MADALGO

CENTER FOR MASSIVE DATA ALGORITHMICS

PhD Defense

Implicit Data Structures, Sorting, and Text Indexing

Jesper Sindahl Nielsen

Danmarks Grundforskningsfond
Danish National Research Foundation

STATSBIBLIOTEKET

AARHUS UNIVERSITY

Overview

Overview

1. Algorithms and Data Structures
   (a) Abstract description
   (b) Example

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2. Implicit Data Structures

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   (a) Finger Search
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   (a) Finger Search
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3. Integer Sorting
4. Text Indexing
5. State and University Library

Algorithms

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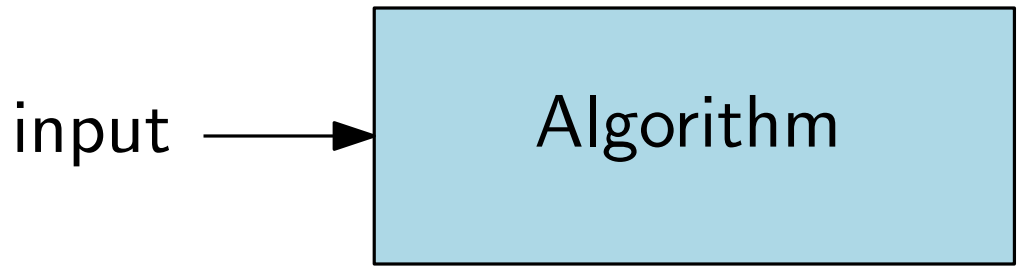
Algorithms



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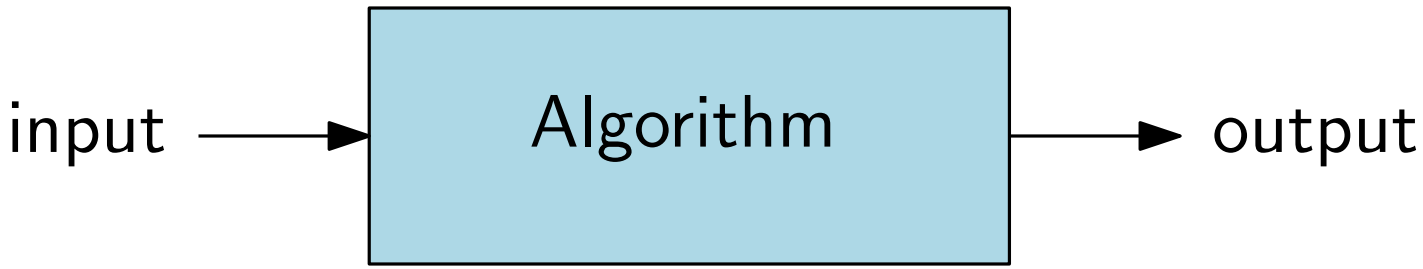

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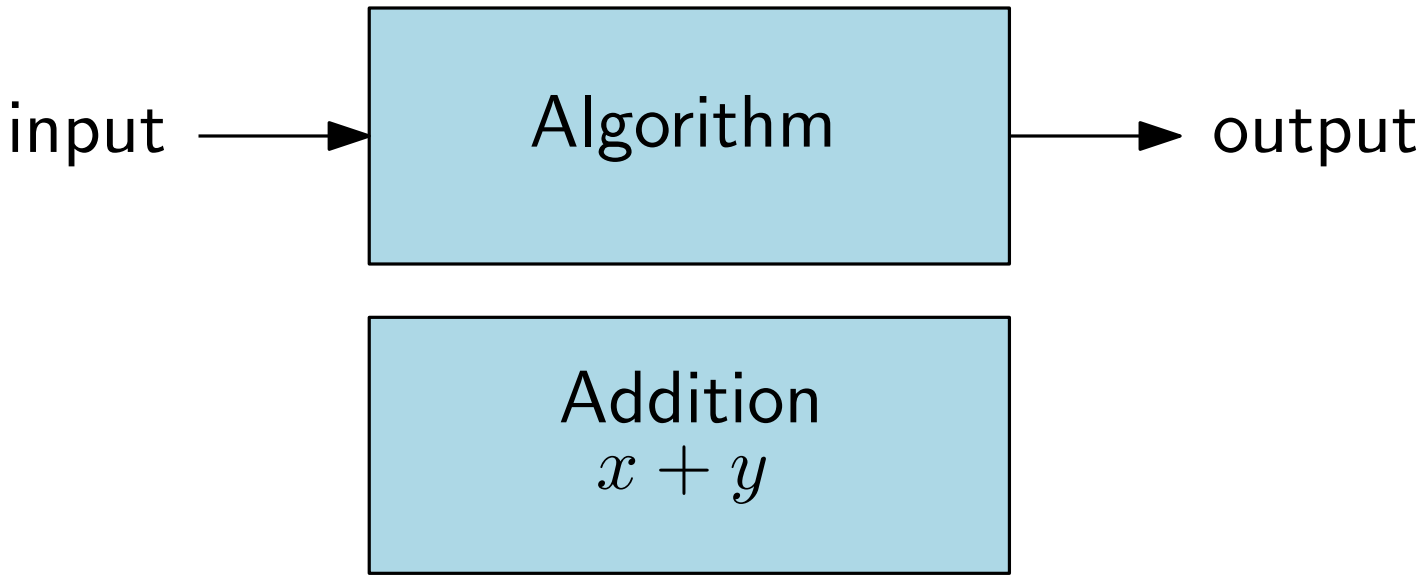

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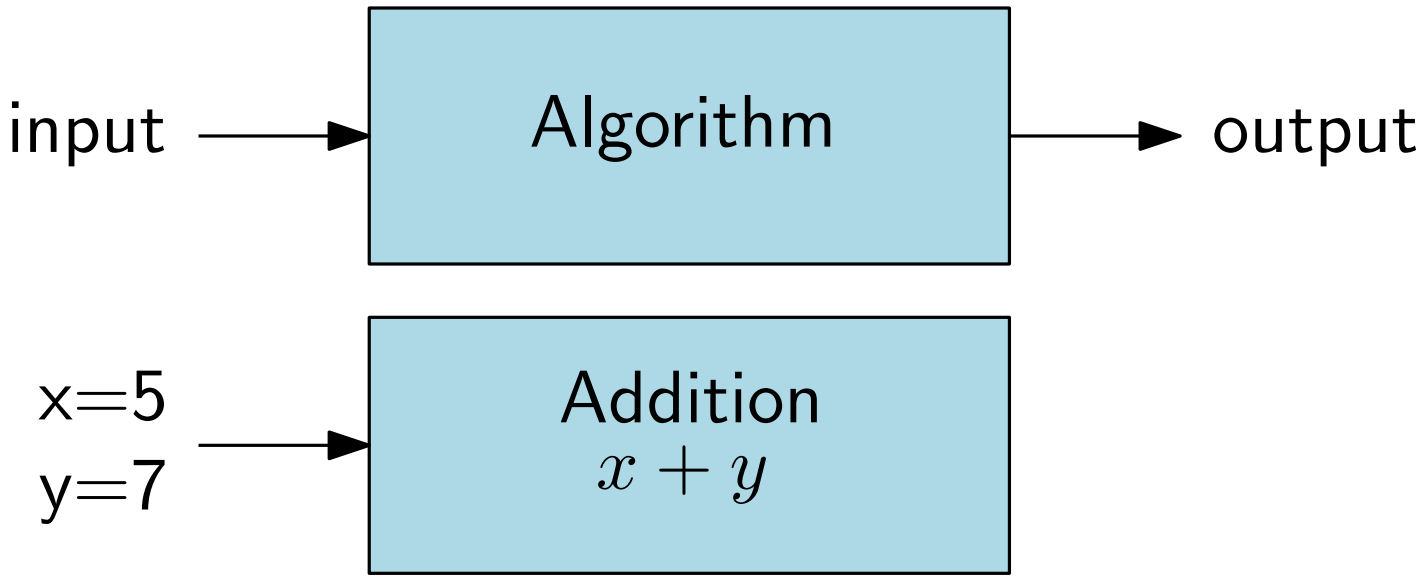

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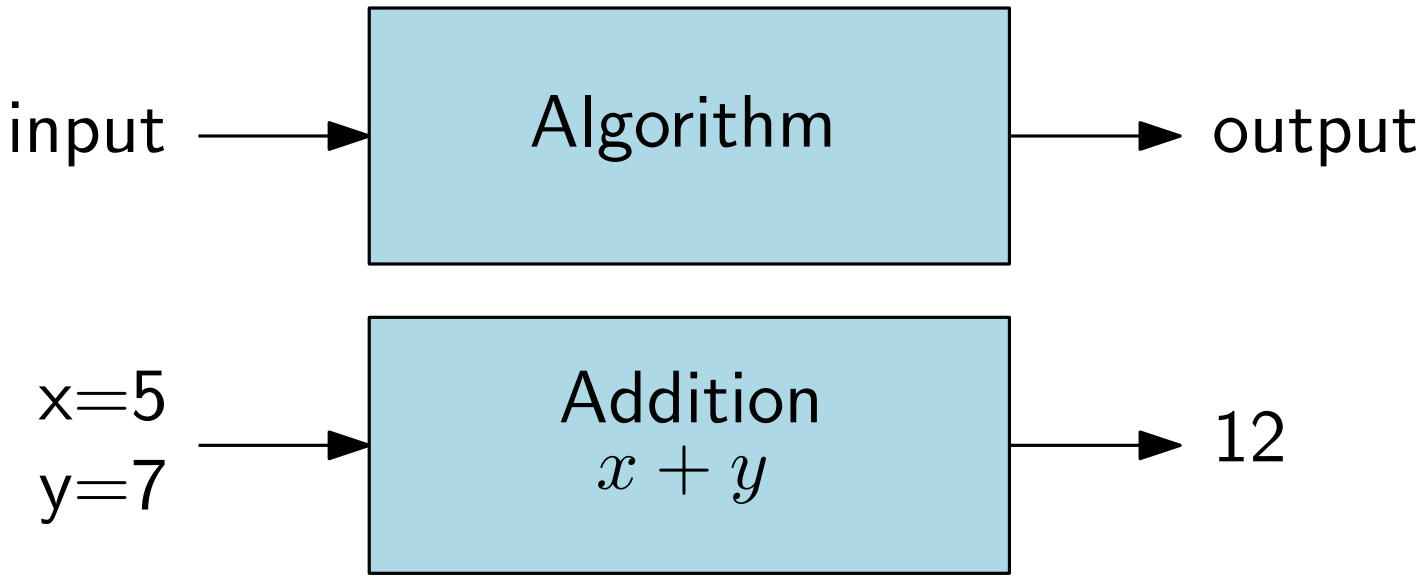

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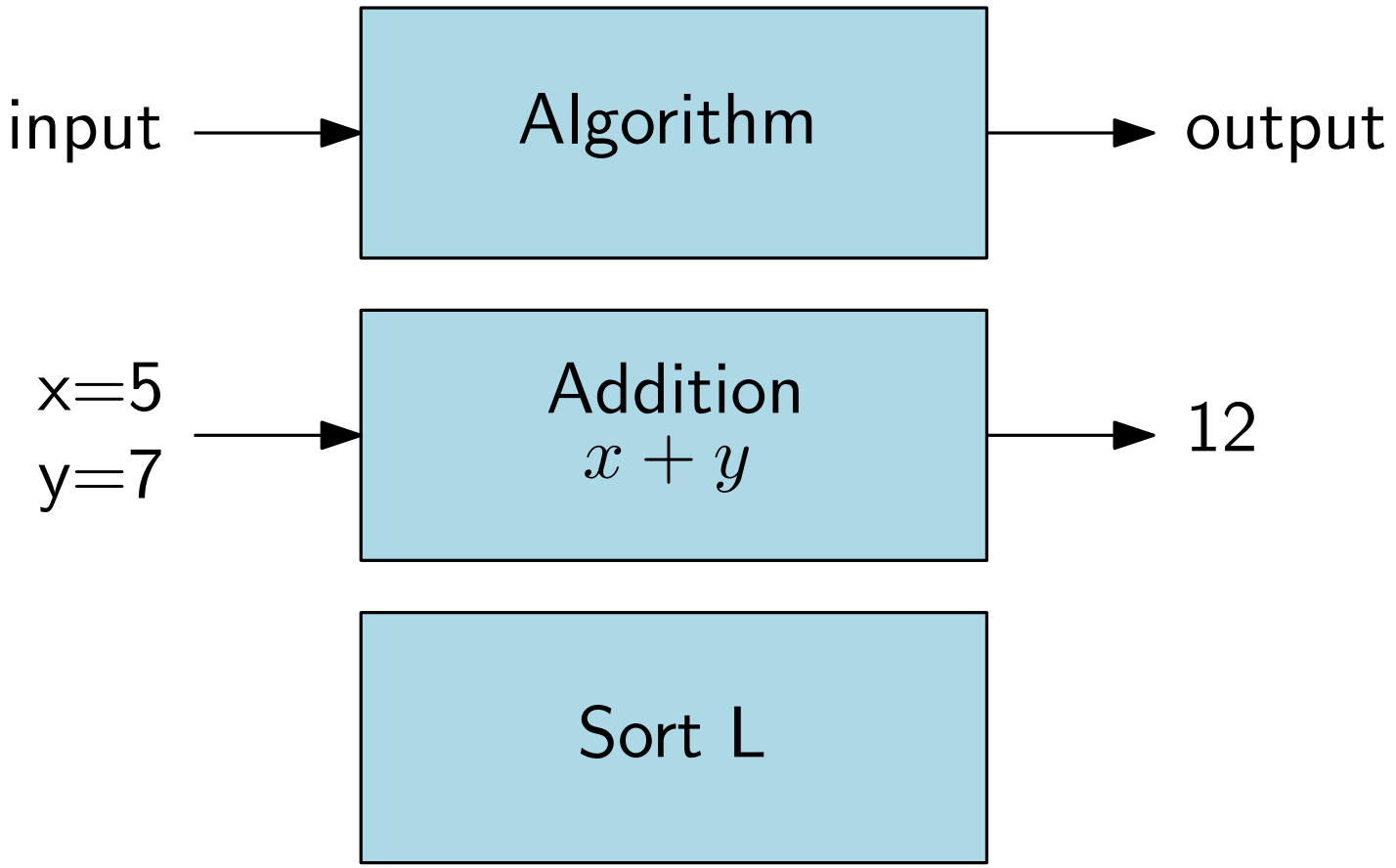

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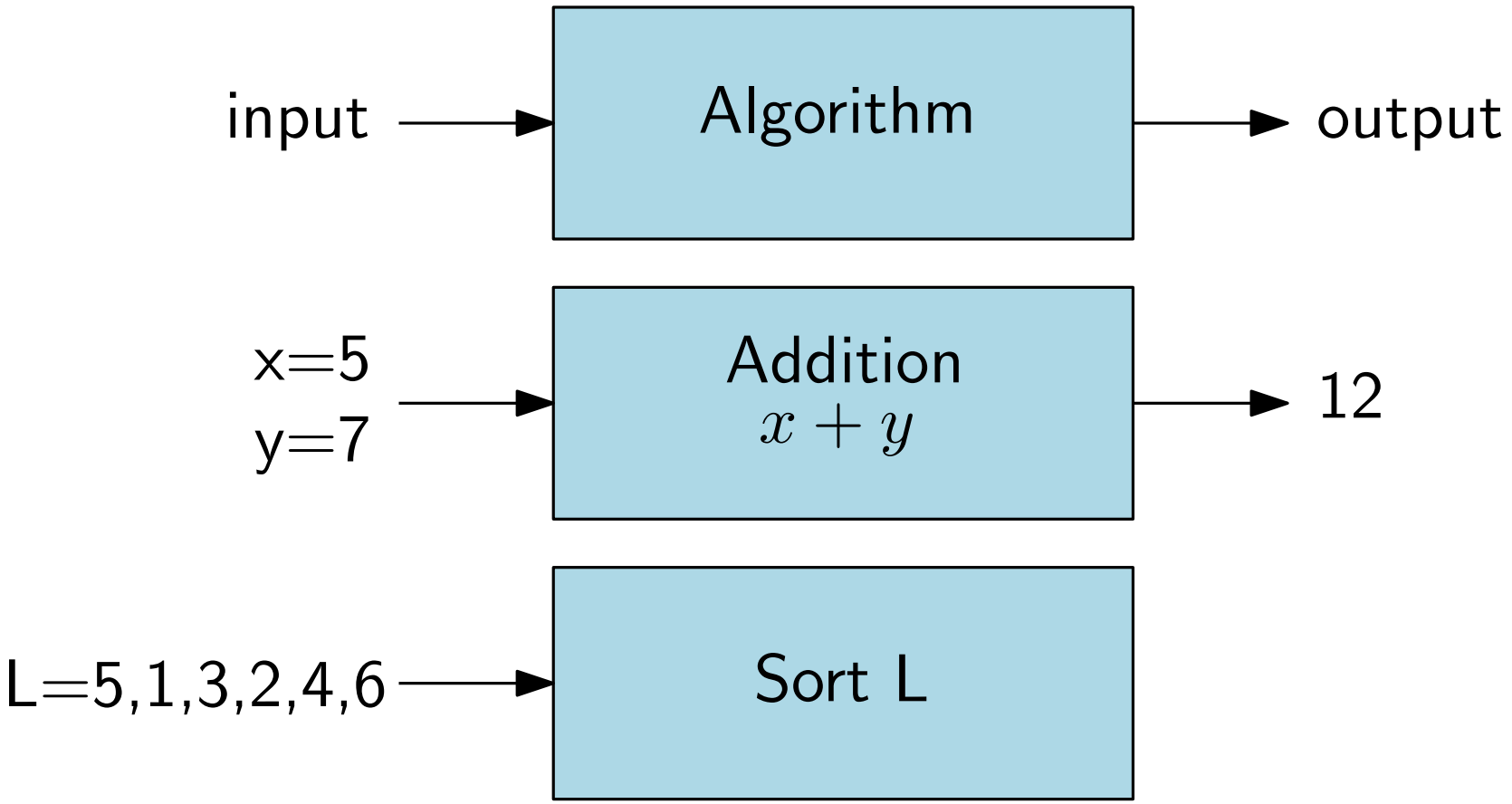

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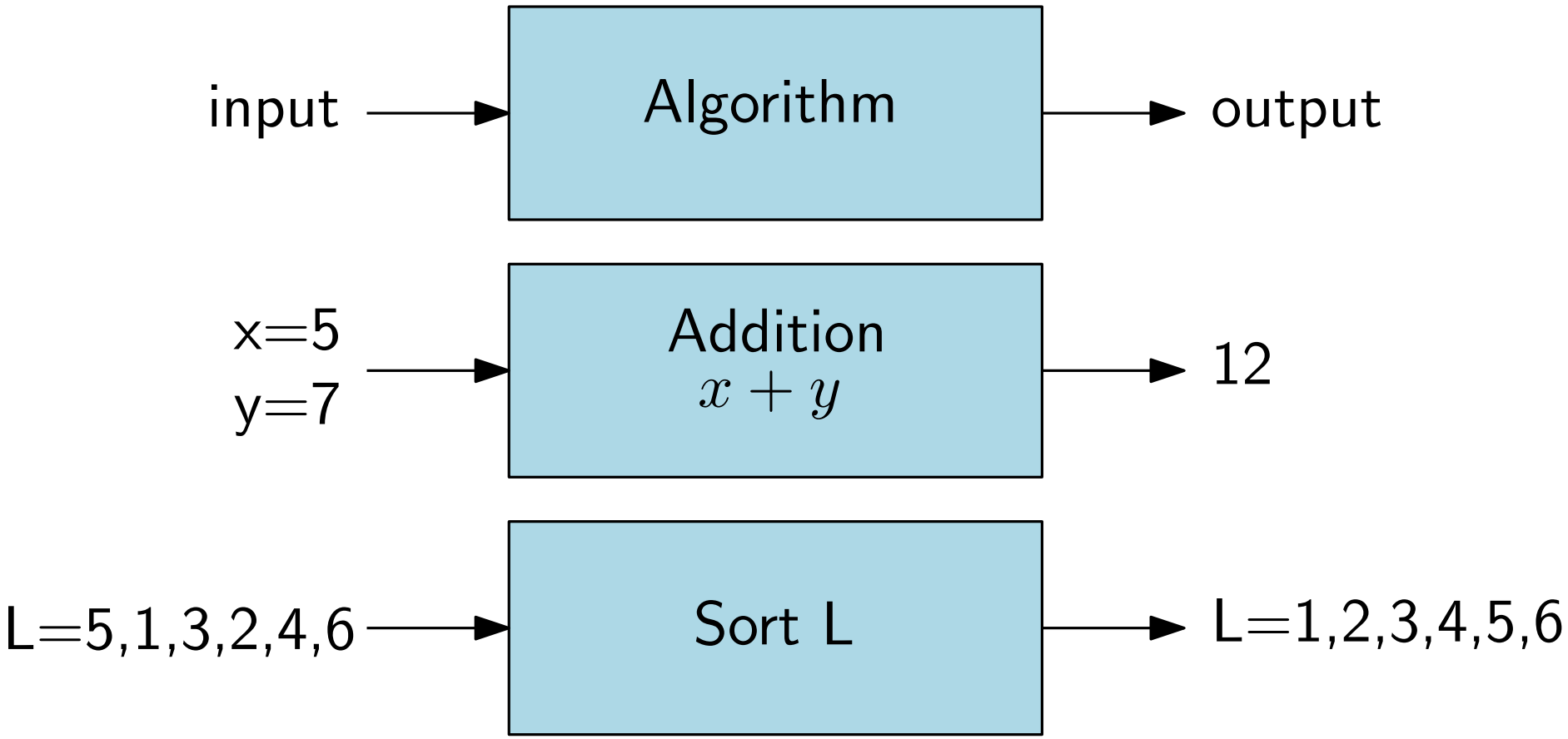

We want to design *fast* algorithms

Data Structures

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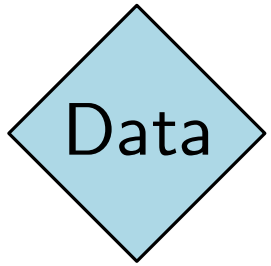

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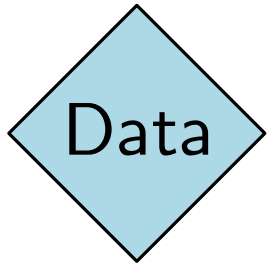

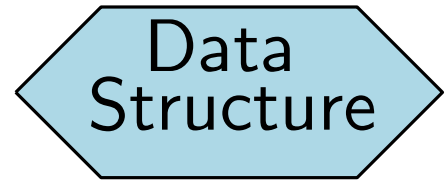

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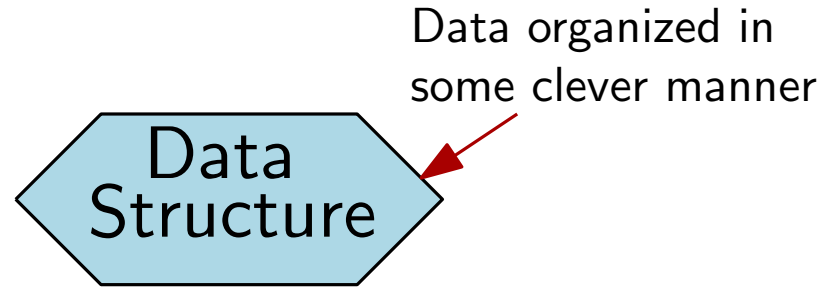

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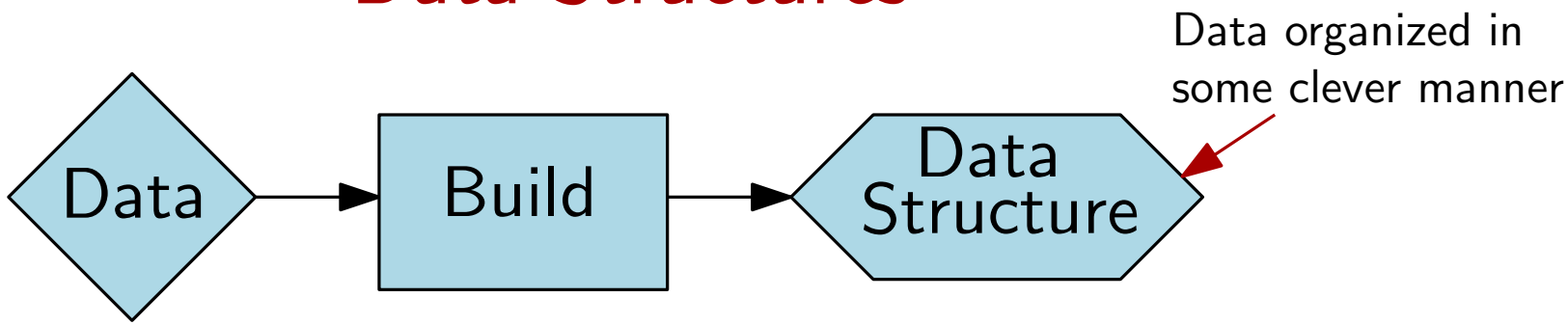

Data Structures

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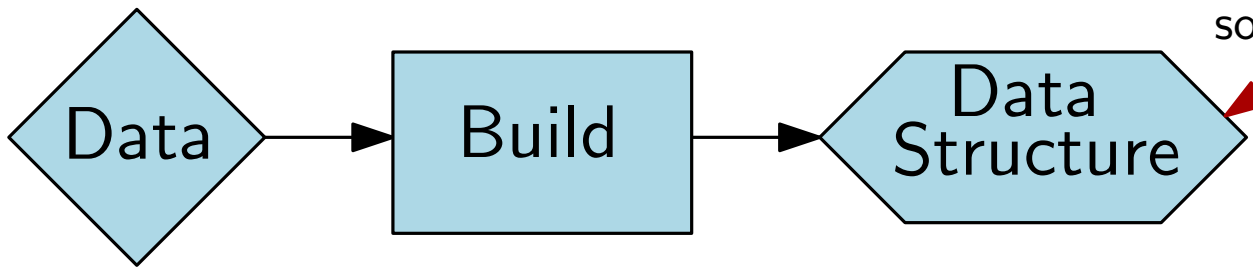

Data organized in some clever manner

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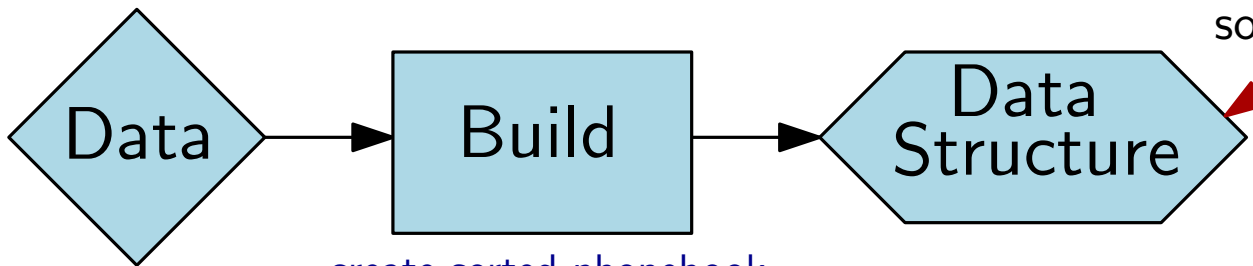

create sorted phonebook

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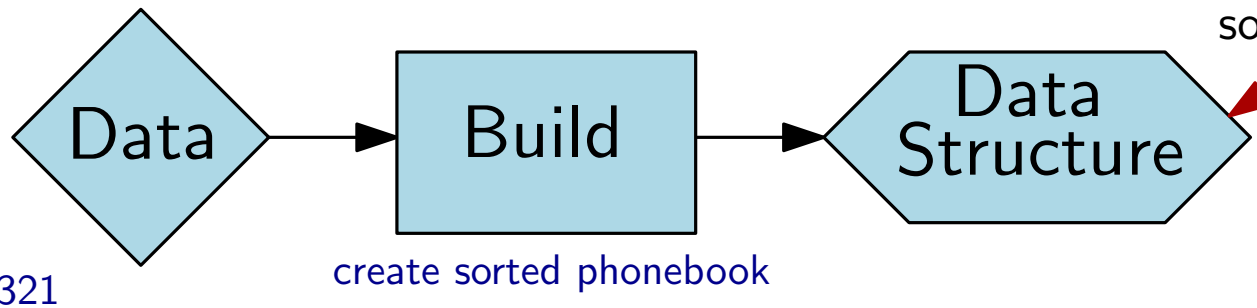

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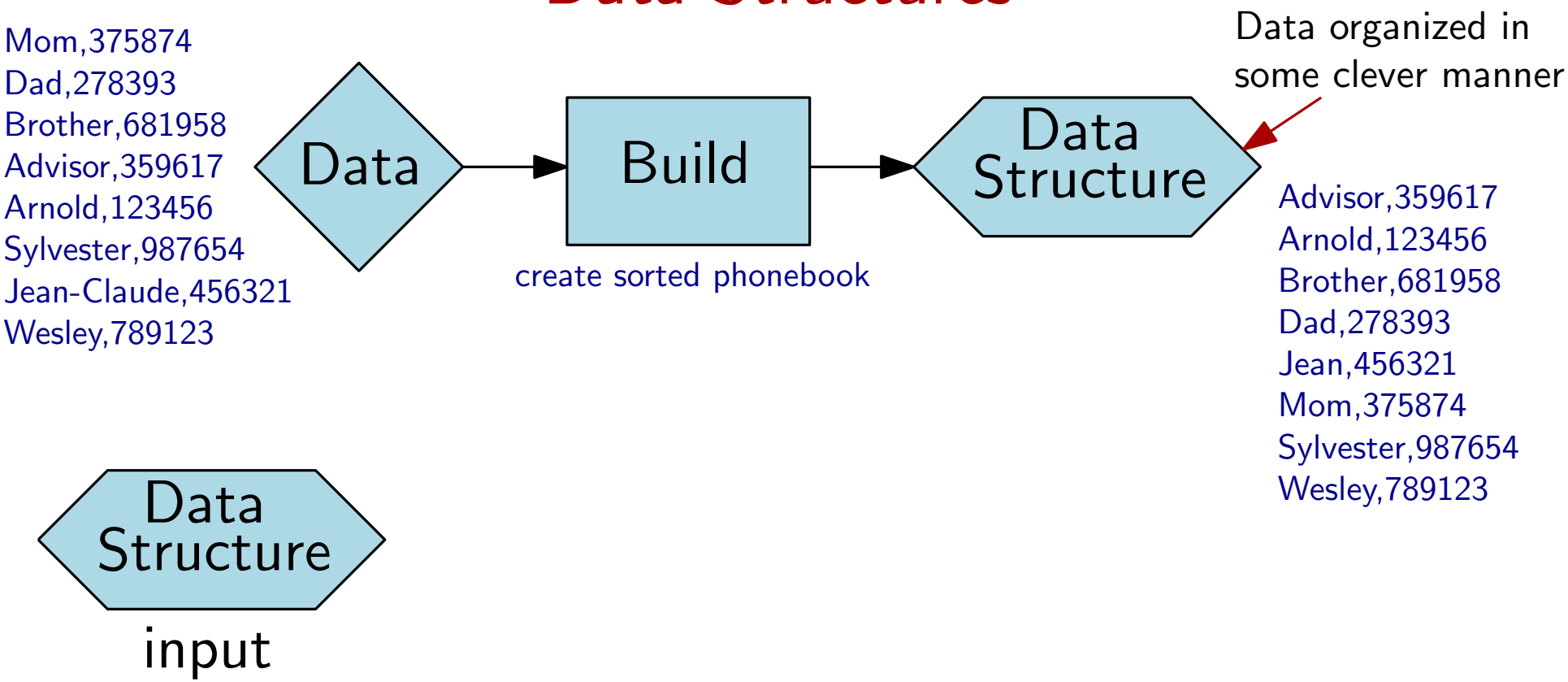

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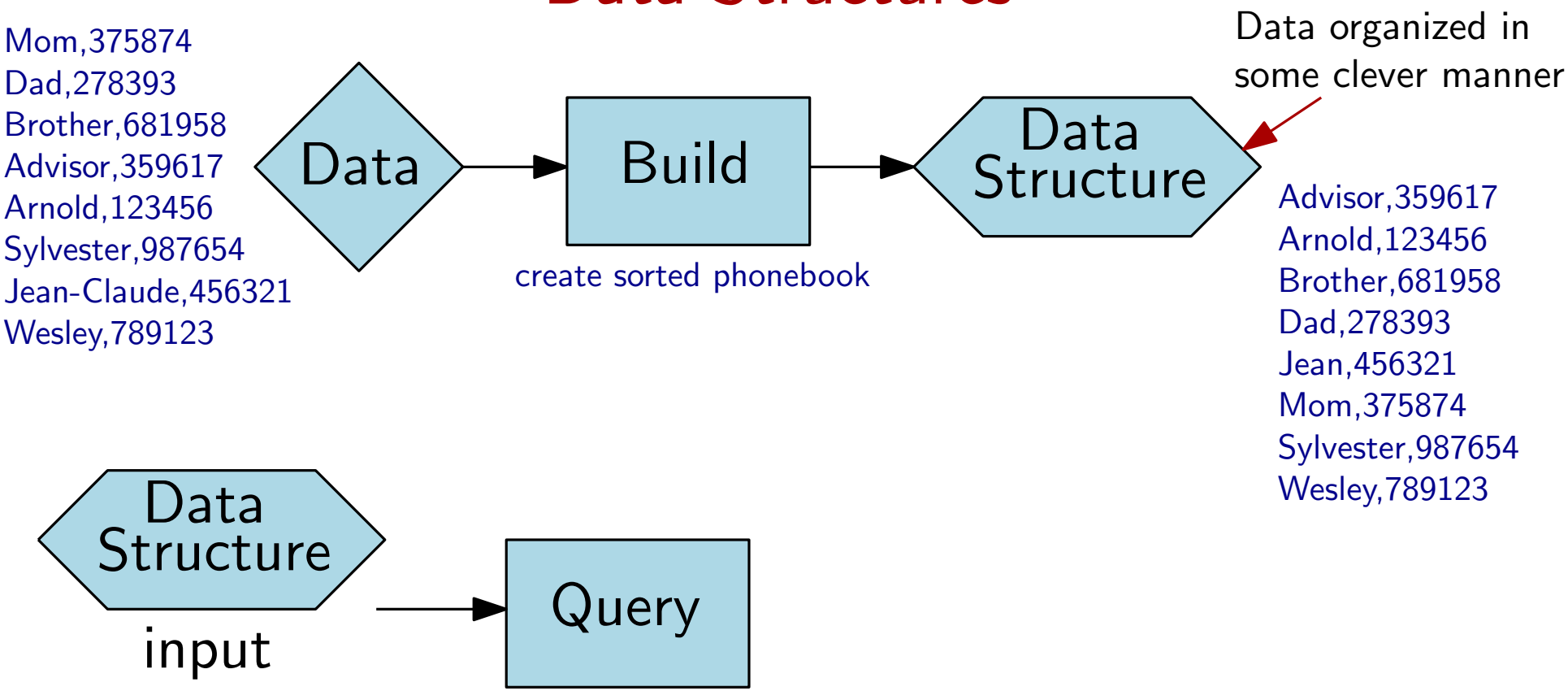

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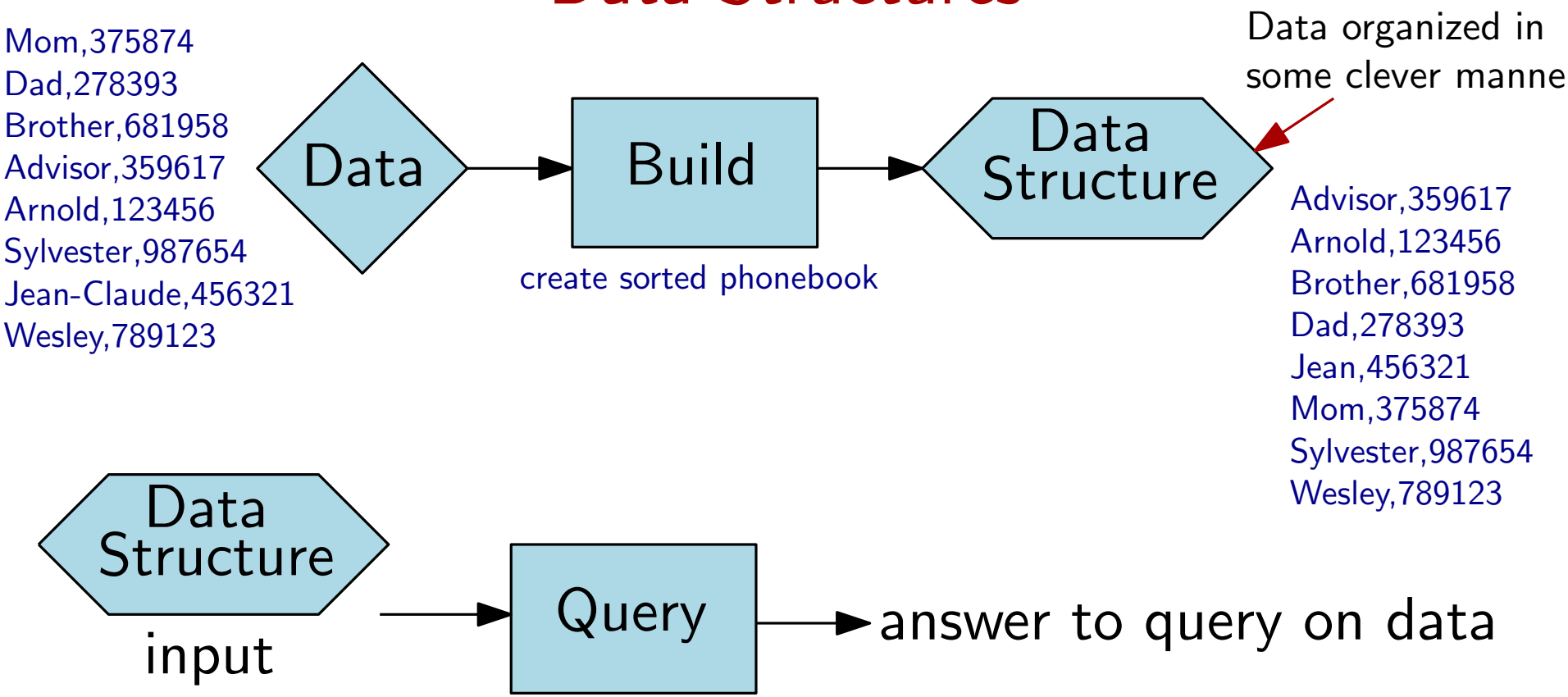

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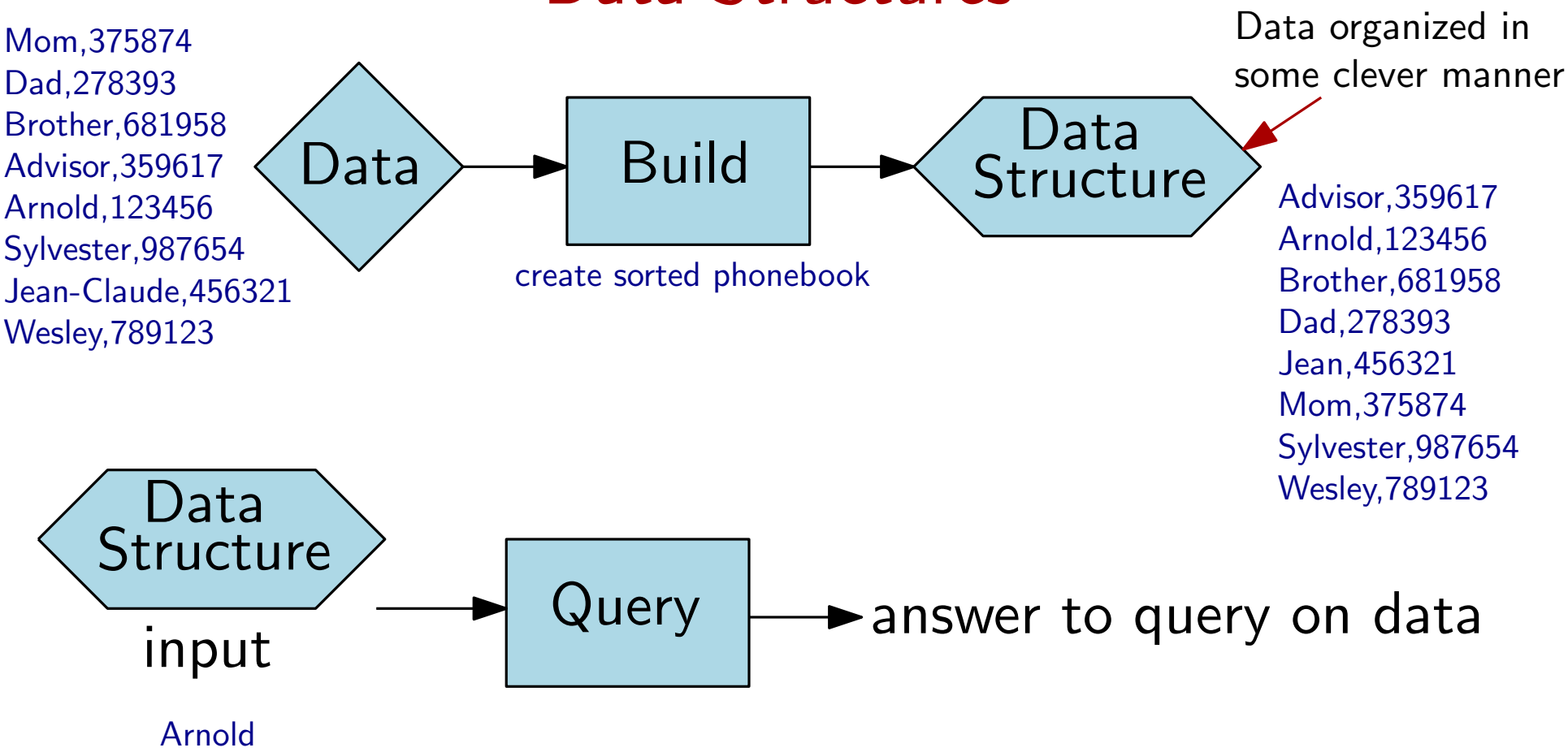

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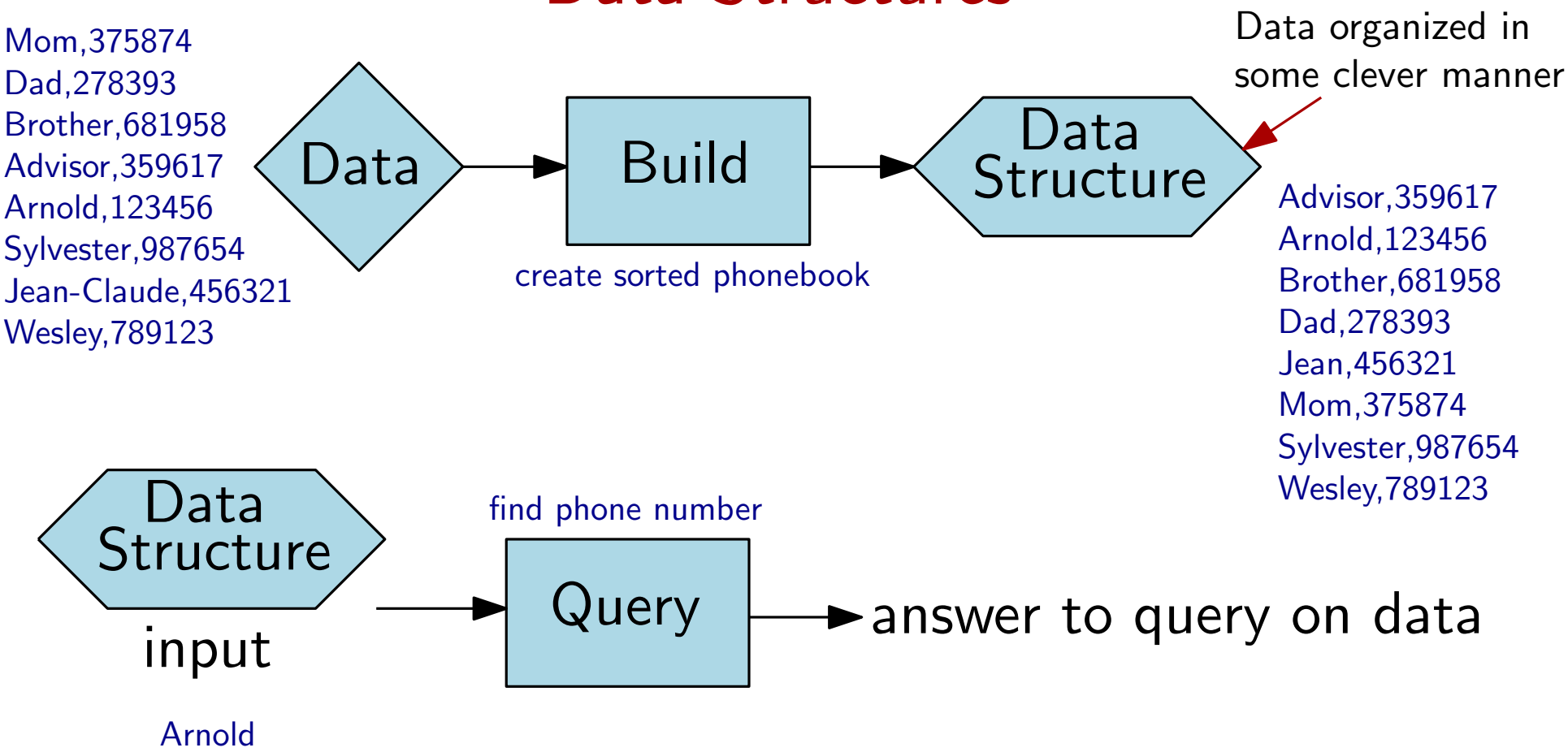

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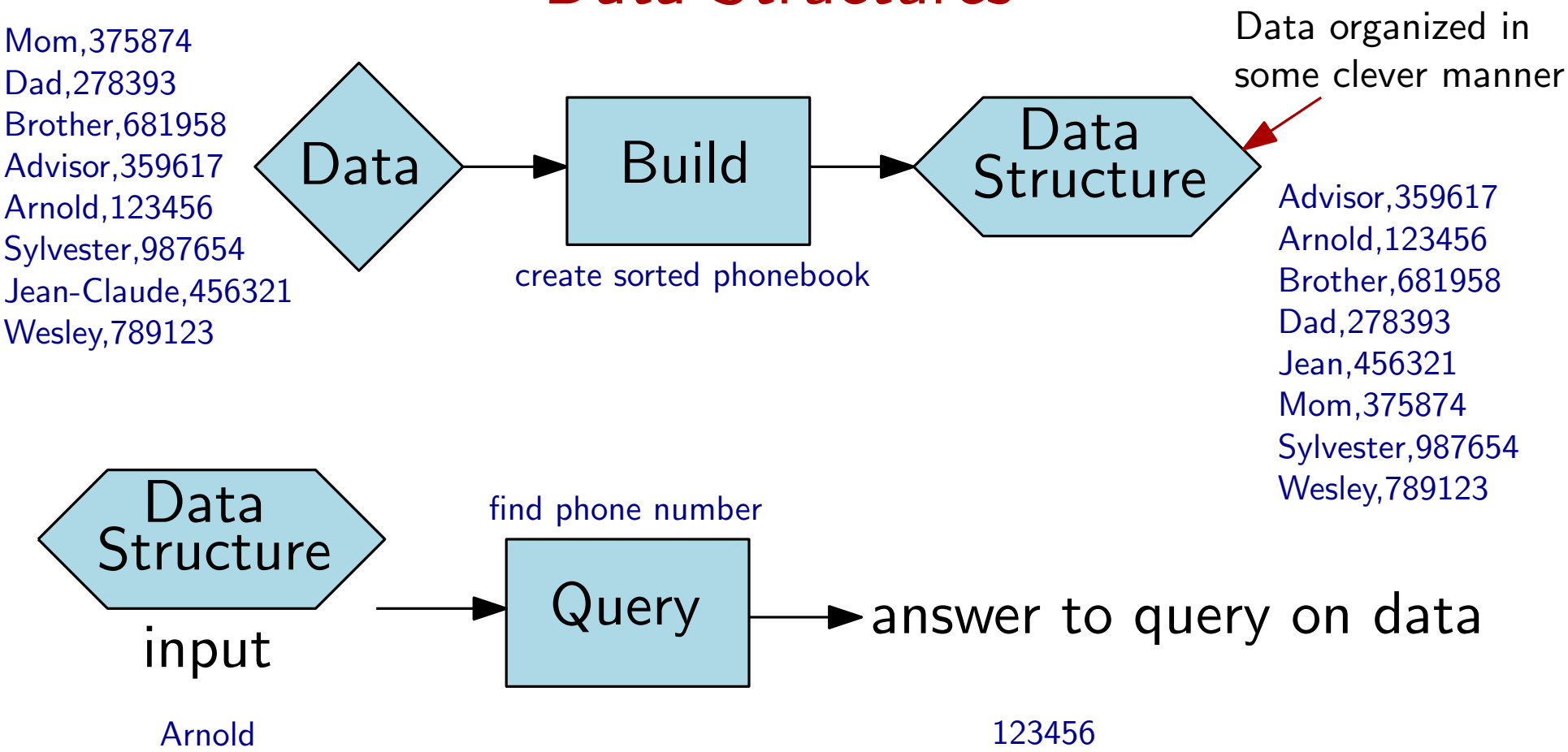

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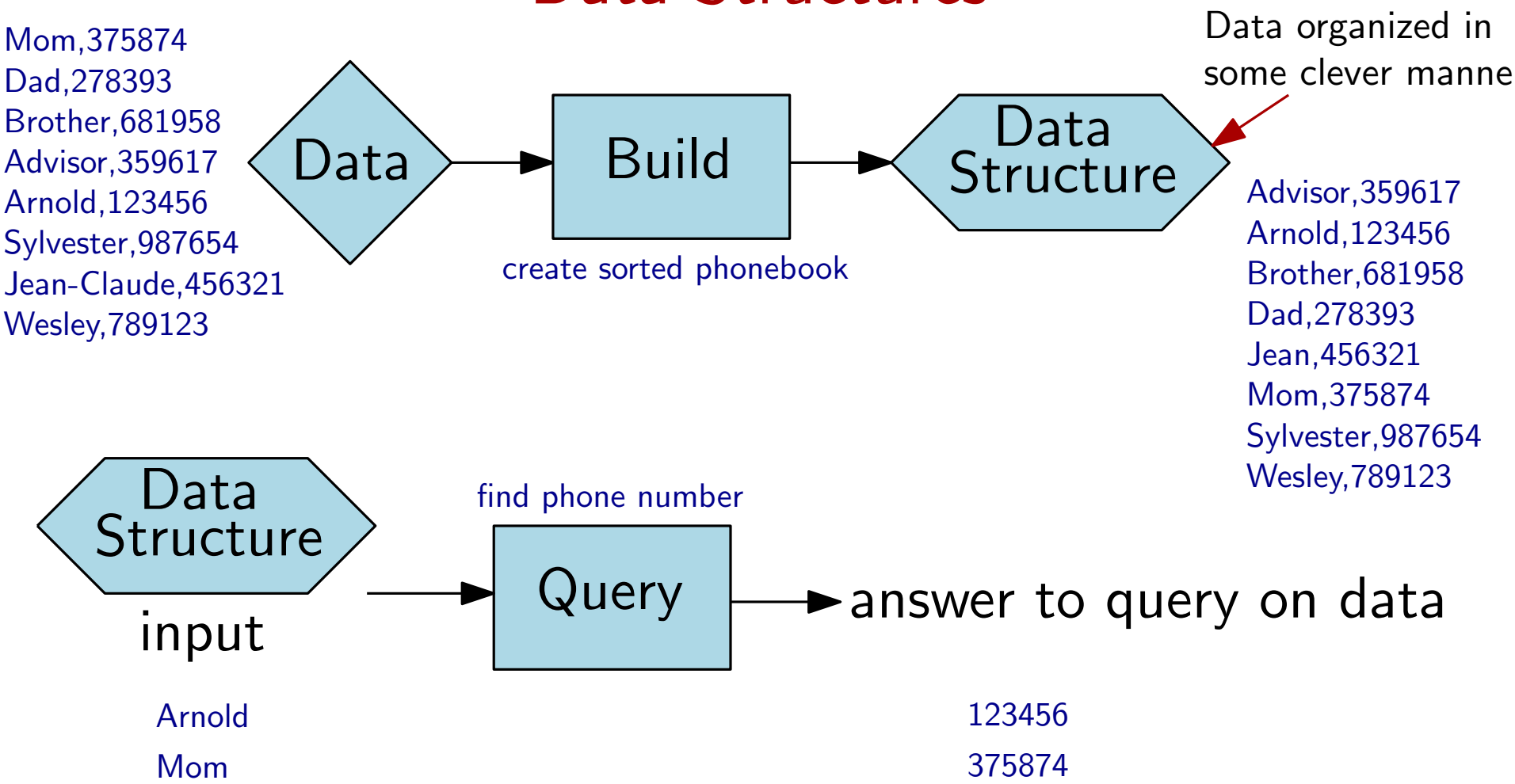

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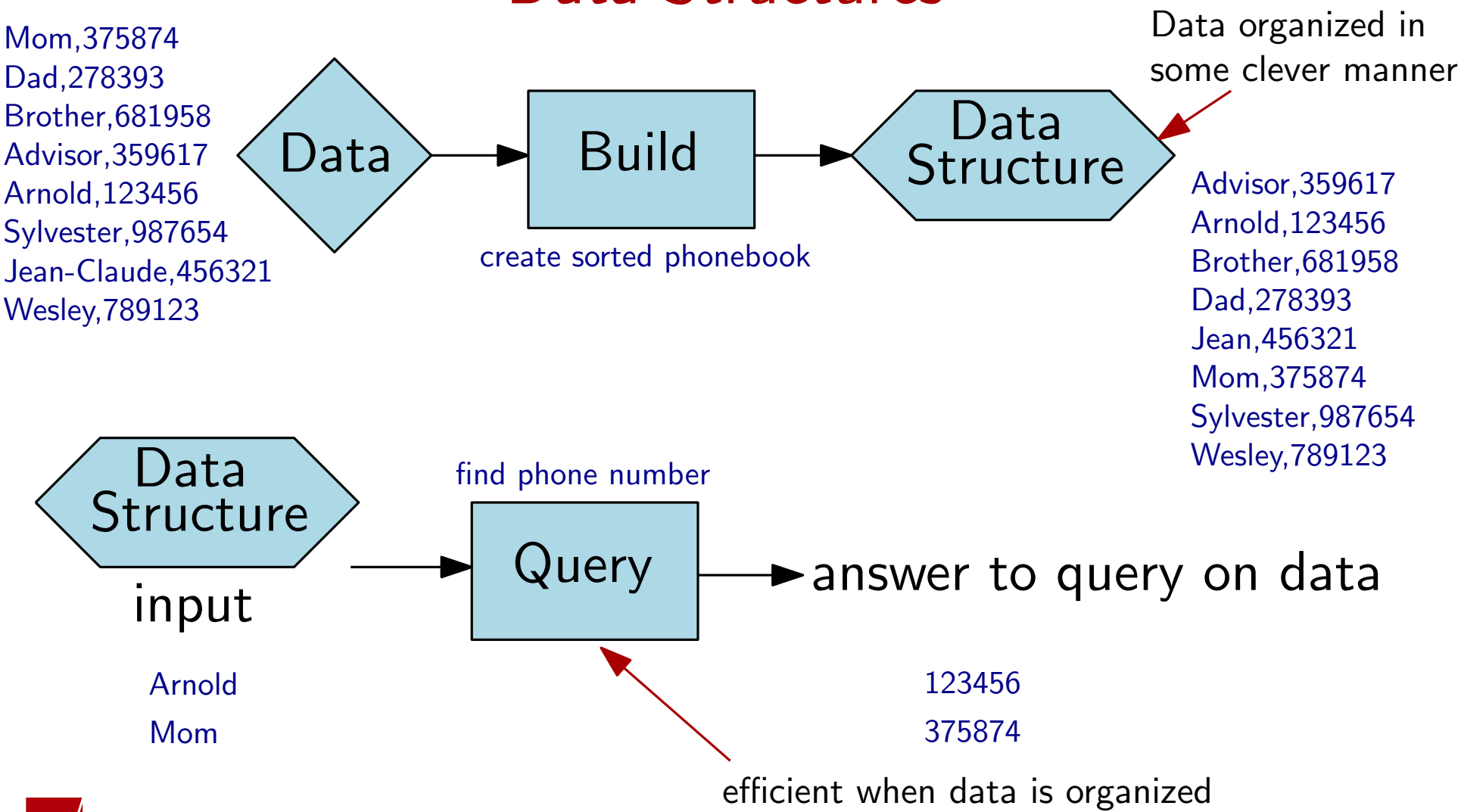

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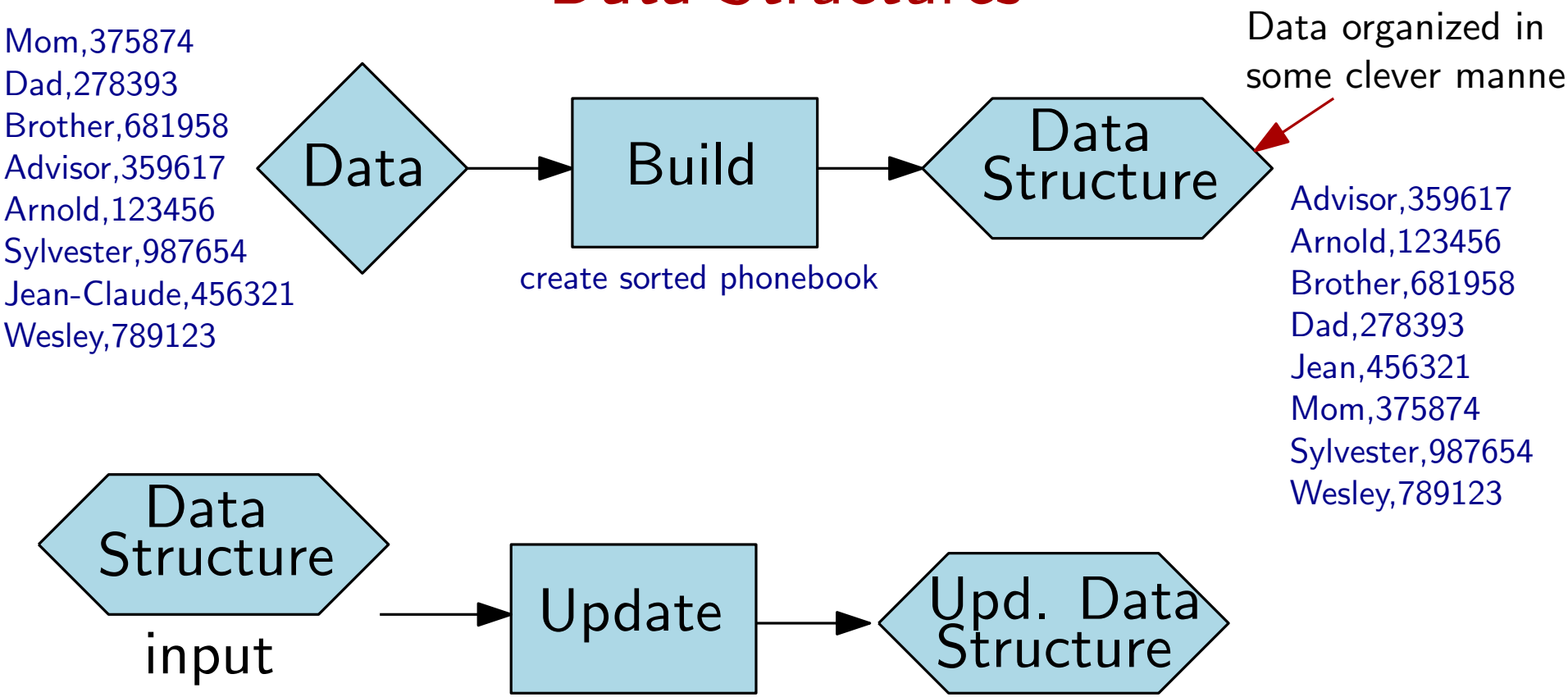

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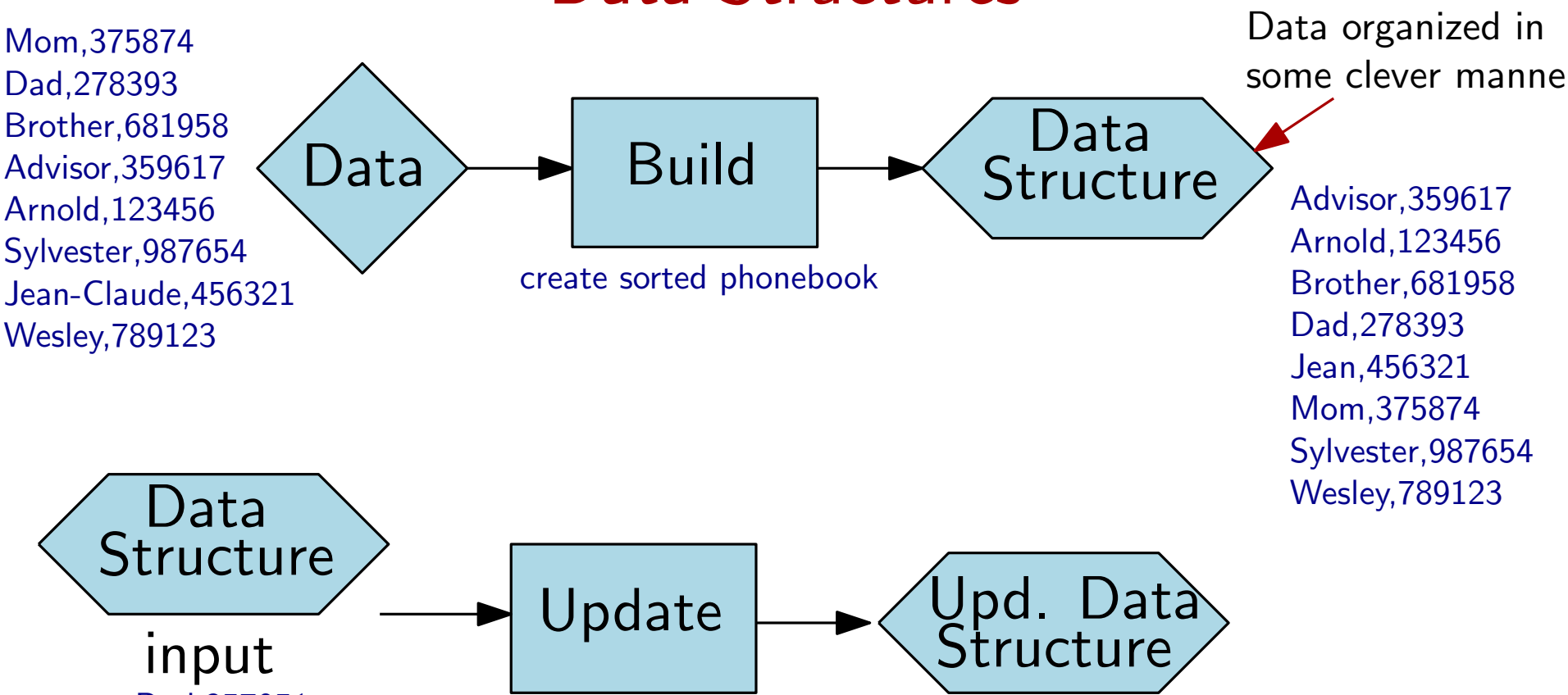

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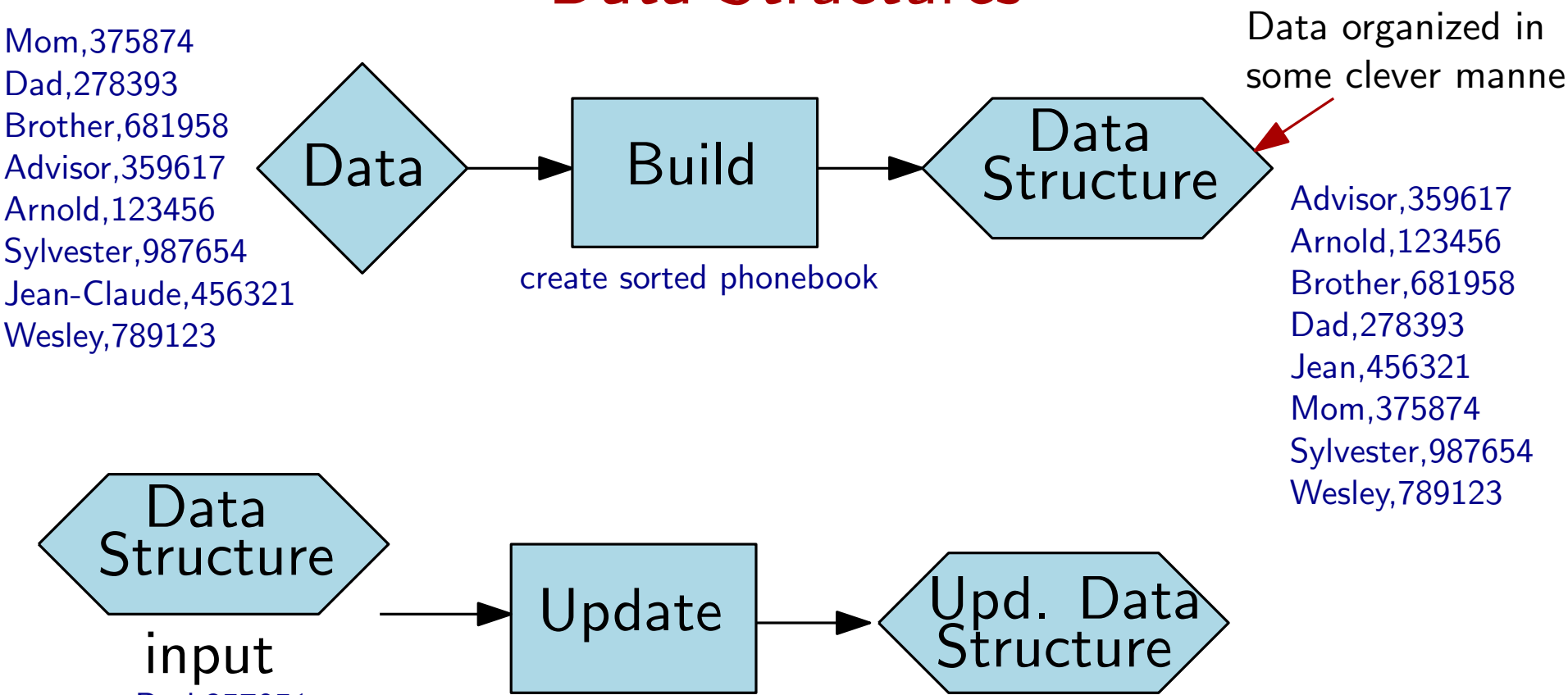

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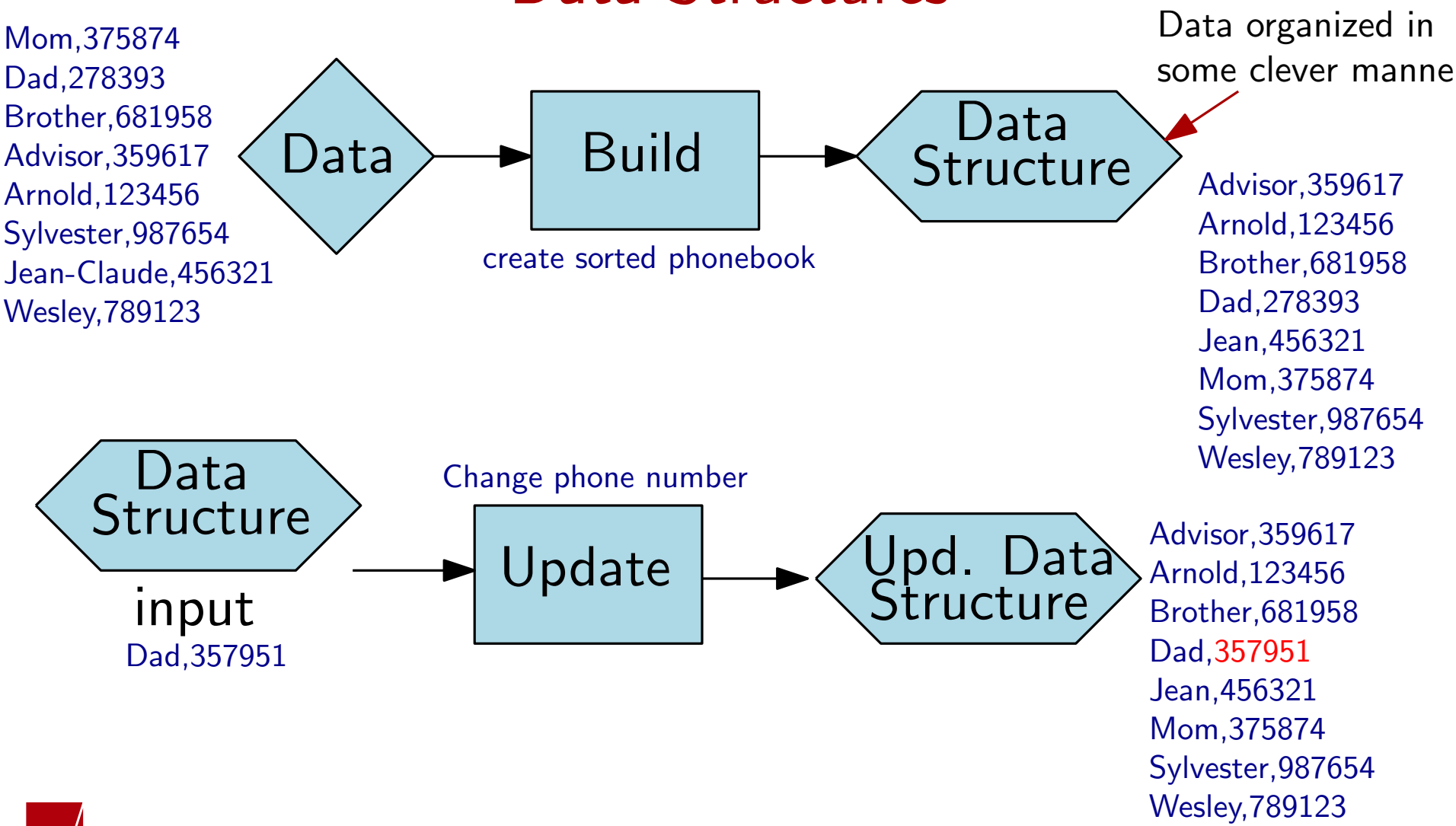

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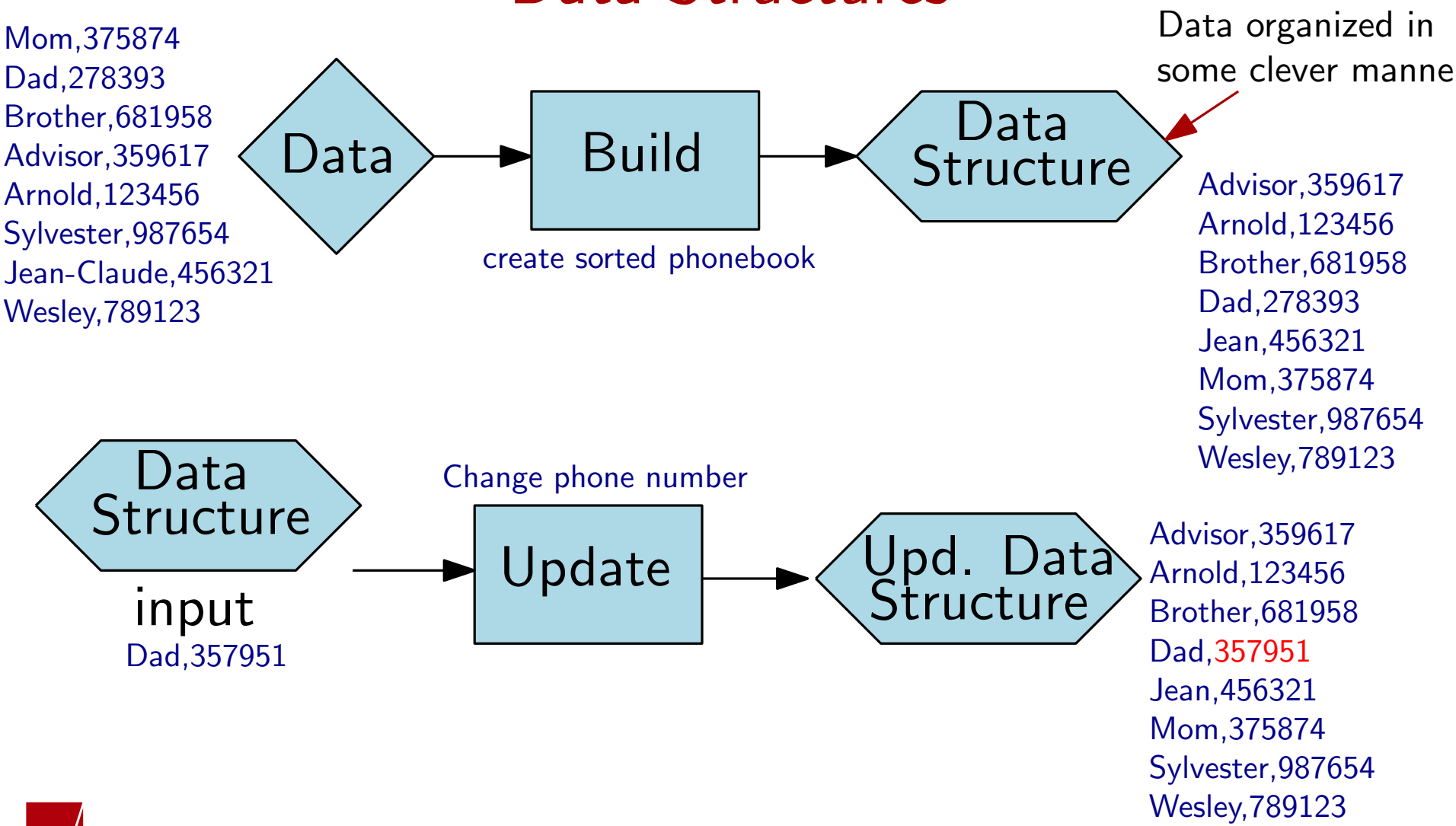

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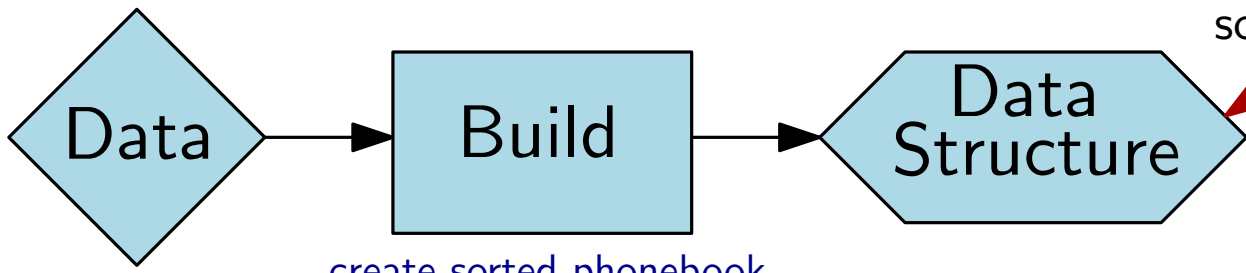

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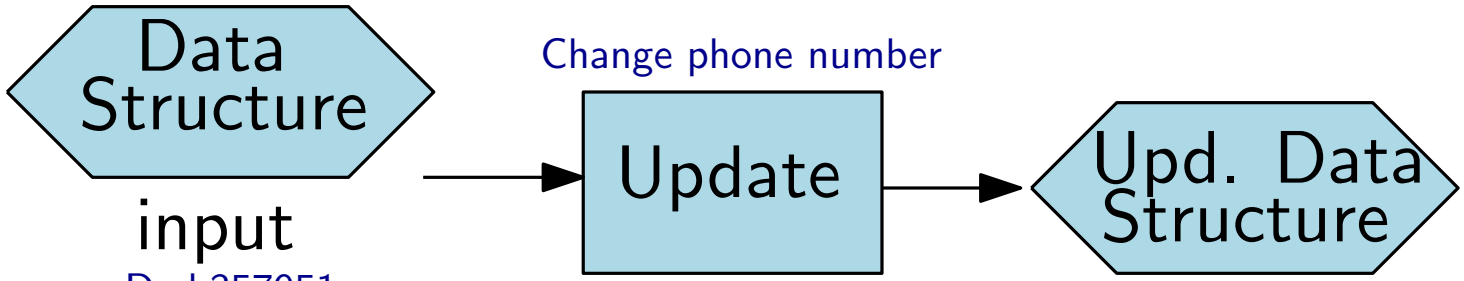
We want space efficient structures, fast queries, and fast updates



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Primary goal: space efficiency

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Data Structure has n comparable elements, laid out in an array of size n.

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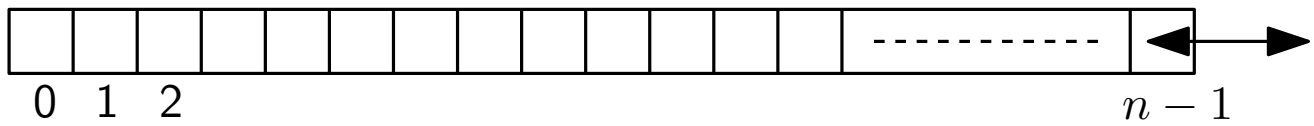

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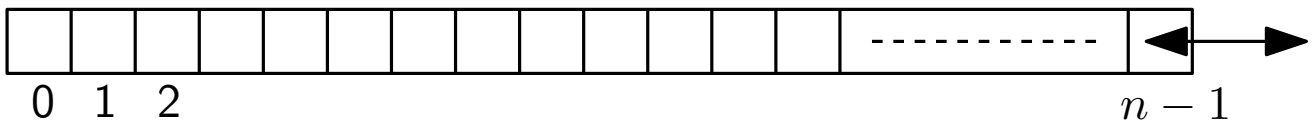

Only the array and n is maintained between operations.

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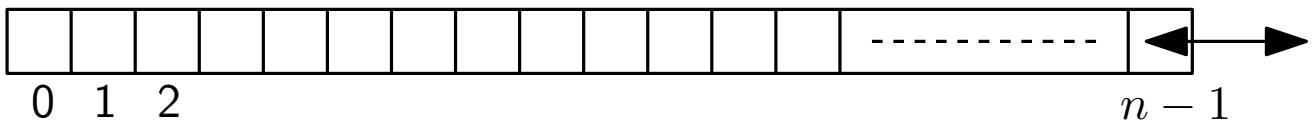

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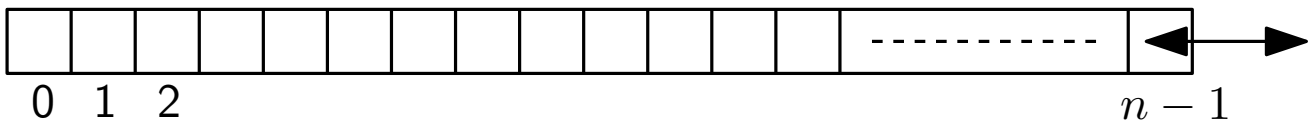

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Weak implicit model: Allow a constant number of extra memory cells

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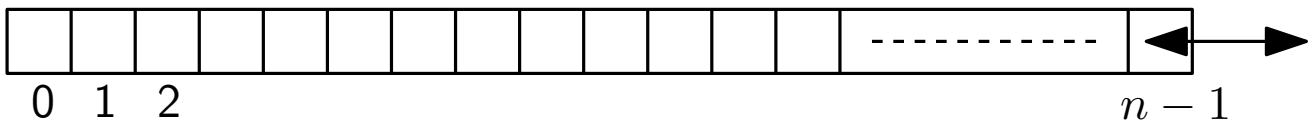

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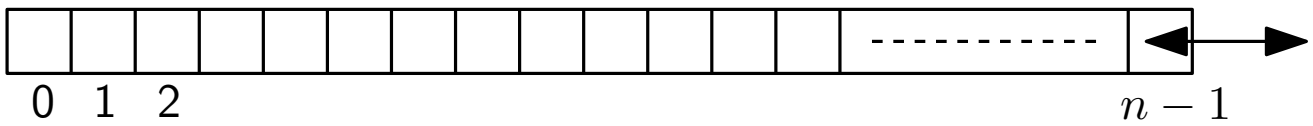

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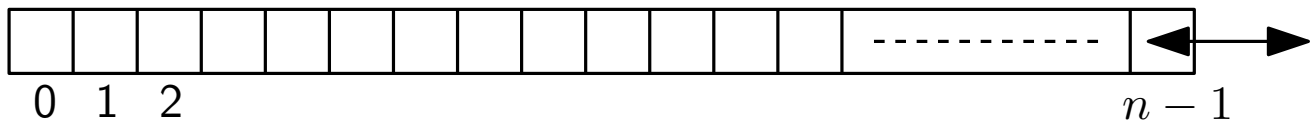

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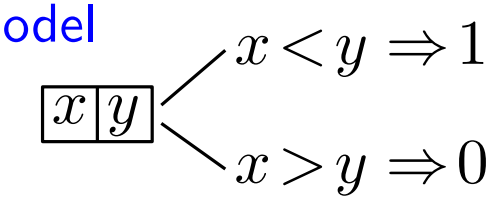

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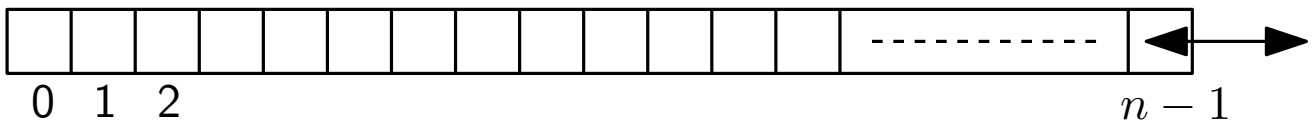

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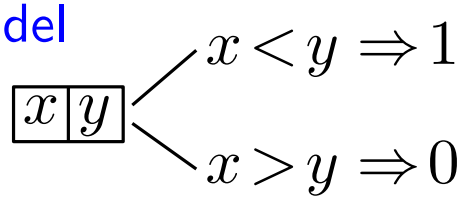

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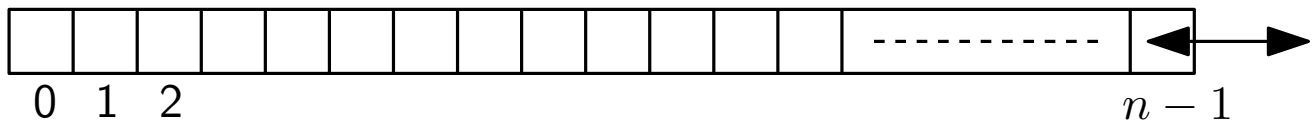

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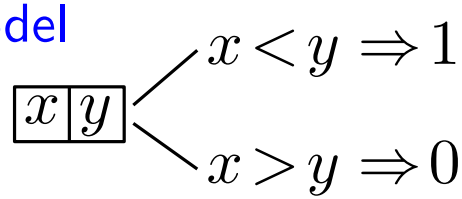

Procedures can use a constant number of working memory/registers

Dictionary and Fingersearch

Dictionary and Fingersearch

Maintain a (dynamic) set of n keys, while supporting:

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Insert(k)

Dictionary and Fingersearch

Maintain a (dynamic) set of n keys, while supporting:

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Dictionary and Fingersearch

Maintain a (dynamic) set of n keys, while supporting:

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Maintain a (dynamic) set of n keys, while supporting:

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Predecessor(k)

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Time: $O(\log n)$

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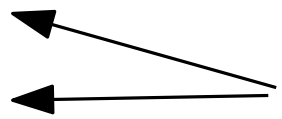

Only in dynamic structures

Special element: the finger element

Time: $O(\log n)$

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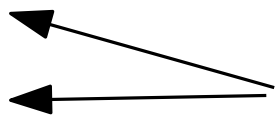
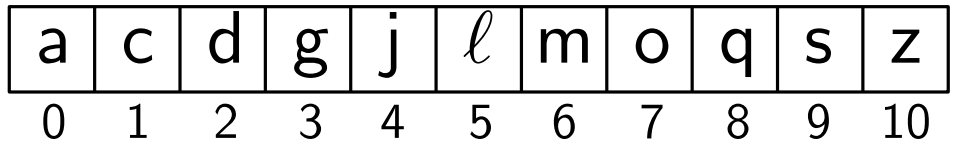


Only in dynamic structures

Special element: the finger element

Time: $O(\log n)$



Dictionary and Fingersearch

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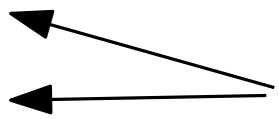

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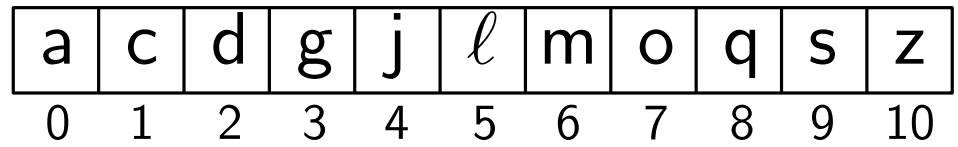

n=11

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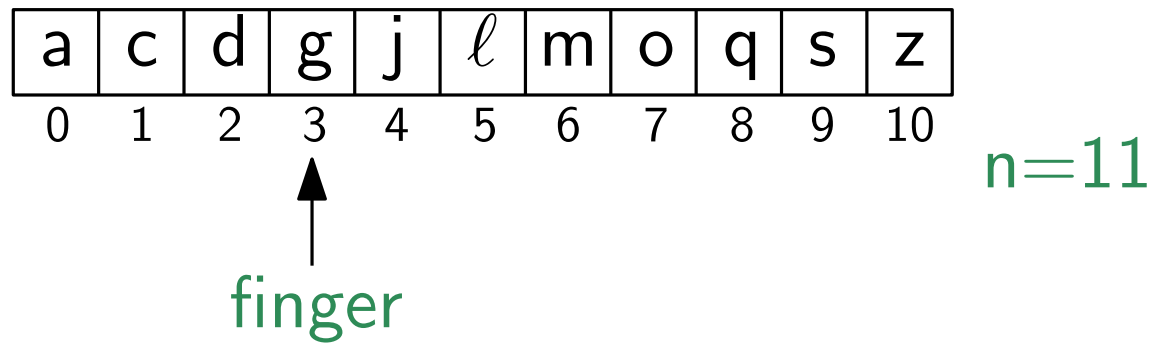

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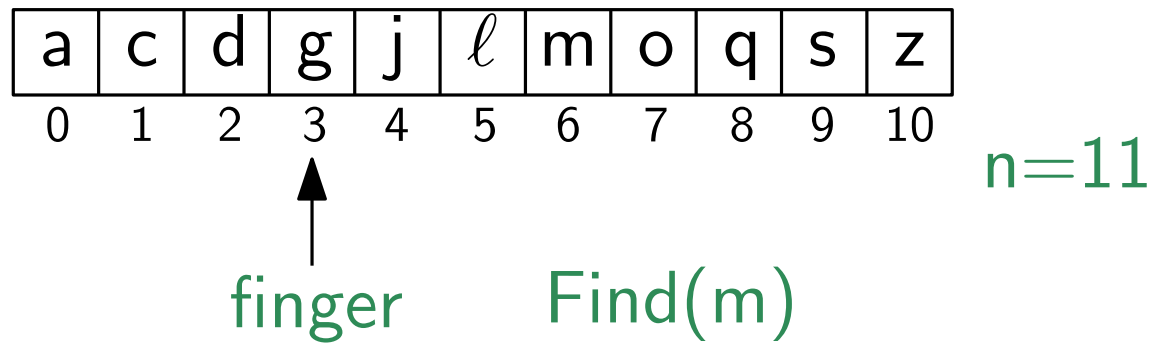

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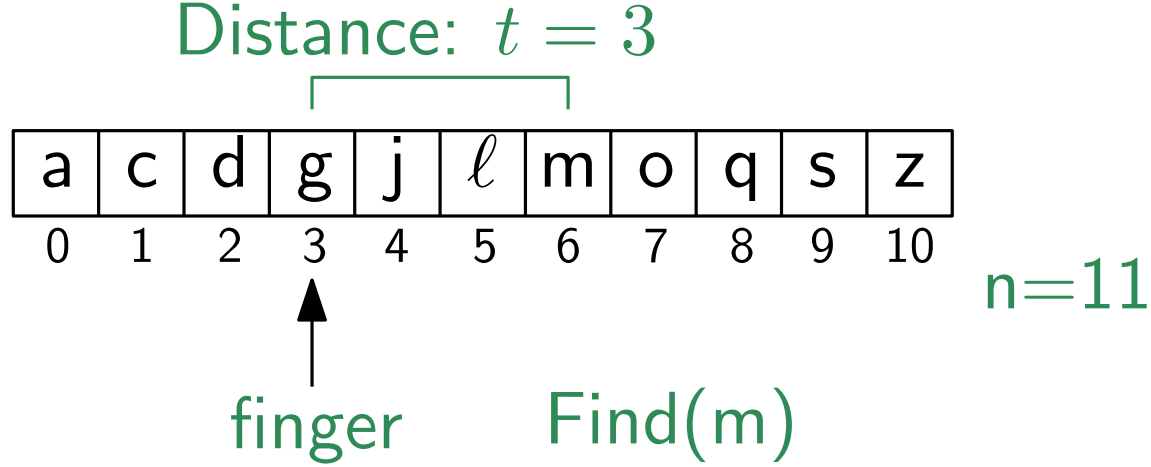

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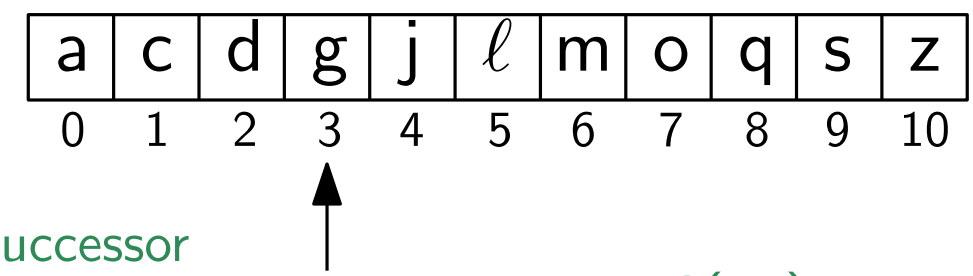
Distance: $t = 3$



Time: $O(\log t)$ for
Find, Predecessor, and Successor

Implicit Finger Search Idea

Implicit Finger Search Idea

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2^{2^0} 2^{2^1} 2^{2^2} 2^{2^ℓ}

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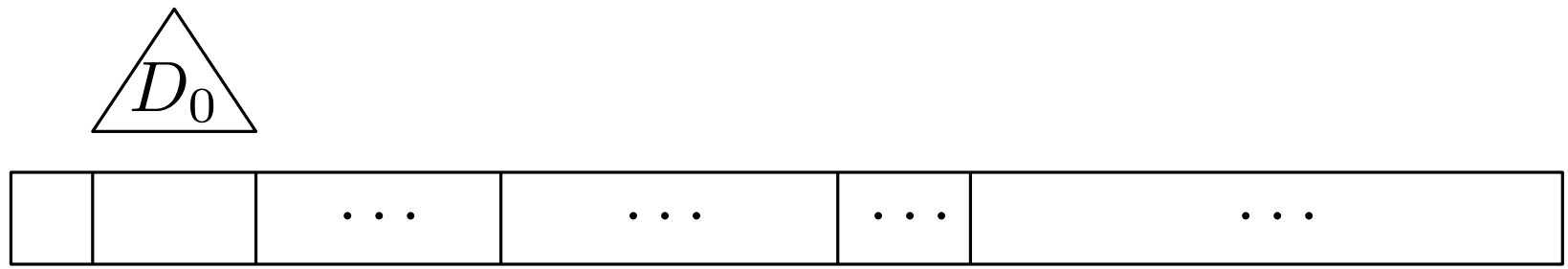

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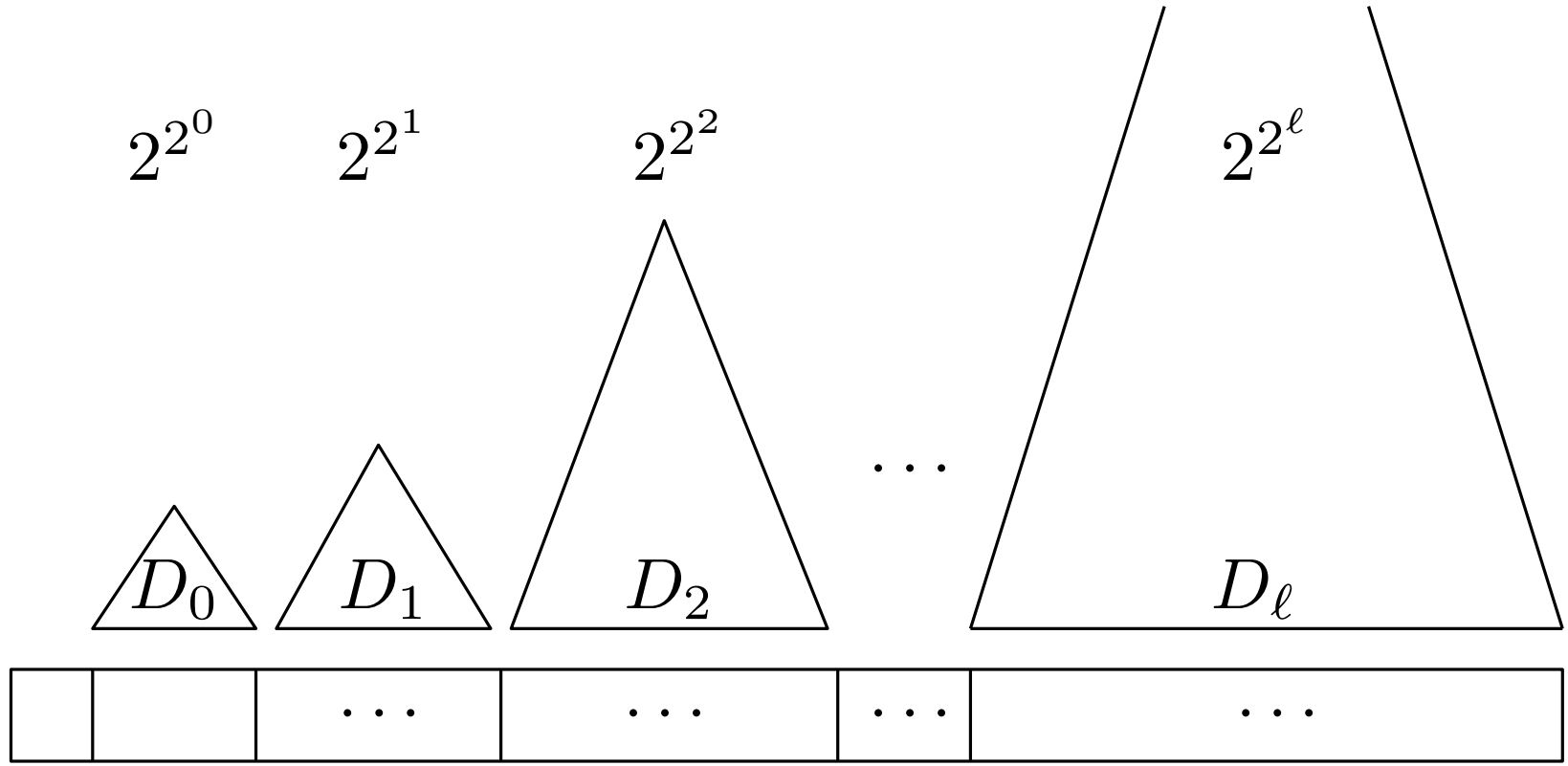

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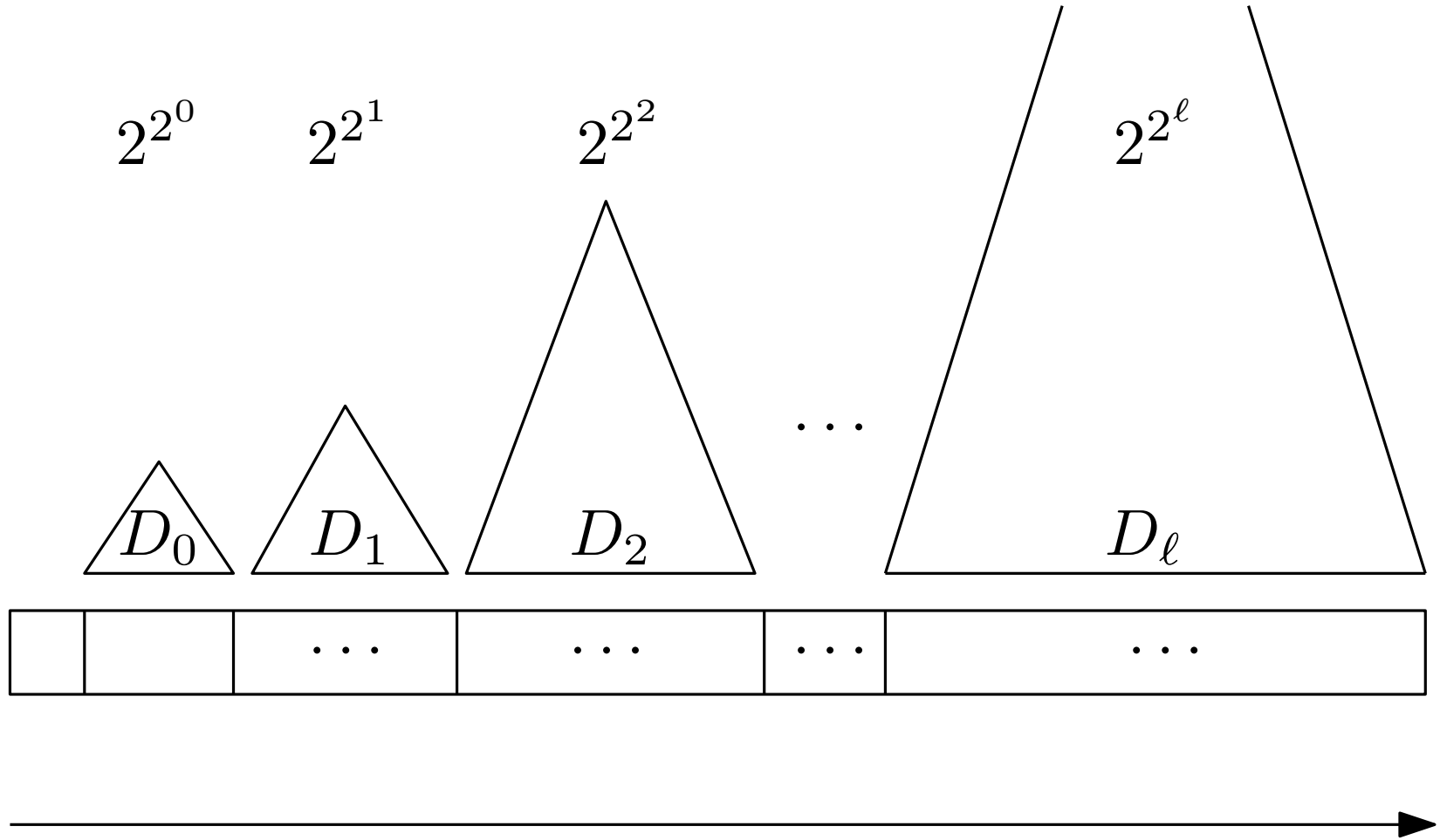

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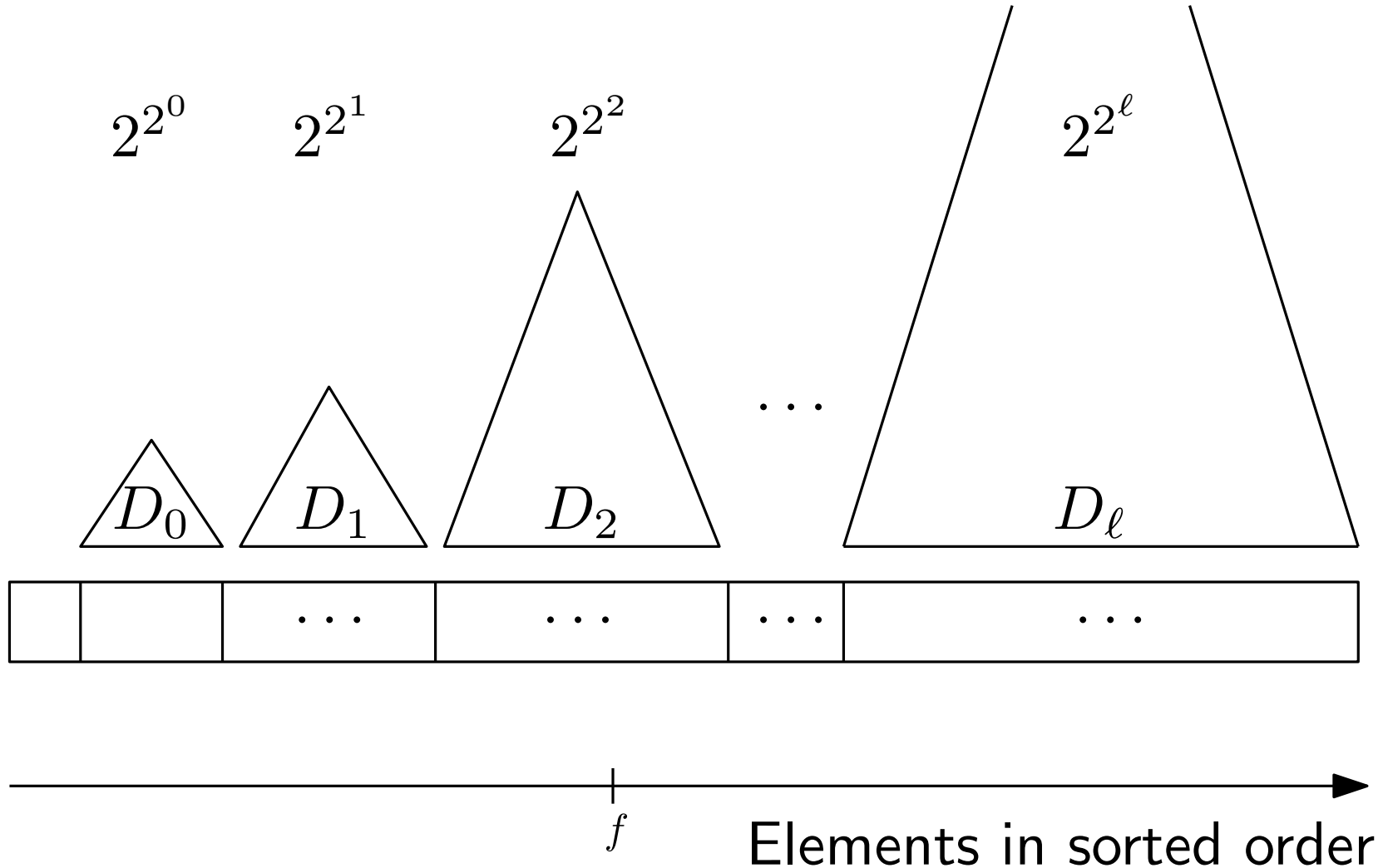

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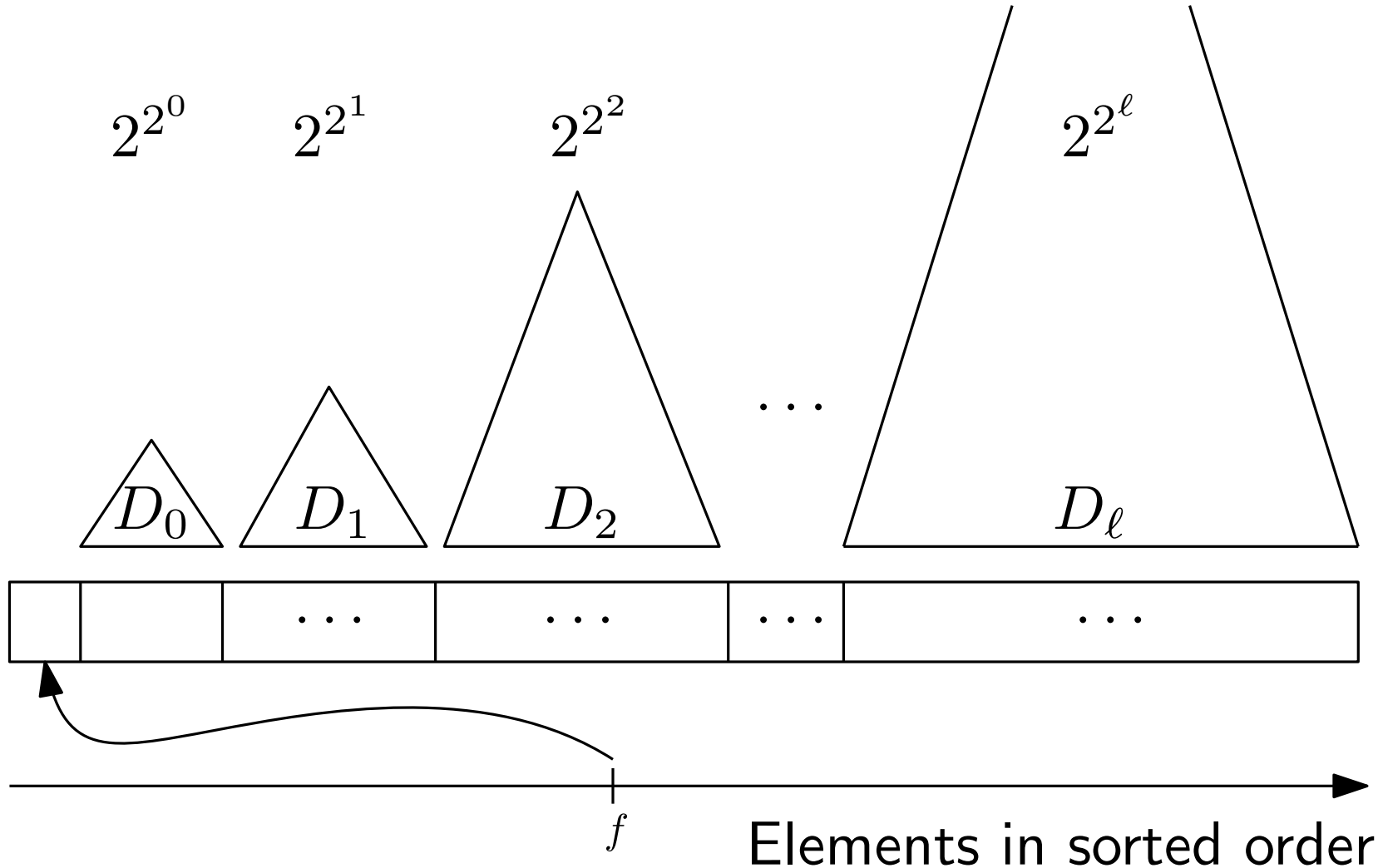

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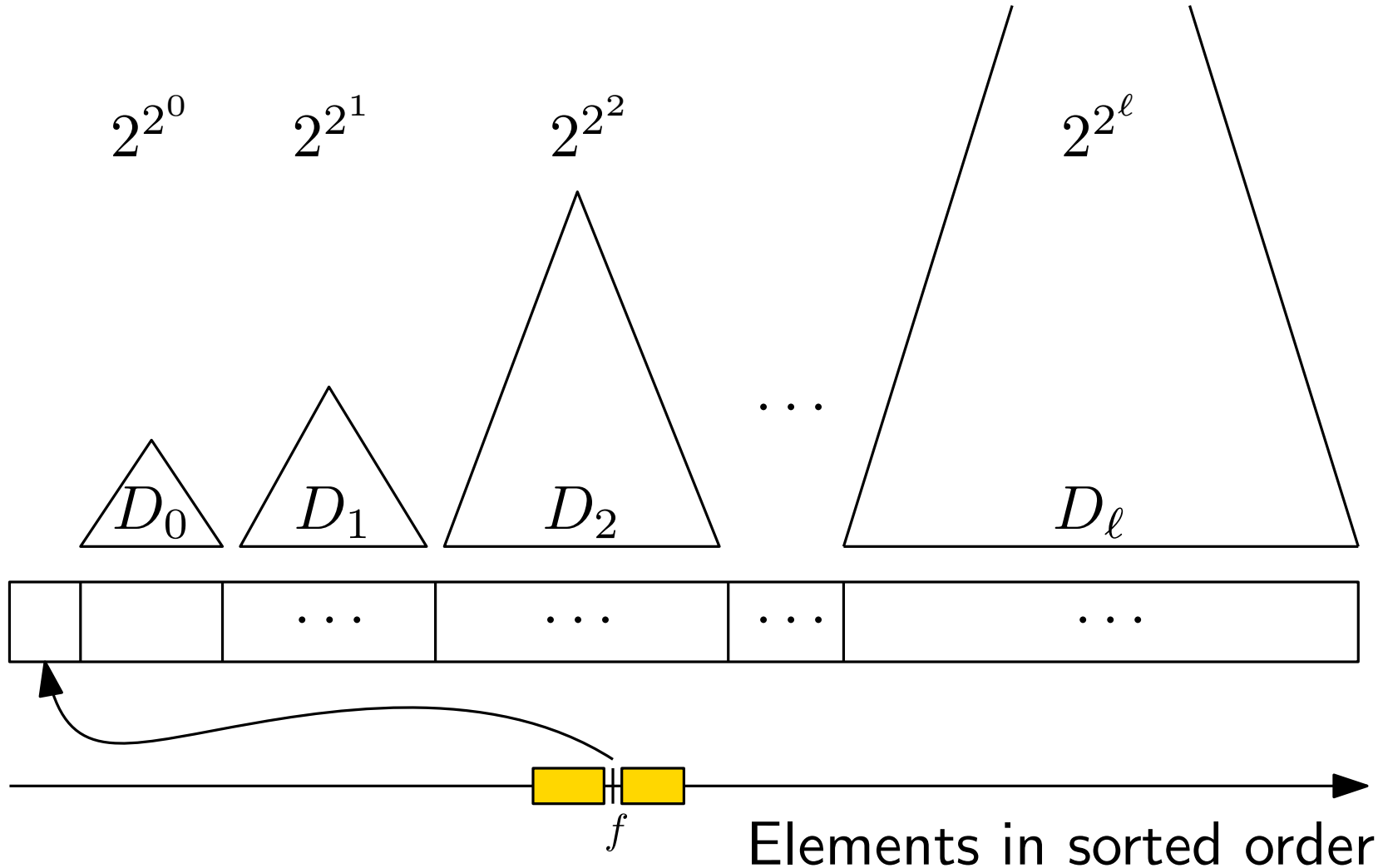

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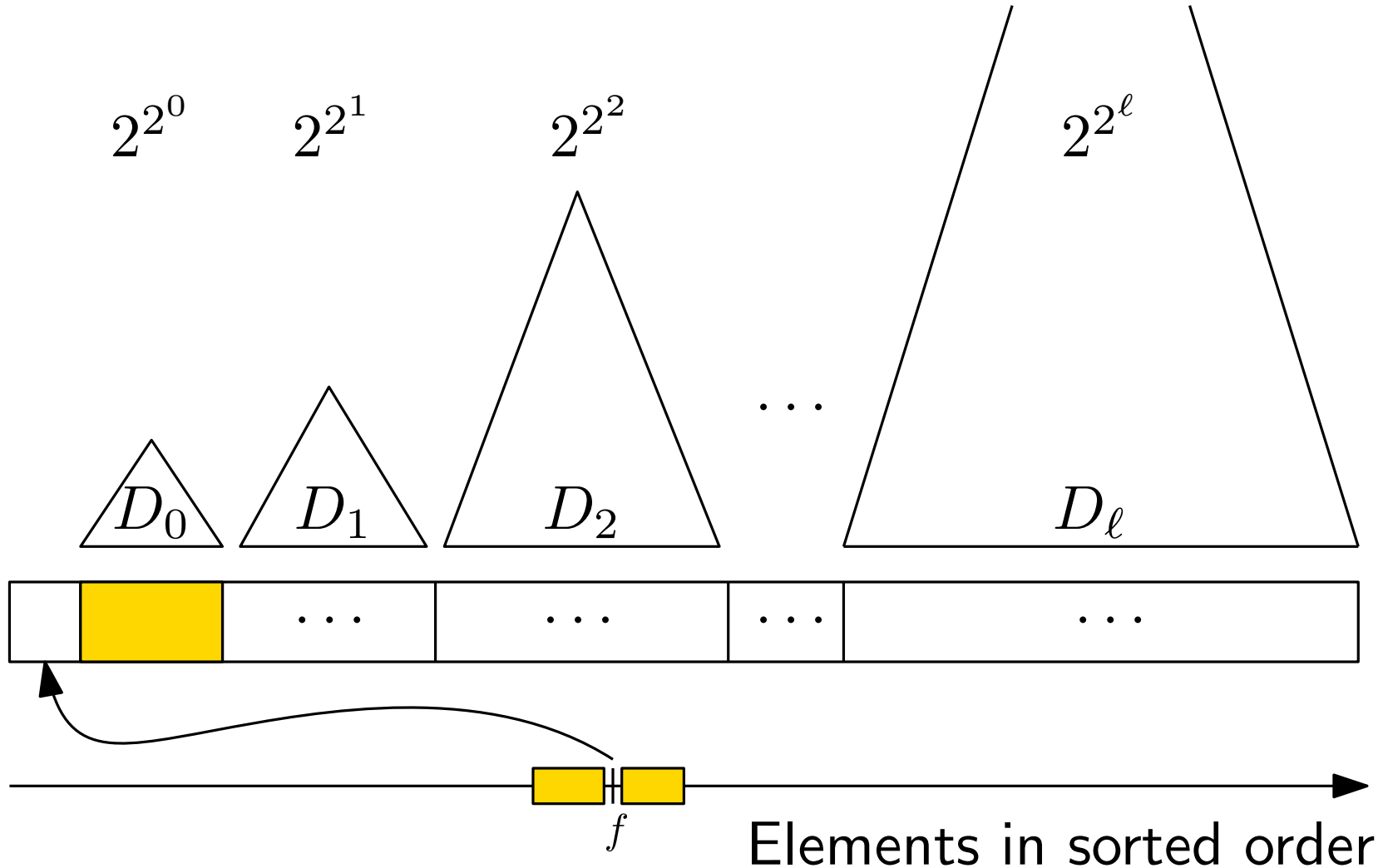

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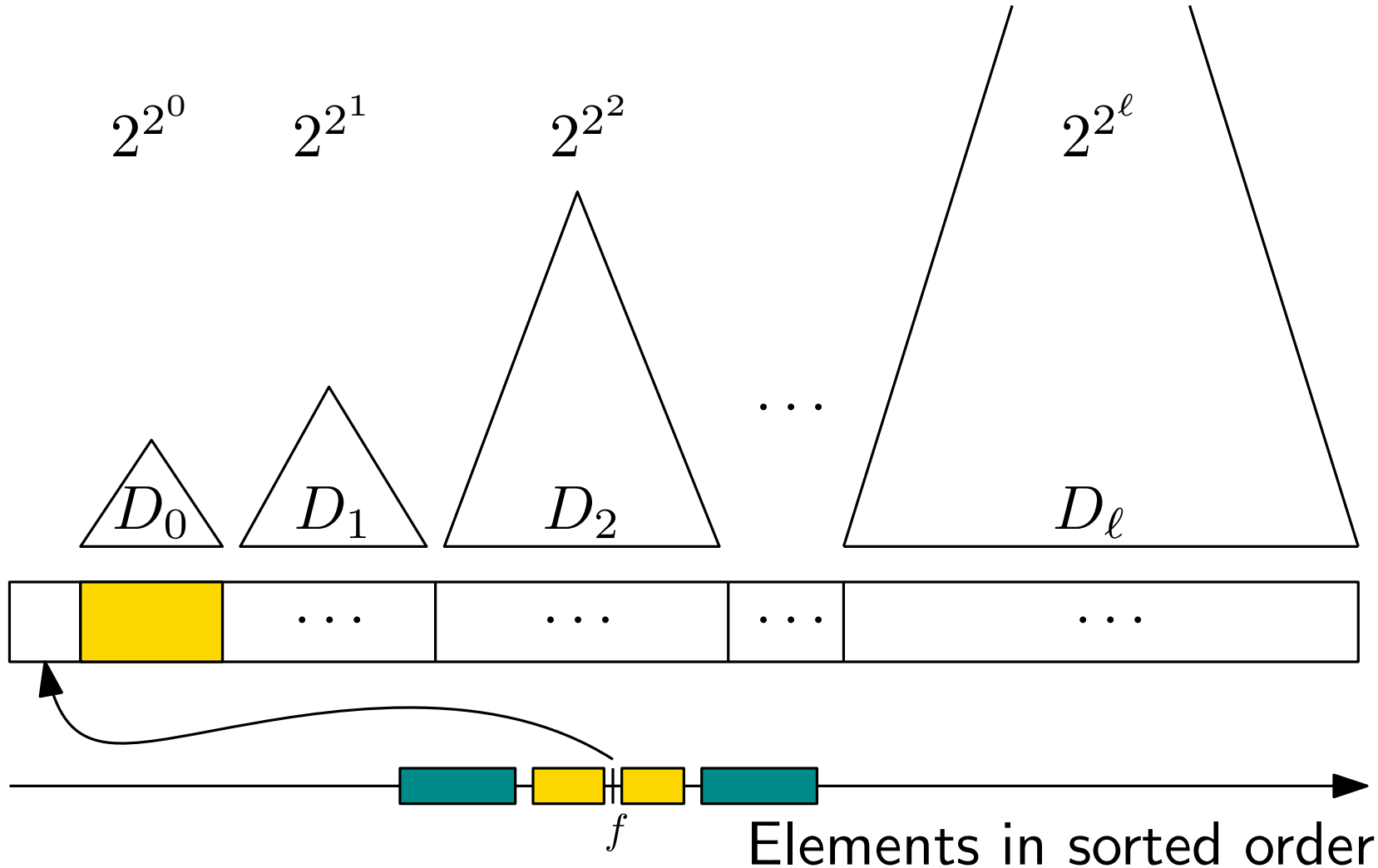

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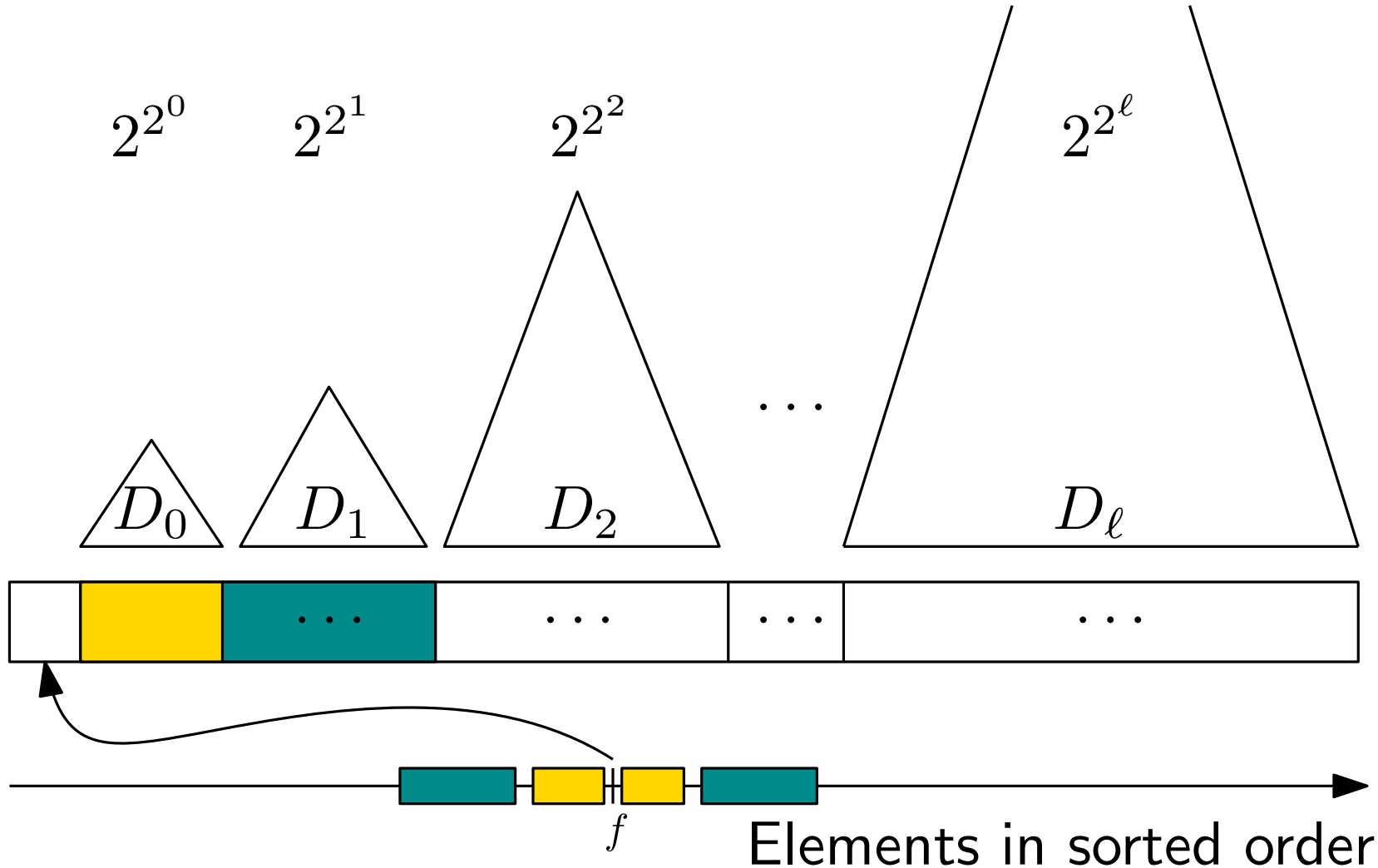

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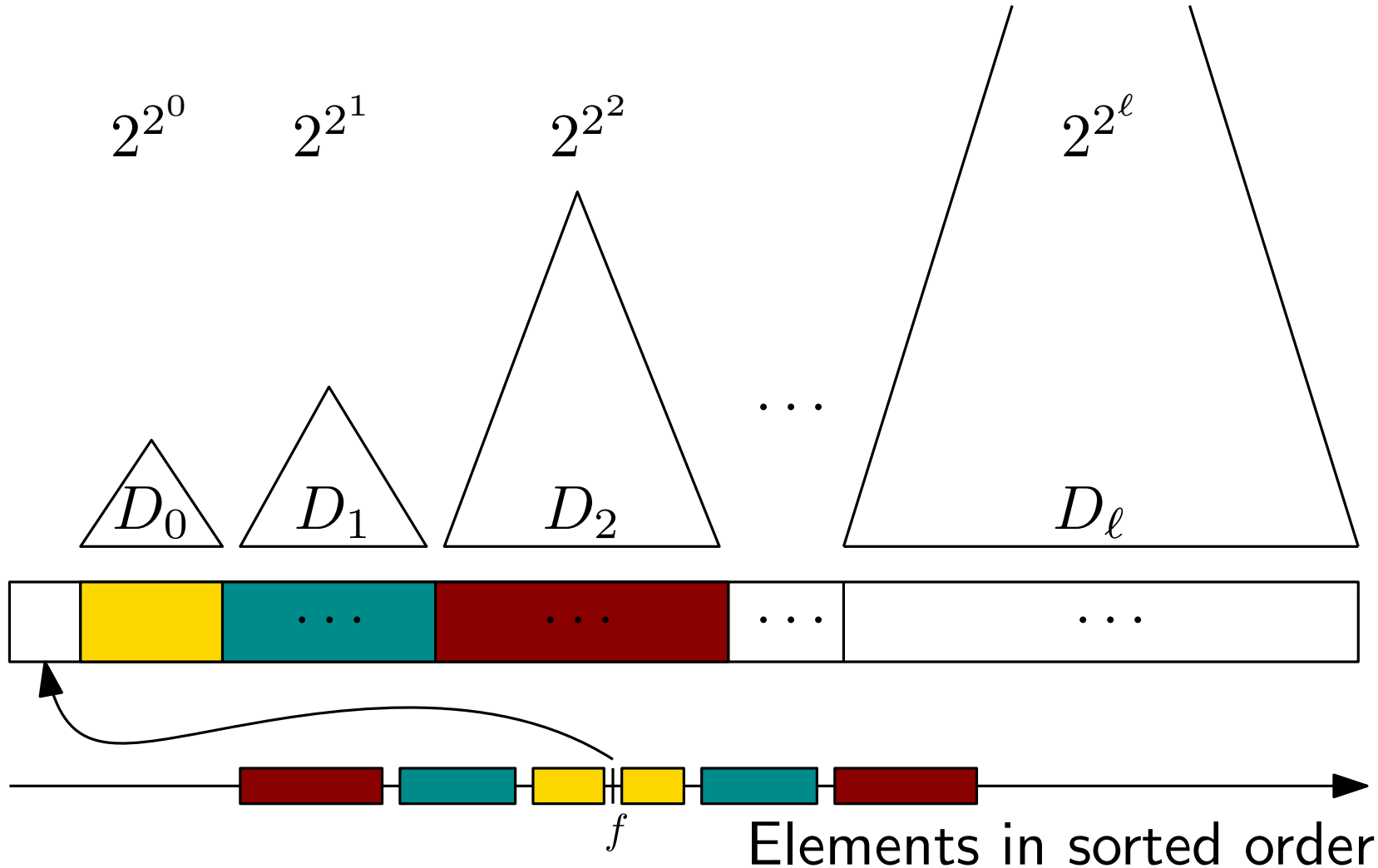

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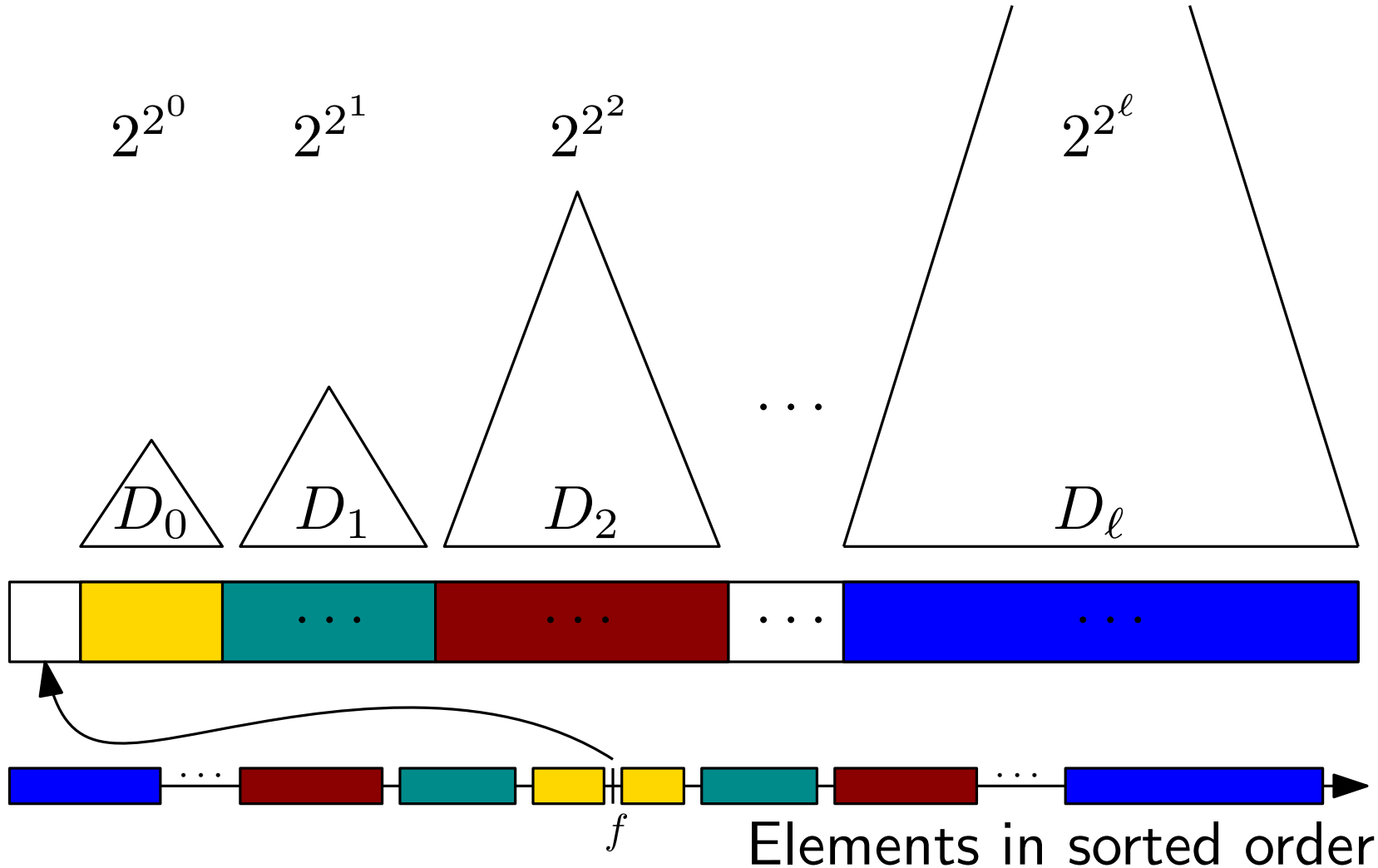
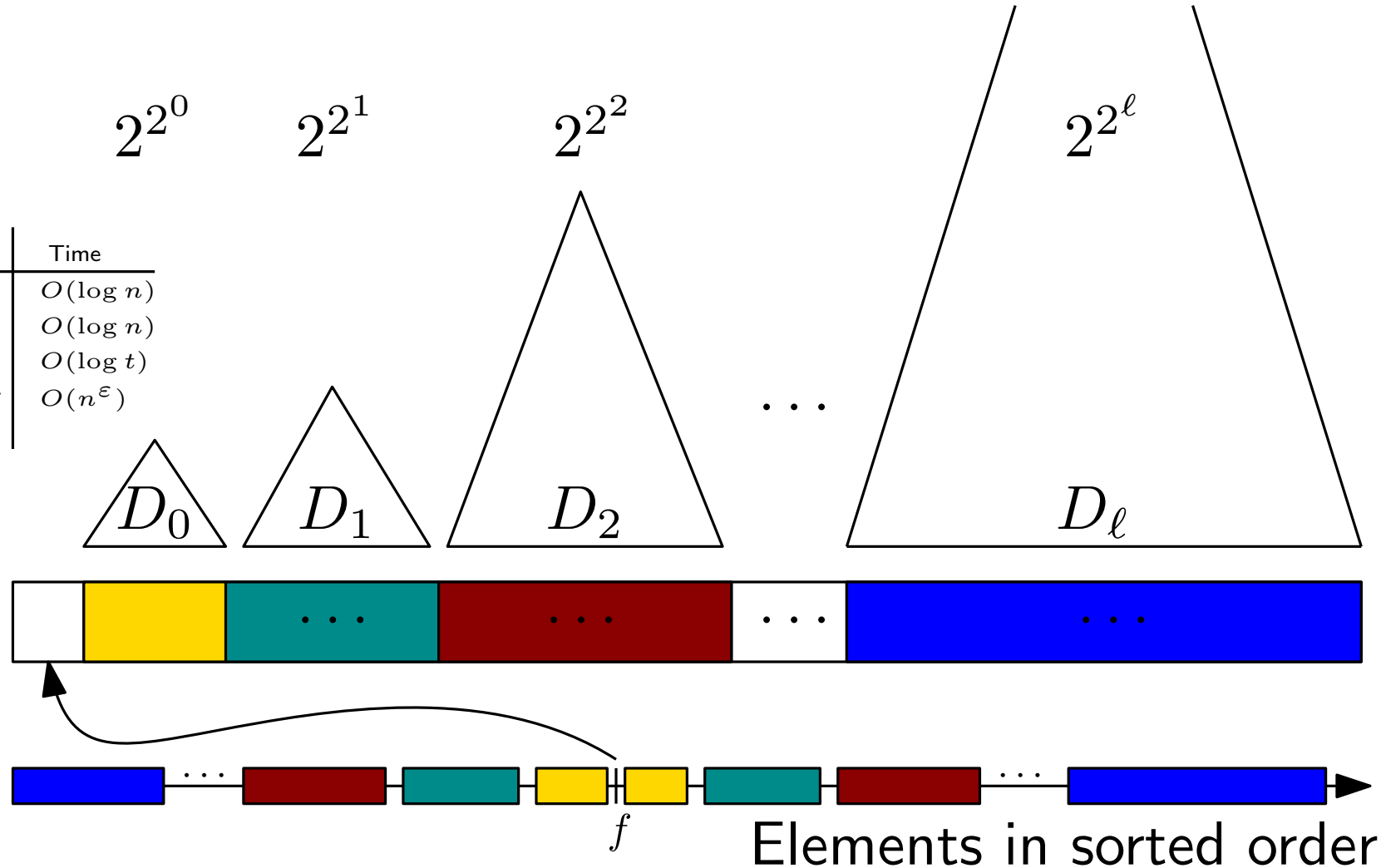


Implicit Finger Search Idea

Operation	Time
Insert	$O(\log n)$
Delete	$O(\log n)$
Find	$O(\log t)$
Change-Finger	$O(n^{\varepsilon})$



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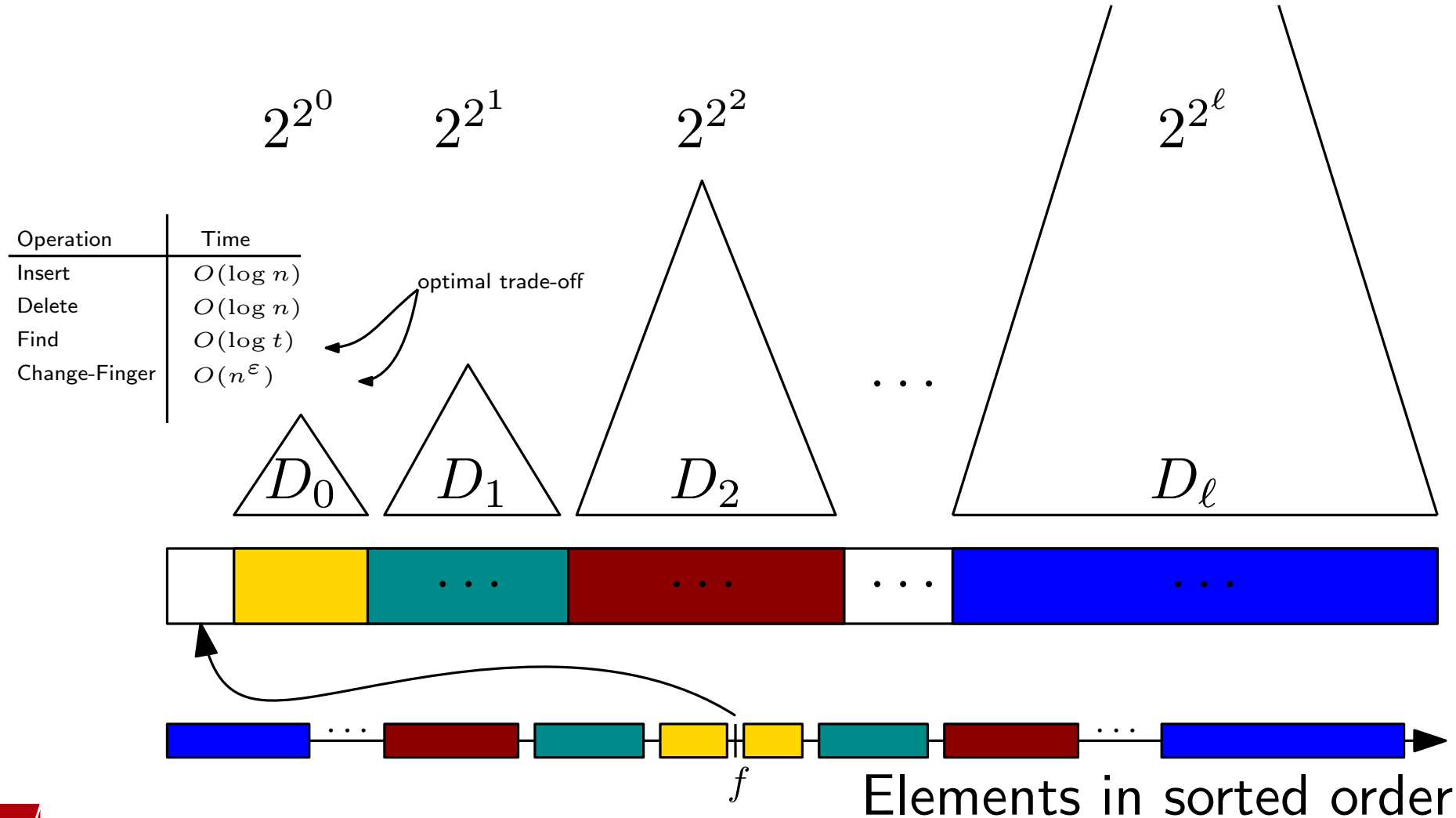

Priority Queues

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Amortized bounds are marked with $\star$

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Comparison based sorting: only allowed operation $\leq$

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Depends on accuracy of the scale

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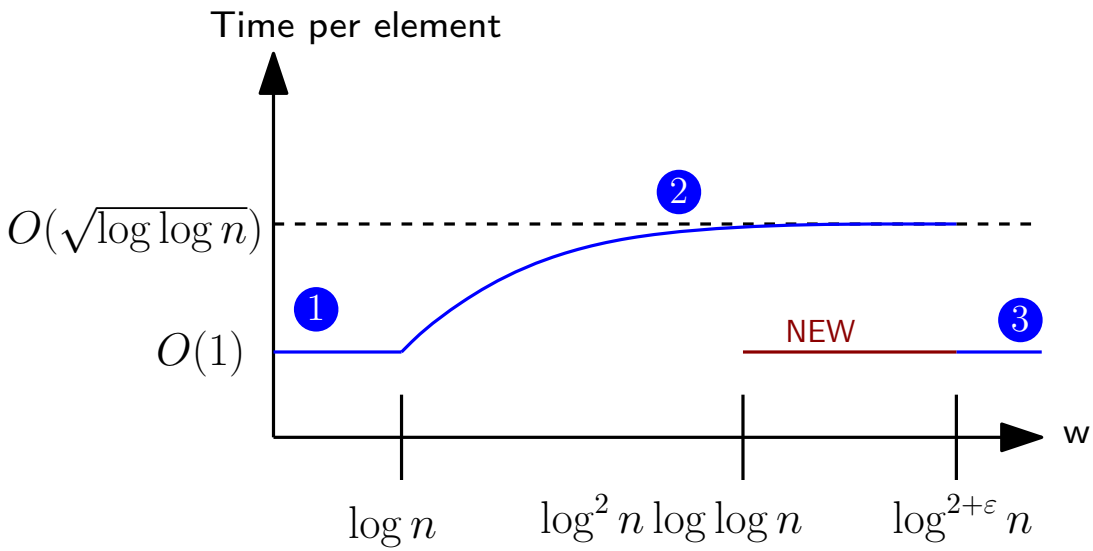

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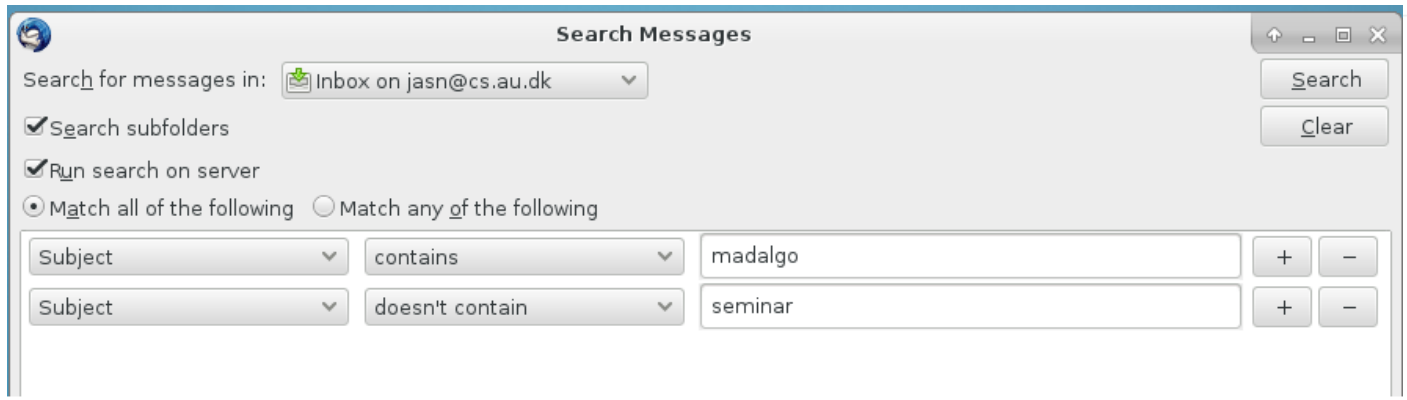

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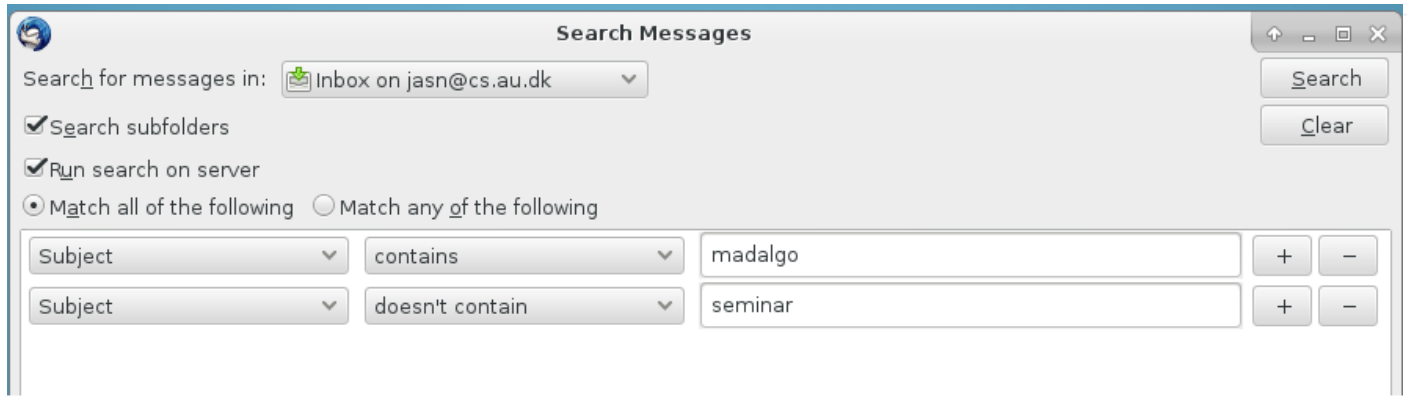

Find all emails sent to madalgo that are not seminars

Text Indexing - Two Patterns

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(The two-pattern problem)

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preprocess and store $\mathcal{D}$ such that

when given two text strings P_1, P_2 (online)

we can report all d_i where both P_1 and P_2 occur

(The two-pattern problem)

we can report all d_i where both P_1 occurs but P_2 does not occur

Text Indexing - Two Patterns

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(The forbidden pattern problem)

Text Indexing - Two Patterns

Problem	Space	Query Time	Authors

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Is $\sqrt{n}$ query time really necessary?

Text Indexing - Two Patterns

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Build suffix tree on concatenation of the documents

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$$S = d_1\$d_2\$\cdots\$d_D\$$$

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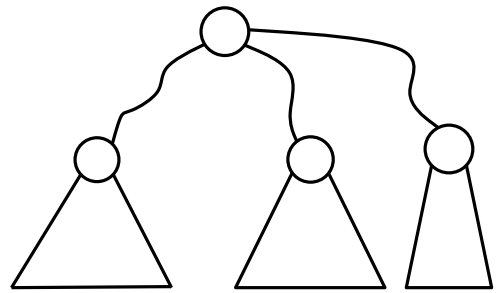

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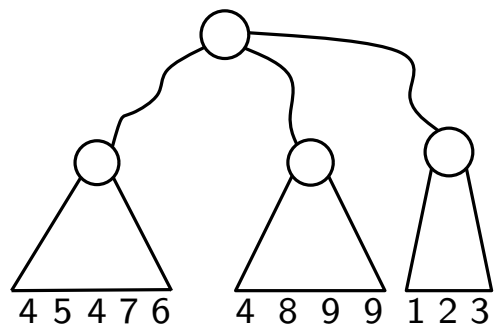

Annotate each leaf with the document id the suffx starts in

Text Indexing - Two Patterns

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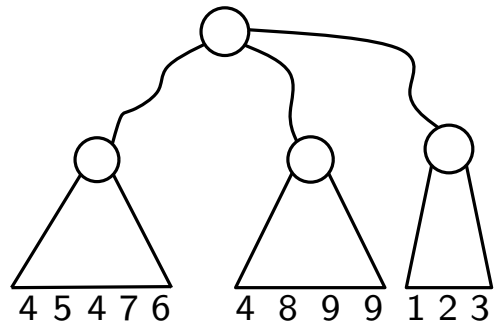

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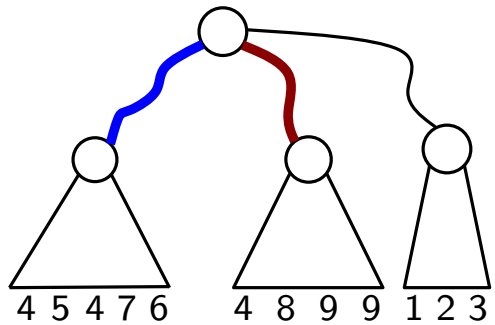

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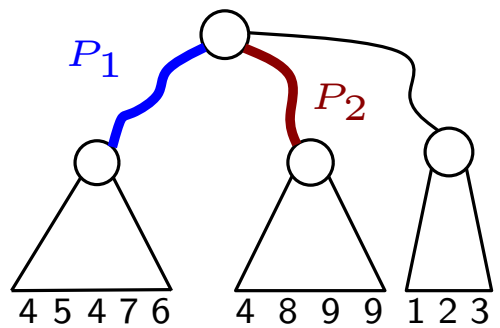

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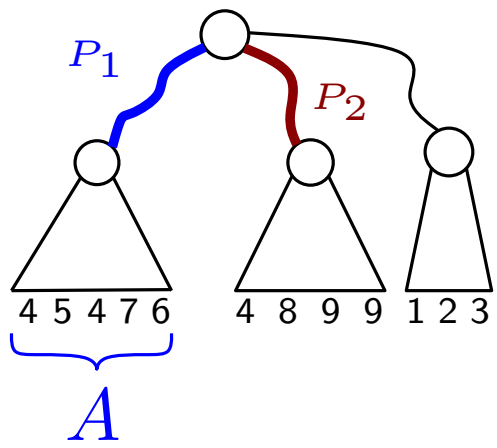

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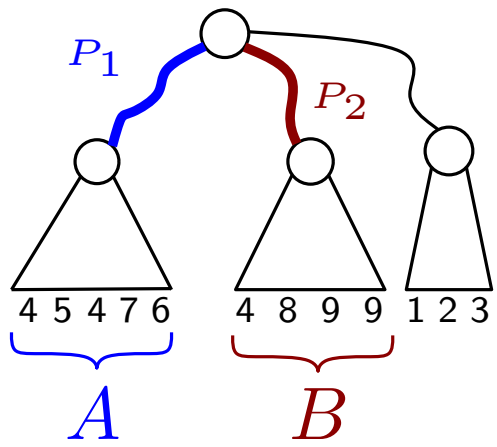

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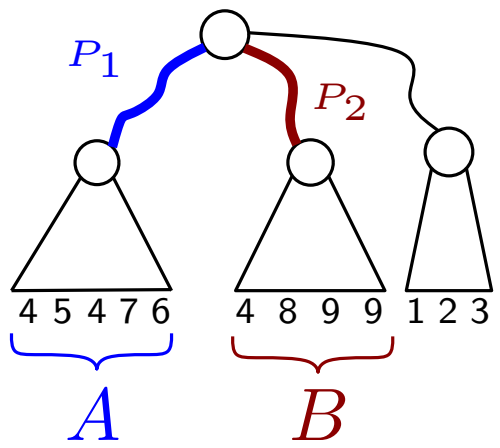

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Query: P_1 and P_2

Report: $A \cap B$

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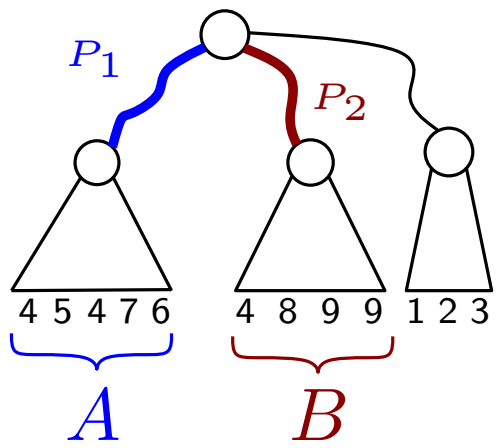

Annotate each leaf with the document id the suffx starts in

Query: P_1 and P_2

Report: $A \cap B = \{4\}$

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Build suffix tree on concatenation of the documents

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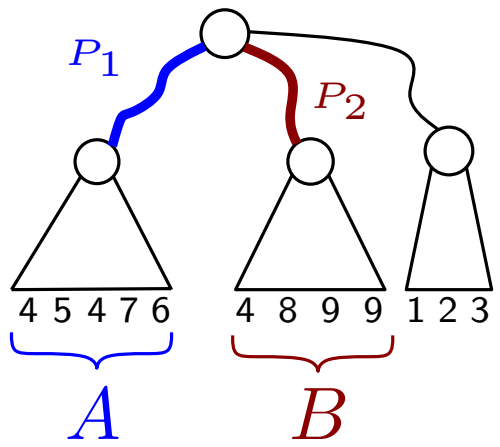

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Query: P_1 and P_2

Report: $A \cap B = \{4\}$

set intersection is usually hard

Text Indexing - Two Patterns

Text Indexing - Two Patterns

Problem	Lower Bound	Notes

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Both 2P and FP	$S(n)Q(n)^2 = \Omega(n^{2-o(1)})$	Semi group model (**new**)

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Both 2P and FP	$Q(n) = (nt)^{\frac{1}{2}-\alpha} \Rightarrow$ $S(n) = \Omega(n^{\frac{1+6\alpha}{1+2\alpha}-o(1)})$	Pointer Machine Model t is the output size (**new**)

Text Indexing - How to prove the lower bounds?

We use two different techniques:

1) Reduction from Matrix Multiplication

2) Find a hard input for a Pointer Machine Data Structure

---

1) Given two matrices, produce a set of documents and queries such that the product of the two matrices can be found

Text Indexing - How to prove the lower bounds?

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$$\begin{matrix} & A & \\ \end{matrix}\begin{bmatrix} T & F & T \\ T & F & F \\ T & T & T \end{bmatrix} \times \begin{matrix} & B & \\ \end{matrix}\begin{bmatrix} F & F & T \\ F & T & T \\ T & F & T \end{bmatrix} = \begin{matrix} & C & \\ \end{matrix}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$C_{i,j} = T \Leftrightarrow \exists \ell$ s.t.
$A_{i,\ell} = B_{\ell,j} = T$

Text Indexing - How to prove the lower bounds?

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} A \\ \left[\begin{array}{ccc} T & F & T \\ T & F & F \\ T & T & T \end{array}\right] \end{array} \times \begin{array}{c} B \\ \left[\begin{array}{ccc} F & F & T \\ F & T & T \\ T & F & T \end{array}\right] \end{array} = \begin{array}{c} C \\ \left[\begin{array}{ccc} & & \\ & & \\ & & \end{array}\right] \end{array}$$

$C_{i,j} = T \Leftrightarrow \exists \ell$ s.t. $A_{i,\ell} = B_{\ell,j} = T$

$$d_1^A = 1\#2\#3\#$$

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$d_2^A = 3\#$

$d_3^A = 1\#3\#$

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$$d_3^A = 1\#3\# \qquad d_3^B = 4\#6\#$$

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$d_2^A = 3\#$ $\quad d_2^B = 5\#6\#$ $\quad d_2 = 3\#5\#6\#$

$d_3^A = 1\#3\#$ $\quad d_3^B = 4\#6\#$ $\quad d_3 = 1\#3\#4\#6\#$

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$$d_3 = 1\#3\#4\#6\# \quad D$$

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$d_2 = 3\#5\#6\#$

$d_3 = 1\#3\#4\#6\#$ D

Query: $P_1 =$ 1 $P_2 =$ 4

Text Indexing - How to prove the lower bounds?

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} A \\ \left[\begin{array}{ccc} T & F & T \\ T & F & F \\ T & T & T \end{array}\right] \end{array} \times \begin{array}{c} B \\ \left[\begin{array}{ccc} F & F & T \\ F & T & T \\ T & F & T \end{array}\right] \end{array} = \begin{array}{c} C \\ \left[\begin{array}{ccc} T & & \\ & & \\ & & \end{array}\right] \end{array}$$

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$d_1 = 1\#2\#3\#6\#$

$d_2 = 3\#5\#6\#$

$d_3 = 1\#3\#4\#6\#$ D

Query: $P_1 =$ 1 $P_2 =$ 5

Text Indexing - How to prove the lower bounds?

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} A \\ \left[\begin{array}{ccc} T & F & T \\ T & F & F \\ T & T & T \end{array}\right] \end{array} \times \begin{array}{c} B \\ \left[\begin{array}{ccc} F & F & T \\ F & T & T \\ T & F & T \end{array}\right] \end{array} = \begin{array}{c} C \\ \left[\begin{array}{cc} T & F \\ & \\ & \end{array}\right] \end{array}$$

$C_{i,j} = T \Leftrightarrow \exists \ell$ s.t. $A_{i,\ell} = B_{\ell,j} = T$

$d_1^A = 1\#2\#3\#$ $\quad$ $d_1^B = 6\#$

$d_2^A = 3\#$ $\quad$ $d_2^B = 5\#6\#$

$d_3^A = 1\#3\#$ $\quad$ $d_3^B = 4\#6\#$

$d_1 = 1\#2\#3\#6\#$

$d_2 = 3\#5\#6\#$

$d_3 = 1\#3\#4\#6\#$ D

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$d_3 = 1\#3\#4\#6\#$ D

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$d_3 = 1\#3\#4\#6\#$ D

Query: $P_1 =$ 3 $P_2 =$ 6

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Text Indexing - How to prove the lower bounds?

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Query: $P_1 =$ 3 $P_2 =$ 6

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Overview

1. Algorithms and Data Structures
   (a) Abstract description
   (b) Example
2. Implicit Data Structures
   (a) Finger Search
   (b) Priority Queues
3. Integer Sorting
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LOTS of data

A nuisance in a collection

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Danish Radio collection 2 hour chunks

Before one runs out another is started

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$\Rightarrow$ overlap

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Task: find overlap and eliminate it

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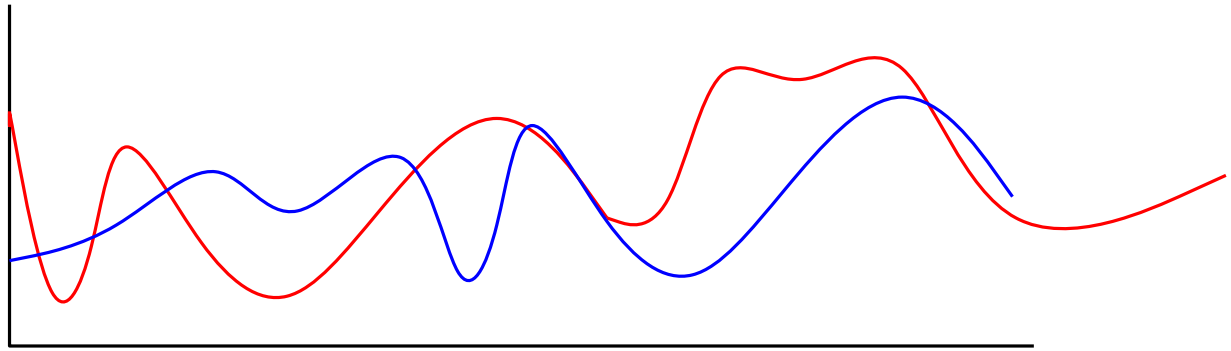

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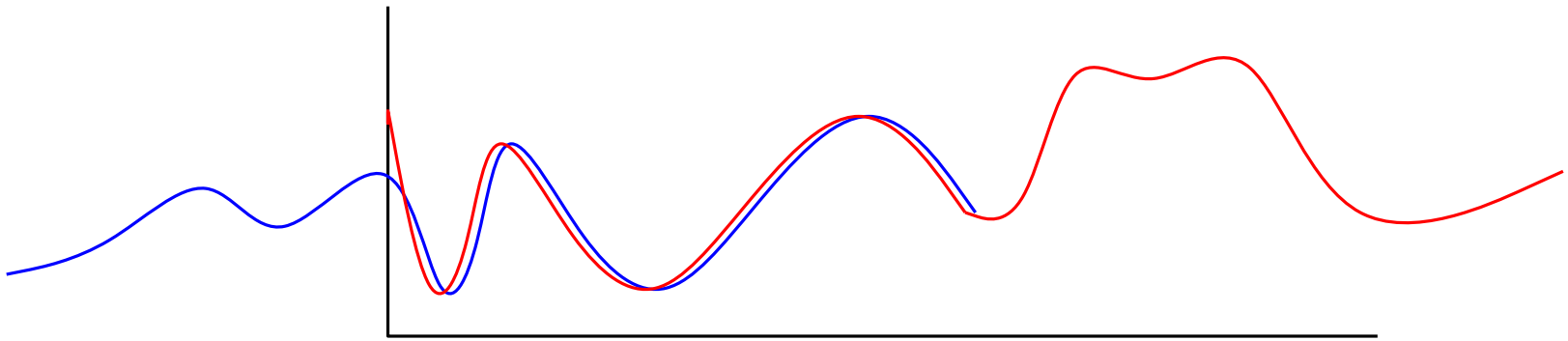

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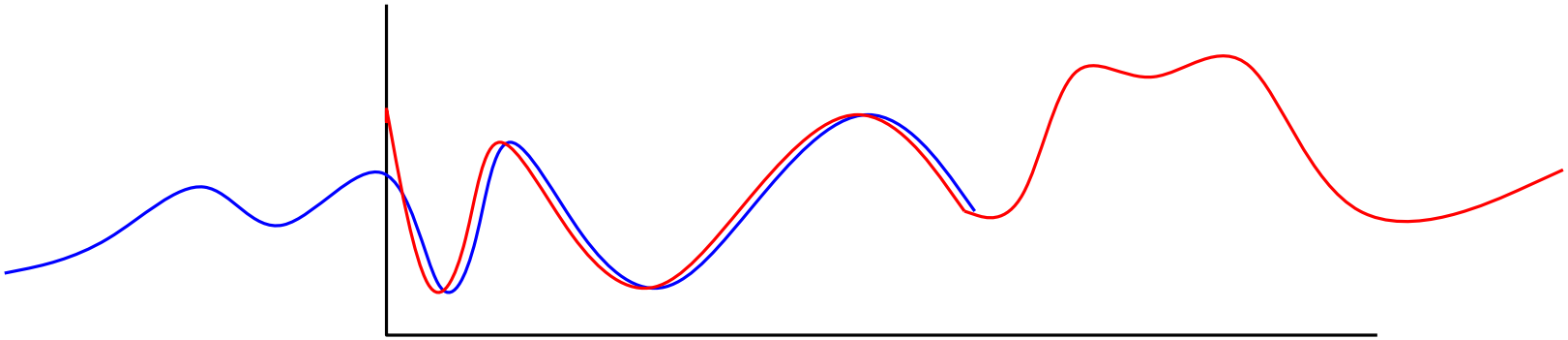
Use Cross Correlation.



Thank You