

Homework Exercises for Lecture 2

Deadline: 23 April

2-1 Consider a range space (X, R) in which X is a set of N points in \mathbb{R}^d .

(i) Assume $d = 1$, $X = \{1, 2, \dots, N\}$ and R is the set of all the intervals $I_{a,b} := [a; b]$ for all $1 \leq a \leq b \leq N$. Find an ε -net of smallest size for this range space.

(ii) Assume $d = 2$, $X = \{(x, y) \mid 1 \leq x, y \leq \sqrt{N}\}$ and R is the set of all the boxes $B_{a,b,c,d} := [a; b] \times [c; d]$ for all $1 \leq a, b, c, d \leq N$ where $a \leq b$ and $c \leq d$. Find an ε -net of size $O(\varepsilon^{-1} \log(\varepsilon^{-1}))$ for this range space.

(optional) (iii) Is the bound obtained in (ii) asymptotically optimal?

(optional) (iii) Assume $d = 3$, $X = \{(x, y, z) \mid 1 \leq x, y, z \leq N^{1/3}\}$ and R is the set of all the boxes $I_{a,b,c,d,e,f} := [a; b] \times [c; d] \times [e; f]$ for all $1 \leq a, b, c, d, e, f \leq N$ where $a \leq b$, $c \leq d$ and $e \leq f$. What is the smallest ε -net that you can find for this range space?