

Exercise 7 - Random Walks

Deadline: 21st May, 2009

7-1 What is the expected length of a random walk of a queen that starts and ends on field b3 of an otherwise empty chessboard.

7-2 A non-aperiodic irreducible transition matrix P can be changed into an aperiodic transition matrix P' as follows:

- $p'_{ij} = \frac{p_{ij}}{2}$, for $i \neq j$
- $p'_{ii} = \frac{1}{2}, \forall i$.

Show that an invariant π of P is also an invariant of P' .

7-3 (optional) Consider a 1-dimensional random walk with a reflection barrier, which is defined as follows. For each natural number i , there is a state i . At state 0, with probability 1 the walk will move to state 1. At every other state $i > 0$, the walk will move to state $i + 1$ with probability q and to state $i - 1$ with probability $1 - q$. Prove the following for the resulting Markov chain:

- For $q > \frac{1}{2}$, each state is transient.
- For $q = \frac{1}{2}$, each state is null persistent.
- For $q < \frac{1}{2}$, each state is non-null persistent.