

Algoritmer og Datastrukturer 2

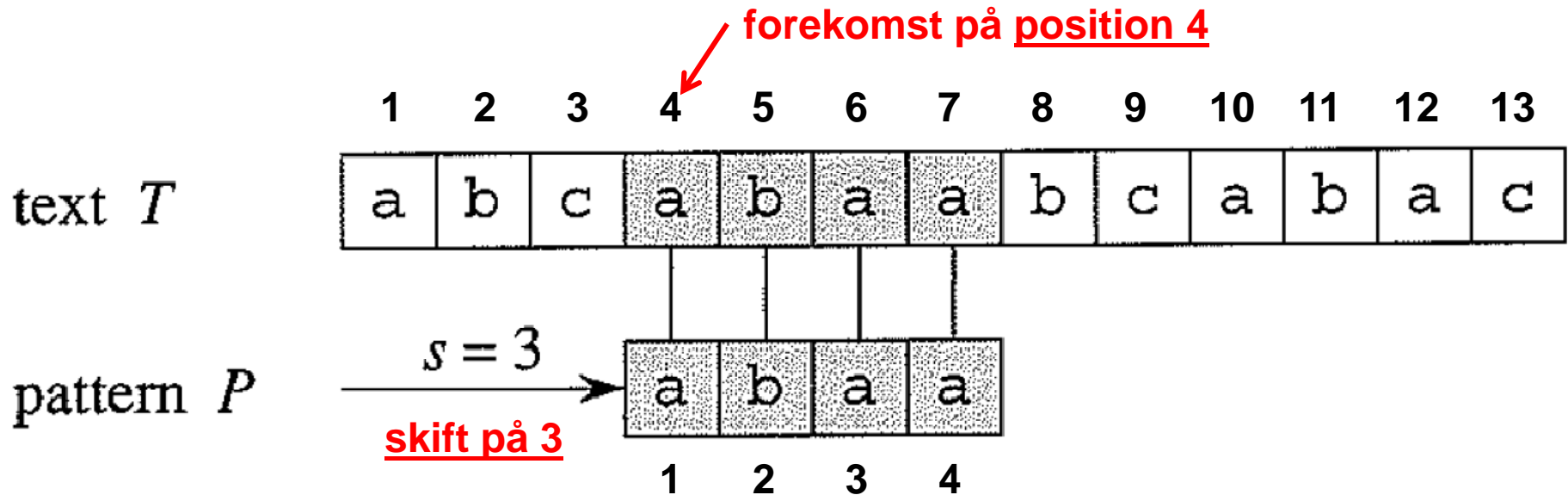
Mønstergenkendelse [CLRS, kapitel 32.1-32.2, 32.4]



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Input: Tekst T af længde n og mønster P af længde m

Output: Alle positioner i T hvor P forekommer

Naive Algorithm

NAIVE-STRING-MATCHER(T, P)

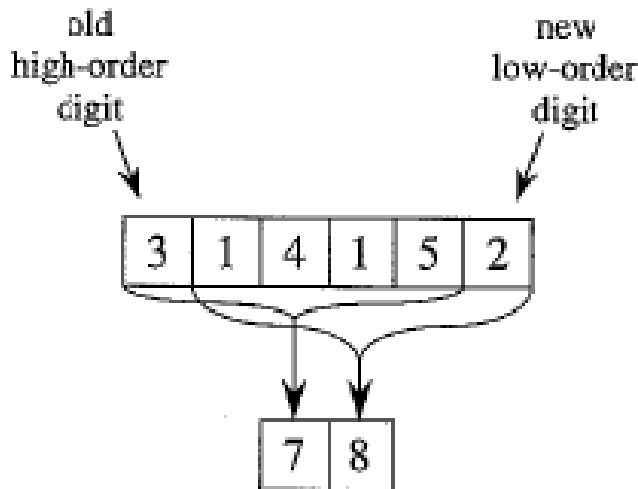
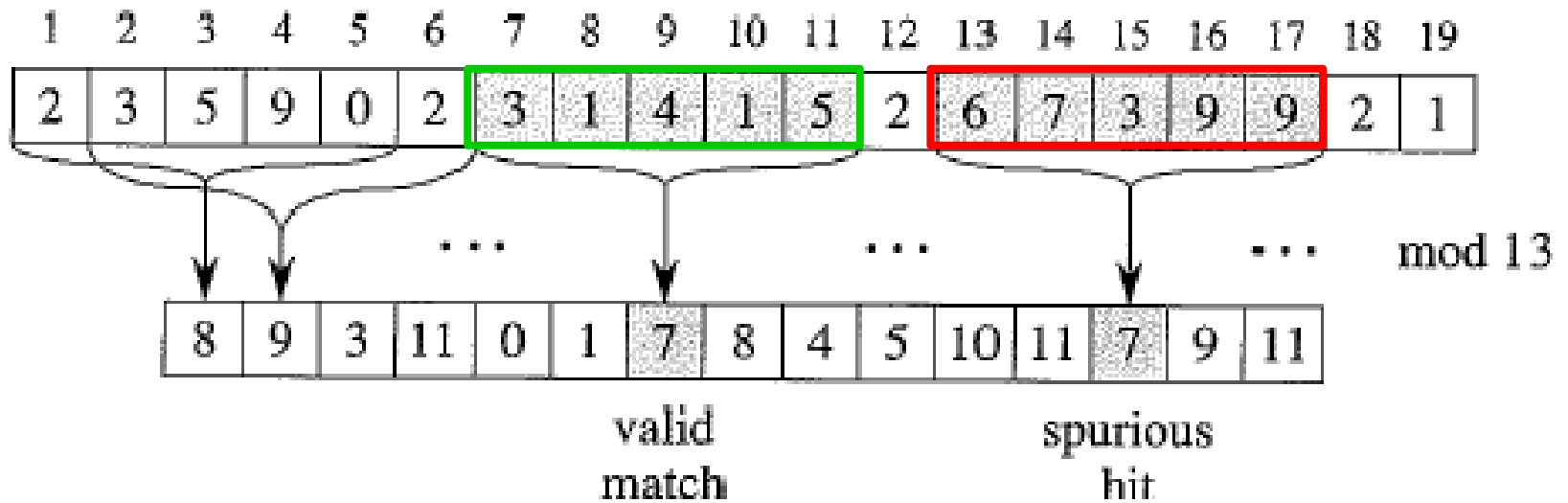
```
1   $n \leftarrow \text{length}[T]$ 
2   $m \leftarrow \text{length}[P]$ 
3  for  $s \leftarrow 0$  to  $n - m$ 
4      do if  $P[1..m] = T[s + 1..s + m]$ 
5          then print "Pattern occurs with shift"
```

Rabin-Karp

RABIN-KARP-MATCHER(T, P, d, q)

```
1   $n \leftarrow \text{length}[T]$ 
2   $m \leftarrow \text{length}[P]$ 
3   $h \leftarrow d^{m-1} \bmod q$ 
4   $p \leftarrow 0$ 
5   $t_0 \leftarrow 0$ 
6  for  $i \leftarrow 1$  to  $m$                                 ▷ Preprocessing.
7      do  $p \leftarrow (dp + P[i]) \bmod q$ 
8          $t_0 \leftarrow (dt_0 + T[i]) \bmod q$ 
9  for  $s \leftarrow 0$  to  $n - m$                             ▷ Matching.
10     do if  $p = t_s$ 
11         then if  $P[1..m] = T[s + 1..s + m]$ 
12             then print "Pattern occurs with shift"  $s$ 
13     if  $s < n - m$ 
14         then  $t_{s+1} \leftarrow (d(t_s - T[s + 1]h) + T[s + m + 1]) \bmod q$ 
```

Rabin-Karp: Eksempel



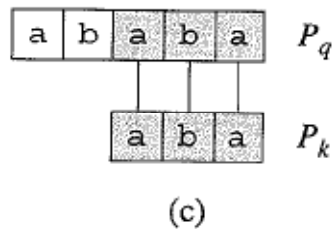
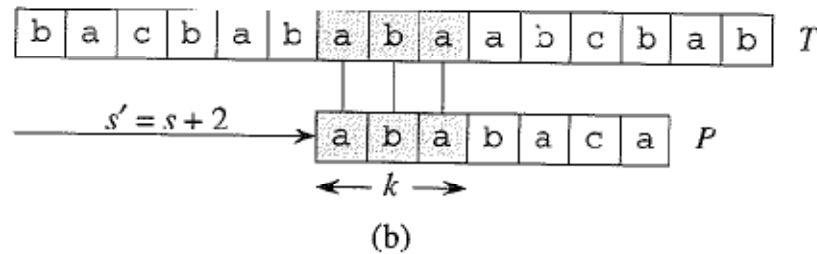
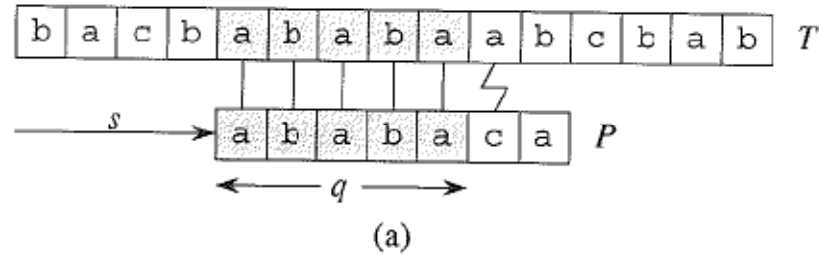
$$\begin{aligned}
 14152 &\equiv (31415 - 3 \cdot 10000) \cdot 10 + 2 \pmod{13} \\
 &\equiv (7 - 3 \cdot 3) \cdot 10 + 2 \pmod{13} \\
 &\equiv 8 \pmod{13}
 \end{aligned}$$

Knuth-Morris-Pratt

KMP-MATCHER(T, P)

```
1   $n \leftarrow \text{length}[T]$ 
2   $m \leftarrow \text{length}[P]$ 
3   $\pi \leftarrow \text{COMPUTE-PREFIX-FUNCTION}(P)$ 
4   $q \leftarrow 0$  ▷ Number of characters matched.
5  for  $i \leftarrow 1$  to  $n$  ▷ Scan the text from left to right.
6      do while  $q > 0$  and  $P[q + 1] \neq T[i]$ 
7          do  $q \leftarrow \pi[q]$  ▷ Next character does not match.
8          if  $P[q + 1] = T[i]$ 
9              then  $q \leftarrow q + 1$  ▷ Next character matches.
10         if  $q = m$  ▷ Is all of  $P$  matched?
11             then print “Pattern occurs with shift”  $i - m$ 
12              $q \leftarrow \pi[q]$  ▷ Look for the next match.
```

Knuth-Morris-Pratt: Eksempel



Knuth-Morris-Pratt: Beregning af prefix funktionen

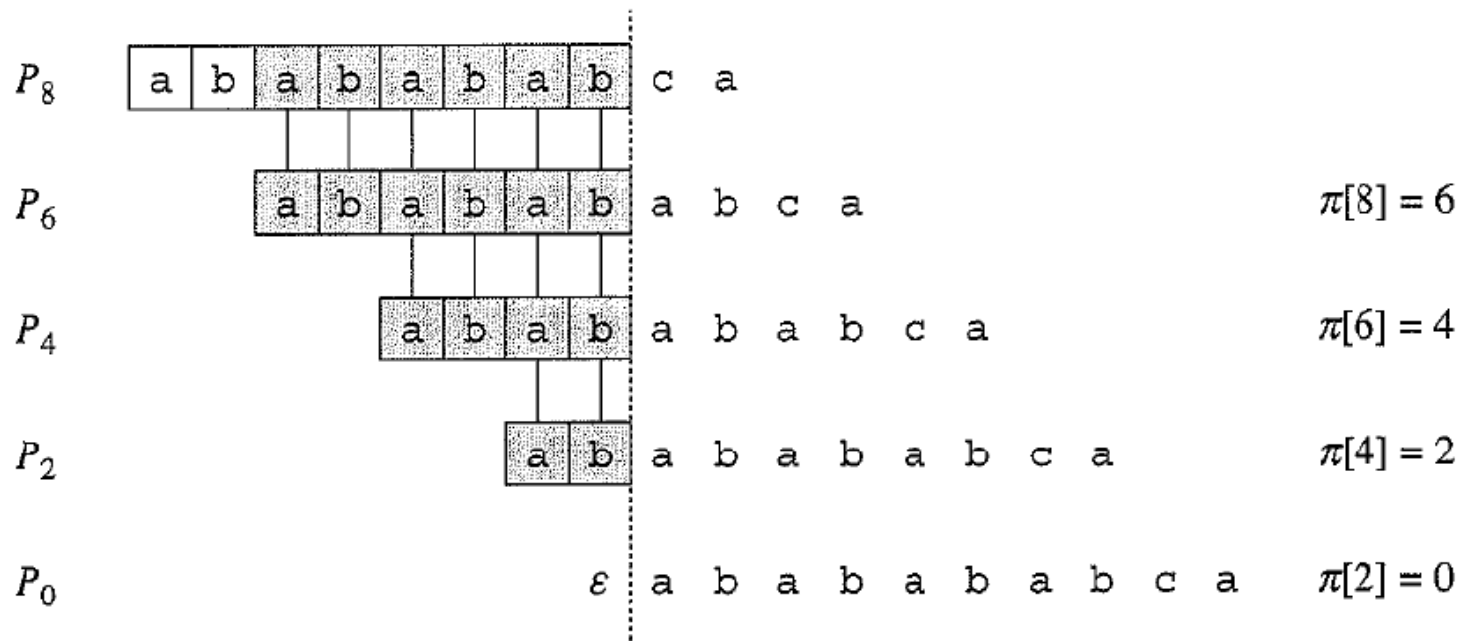
COMPUTE-PREFIX-FUNCTION(P)

```
1   $m \leftarrow \text{length}[P]$ 
2   $\pi[1] \leftarrow 0$ 
3   $k \leftarrow 0$ 
4  for  $q \leftarrow 2$  to  $m$ 
5      do while  $k > 0$  and  $P[k + 1] \neq P[q]$ 
6          do  $k \leftarrow \pi[k]$ 
7          if  $P[k + 1] = P[q]$ 
8              then  $k \leftarrow k + 1$ 
9           $\pi[q] \leftarrow k$ 
10 return  $\pi$ 
```


Knuth-Morris-Pratt: Beregning af prefix funktionen

i	1	2	3	4	5	6	7	8	9	10
$P[i]$	a	b	a	b	a	b	a	b	c	a
$\pi[i]$	0	0	1	2	3	4	5	6	0	1

(a)



(b)

Worst-case tider

Algorithm	Preprocessing time	Matching time	[CLRS]
Naive	0	$O((n - m + 1)m)$	32.1
Rabin-Karp	$\Theta(m)$	$O((n - m + 1)m)$	32.2
Finite automaton	$O(m \Sigma)$	$\Theta(n)$	(32.3)
Knuth-Morris-Pratt	$\Theta(m)$	$\Theta(n)$	32.4