Algoritmer og Datastrukturer 1

Elementære Datastrukturer [CLRS, kapitel 10]

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[CLRS, Del 3] : Datastrukturer

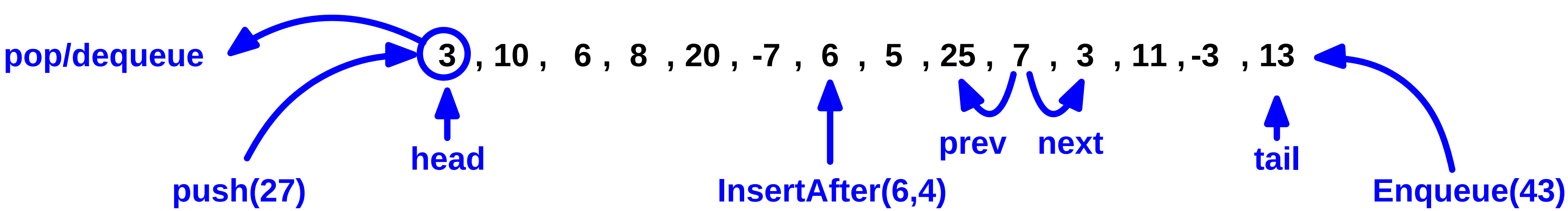
Opretholde en struktur for en **dynamisk** mængde data

Abstrakte Datastrukturer for Mængder

			-Min-prioritetskø	-Max-prioritetskø	- Ordbog
Forespørgsel	**Minimum(*S*)**	pointer til element	●		
	Maximum(*S*)	pointer til element		●	
	Search(*S*,*x*)	pointer til element			●
	Member(*S*,*x*)	TRUE eller FALSE			
	Successor(*S*,*x*)	pointer til element			
	Predecessor(*S*,*x*)	pointer til element			
Opdateringer	**Insert(*S*,*x*)**	pointer til element	●	●	●
	Delete(*S*,*x*)	-			●
	DeleteMin(*S*)	element	●		
	DeleteMax(*S*)	element		●	
	Join(S_1,S_2)	mængde *S*			
	Split(*S*,*x*)	mængder S_1 og S_2			

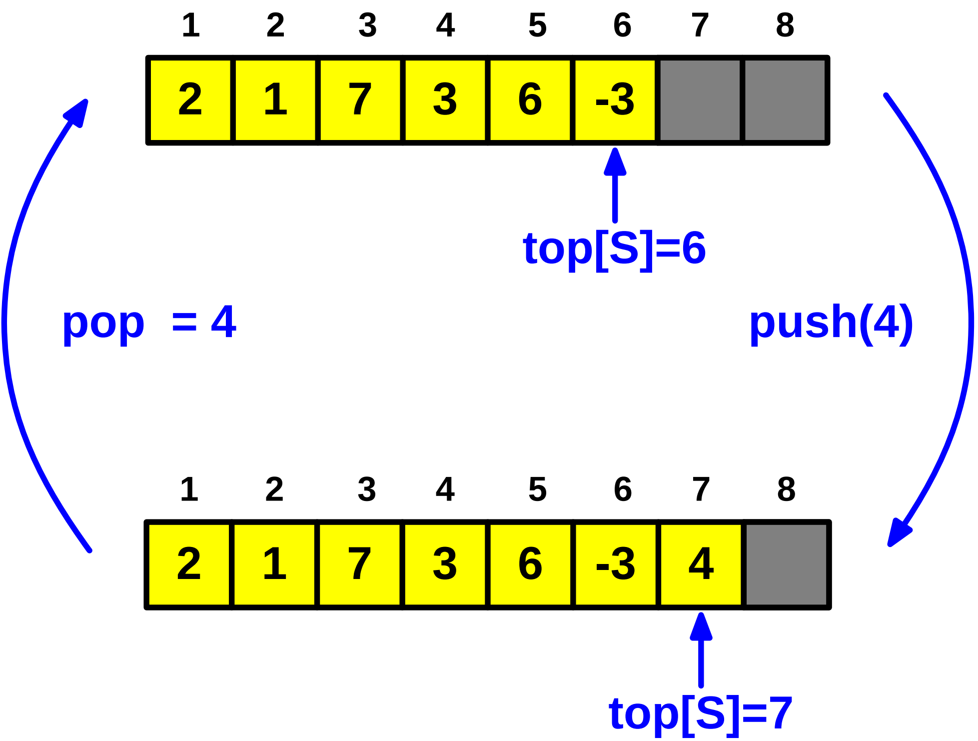
Abstrakte Datastrukturer for Lister

			-Stak	-Kø
Forespørgsel	Empty(*S*)	TRUE eller FALSE	●	●
	Head(*S*), Tail(*S*)	pointer til element		
	Next(*S*,*x*), Prev(*S*,*x*)	pointer til element		
	Search(*S*,*x*)	pointer til element		
Opdateringer	Push(*S*,*x*)	-	●	
	Pop/Dequeue(*S*)	element	●	●
	Enqueue(*S*)	-		●
	Delete(*S*,*x*)	element		
	InsertAfter(*S*,*x*,*y*)	pointer til element		



Stak

Stak : Array Implementation



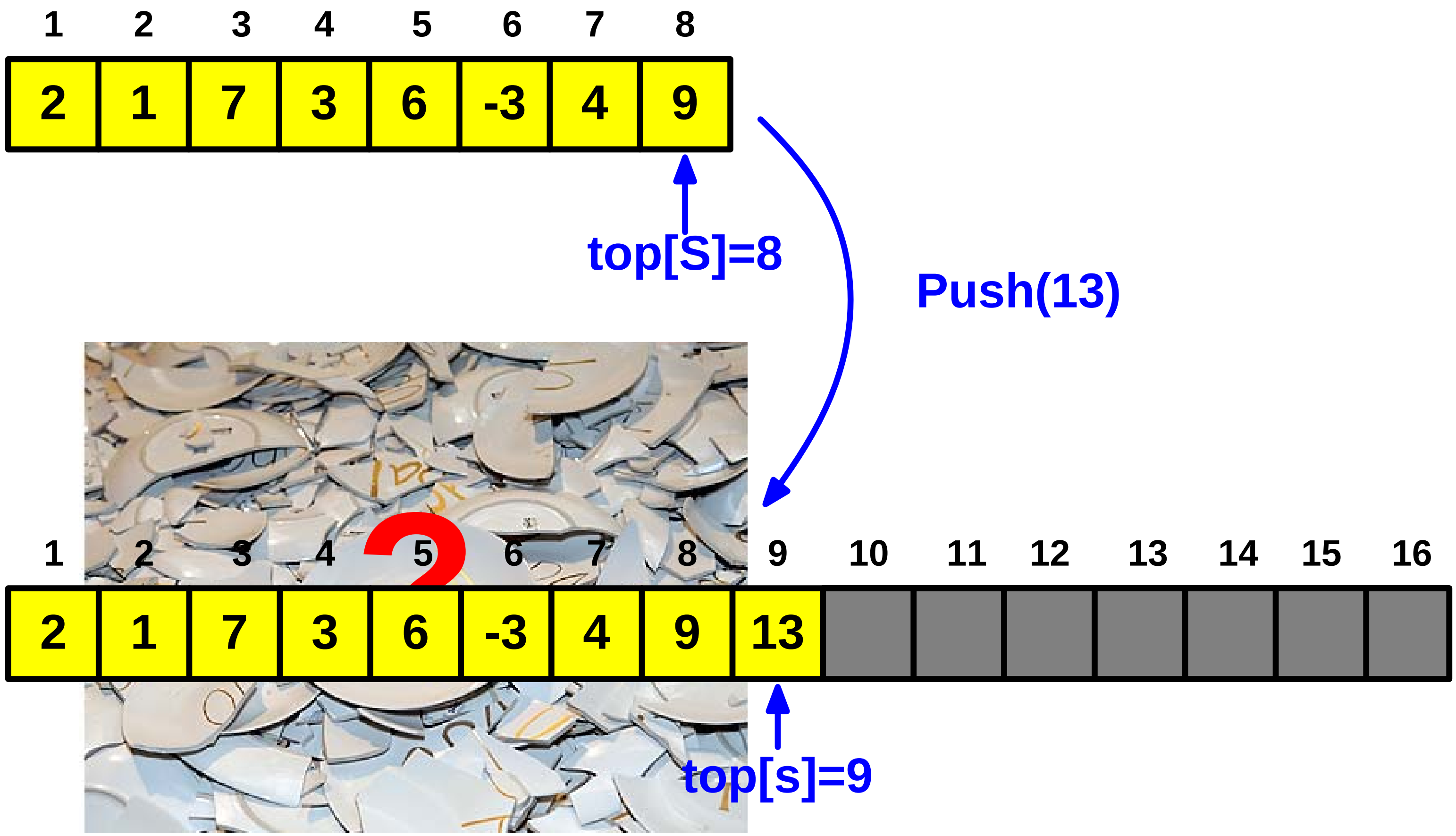

```
STACK-EMPTY(S)
if top[S] = 0
    then return TRUE
    else return FALSE

PUSH(S, x)
top[S] ← top[S] + 1
S[top[S]] ← x

POP(S)
if STACK-EMPTY(S)
    then error "underflow"
    else top[S] ← top[S] − 1
         return S[top[S] + 1]
```

Stack-Empty, Push, Pop : O(1) tid

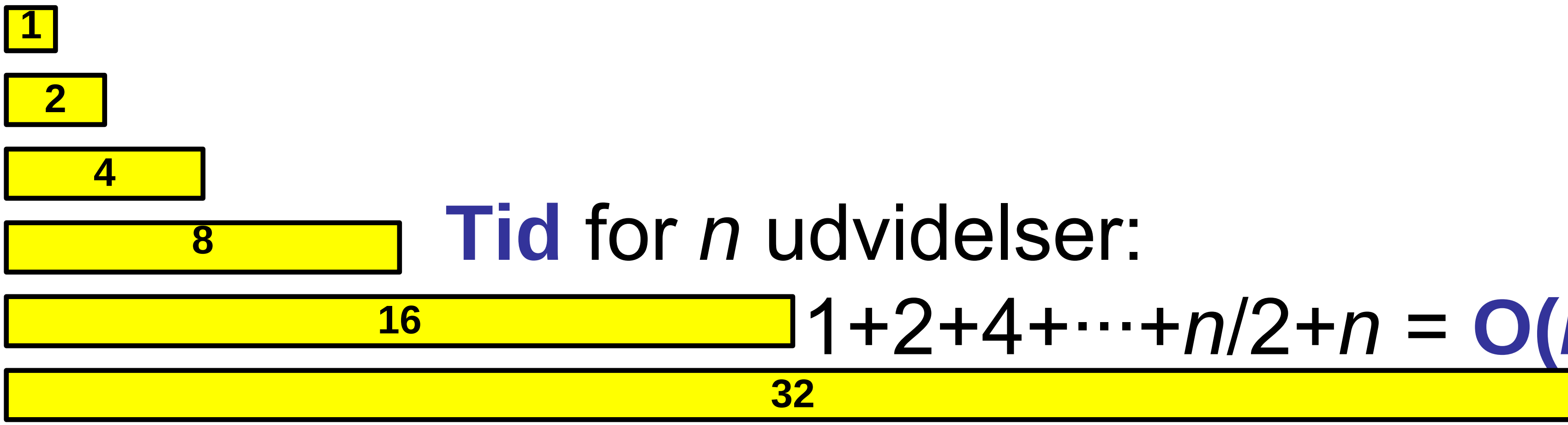
Stak : Overløb



Array fordobling : O(*n*) tid

Array Fordobling

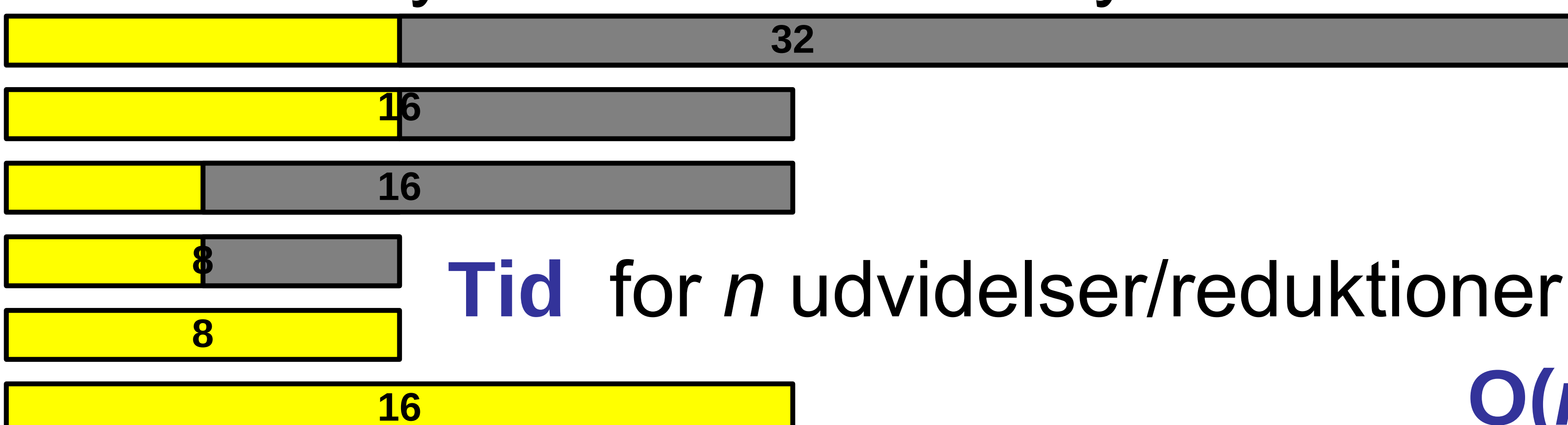
Fordoble arrayet når det er fuld



Tid for *n* udvidelser:

$1+2+4+\cdots+n/2+n = \mathbf{O}(n)$

Halver arrayet når det er <1/4 fyldt



Tid for *n* udvidelser/reduktioner:

$\mathbf{O}(n)$

Array Fordobling + Halvering

– en generel teknik

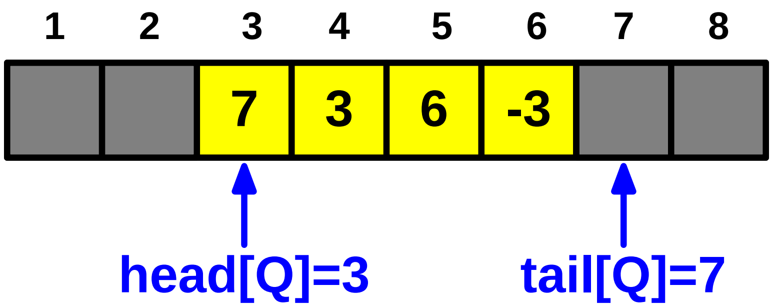
Tid for n udvidelser/reduktioner er **O(n)**

Plads $\leq$ 4 · aktuelle antal elementer

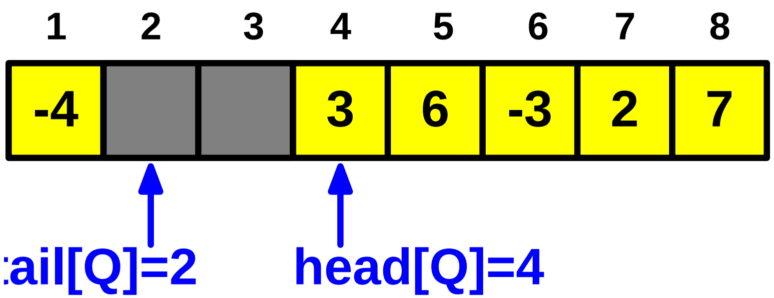
Array implementation af Stak:
n push og pop operationer tager O(n) tid

Kø

Kø : Array Implementation



Enqueue(2)
Enqueue(7)
Engueue(-4)
Dequeue = 7

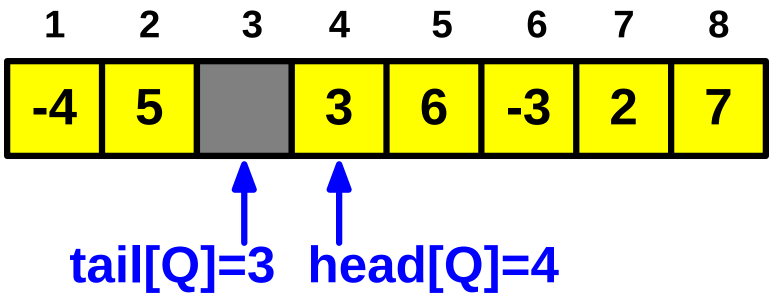



```
ENQUEUE(Q, x)
Q[tail[Q]] ← x
if tail[Q] = length[Q]
    then tail[Q] ← 1
    else tail[Q] ← tail[Q] + 1

DEQUEUE(Q)
x ← Q[head[Q]]
if head[Q] = length[Q]
    then head[Q] ← 1
    else head[Q] ← head[Q] + 1
return x
```

Enqueue, dequeue : O(1) tid

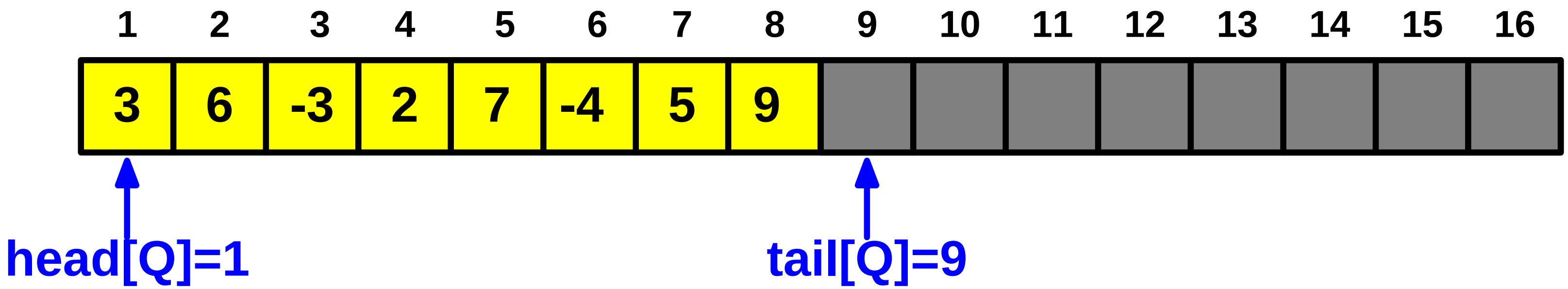
Kø : Array Implementation



Empty : tail[Q]=head[Q] ?

Overløb : array fordobling/ halvering

Enqueue(9)



Array implementation af Kø:
***n* enqueue og dequeue operationer tager O(*n*) tid**

Arrays (med Fordobling/Halvering)

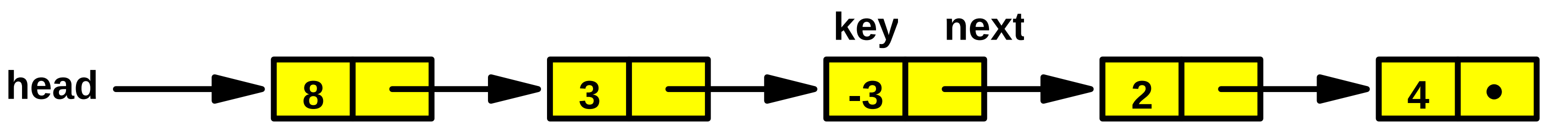
Stak	**Push(*S*,*x*)**	**O(1)***
	Pop(*S*)	**O(1)***
Kø	**Enqueue(*S*,*x*)**	**O(1)***
	Dequeue(*S*)	**O(1)***

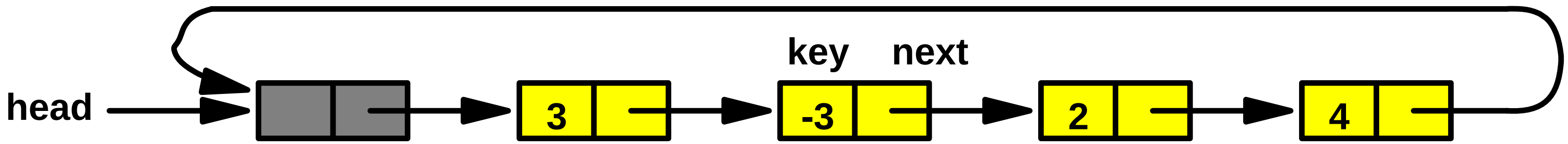
*** Worst-case uden fordobling/halvering**
Amortiseret ([CLRS, Kap. 17]) med fordobling/halvering

Kædede lister

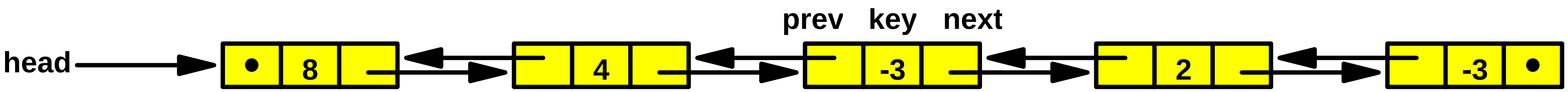
Kædede Lister

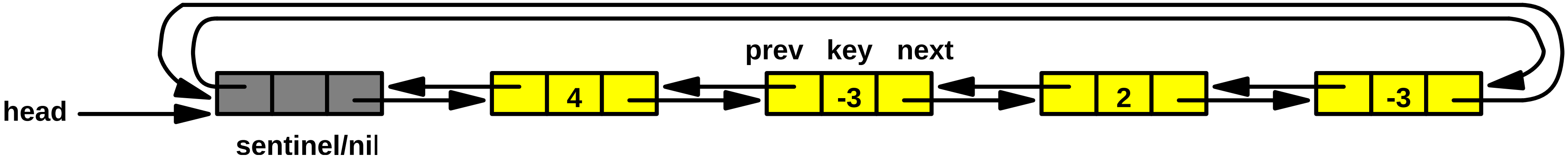
Enkelt kædede (ikke-cyklisk og cyklisk)



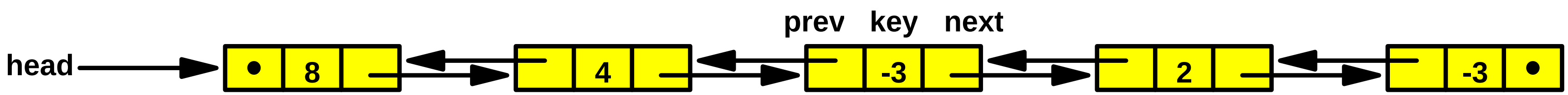


Dobbelt kædede (ikke-cyklisk og cyklisk)





Dobbelt Kædede Lister



LIST-INSERT(L, x)

```
next[x] ← head[L]
if head[L] ≠ NIL
    then prev[head[L]] ← x
head[L] ← x
prev[x] ← NIL
```

LIST-SEARCH(L, k)

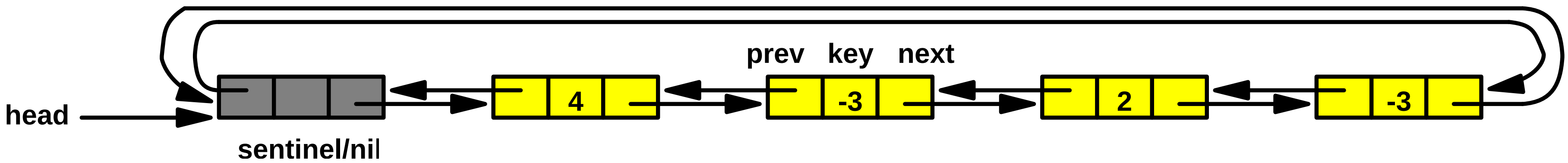
```
x ← head[L]
while x ≠ NIL and key[x] ≠ k
    do x ← next[x]
return x
```

LIST-DELETE(L, x)

```
if prev[x] ≠ NIL
    then next[prev[x]] ← next[x]
    else head[L] ← next[x]
if next[x] ≠ NIL
    then prev[next[x]] ← prev[x]
```

List-Search	O(n)
List-Insert	O(1)
List-Delete	O(1)

Dobbelt Kædede Cykliske Lister



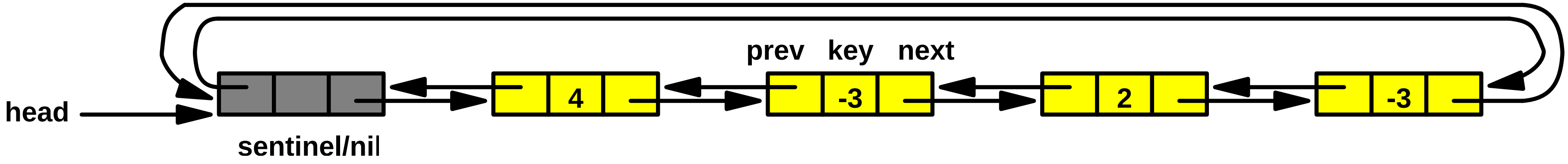

```
LIST-INSERT'(L, x)
next[x] ← next[nil[L]]
prev[next[nil[L]]] ← x
next[nil[L]] ← x
prev[x] ← nil[L]
```

```
LIST-SEARCH'(L, k)
x ← next[nil[L]]
while x ≠ nil[L] and key[x] ≠ k
      do x ← next[x]
return x
```

```
LIST-DELETE'(L, x)
next[prev[x]] ← next[x]
prev[next[x]] ← prev[x]
```

List-Search'	O(n)
List-Insert'	O(1)
List-Delete'	O(1)

Dobbelt Kædede Cykliske Lister



Stak	**Push(*S*,*x*)**	**O(1)**
	Pop(*S*)	**O(1)**
Kø	**Enqueue(*S*,*x*)**	**O(1)**
	Dequeue(*S*)	**O(1)**

Dancing Links

Donald E. Knuth, Stanford University

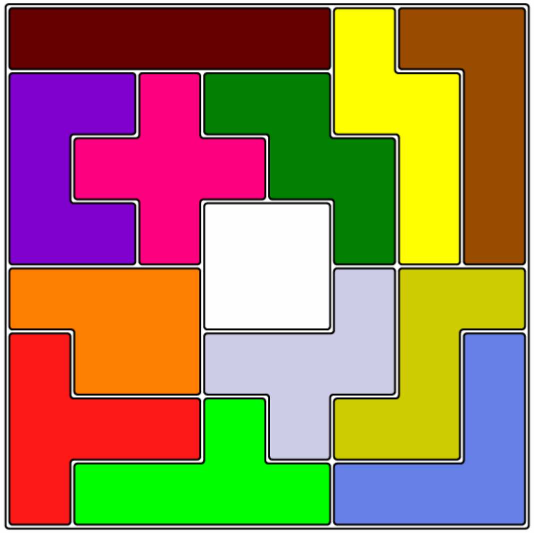
My purpose is to discuss an extremely simple technique that deserves to be better known. Suppose x points to an element of a doubly linked list; let $L[x]$ and $R[x]$ point to the predecessor and successor of that element. Then the operations

$$L\big[R[x]\big] \leftarrow L[x], \qquad R\big[L[x]\big] \leftarrow R[x] \tag{1}$$

remove x from the list; every programmer knows this. But comparatively few programmers have realized that the subsequent operations

$$L\big[R[x]\big] \leftarrow x, \qquad R\big[L[x]\big] \leftarrow x \tag{2}$$

will put x back into the list again.



"The Challenge Puzzle"

?

”The Challenge Puzzle”

L := Tomt bræt
B := Alle brikker
Solve(*L*,*B*)

```
procedure Solve(Delløsning L, Brikker B)
   for alle b i B
      for alle orienteringer af b  (* max 8 forskellige *)
         if b kan placeres i nederste venstre fri then
             fjern b fa B
             indsæt b i L
             if |B|=0 then
               rapporter L er en løsning
             else
               Solve(L,B)
             fi
             slet b fra L
             genindsæt b i B
         fi
```

Nederste-venstre fri

Før

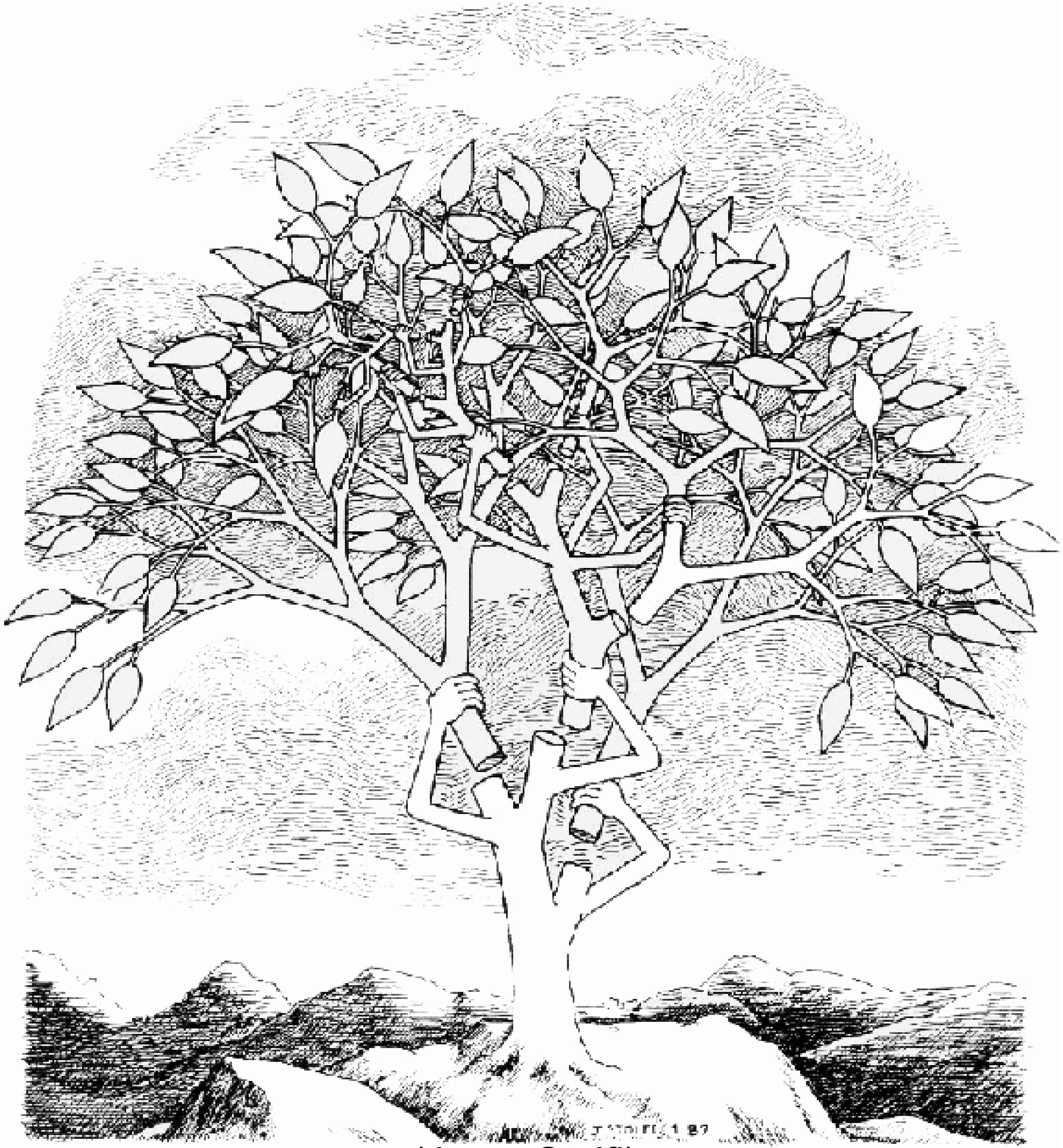
Efter

”The Challenge Puzzle”

4.040 løsninger

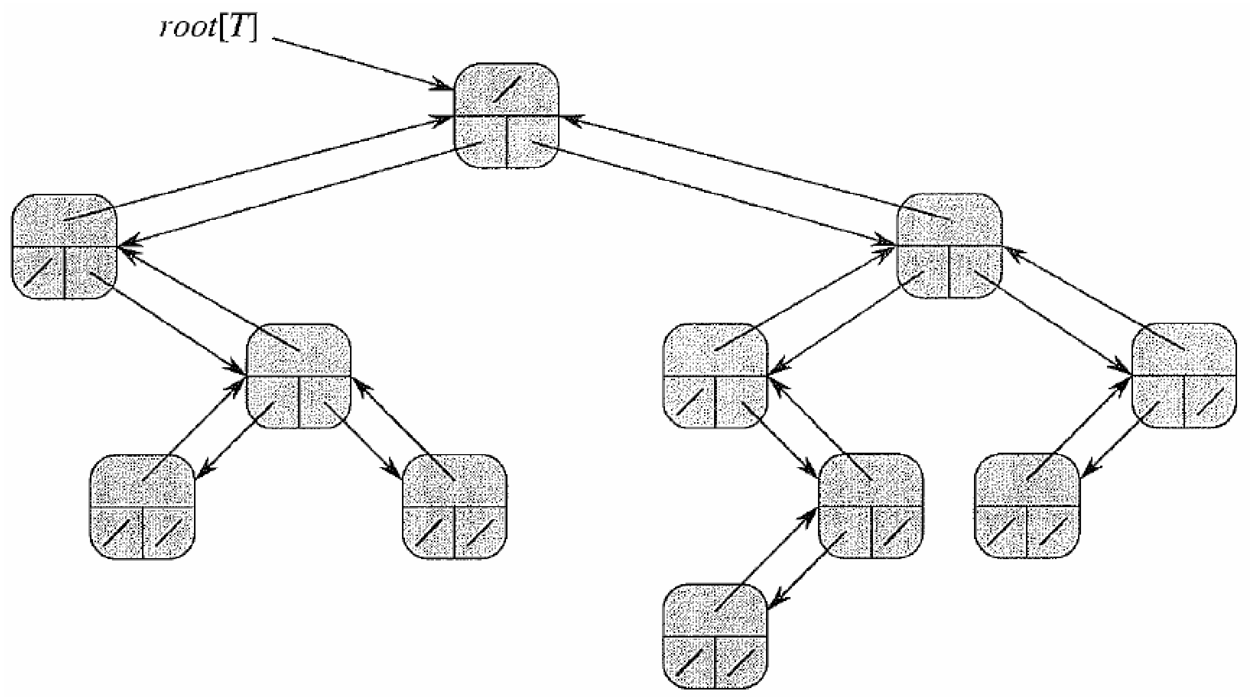
Solve placerer

8.387.259 brikker



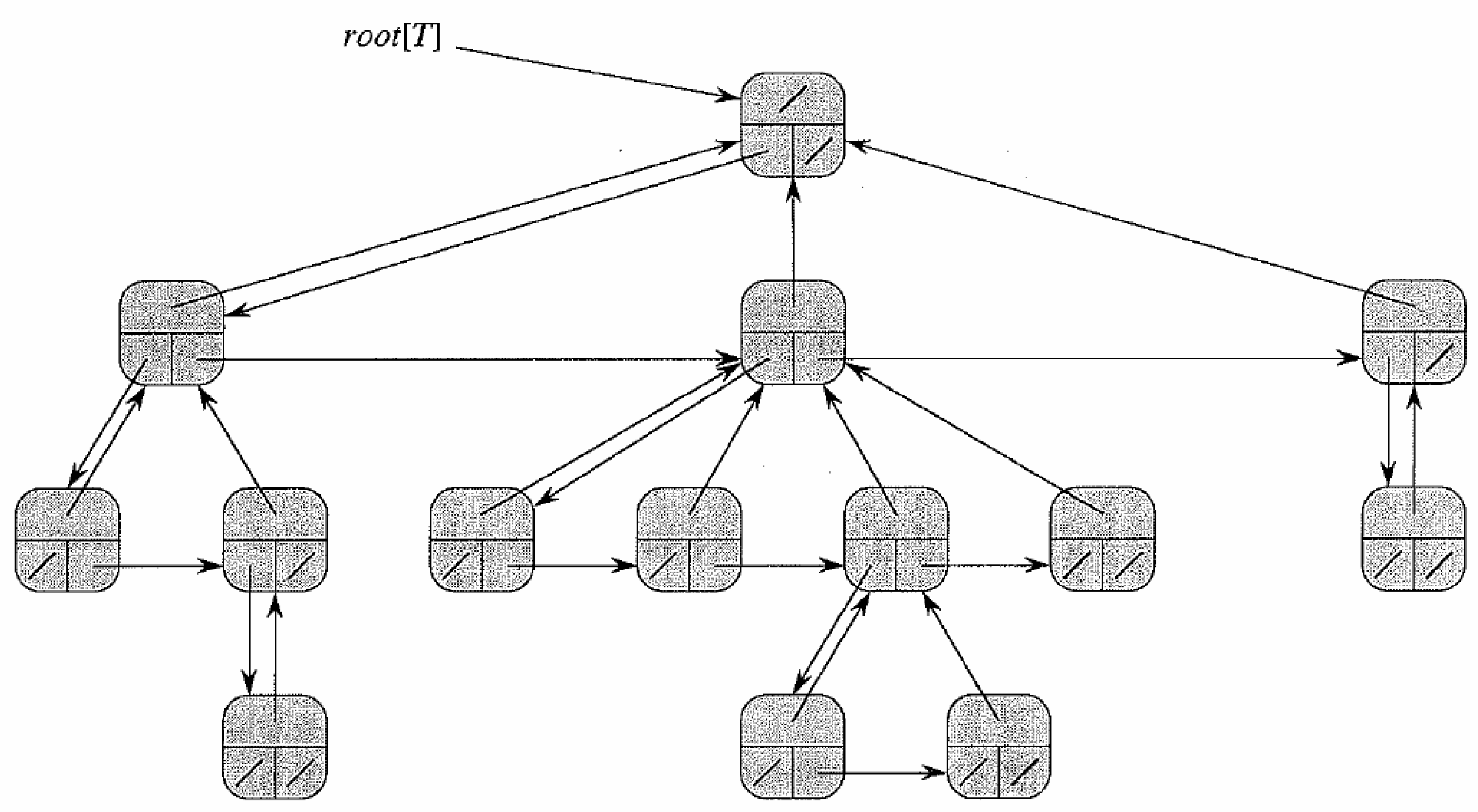
(Jorge Stolfi)

Binær Træ Repræsentation



Felter: **Left**, **right**, **parent**

Træ Repræsentation



Felter: **Left**, **right sibling**, **parent**