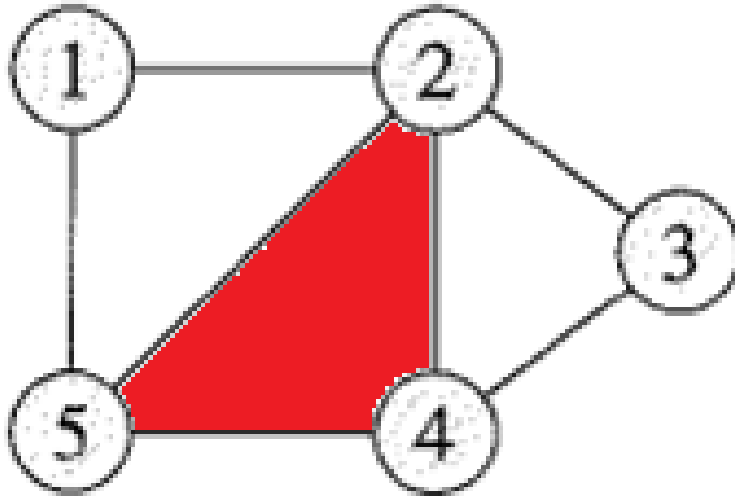
$E : \{u, v\}$ kant mellem u og v i en uorienteret graf og

(u, v) en orienteret kant fra u til v .

$n = |V| =$ antal knuder

$m = |E| =$ antal kanter (forbindelser mellem knuder)

Planar Grafer - Eulers formel



$$V = 5$$

$$E = 7$$

$$\# \text{ flader} = 4$$

For en sammenhængende planar graf gælder:

Eulers formel:

$$|V| - |E| + \# \text{ flader} = 2$$

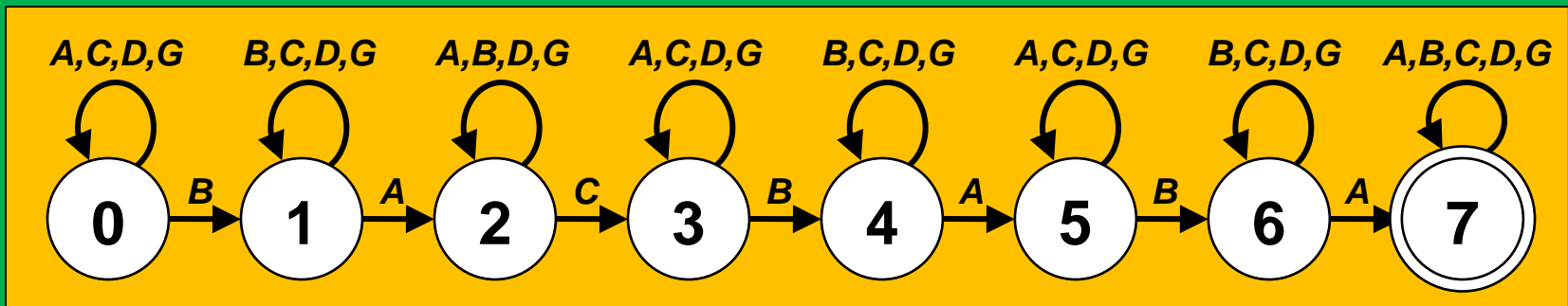
Korollar:

$$|E| \leq 3|V| - 6$$

(for $|V| \geq 3$, ingen
selvløkker, ingen
parallelle kanter)

Hvilken løsning finder den grådige algoritme?

- a) ABABGACBABAD
- 😊 b) ABABGACBABAD
- c) ABABGACBABAD
- d) Ved ikke



Microsoft Excel - Copy of SheepFlock

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

H18 fx =B18*G18

	A	B	C	D	E	F	G	H	I
3	I. Description of animals in flock during the year.								
4	Ewes in flock:	700			[Green cells are those you can change.]				
5	Lambing rate:	4	times per	3	years =	1.33	times/year.		
6	Lambs weaned/lambing:	1.5	Days of lactation/lambing:		60				
7	Adult death loss per year:	3%	Days in lactation/year:		80				
8	Postweaning lamb loss:	2%	Lambs weaned per ewe per year:		2.0				
9	Ewe culling rate:	15%	Ram culling rate:		50%				
10	Rams/100 ewes:	1	(Only 1/3 of ewes bred per season under STAR system.)						Inventory
11			Weaning	Market	Final	Price	Value	or sale	
12		Number	wt, lb	wt, lb	wt, lb	\$/lb	per head	value	
13	Ewes	700			150	\$1.00	\$150	\$105,000	
14	Rams	8			200	\$2.00	\$400	\$3,200	
15	Ewe lamb rplcmnts	126	30		100	\$1.25	\$125	\$15,750	
16	Ram lamb rplcmnts	5	40		130	\$2.00	\$260	\$1,300	
17	Ewe lambs sold	560	30	70		\$1.10	\$77	\$43,120	
18	Ram lambs sold	681	40	70		\$1.10	\$77	\$52,437	
19	Cull ewes sold	105		150		\$0.30	\$45	\$4,725	
20	Cull rams sold	5		200		\$0.30	\$60	\$300	
21	Fleece weight per adult	708			6	\$0.30	\$1.80	\$1,274	
22							Inventory:	\$125,250	
23							Sales:	\$101,856	

Sheep flock /

Ready

Hvilken beregningsrækkefølge ?

	A	B	C
1	10	20	= A1+B1
2	50	30	= A2+B2
3	= (A1+A2)/C3	= (B1+B2)/C3	= C1+C2

14% a) C1 C2 A3 B3 C3

10% b) A3 B3 C2 C1 C3

75% c) C2 C1 C3 B3 A3



1% d) Ved ikke

Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

B3 fx =A1+1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	3	2	1																							
2																										
3		0																								
4																										
5																										
6																										
7																										
8																										
9																										
10																										
11																										
12																										
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16																										
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18																										
19																										
20																										
21																										
22																										
23																										

Circular Reference

\$B\$3

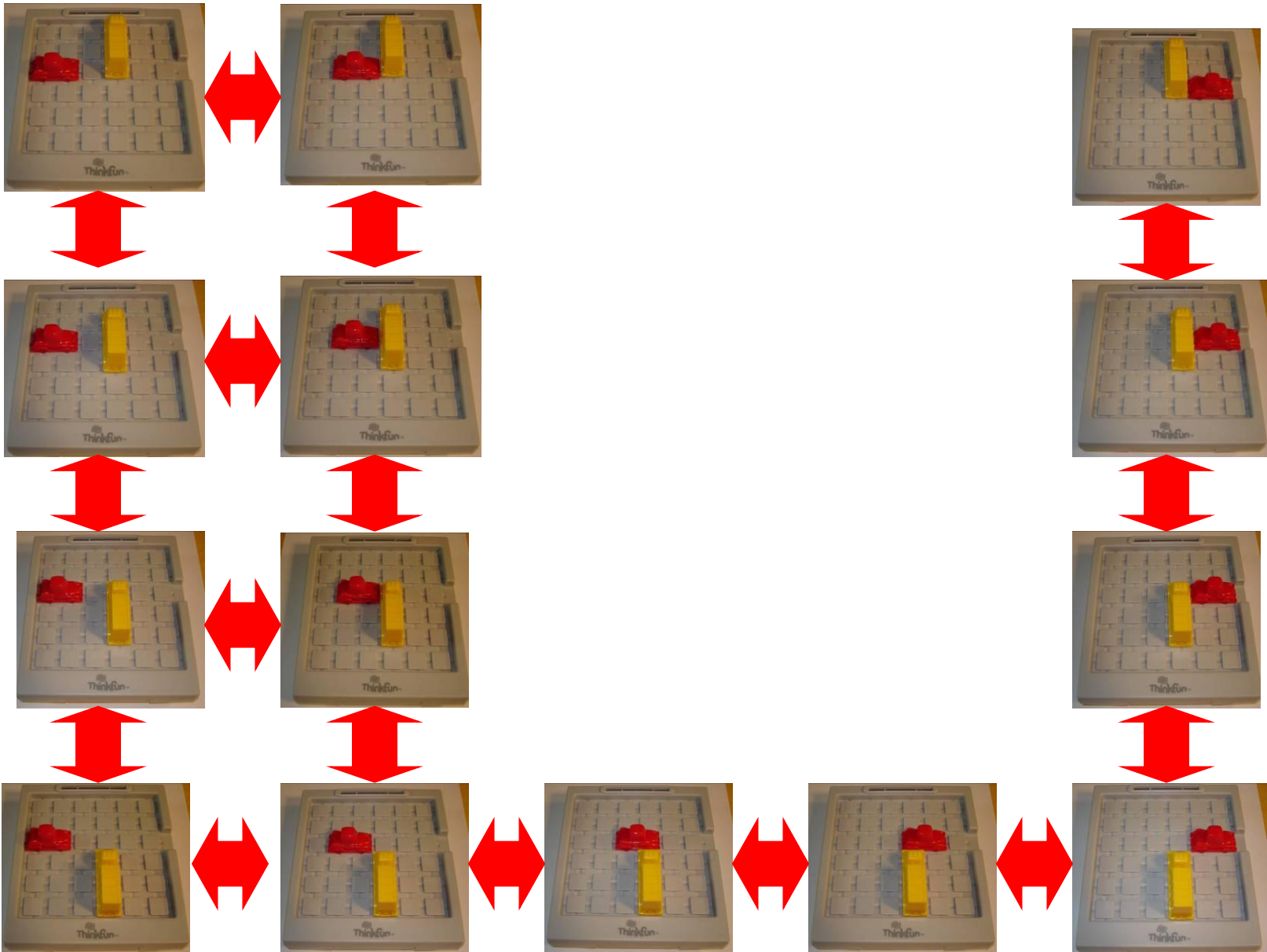
Microsoft Excel Help

Allow or correct a circular reference

When a formula refers back to its own cell, either directly or indirectly, it is called a circular reference. Microsoft Excel cannot automatically calculate all open workbooks when one of them contains a circular reference. You can remove a circular reference, or you can have Excel calculate each cell involved in the circular reference once by using the results of the previous iteration. Unless you change the default settings for iteration, Excel stops calculating after 100 iterations or after all values in the circular reference change by less than 0.001 between iterations, whichever comes first.

- ▶ [Locate and remove a circular reference](#)
- ▶ [Make a circular reference work by changing the number of times Microsoft Excel iterates formulas](#)

Ready Circular: C1





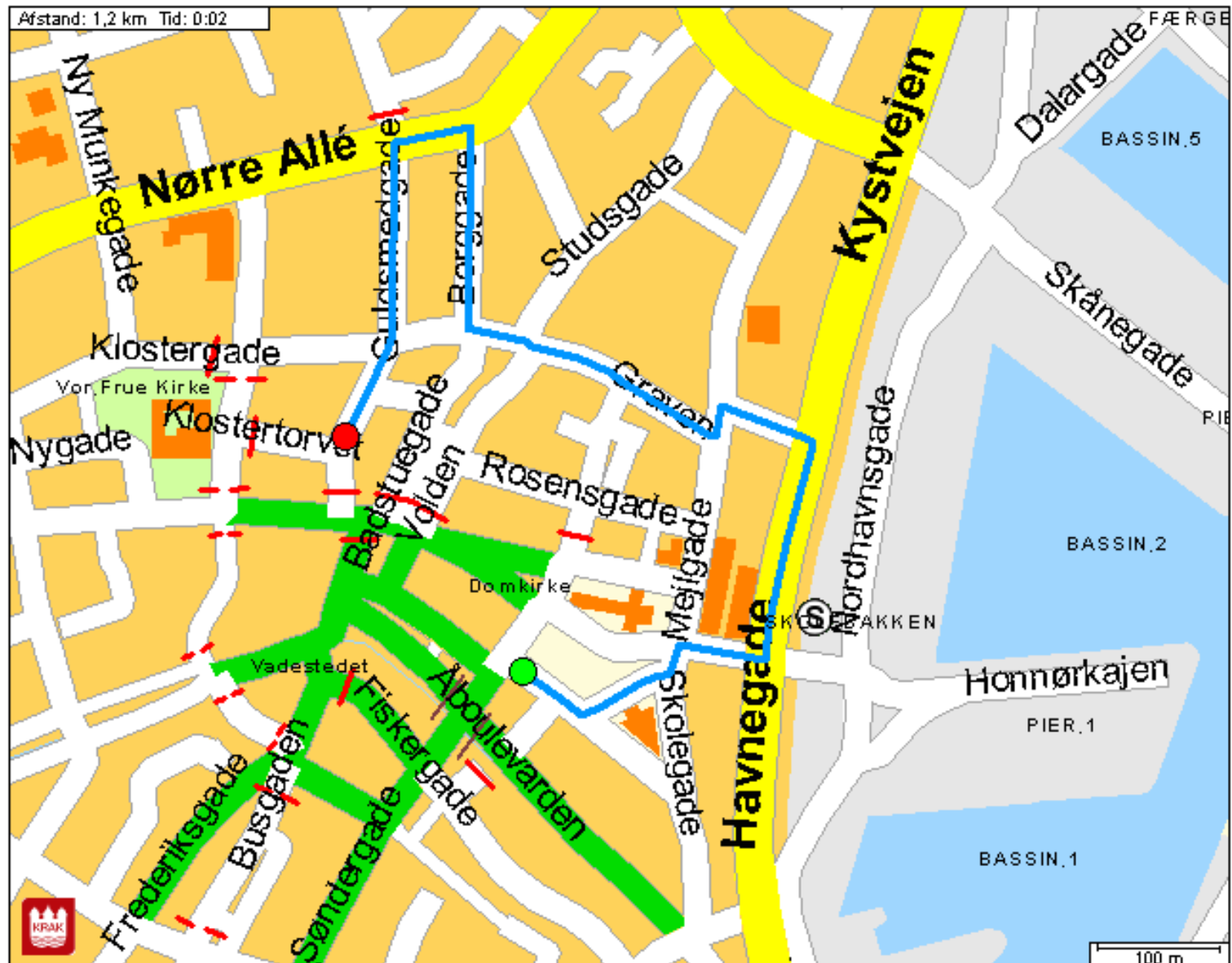
Rute på kort

Fra Kannikegade 1 , 8000 Århus C

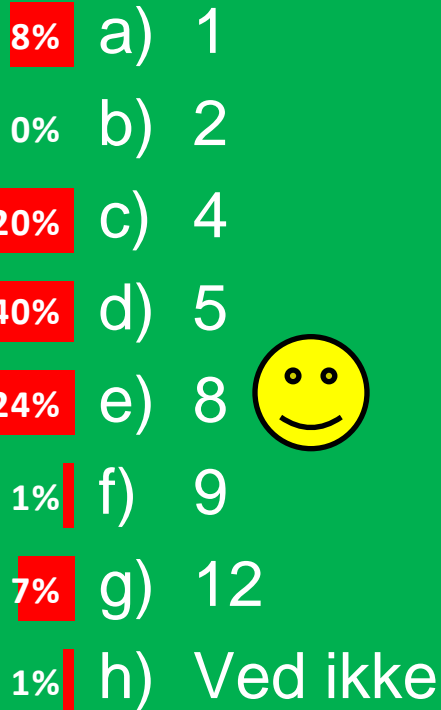
Til Guldsmedgade 1 , 8000 Århus C

Via

Årstand: 1,2 km Tid: 0:02



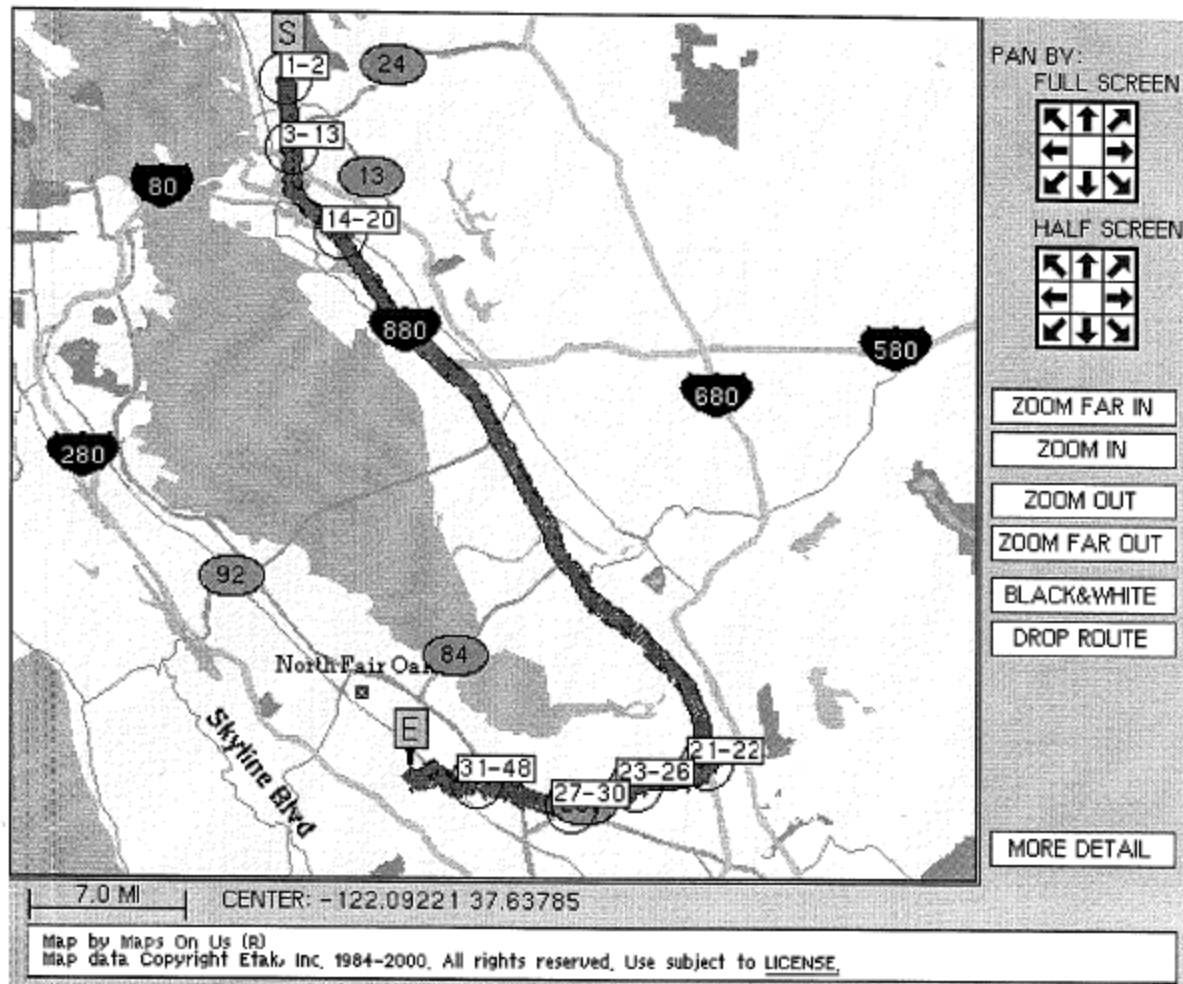
Hvor mange knuder skal man bruge for at repræsentere et vejkryds?



Kort over Vest-Europa

- 18.029.721 knuder
- 42.199.587 orienterede kanter





“However, because of the size of the routing data, we have to use heuristics when planning routes. As a result, sometimes a Favor Highways route will be slightly faster than the Fastest route.”

— MapsOnUs

Dine valg

Fra: Skagen st
Til: Rødby Færge Vælg anden Fra/Til
Udrejse: 27.04.07
Kl.: 10:00 (Afgang)

Oversigt

Tidligere forbindelser

	Station/Stop	Dato	Kl.	Varighed	Skift	Transportmidler
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 08:56 Ank. 17:35	8:39	2	Lyn EC
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 09:56 Ank. 17:35	7:39	3	Lyn IC Re
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 09:56 Ank. 18:30	8:34	2	Lyn Re
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 11:54 Ank. 19:35	7:41	3	Lyn IC EC
<input checked="" type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 13:54 Ank. 22:31	8:37	3	Lyn Re

Senere forbindelser

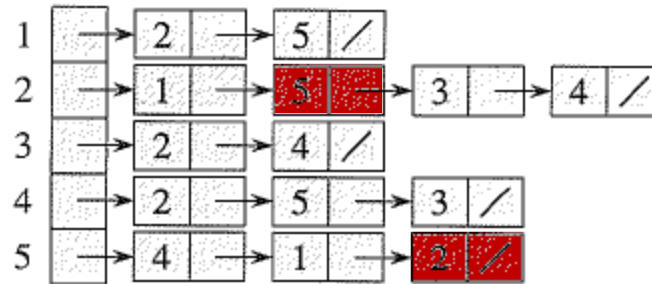
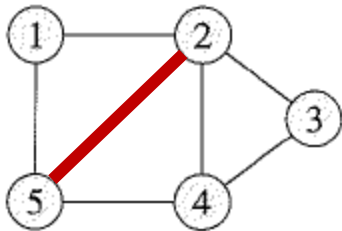
▶ Vis valgte ▶ Vis alle

Din rejseplan

Station/Stop	Dato	Kl.	Spor	Transportmidler	Bemærkninger
Skagen st Frederikshavn st	27.04.07 27.04.07	Afg. 13:54 Ank. 14:31		▶ PP 79	Privatbane Retning: Frederikshavn st
Frederikshavn st Frederikshavn Busterminal	27.04.07 27.04.07			▶ Til fods Se kort	0 min.
Frederikshavn Busterminal Aalborg Busterminal	27.04.07 27.04.07	Afg. 14:35 Ank. 15:48		▶ X-B 973X	X-BUS Retning: Aalborg Busterminal
Aalborg Busterminal Aalborg st	27.04.07 27.04.07			▶ Til fods Se kort	5 min.
Aalborg st Høje Taastrup st	27.04.07 27.04.07	Afg. 15:59 Ank. 20:14	3 2	▶ ICL 54	IC Lyntog Retning: København H Spornummeret er kun vejledende.
Høje Taastrup st Rødby Færge	27.04.07 27.04.07	Afg. 20:23 Ank. 22:31		▶ RE 82273	Regionaltog Retning: Rødby Færge

Varighed: 8:37; kører 27. apr, 11. maj
 Bemærkning: En station/stop er passeret flere gange, hvilket kan have betydning for prisudregningen af billetten.

Graf repræsentationer: Incidenslister og incidensmatricer

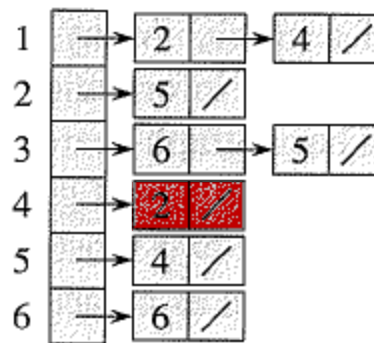
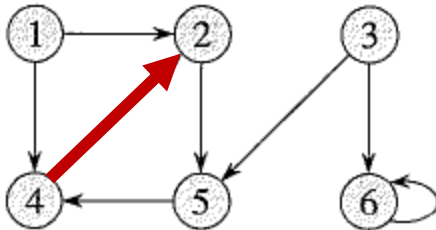


	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Uorienterede grafer

Plads $O(n+m)$

Plads $O(n^2)$



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

Orienterede grafer

Bredde først søgning (BFS)

BFS(G, s)

```
1 for each vertex  $u \in G.V - \{s\}$ 
2    $u.color = WHITE$ 
3    $u.d = \infty$ 
4    $u.\pi = NIL$ 
5  $s.color = GRAY$ 
6  $s.d = 0$ 
7  $s.\pi = NIL$ 
8  $Q = \emptyset$ 
9 ENQUEUE( $Q, s$ )
10 while  $Q \neq \emptyset$ 
11    $u = DEQUEUE(Q)$ 
12   for each  $v \in G.Adj[u]$ 
13     if  $v.color == WHITE$ 
14        $v.color = GRAY$ 
15        $v.d = u.d + 1$ 
16        $v.\pi = u$ 
17       ENQUEUE( $Q, v$ )
18    $u.color = BLACK$ 
```

$u.color$:

WHITE = knuderne endnu ikke besøgt

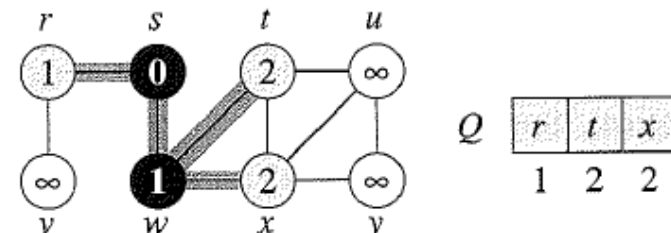
GRAY = knuderne i køen Q

BLACK = knuderne besøgt

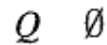
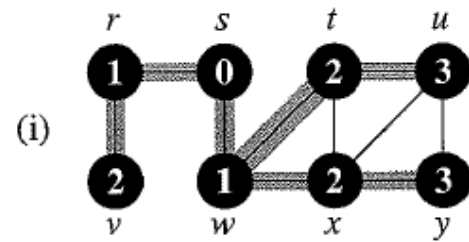
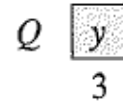
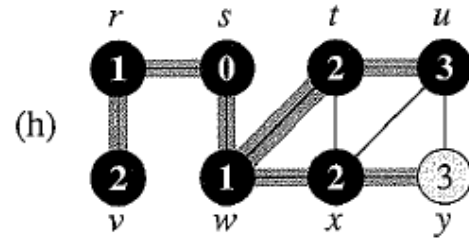
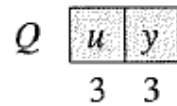
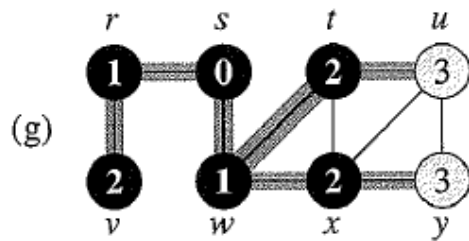
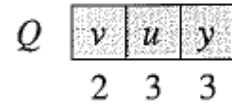
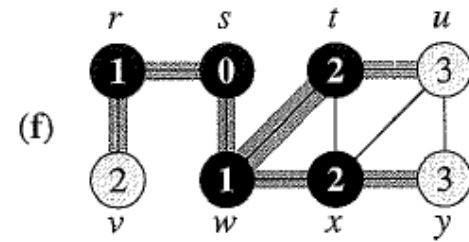
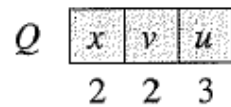
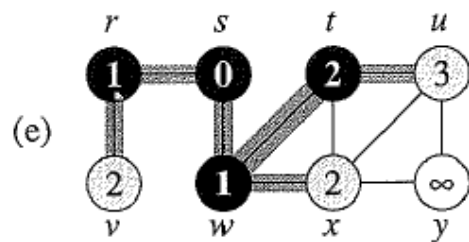
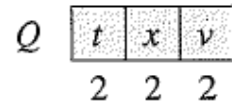
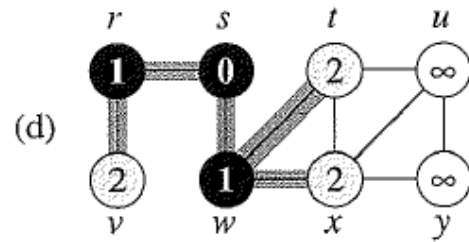
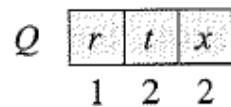
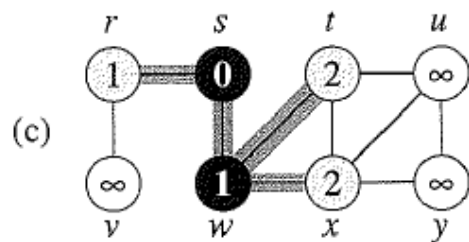
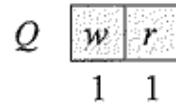
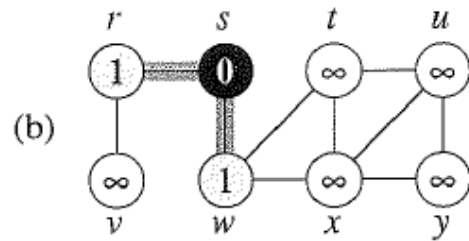
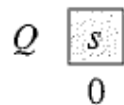
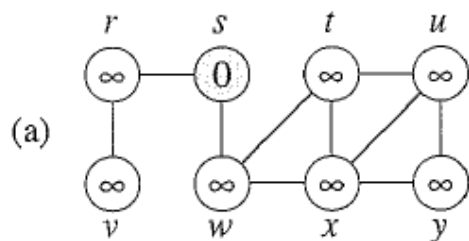
$u.d =$ afstand til s

$u.\pi =$ faderen til u i BFS træet

$Q =$ kø af grå knuder (som er forbundet til sorte knuder)

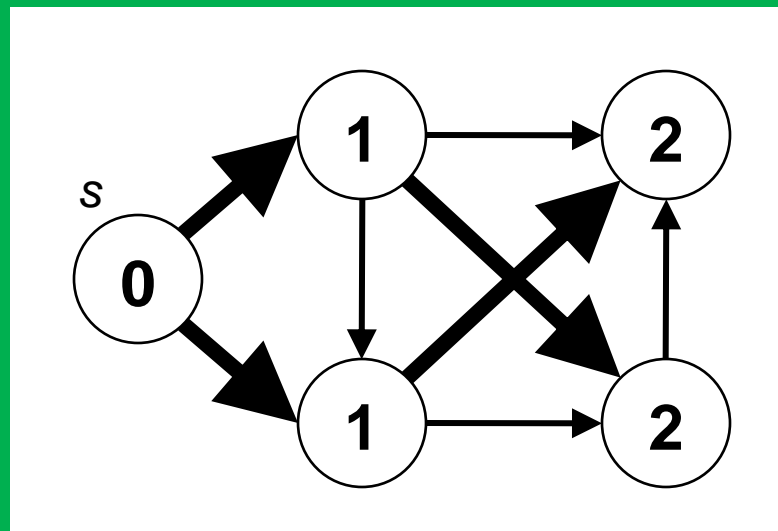


Tid $O(n+m)$



Er nedenstående et BFS træ?

- 35% a) Ja
- 61% b) Nej 😊
- 4% c) Ved ikke



BFS : Udskrivning af sti fra s til v

PRINT-PATH(G, s, v)

1 **if** $v == s$

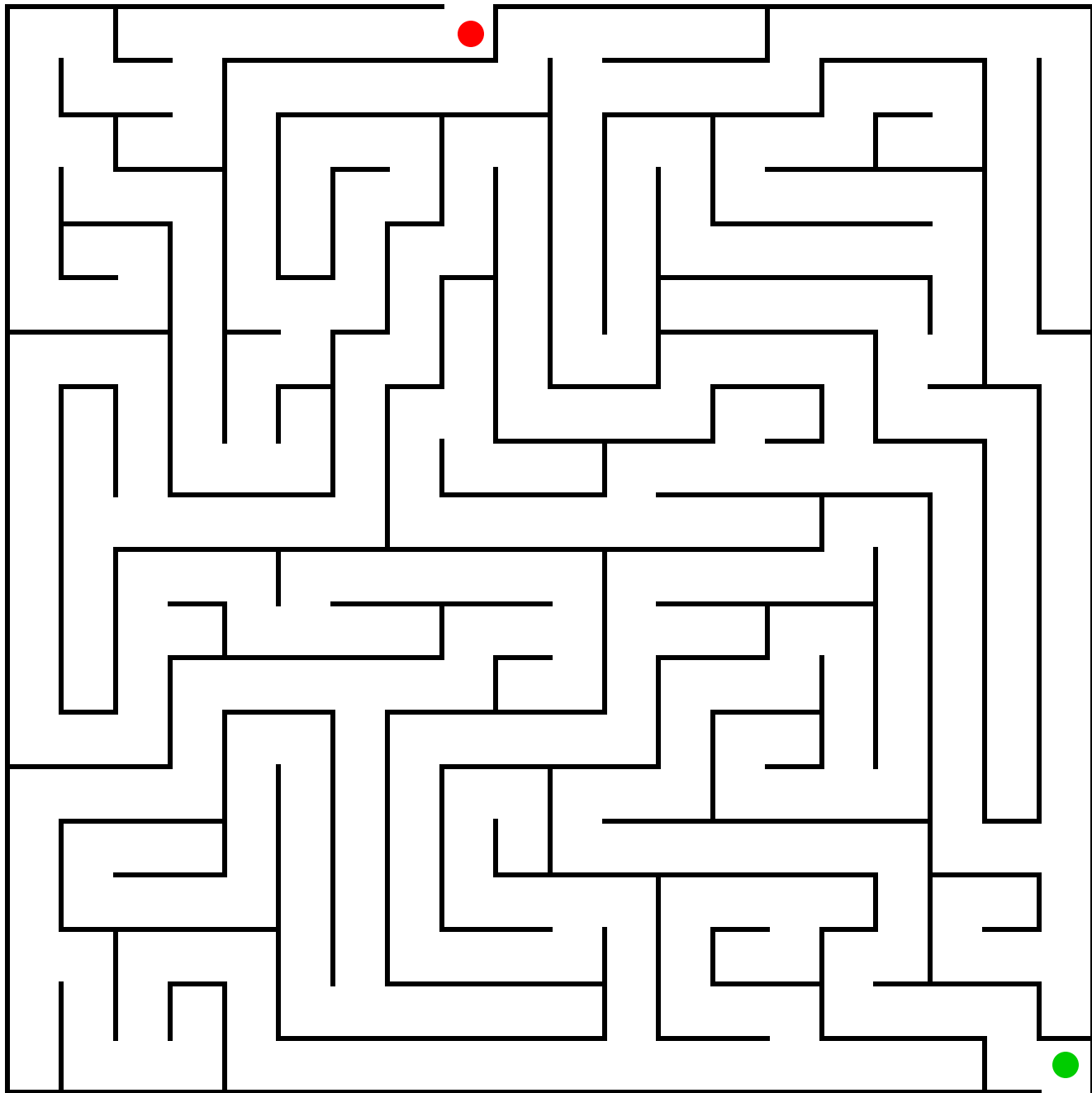
2 print s

3 **elseif** $v.\pi == \text{NIL}$

4 print “no path from” s “to” v “exists”

5 **else** PRINT-PATH($G, s, v.\pi$)

6 print v



Dybde Først Søgning (DFS)

DFS(G)

```
1 for each vertex  $u \in G.V$ 
2    $u.color = WHITE$ 
3    $u.\pi = NIL$ 
4    $time = 0$ 
5 for each vertex  $u \in G.V$ 
6   if  $u.color == WHITE$ 
7     DFS-VISIT( $G, u$ )
```

$u.color$

WHITE = knuderne endnu ikke besøgt
GRAY = knuder på rekursionsstakken
BLACK = knuderne besøgt

$u.\pi$ = faderen til u i DFS træet

$u.d$ = "discover time" for u
 $u.f$ = "finishing time" for u

DFS-VISIT(G, u)

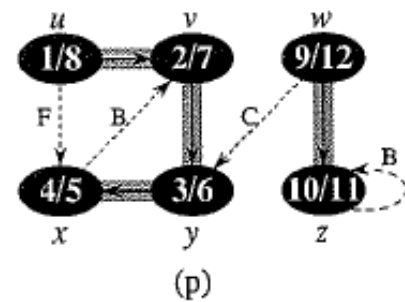
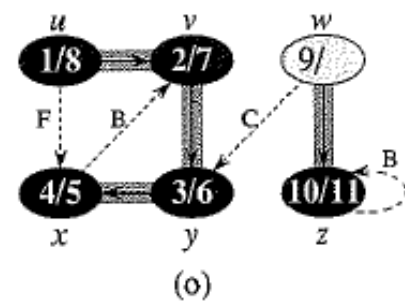
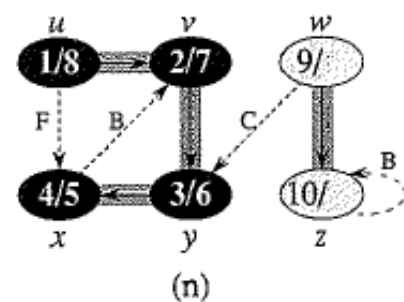
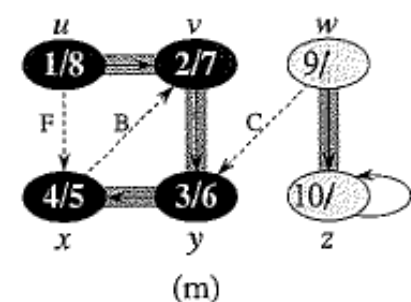
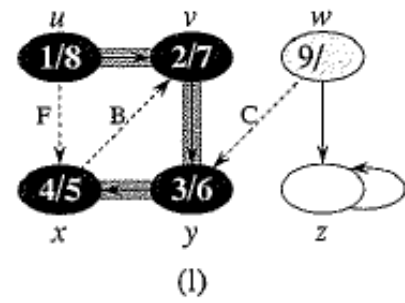
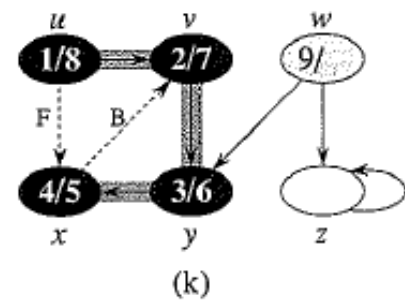
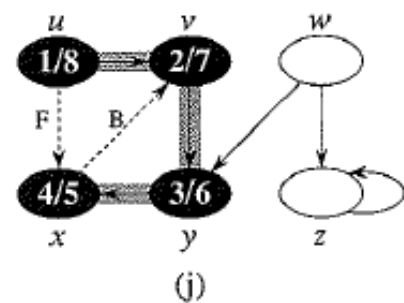
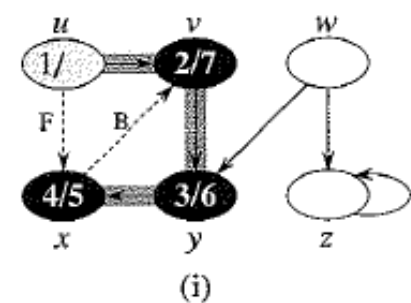
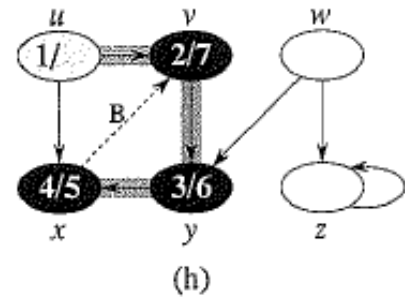
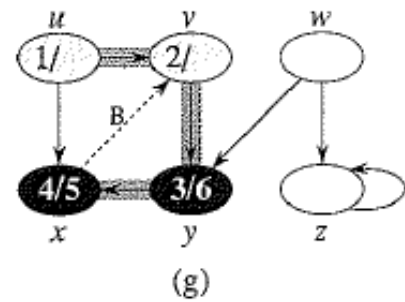
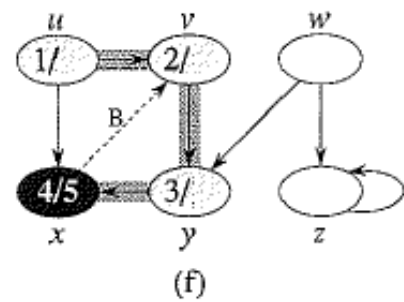
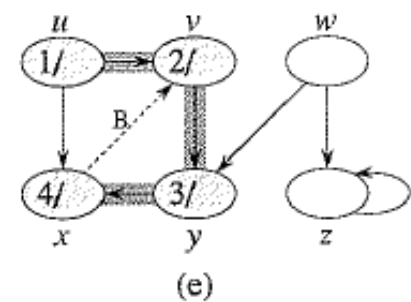
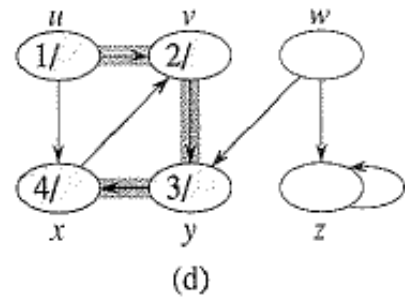
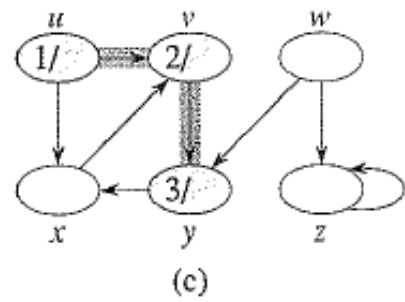
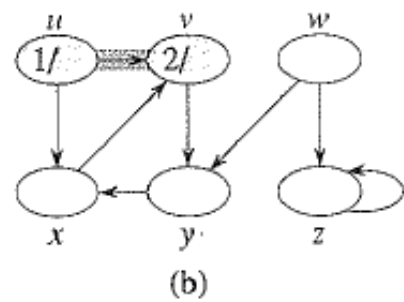
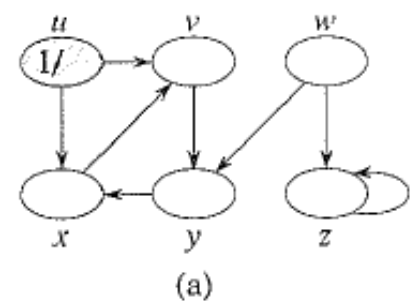
```
1  $time = time + 1$ 
2  $u.d = time$ 
3  $u.color = GRAY$ 
4 for each  $v \in G.Adj[u]$ 
5   if  $v.color == WHITE$ 
6      $v.\pi = u$ 
7     DFS-VISIT( $G, v$ )
8  $u.color = BLACK$ 
9  $time = time + 1$ 
10  $u.f = time$ 
```

// white vertex u has just been discovered

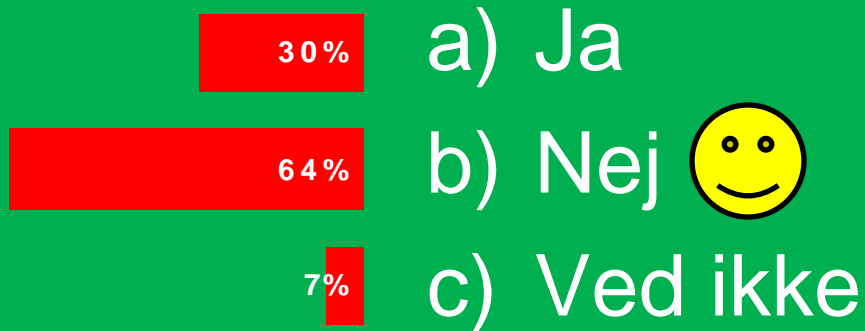
// explore edge (u, v)

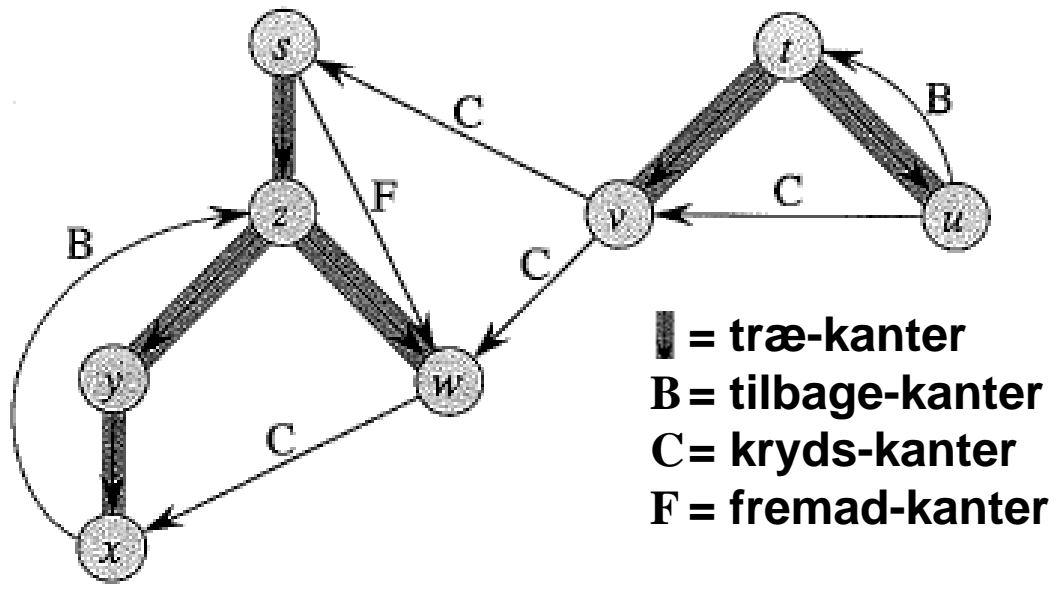
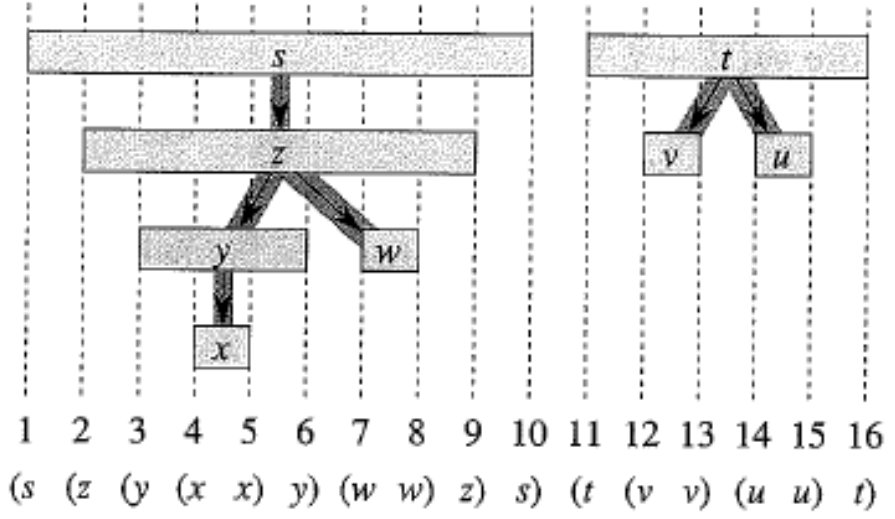
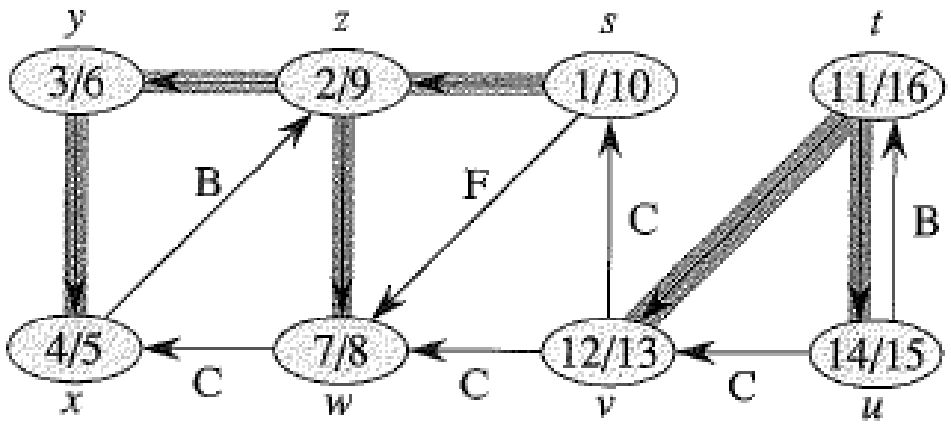
// blacken u ; it is finished

Tid $O(n+m)$



Kan en knude have 13/17 som DFS discover/finishing tider ?





█ = træ-kanter
B = tilbage-kanter
C = kryds-kanter
F = fremad-kanter

BFS og DFS anvendelser

BFS Finde afstande til startknuden
(afstand = antal kanter, f.eks. RushHour)

DFS *Topologisk sortering*
Stærke sammenhængskomponenter
(næste forelæsning)