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## Asymptotic notation

### Problem 1 (6 %)

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$(\log n)^3 = O(n^2)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.1
$4 \log(n^2) = O(n^{3/2})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.2
$n^2 + 3 \log n = O(\log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	1.3
$n^2 = O(n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	1.4
$2n \cdot \log n = O(2^n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.5
$3^3 + \sqrt{n} = O(n!)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.6
$3\sqrt{n} + 7n^{1/3} = O(n^{0.1})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	1.7
$4(\log n)^4 = O(2^n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.8
$n \cdot \log n = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	1.9
$\log n = \Omega(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	1.10
$n \cdot \log n = \Omega(n^{0.1})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.11
$n^{0.01}/5 = \Omega(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	1.12

### Problem 2 (6 %)

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$6\sqrt{n} \cdot \log n = O(n^{2/3})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	2.1
$n^{0.1} = O(n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	2.2
$7(\log n)^2 = O(n^{0.001})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	2.3
$n \cdot \log n = O(\sqrt{n} \cdot \log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	2.4
$2^n = O(n^3)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	2.5
$n^2 \log n = O(\log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	2.6
$n^3 = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	2.7
$n = O(\log(n!))$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	2.8
$7n \cdot \log n = O(n^{0.001})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	2.9
$n \cdot \log n = \Omega(8^{\log n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	2.10
$\sqrt{n} = \Omega((\log n)^2)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	2.11
$n \cdot \log n = \Omega(n^{2/3})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	2.12

**Problem 3 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$\sqrt{n} = O(n^{2/3})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.1
$\log(n^2) = O(n \cdot \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.2
$n = O(n^{0.01})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	3.3
$4n = O(n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.4
$8^{\log n} = O(\log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	3.5
$2^n = O(\log(n^2))$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	3.6
$\log(n!) = O(n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	3.7
$n^2 = O(n^{0.001})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	3.8
$(\log n)^2 = O(n^{0.01})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.9
$n^{0.001} = \Omega(\log(n^2))$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.10
$n^2 \log n = \Omega(\sqrt{n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.11
$3^n = \Omega(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	3.12

**Problem 4 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$n + \sqrt{n} = O(n \cdot \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.1
$2^{3 \log n} = O(n^n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.2
$7n^3 = O(3^3)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	4.3
$n^2 \log n = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	4.4
$n \cdot \log n = O(n^3)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.5
$\log(n!) = O(n^3)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.6
$2^n = O(2^{3 \log n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	4.7
$5n^{2/3} \cdot n^{1/3} = O(n^n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.8
$\sqrt{n} = O(2^{\log n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.9
$\log n + \log n = \Theta(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	4.10
$4\sqrt{n} \cdot \log n = \Omega(\sum_{i=1}^n i)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	4.11
$(\log n)^3 = \Theta((\log n)^6)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	4.12

**Problem 5 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$\log(n!) = O(n!)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	5.1
$2^n = O(n^{0.1})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.2
$n^{3/2} = O(n^2 \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	5.3
$6n^2 = O(2^{\log n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.4
$\log n = O(\sqrt{n} \cdot \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	5.5
$5 \log n + n^2 = O(\log(n^2))$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.6
$2^{3 \log n} = O(n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.7
$1 = O(n^{0.1})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	5.8
$n \cdot \log n = O(n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.9
$1 = \Omega(n^{1/3})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.10
$6 \log n = \Theta(4^{\log n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.11
$n^{0.001} = \Omega(\sqrt{n} \cdot \log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	5.12

**Problem 6 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$\log n = O(2^{\log n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	6.1
$8^{\log n} = O(n^{2/3})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	6.2
$\sqrt{n} = O(n^{2/3})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	6.3
$2^{\log n} = O(2^{2 \log n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	6.4
$\log(n^2) = O(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	6.5
$\sqrt{n} \cdot \log n = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	6.6
$\log(n!)/7 = O(n^2 \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	6.7
$n! + \sqrt{n} \cdot (\log n)/3 = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	6.8
$\sqrt{n} = O((\log n)^3)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	6.9
$5\sqrt{n} + 2 \cdot 8^{\log n} = \Omega(n!)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	6.10
$8^{\log n} = \Theta(2^{3 \log n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	6.11
$n \cdot \log n = \Omega(2^{3 \log n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	6.12

**Problem 7 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$1 + \log(n^2)/2 = O(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	7.1
$2n = O(n \cdot \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	7.2
$1 + 3^3 = O(2^{3 \log n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	7.3
$7n \cdot \log n = O(\log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.4
$n! = O((\log n)^3)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.5
$n^2 \log n = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.6
$n \cdot \log n = O(\log(n^2))$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.7
$n^{2/3} = O(2^{3 \log n})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	7.8
$3n \cdot \log n = O(1)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.9
$n = \Theta(n^{0.1})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.10
$\log n = \Omega(n \cdot \log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	7.11
$4 \log n = \Theta(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	7.12

**Problem 8 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$n \cdot \log n = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	8.1
$n^{0.01} = O(\sqrt{n} \cdot \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	8.2
$4 \cdot 3^n = O(\log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	8.3
$n^{2/3} = O(n^{2/3} \cdot n^{1/3})$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	8.4
$5^5 + n^{0.001} = O(n \cdot \log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	8.5
$\log(n^2) = O(\log n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	8.6
$5n^{1/3} = O(\log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	8.7
$2^{3 \log n} = O(n^3)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	8.8
$6\sqrt{n} \cdot \log n = O(\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	8.9
$n/6 = \Theta(n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	8.10
$2^{2 \log n} = \Theta(n\sqrt{n})$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	8.11
$n^3 = \Theta(n \cdot \log n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	8.12

**Problem 9 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$\sqrt{n}/2 = O(\log(n^2))$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	9.1
$2^n = O(n \cdot \log n)$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	9.2
$6 \log(n!) + n \cdot \log n = O(2^{\log n})$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	9.3
$\log n = O(\sqrt{n} \cdot \log n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.4
$2 = O((\log n)^2)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.5
$(\log n)^3 = O(n \cdot \log n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.6
$n \cdot \log n = O(2^n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.7
$n^3 + n \cdot \log n = O(n \cdot \log n)$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	9.8
$8^{\log n} = O(2^n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.9
$n^{0.1}/6 = \Theta(n^n)$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	9.10
$4^{\log n} = \Theta(\sum_{i=1}^n i)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.11
$1 = \Theta(5^5)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	9.12

**Problem 10 (6 %)**

In the following,  $\log n$  denotes the base-2 logarithm of  $n$ .

	Yes	No	
$n \cdot \log n = O(2^{2 \log n})$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.1
$2^{3 \log n} = O(n!)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.2
$n = O(n \cdot \log n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.3
$n = O(n\sqrt{n})$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.4
$3n = O(n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.5
$\sqrt{n} + \sqrt{n} = O(n^{0.1})$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	10.6
$\log(n!) + 6n^{3/2} = O(\log n)$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	10.7
$n^{0.1} = O(\sqrt{n} \cdot \log n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.8
$n \cdot (\log n)/3 = O(2^n)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.9
$n \cdot \log n = \Theta(\log(n!))$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.10
$1 + 4 = \Theta(4^4)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.11
$\sqrt{n} \cdot \log n = \Omega(\sqrt{n})$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	10.12

## Analysis of loops

### Problem 11 (6 %)

<b>Algorithm loop1(<math>n</math>)</b> $s = 1$ <b>while</b> $s \leq n$ $s = s + 1$	<b>Algorithm loop2(<math>n</math>)</b> $s = 0$ <b>for</b> $i = 1$ <b>to</b> $n$ <b>for</b> $j = 1$ <b>to</b> $n$ <b>for</b> $k = 1$ <b>to</b> $n$ $s = s + 1$
<b>Algorithm loop3(<math>n</math>)</b> $i = 0$ $j = n$ <b>while</b> $i < j$ $i = i + 2$ $j = j + 1$	<b>Algorithm loop4(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = i$ <b>while</b> $j \leq n$ $j = 2 * j$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta((\log n)^2)$	$\Theta(n^3)$	$\Theta(n \log n)$	$\Theta(n)$	$\Theta(\log \log n)$	$\Theta(\log n)$	$\Theta(n^2)$	$\Theta(n^2 \cdot \log n)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	11.1
loop2	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	11.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	11.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	11.4

### Problem 12 (6 %)

<b>Algorithm loop1(<math>n</math>)</b> $i = 1$ <b>while</b> $i \leq n$ $i = 2 * i$	<b>Algorithm loop2(<math>n</math>)</b> $s = 1$ <b>for</b> $i = 1$ <b>to</b> $n$ <b>for</b> $j = i$ <b>to</b> $n$ $s = s + 1$
<b>Algorithm loop3(<math>n</math>)</b> $i = 1$ $j = n$ <b>while</b> $i \leq j$ $i = i * 2$ $j = \lfloor j/2 \rfloor$	<b>Algorithm loop4(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = 0$ <b>while</b> $j \leq n$ $j = j + i$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(\log n)$	$\Theta((\log n)^2)$	$\Theta(\log \log n)$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^3)$	$\Theta(n^2)$	$\Theta(n^2 \cdot \log n)$	
loop1	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	12.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	12.2
loop3	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	12.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	12.4

**Problem 13 (6 %)**

```

Algorithm loop1(n)
    s = 0
    for i = 1 to n
        for j = 1 to i * i
            s = s + 1

Algorithm loop2(n)
    i = 1
    while i ≤ n
        i = 2 * i

Algorithm loop3(n)
    i = 1
    while i ≤ n * n
        i = 2 * i

Algorithm loop4(n)
    i = 1
    while i ≤ n
        j = 1
        while j ≤ i
            j = 2 * j
        i = 2 * i
    
```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n^2)$	$\Theta((\log n)^2)$	$\Theta(\log n)$	$\Theta(n^3)$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(\sqrt{n})$	$\Theta(n^2 \cdot \log n)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	13.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	13.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	13.3
loop4	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	13.4

**Problem 14 (6 %)**

```

Algorithm loop1(n)
    i = 1
    while i ≤ n
        i = 2 * i

Algorithm loop2(n)
    i = n
    while i > 0
        i = i - 1

Algorithm loop3(n)
    i = 1
    while i ≤ n * n
        i = 3 * i

Algorithm loop4(n)
    i = 1
    j = 1
    s = 0
    while s ≤ n
        while j ≤ s
            j = 2 * j
        s = s + i
        i = i + 1
    
```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n^2)$	$\Theta(2^n)$	$\Theta(\sqrt{n})$	$\Theta(n^3)$	$\Theta(\log n)$	$\Theta(n \log n)$	$\Theta(n)$	$\Theta((\log n)^2)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	14.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	14.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	14.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	14.4

## Problem 15 (6 %)

```

Algorithm loop1(n)
  s = 0
  for i = 1 to n
    for j = 1 to n
      for k = 1 to n
        s = s + 1

```

```

Algorithm loop2(n)
  for i = 1 to n
    j = i
    while j > 0
      j = j - 1

```

```

Algorithm loop3(n)
  i = 1
  while i ≤ n
    j = 0
    while j ≤ n
      j = j + 1
    i = 2 * i

```

```

Algorithm loop4(n)
  i = n
  j = 0
  while i > 0
    if j < i
      j = j + 1
    else
      j = 0
      i = i - 1

```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n \log n)$	$\Theta((\log n)^2)$	$\Theta(n)$	$\Theta(n^3)$	$\Theta(\sqrt{n})$	$\Theta(\frac{\log n}{\log \log n})$	$\Theta(n^2)$	$\Theta(\log n)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	15.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	15.2
loop3	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	15.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	15.4

## Problem 16 (6 %)

<b>Algorithm</b> loop1( $n$ ) $i = 1$ <b>while</b> $i \leq n$ $j = 1$ <b>while</b> $j \leq n$ $j = j + 1$ $i = i + 1$	<b>Algorithm</b> loop2( $n$ ) $s = 0$ <b>for</b> $i = 1$ <b>to</b> $n$ <b>for</b> $j = 1$ <b>to</b> $i * i$ $s = s + 1$
<b>Algorithm</b> loop3( $n$ ) $i = n$ <b>while</b> $i \leq n * n$ $i = 2 * i$	<b>Algorithm</b> loop4( $n$ ) $i = 1$ <b>while</b> $i \leq n$ $j = i$ <b>while</b> $j \leq n$ $j = j + 1$ $i = 2 * i$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n \log n)$	$\Theta(\log \log n)$	$\Theta(n^3)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta((\log n)^2)$	$\Theta(n^2)$	$\Theta(n\sqrt{n})$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	16.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	16.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	16.3
loop4	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	16.4

**Problem 17 (6 %)**

<b>Algorithm loop1(<math>n</math>)</b> $s = 1$ <b>for</b> $i = 1$ <b>to</b> $n * n$ <b>for</b> $j = 1$ <b>to</b> $n$ $s = s + 1$	<b>Algorithm loop2(<math>n</math>)</b> $s = 1$ <b>for</b> $i = 1$ <b>to</b> $n$ $s = s + 1$
<b>Algorithm loop3(<math>n</math>)</b> $s = 0$ $i = n$ <b>while</b> $i > 0$ <b>for</b> $j = 1$ <b>to</b> $i$ $s = s + 1$ $i = i - 1$	<b>Algorithm loop4(<math>n</math>)</b> $i = n$ $j = 0$ <b>while</b> $i > 0$ <b>if</b> $j < i$ $j = j + 1$ <b>else</b> $j = 0$ $i = i - 1$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(\sqrt{n} \log n)$	$\Theta((\log n)^2)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^3)$	$\Theta(n)$	$\Theta(\log n)$	$\Theta(\sqrt{n})$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	17.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	17.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	17.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	17.4

**Problem 18 (6 %)**

<b>Algorithm loop1(<math>n</math>)</b> $s = 0$ <b>for</b> $i = 1$ <b>to</b> $n$ <b>for</b> $j = 1$ <b>to</b> $n$ <b>for</b> $k = 1$ <b>to</b> $n$ $s = s + 1$	<b>Algorithm loop2(<math>n</math>)</b> $i = n$ <b>while</b> $i > 0$ $i = i - 1$
<b>Algorithm loop3(<math>n</math>)</b> $i = 2$ <b>while</b> $i \leq n$ $i = i * i$	<b>Algorithm loop4(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = i$ <b>while</b> $j \leq n$ $j = 2 * j$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n^2)$	$\Theta(n)$	$\Theta(\log \log n)$	$\Theta(\sqrt{n})$	$\Theta((\log n)^2)$	$\Theta(n \log n)$	$\Theta(n^3)$	$\Theta(\log n)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	18.1
loop2	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	18.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	18.3
loop4	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	18.4

**Problem 19 (6 %)**

```

Algorithm loop1(n)   Algorithm loop2(n)
for i = 1 to n       i = 1
    j = i               while i ≤ n
    while j > 0         i = 2 * i
        j = j - 1
    
```

```

Algorithm loop3(n)   Algorithm loop4(n)
s = 0                 i = 1
i = 1                 while i ≤ n
while s ≤ n           j = i
    s = s + i           while j ≤ n
    i = i + 1           j = j + 1
                       i = 2 * i
    
```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta((\log n)^2)$	$\Theta(\sqrt{n})$	$\Theta(n^2 \cdot \log n)$	$\Theta(n^3)$	$\Theta(n^2)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n \log n)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	19.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	19.2
loop3	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	19.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	19.4

**Problem 20 (6 %)**

```

Algorithm loop1(n)   Algorithm loop2(n)
for i = 1 to n       s = 1
    j = i               for i = n to 1 step -1
    while j > 0         for j = n to 1 step -1
        j = j - 1           s = s + 1
    
```

```

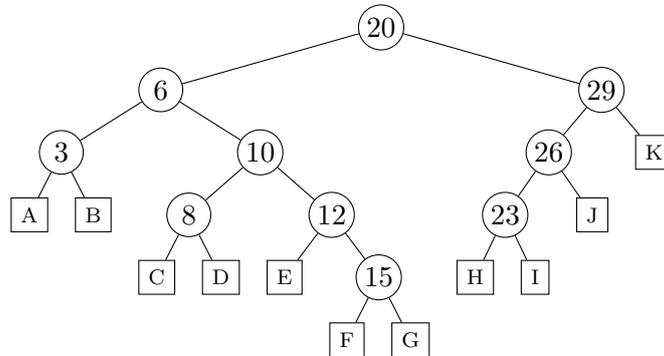
Algorithm loop3(n)   Algorithm loop4(n)
i = 1                 i = 1
while i ≤ n           j = n
    j = 1               while i ≤ j
    while j ≤ i         i = 4 * i
        j = 2 * j       j = 2 * j
    i = i + 1
    
```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n)$	$\Theta(n^3)$	$\Theta(\log n)$	$\Theta(\frac{\log n}{\log \log n})$	$\Theta(2^n)$	$\Theta((\log n)^2)$	$\Theta(n \log n)$	$\Theta(n^2)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	20.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	20.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	20.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	20.4

### Insertions into search trees

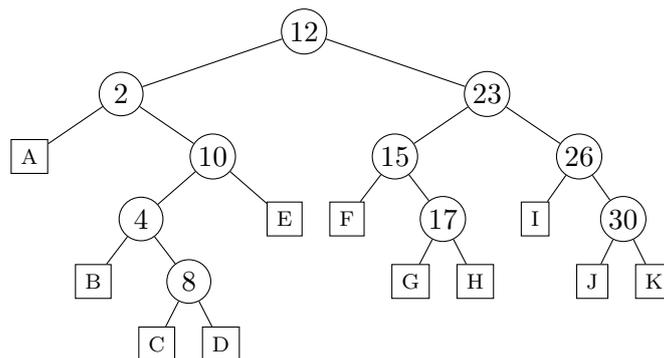
**Problem 21 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 18, 2, 11, 25 and 22 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(18)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	21.1
INSERT(2)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	21.2
INSERT(11)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	21.3
INSERT(25)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	21.4
INSERT(22)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	21.5

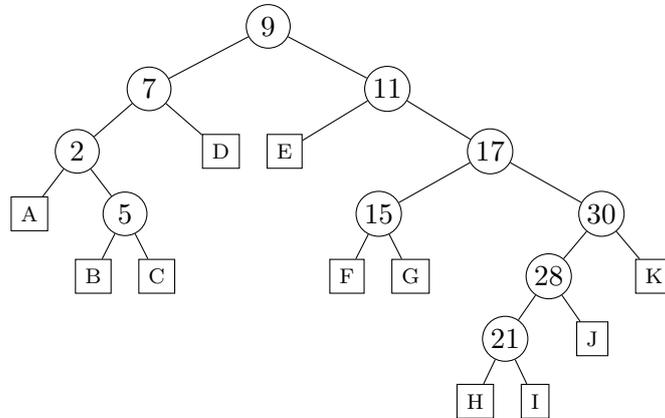
**Problem 22 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 19, 28, 27, 1 and 21 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(19)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	22.1
INSERT(28)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	22.2
INSERT(27)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	22.3
INSERT(1)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	22.4						
INSERT(21)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	22.5

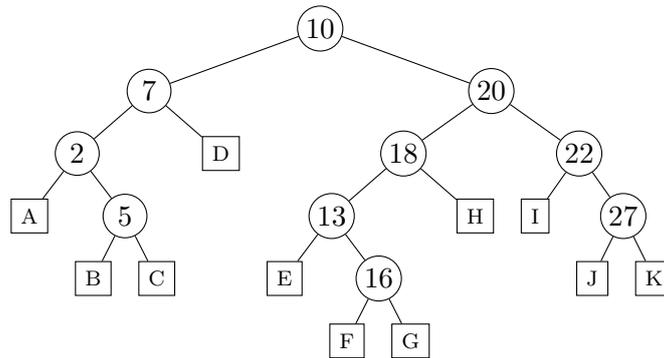
**Problem 23 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 10, 3, 22, 25 and 20 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(10)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	23.1
INSERT(3)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	23.2
INSERT(22)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	23.3
INSERT(25)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	23.4
INSERT(20)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	23.5

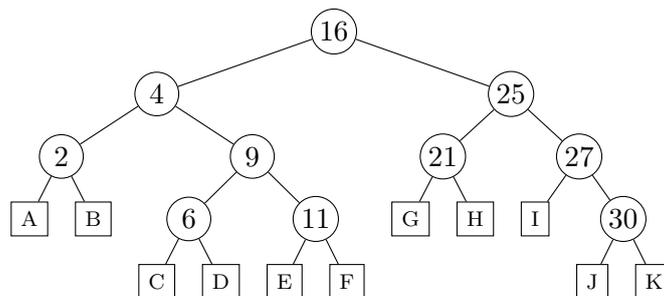
**Problem 24 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 11, 19, 17, 28 and 1 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(11)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	24.1
INSERT(19)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	24.2
INSERT(17)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	24.3
INSERT(28)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	24.4
INSERT(1)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	24.5

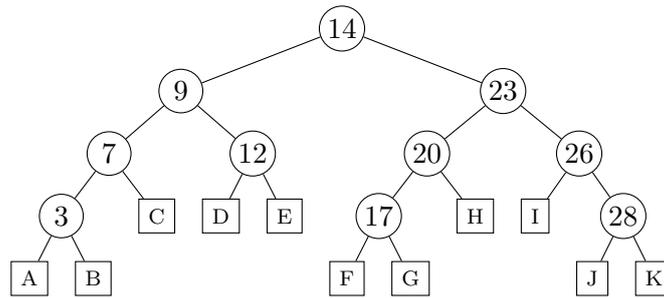
**Problem 25 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 18, 5, 12, 19 and 22 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(18)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	25.1
INSERT(5)	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	25.2
INSERT(12)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	25.3
INSERT(19)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	25.4
INSERT(22)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	25.5

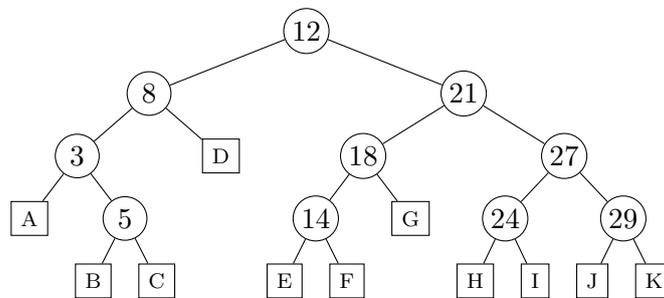
**Problem 26 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 16, 2, 4, 19 and 22 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(16)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	26.1
INSERT(2)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	26.2
INSERT(4)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	26.3
INSERT(19)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	26.4
INSERT(22)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	26.5

**Problem 27 (4 %)**

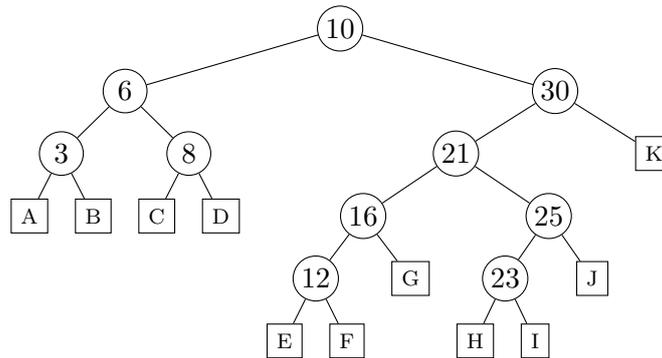


Indicate in which leaves A–K in the above unbalanced binary search tree the elements 17, 10, 19, 15 and 23 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(17)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	27.1
INSERT(10)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	27.2
INSERT(19)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	27.3
INSERT(15)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	27.4
INSERT(23)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	27.5



**Problem 30 (4 %)**



Indicate in which leaves A–K in the above unbalanced binary search tree the elements 18, 7, 4, 24 and 9 should be inserted (it is assumed that before each insertion the tree contains only the above ten elements).

	A	B	C	D	E	F	G	H	I	J	K	
INSERT(18)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	30.1
INSERT(7)	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	30.2
INSERT(4)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	30.3
INSERT(24)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	30.4
INSERT(9)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	30.5

**Max-Heap-Insert**

**Problem 31 (4 %)**

What is the binary max-heap after inserting the elements 3, 11, 13, 14, 1, 8 and 5 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

1	2	3	4	5	6	7	
14	13	11	3	1	8	5	<input checked="" type="checkbox"/>
1	2	3	4	5	6	7	
14	11	13	3	1	8	5	<input type="checkbox"/>
1	2	3	4	5	6	7	
3	11	13	14	1	8	5	<input type="checkbox"/>
1	2	3	4	5	6	7	
14	13	11	8	5	3	1	<input type="checkbox"/>
1	2	3	4	5	6	7	
13	14	8	11	1	3	5	<input type="checkbox"/>

**Problem 32 (4 %)**

What is the binary max-heap after inserting the elements 2, 7, 4, 9, 8, 11 and 3 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| 11 | 8 | 9 | 2 | 7 | 4 | 3 |
- 
- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| 11 | 9 | 8 | 7 | 4 | 3 | 2 |
- 
- |   |   |    |   |   |   |   |
|---|---|----|---|---|---|---|
| 1 | 2 | 3  | 4 | 5 | 6 | 7 |
| 7 | 9 | 11 | 2 | 8 | 4 | 3 |
- 
- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| 11 | 9 | 4 | 7 | 8 | 2 | 3 |
- 
- |   |   |   |   |   |    |   |
|---|---|---|---|---|----|---|
| 1 | 2 | 3 | 4 | 5 | 6  | 7 |
| 2 | 7 | 4 | 9 | 8 | 11 | 3 |
- 

**Problem 33 (4 %)**

What is the binary max-heap after inserting the elements 10, 2, 1, 13, 12, 14 and 6 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |    |    |   |    |   |   |
|----|----|----|---|----|---|---|
| 1  | 2  | 3  | 4 | 5  | 6 | 7 |
| 14 | 13 | 10 | 2 | 12 | 1 | 6 |
- 
- |    |    |    |   |    |   |   |
|----|----|----|---|----|---|---|
| 1  | 2  | 3  | 4 | 5  | 6 | 7 |
| 14 | 12 | 13 | 2 | 10 | 1 | 6 |
- 
- |    |    |    |    |   |   |   |
|----|----|----|----|---|---|---|
| 1  | 2  | 3  | 4  | 5 | 6 | 7 |
| 14 | 13 | 12 | 10 | 6 | 2 | 1 |
- 
- |    |    |    |   |    |   |   |
|----|----|----|---|----|---|---|
| 1  | 2  | 3  | 4 | 5  | 6 | 7 |
| 10 | 13 | 14 | 2 | 12 | 1 | 6 |
- 
- |    |   |   |    |    |    |   |
|----|---|---|----|----|----|---|
| 1  | 2 | 3 | 4  | 5  | 6  | 7 |
| 10 | 2 | 1 | 13 | 12 | 14 | 6 |
- 

**Problem 34 (4 %)**

What is the binary max-heap after inserting the elements 10, 12, 1, 14, 6, 11 and 13 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |    |   |    |   |    |    |
|----|----|---|----|---|----|----|
| 1  | 2  | 3 | 4  | 5 | 6  | 7  |
| 10 | 12 | 1 | 14 | 6 | 11 | 13 |
- 
- |    |    |    |    |    |   |   |
|----|----|----|----|----|---|---|
| 1  | 2  | 3  | 4  | 5  | 6 | 7 |
| 14 | 13 | 12 | 11 | 10 | 6 | 1 |
- 
- |    |    |    |    |   |    |   |
|----|----|----|----|---|----|---|
| 1  | 2  | 3  | 4  | 5 | 6  | 7 |
| 12 | 14 | 13 | 10 | 6 | 11 | 1 |
- 
- |    |    |    |    |   |   |    |
|----|----|----|----|---|---|----|
| 1  | 2  | 3  | 4  | 5 | 6 | 7  |
| 14 | 12 | 13 | 10 | 6 | 1 | 11 |
- 
- |    |    |    |    |   |    |   |
|----|----|----|----|---|----|---|
| 1  | 2  | 3  | 4  | 5 | 6  | 7 |
| 14 | 12 | 13 | 10 | 6 | 11 | 1 |
-

**Problem 35 (4 %)**

What is the binary max-heap after inserting the elements 4, 10, 1, 2, 5, 12 and 3 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |    |   |   |   |   |   |
|----|----|---|---|---|---|---|
| 1  | 2  | 3 | 4 | 5 | 6 | 7 |
| 12 | 10 | 5 | 4 | 3 | 2 | 1 |
- 
- A
- |    |    |   |   |   |   |   |
|----|----|---|---|---|---|---|
| 1  | 2  | 3 | 4 | 5 | 6 | 7 |
| 12 | 10 | 4 | 2 | 5 | 1 | 3 |
- 
- B
- |    |   |    |   |   |   |   |
|----|---|----|---|---|---|---|
| 1  | 2 | 3  | 4 | 5 | 6 | 7 |
| 12 | 5 | 10 | 2 | 4 | 1 | 3 |
- 
- C
- |    |   |    |   |   |   |   |
|----|---|----|---|---|---|---|
| 1  | 2 | 3  | 4 | 5 | 6 | 7 |
| 10 | 5 | 12 | 2 | 4 | 1 | 3 |
- 
- D
- |   |    |   |   |   |    |   |
|---|----|---|---|---|----|---|
| 1 | 2  | 3 | 4 | 5 | 6  | 7 |
| 4 | 10 | 1 | 2 | 5 | 12 | 3 |
- 
- E

**Problem 36 (4 %)**

What is the binary max-heap after inserting the elements 6, 3, 9, 11, 12, 8 and 14 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |   |   |   |    |    |   |    |
|---|---|---|----|----|---|----|
| 1 | 2 | 3 | 4  | 5  | 6 | 7  |
| 6 | 3 | 9 | 11 | 12 | 8 | 14 |
- 
- A
- |   |    |    |    |   |   |   |
|---|----|----|----|---|---|---|
| 1 | 2  | 3  | 4  | 5 | 6 | 7 |
| 9 | 12 | 14 | 11 | 3 | 8 | 6 |
- 
- B
- |    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 |
| 14 | 12 | 11 | 9 | 8 | 6 | 3 |
- 
- C
- |    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 |
| 14 | 11 | 12 | 3 | 9 | 6 | 8 |
- 
- D
- |    |    |   |    |   |   |   |
|----|----|---|----|---|---|---|
| 1  | 2  | 3 | 4  | 5 | 6 | 7 |
| 14 | 12 | 9 | 11 | 3 | 8 | 6 |
- 
- E

**Problem 37 (4 %)**

What is the binary max-heap after inserting the elements 3, 12, 4, 10, 11, 2 and 14 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |    |   |    |    |   |   |
|----|----|---|----|----|---|---|
| 1  | 2  | 3 | 4  | 5  | 6 | 7 |
| 14 | 12 | 4 | 10 | 11 | 2 | 3 |
- 
- A
- |   |    |   |    |    |   |    |
|---|----|---|----|----|---|----|
| 1 | 2  | 3 | 4  | 5  | 6 | 7  |
| 3 | 12 | 4 | 10 | 11 | 2 | 14 |
- 
- B
- |    |    |    |   |    |   |   |
|----|----|----|---|----|---|---|
| 1  | 2  | 3  | 4 | 5  | 6 | 7 |
| 14 | 11 | 12 | 3 | 10 | 2 | 4 |
- 
- C
- |    |    |    |    |   |   |   |
|----|----|----|----|---|---|---|
| 1  | 2  | 3  | 4  | 5 | 6 | 7 |
| 12 | 11 | 14 | 10 | 3 | 2 | 4 |
- 
- D
- |    |    |    |    |   |   |   |
|----|----|----|----|---|---|---|
| 1  | 2  | 3  | 4  | 5 | 6 | 7 |
| 14 | 12 | 11 | 10 | 4 | 3 | 2 |
- 
- E

**Problem 38 (4%)**

What is the binary max-heap after inserting the elements 6, 8, 10, 14, 7, 1 and 12 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |   |    |   |   |   |    |
|----|---|----|---|---|---|----|
| 1  | 2 | 3  | 4 | 5 | 6 | 7  |
| 14 | 8 | 12 | 6 | 7 | 1 | 10 |
- 
- A
- |    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 |
| 14 | 12 | 10 | 8 | 7 | 6 | 1 |
- 
- B
- |    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 |
| 10 | 14 | 12 | 8 | 7 | 1 | 6 |
- 
- C
- |    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 |
| 14 | 10 | 12 | 6 | 7 | 1 | 8 |
- 
- D
- |   |   |    |    |   |   |    |
|---|---|----|----|---|---|----|
| 1 | 2 | 3  | 4  | 5 | 6 | 7  |
| 6 | 8 | 10 | 14 | 7 | 1 | 12 |
- 
- E

**Problem 39 (4%)**

What is the binary max-heap after inserting the elements 2, 8, 1, 5, 10, 9 and 6 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| 10 | 8 | 9 | 2 | 5 | 1 | 6 |
- 
- A
- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| 10 | 8 | 9 | 5 | 2 | 1 | 6 |
- 
- B
- |   |    |   |   |   |   |   |
|---|----|---|---|---|---|---|
| 1 | 2  | 3 | 4 | 5 | 6 | 7 |
| 8 | 10 | 9 | 5 | 2 | 1 | 6 |
- 
- C
- |   |   |   |   |    |   |   |
|---|---|---|---|----|---|---|
| 1 | 2 | 3 | 4 | 5  | 6 | 7 |
| 2 | 8 | 1 | 5 | 10 | 9 | 6 |
- 
- D
- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| 10 | 9 | 8 | 6 | 5 | 2 | 1 |
- 
- E

**Problem 40 (4%)**

What is the binary max-heap after inserting the elements 10, 5, 6, 9, 8, 11 and 14 in the given order with MAX-HEAP-INSERT, starting with the empty heap.

- |    |   |    |   |   |    |   |
|----|---|----|---|---|----|---|
| 1  | 2 | 3  | 4 | 5 | 6  | 7 |
| 10 | 9 | 14 | 5 | 8 | 11 | 6 |
- 
- A
- |    |   |    |   |   |   |    |
|----|---|----|---|---|---|----|
| 1  | 2 | 3  | 4 | 5 | 6 | 7  |
| 14 | 9 | 11 | 5 | 8 | 6 | 10 |
- 
- B
- |    |   |   |   |   |    |    |
|----|---|---|---|---|----|----|
| 1  | 2 | 3 | 4 | 5 | 6  | 7  |
| 10 | 5 | 6 | 9 | 8 | 11 | 14 |
- 
- C
- |    |   |    |   |   |    |   |
|----|---|----|---|---|----|---|
| 1  | 2 | 3  | 4 | 5 | 6  | 7 |
| 14 | 9 | 11 | 5 | 8 | 10 | 6 |
- 
- D
- |    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 |
| 14 | 11 | 10 | 9 | 8 | 6 | 5 |
- 
- E

## Build-Max-Heap

### Problem 41 (4%)

1	2	3	4	5	6	7	8	9
2	5	6	1	7	9	4	8	3

What is the result of applying BUILD-MAX-HEAP on the above array?

- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 6 | 5 | 7 | 2 | 4 | 1 | 3 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 9 | 8 | 5 | 2 | 4 | 1 | 3 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 2 | 5 | 4 | 1 | 3 |
- 

### Problem 42 (4%)

1	2	3	4	5	6	7	8	9
4	3	1	8	2	7	9	5	6

What is the result of applying BUILD-MAX-HEAP on the above array?

- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 2 | 4 | 1 | 5 | 3 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 4 | 8 | 9 | 6 | 2 | 7 | 1 | 5 | 3 |
- 
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 6 | 8 | 5 | 2 | 1 | 7 | 3 | 4 |
-

**Problem 43 (4%)**

1	2	3	4	5	6	7	8	9
5	6	9	2	7	3	1	8	4

What is the result of applying BUILD-MAX-HEAP on the above array?

1	2	3	4	5	6	7	8	9
9	7	5	8	6	3	1	2	4

 A

1	2	3	4	5	6	7	8	9
9	8	5	6	7	3	1	2	4

 B

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9

 C

1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1

 D

1	2	3	4	5	6	7	8	9
9	8	6	7	5	3	1	2	4

 E
**Problem 44 (4%)**

1	2	3	4	5	6	7	8	9
9	3	6	5	7	2	1	8	4

What is the result of applying BUILD-MAX-HEAP on the above array?

1	2	3	4	5	6	7	8	9
9	8	6	7	5	2	1	3	4

 A

1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1

 B

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9

 C

1	2	3	4	5	6	7	8	9
9	8	6	5	7	2	1	3	4

 D

1	2	3	4	5	6	7	8	9
9	7	6	8	3	2	1	5	4

 E

**Problem 45 (4%)**

1	2	3	4	5	6	7	8	9
3	7	5	8	1	9	2	6	4

What is the result of applying BUILD-MAX-HEAP on the above array?

- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
- 
- A
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 9 | 6 | 1 | 5 | 2 | 3 | 4 |
- 
- B
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 7 | 8 | 6 | 1 | 5 | 2 | 3 | 4 |
- 
- C
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 5 | 7 | 1 | 3 | 2 | 6 | 4 |
- 
- D
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
- 
- E

**Problem 46 (4%)**

1	2	3	4	5	6	7	8	9
8	1	7	6	5	2	9	3	4

What is the result of applying BUILD-MAX-HEAP on the above array?

- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
- 
- A
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 8 | 6 | 9 | 4 | 5 | 2 | 7 | 3 | 1 |
- 
- B
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
- 
- C
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 6 | 8 | 4 | 5 | 2 | 7 | 1 | 3 |
- 
- D
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 6 | 8 | 4 | 5 | 2 | 7 | 3 | 1 |
- 
- E

**Problem 47 (4%)**

1	2	3	4	5	6	7	8	9
1	4	7	6	8	9	2	3	5

What is the result of applying BUILD-MAX-HEAP on the above array?

1	2	3	4	5	6	7	8	9
9	7	8	5	6	4	2	1	3

 A

1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1

 B

1	2	3	4	5	6	7	8	9
9	8	7	6	4	1	2	3	5

 C

1	2	3	4	5	6	7	8	9
7	8	9	6	4	1	2	3	5

 D

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9

 E
**Problem 48 (4%)**

1	2	3	4	5	6	7	8	9
2	1	5	6	7	3	9	8	4

What is the result of applying BUILD-MAX-HEAP on the above array?

1	2	3	4	5	6	7	8	9
9	8	7	6	5	2	3	1	4

 A

1	2	3	4	5	6	7	8	9
5	7	9	8	1	3	2	6	4

 B

1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1

 C

1	2	3	4	5	6	7	8	9
9	8	5	6	7	3	2	1	4

 D

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9

 E

**Problem 49 (4%)**

1	2	3	4	5	6	7	8	9
7	3	5	8	6	9	1	4	2

What is the result of applying BUILD-MAX-HEAP on the above array?

- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
- 
- A
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 9 | 4 | 6 | 5 | 1 | 3 | 2 |
- 
- B
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
- 
- C
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 4 | 6 | 5 | 1 | 3 | 2 |
- 
- D
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 7 | 8 | 4 | 6 | 5 | 1 | 3 | 2 |
- 
- E

**Problem 50 (4%)**

1	2	3	4	5	6	7	8	9
7	8	4	3	6	1	2	5	9

What is the result of applying BUILD-MAX-HEAP on the above array?

- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 4 | 7 | 6 | 1 | 2 | 3 | 5 |
- 
- A
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
- 
- B
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 8 | 7 | 4 | 9 | 6 | 1 | 2 | 5 | 3 |
- 
- C
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 4 | 7 | 6 | 1 | 2 | 5 | 3 |
- 
- D
- 
- | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
- 
- E

## Heap-Extract-Max

### Problem 51 (4%)

1	2	3	4	5	6	7	8	9	10	11	12	13
25	24	19	18	23	10	16	14	6	1	7	4	8

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |    |   |    |    |    |   |    |    |    |                                     |
|----|----|----|----|---|----|----|----|---|----|----|----|-------------------------------------|
| 1  | 2  | 3  | 4  | 5 | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input checked="" type="checkbox"/> |
| 24 | 23 | 19 | 18 | 8 | 10 | 16 | 14 | 6 | 1  | 7  | 4  |                                     |
- |    |    |    |    |   |    |    |    |   |    |    |    |    |                          |
|----|----|----|----|---|----|----|----|---|----|----|----|----|--------------------------|
| 1  | 2  | 3  | 4  | 5 | 6  | 7  | 8  | 9 | 10 | 11 | 12 | 13 | <input type="checkbox"/> |
| 24 | 23 | 19 | 18 | 7 | 10 | 16 | 14 | 6 | 1  |    | 4  | 8  |                          |
- |    |    |    |    |   |    |    |    |   |    |    |    |                          |
|----|----|----|----|---|----|----|----|---|----|----|----|--------------------------|
| 1  | 2  | 3  | 4  | 5 | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input type="checkbox"/> |
| 24 | 23 | 19 | 18 | 7 | 10 | 16 | 14 | 6 | 1  | 8  | 4  |                          |
- |    |    |    |    |   |    |    |    |   |    |    |    |                          |
|----|----|----|----|---|----|----|----|---|----|----|----|--------------------------|
| 1  | 2  | 3  | 4  | 5 | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input type="checkbox"/> |
| 24 | 23 | 19 | 18 | 7 | 10 | 16 | 14 | 6 | 1  | 4  | 8  |                          |
- |    |    |    |    |    |    |    |   |   |    |    |    |                          |
|----|----|----|----|----|----|----|---|---|----|----|----|--------------------------|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 | <input type="checkbox"/> |
| 24 | 23 | 18 | 19 | 10 | 16 | 14 | 6 | 1 | 7  | 4  | 8  |                          |

### Problem 52 (4%)

1	2	3	4	5	6	7	8	9	10	11	12	13
25	24	23	8	16	18	9	4	3	7	10	13	15

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |   |    |    |   |   |   |    |    |    |                          |
|----|----|----|---|----|----|---|---|---|----|----|----|--------------------------|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | <input type="checkbox"/> |
| 24 | 16 | 23 | 8 | 10 | 18 | 9 | 4 | 3 | 7  | 15 | 13 |                          |
- |    |    |    |    |    |   |   |   |   |    |    |    |                          |
|----|----|----|----|----|---|---|---|---|----|----|----|--------------------------|
| 1  | 2  | 3  | 4  | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 | <input type="checkbox"/> |
| 24 | 23 | 15 | 16 | 18 | 9 | 4 | 3 | 7 | 10 | 13 | 8  |                          |
- |    |    |    |   |    |    |   |   |   |    |    |    |                          |
|----|----|----|---|----|----|---|---|---|----|----|----|--------------------------|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | <input type="checkbox"/> |
| 24 | 16 | 23 | 8 | 10 | 18 | 9 | 4 | 3 | 7  | 13 | 15 |                          |
- |    |    |    |   |    |    |   |   |   |    |    |    |    |                          |
|----|----|----|---|----|----|---|---|---|----|----|----|----|--------------------------|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | 13 | <input type="checkbox"/> |
| 24 | 16 | 23 | 8 | 10 | 18 | 9 | 4 | 3 | 7  |    | 13 | 15 |                          |
- |    |    |    |   |    |    |   |   |   |    |    |    |                                     |
|----|----|----|---|----|----|---|---|---|----|----|----|-------------------------------------|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | <input checked="" type="checkbox"/> |
| 24 | 16 | 23 | 8 | 15 | 18 | 9 | 4 | 3 | 7  | 10 | 13 |                                     |

**Problem 53 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
25	24	22	19	17	13	20	7	6	11	5	9	8

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |   |    |    |    |   |   |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 |
| 24 | 19 | 22 | 7 | 17 | 13 | 20 | 8 | 6 | 11 | 5  | 9  |
- 
- A
- 
- |    |    |    |   |    |    |    |   |   |    |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 | 13 |
| 24 | 19 | 22 | 7 | 17 | 13 | 20 |   | 6 | 11 | 5  | 9  | 8  |
- 
- B
- 
- |    |    |    |   |    |    |    |   |   |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 |
| 24 | 19 | 22 | 8 | 17 | 13 | 20 | 7 | 6 | 11 | 5  | 9  |
- 
- C
- 
- |    |    |    |    |    |    |   |   |    |    |    |    |
|----|----|----|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 22 | 20 | 17 | 13 | 19 | 7 | 6 | 11 | 5  | 9  | 8  |
- 
- D
- 
- |    |    |    |   |    |    |    |   |    |    |    |    |
|----|----|----|---|----|----|----|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9  | 10 | 11 | 12 |
| 24 | 19 | 22 | 7 | 17 | 13 | 20 | 6 | 11 | 5  | 9  | 8  |
- 
- E

**Problem 54 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
25	18	16	13	12	15	8	5	4	7	1	14	6

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |   |    |    |   |   |   |    |    |    |
|----|----|----|---|----|----|---|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 13 | 16 | 5 | 12 | 15 | 8 | 6 | 4 | 7  | 1  | 14 |
- 
- A
- 
- |    |    |    |   |    |    |   |   |   |    |    |    |
|----|----|----|---|----|----|---|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 13 | 16 | 5 | 12 | 15 | 8 | 4 | 7 | 1  | 14 | 6  |
- 
- B
- 
- |    |    |    |   |    |    |   |   |   |    |    |    |
|----|----|----|---|----|----|---|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 13 | 16 | 6 | 12 | 15 | 8 | 5 | 4 | 7  | 1  | 14 |
- 
- C
- 
- |    |    |    |   |    |    |   |   |   |    |    |    |    |
|----|----|----|---|----|----|---|---|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 18 | 13 | 16 | 5 | 12 | 15 | 8 |   | 4 | 7  | 1  | 14 | 6  |
- 
- D
- 
- |    |    |    |    |    |   |   |   |   |    |    |    |
|----|----|----|----|----|---|---|---|---|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 16 | 13 | 12 | 15 | 8 | 5 | 4 | 7 | 1  | 14 | 6  |
- 
- E

**Problem 55 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
25	24	22	21	19	20	5	1	16	14	11	3	18

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |    |    |    |   |    |    |    |    |    |
|----|----|----|----|----|----|---|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 |
| 24 | 22 | 21 | 19 | 20 | 18 | 1 | 16 | 14 | 11 | 3  | 5  |

 A
- |    |    |    |    |    |    |   |   |    |    |    |    |
|----|----|----|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 21 | 22 | 16 | 19 | 20 | 5 | 1 | 14 | 11 | 3  | 18 |

 B
- |    |    |    |    |    |    |   |   |    |    |    |    |
|----|----|----|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 21 | 22 | 18 | 19 | 20 | 5 | 1 | 16 | 14 | 11 | 3  |

 C
- |    |    |    |    |    |    |   |   |   |    |    |    |    |
|----|----|----|----|----|----|---|---|---|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 24 | 21 | 22 | 16 | 19 | 20 | 5 | 1 |   | 14 | 11 | 3  | 18 |

 D
- |    |    |    |    |    |    |   |   |    |    |    |    |
|----|----|----|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 21 | 22 | 16 | 19 | 20 | 5 | 1 | 18 | 14 | 11 | 3  |

 E

**Problem 56 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
25	24	18	22	19	13	4	7	9	17	12	6	10

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |   |    |    |   |   |    |    |    |    |
|----|----|----|---|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 22 | 18 | 9 | 19 | 13 | 4 | 7 | 17 | 12 | 6  | 10 |

 A
- |    |    |    |    |    |    |   |   |   |    |    |    |
|----|----|----|----|----|----|---|---|---|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 |
| 24 | 22 | 18 | 10 | 19 | 13 | 4 | 7 | 9 | 17 | 12 | 6  |

 B
- |    |    |    |    |    |    |   |   |    |    |    |    |
|----|----|----|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 19 | 22 | 18 | 13 | 10 | 7 | 9 | 17 | 12 | 6  | 4  |

 C
- |    |    |    |   |    |    |   |   |    |    |    |    |
|----|----|----|---|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 |
| 24 | 22 | 18 | 9 | 19 | 13 | 4 | 7 | 10 | 17 | 12 | 6  |

 D
- |    |    |    |   |    |    |   |   |   |    |    |    |    |
|----|----|----|---|----|----|---|---|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 24 | 22 | 18 | 9 | 19 | 13 | 4 | 7 |   | 17 | 12 | 6  | 10 |

 E

**Problem 57 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
25	24	21	18	14	19	11	8	4	9	6	2	13

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |    |    |    |    |    |   |    |    |    |                                     |                          |
|----|----|----|----|----|----|----|----|---|----|----|----|-------------------------------------|--------------------------|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input type="checkbox"/>            |                          |
| 24 | 21 | 18 | 14 | 19 | 13 | 8  | 4  | 9 | 6  | 2  | 11 | A                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input checked="" type="checkbox"/> |                          |
| 24 | 18 | 21 | 13 | 14 | 19 | 11 | 8  | 4 | 9  | 6  | 2  | X                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input type="checkbox"/>            |                          |
| 24 | 18 | 21 | 8  | 14 | 19 | 11 | 4  | 9 | 6  | 2  | 13 | C                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | <input type="checkbox"/>            |                          |
| 24 | 18 | 21 | 8  | 14 | 19 | 11 | 13 | 4 | 9  | 6  | 2  | D                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | 13                                  | <input type="checkbox"/> |
| 24 | 18 | 21 | 8  | 14 | 19 | 11 |    | 4 | 9  | 6  | 2  | 13                                  | E                        |

**Problem 58 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
23	22	18	21	15	16	13	6	9	11	14	5	12

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |    |    |    |    |   |    |    |    |    |                                     |                          |
|----|----|----|----|----|----|----|---|----|----|----|----|-------------------------------------|--------------------------|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9  | 10 | 11 | 12 | 13                                  | <input type="checkbox"/> |
| 22 | 21 | 18 | 9  | 15 | 16 | 13 | 6 |    | 11 | 14 | 5  | 12                                  | A                        |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9  | 10 | 11 | 12 | <input type="checkbox"/>            |                          |
| 22 | 21 | 18 | 9  | 15 | 16 | 13 | 6 | 12 | 11 | 14 | 5  | B                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9  | 10 | 11 | 12 | <input type="checkbox"/>            |                          |
| 22 | 18 | 21 | 15 | 16 | 13 | 6  | 9 | 11 | 14 | 5  | 12 | C                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9  | 10 | 11 | 12 | <input type="checkbox"/>            |                          |
| 22 | 21 | 18 | 9  | 15 | 16 | 13 | 6 | 11 | 14 | 5  | 12 | D                                   |                          |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9  | 10 | 11 | 12 | <input checked="" type="checkbox"/> |                          |
| 22 | 21 | 18 | 12 | 15 | 16 | 13 | 6 | 9  | 11 | 14 | 5  | X                                   |                          |

**Problem 59 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
26	25	22	19	10	13	20	8	7	5	1	6	9

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |   |    |    |    |   |   |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 |
| 25 | 19 | 22 | 8 | 10 | 13 | 20 | 9 | 7 | 5  | 1  | 6  |

 A
- |    |    |    |   |    |    |    |   |   |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 |
| 25 | 19 | 22 | 9 | 10 | 13 | 20 | 8 | 7 | 5  | 1  | 6  |

 B
- |    |    |    |    |    |    |   |   |   |    |    |    |
|----|----|----|----|----|----|---|---|---|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 | 10 | 11 | 12 |
| 25 | 22 | 20 | 10 | 13 | 19 | 8 | 7 | 5 | 1  | 6  | 9  |

 C
- |    |    |    |   |    |    |    |   |   |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 |
| 25 | 19 | 22 | 8 | 10 | 13 | 20 | 7 | 5 | 1  | 6  | 9  |

 D
- |    |    |    |   |    |    |    |   |   |    |    |    |    |
|----|----|----|---|----|----|----|---|---|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 | 13 |
| 25 | 19 | 22 | 8 | 10 | 13 | 20 |   | 7 | 5  | 1  | 6  | 9  |

 E

**Problem 60 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13
26	24	22	13	23	19	8	11	12	2	10	4	15

What is the result of applying HEAP-EXTRACT-MAX to the above max-heap?

- |    |    |    |    |    |    |   |    |    |    |    |    |    |
|----|----|----|----|----|----|---|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 | 13 |
| 24 | 23 | 22 | 13 | 10 | 19 | 8 | 11 | 12 | 2  |    | 4  | 15 |

 A
- |    |    |    |    |    |    |   |    |    |    |    |    |
|----|----|----|----|----|----|---|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 |
| 24 | 23 | 22 | 13 | 10 | 19 | 8 | 11 | 12 | 2  | 4  | 15 |

 B
- |    |    |    |    |    |    |    |    |   |    |    |    |
|----|----|----|----|----|----|----|----|---|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 |
| 24 | 23 | 15 | 22 | 19 | 13 | 11 | 12 | 2 | 10 | 4  | 8  |

 C
- |    |    |    |    |    |    |   |    |    |    |    |    |
|----|----|----|----|----|----|---|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 |
| 24 | 23 | 22 | 13 | 15 | 19 | 8 | 11 | 12 | 2  | 10 | 4  |

 D
- |    |    |    |    |    |    |   |    |    |    |    |    |
|----|----|----|----|----|----|---|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 |
| 24 | 23 | 22 | 13 | 10 | 19 | 8 | 11 | 12 | 2  | 15 | 4  |

 E

## Partition

### Problem 61 (4%)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
14	17	1	25	3	13	22	24	9	7	29	30	10	23	12

Indicate the result of applying  $\text{PARTITION}(A, 3, 14)$  to the above array  $A$ .

- | 1  | 2  | 3 | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|----|----|---|---|---|---|----|----|----|----|----|----|----|----|----|
| 14 | 17 | 1 | 3 | 7 | 9 | 10 | 13 | 22 | 23 | 24 | 25 | 29 | 30 | 12 |
- 
- A
- 
- | 1  | 2  | 3 | 4 | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|----|----|---|---|----|----|---|---|----|----|----|----|----|----|----|
| 14 | 17 | 1 | 3 | 13 | 22 | 9 | 7 | 10 | 23 | 29 | 30 | 25 | 24 | 12 |
- 
- B
- 
- | 1  | 2  | 3 | 4 | 5  | 6  | 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|----|----|---|---|----|----|---|---|----|----|----|----|----|----|----|
| 14 | 17 | 1 | 3 | 13 | 22 | 9 | 7 | 10 | 23 | 25 | 24 | 29 | 30 | 12 |
- 
- C
- 
- | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 3 | 7 | 9 | 10 | 12 | 13 | 14 | 17 | 22 | 23 | 24 | 25 | 29 | 30 |
- 
- D

### Problem 62 (4%)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15	21	26	23	28	18	22	19	8	4	20	6	24	12	3

Indicate the result of applying  $\text{PARTITION}(A, 2, 12)$  to the above array  $A$ .

- | 1  | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|----|---|---|----|----|----|----|----|---|----|----|----|----|----|----|
| 15 | 4 | 6 | 23 | 28 | 18 | 22 | 19 | 8 | 21 | 20 | 26 | 24 | 12 | 3  |
- 
- A
- 
- | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 3 | 4 | 6 | 8 | 12 | 15 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 26 | 28 |
- 
- B
- 
- | 1  | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|----|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 15 | 4 | 6 | 8 | 18 | 19 | 20 | 21 | 22 | 23 | 26 | 28 | 24 | 12 | 3  |
- 
- C
- 
- | 1  | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|----|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| 15 | 4 | 6 | 21 | 26 | 23 | 28 | 18 | 22 | 19 | 8  | 20 | 24 | 12 | 3  |
- 
- D

**Problem 63 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
21	17	8	23	29	2	18	26	22	20	25	11	1	9	28

Indicate the result of applying  $\text{PARTITION}(A, 3, 13)$  to the above array  $A$ .

- |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 2 | 8 | 9 | 11 | 17 | 18 | 20 | 21 | 22 | 23 | 25 | 26 | 28 | 29 |
- 
- A
- 
- |    |    |   |   |    |    |   |    |    |    |    |    |    |    |    |
|----|----|---|---|----|----|---|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 21 | 17 | 1 | 8 | 23 | 29 | 2 | 18 | 26 | 22 | 20 | 25 | 11 | 9  | 28 |
- 
- B
- 
- |    |    |   |   |   |    |    |    |    |    |    |    |    |    |    |
|----|----|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5 | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 21 | 17 | 1 | 2 | 8 | 11 | 18 | 20 | 22 | 23 | 25 | 26 | 29 | 9  | 28 |
- 
- C
- 
- |    |    |   |    |    |   |    |    |    |    |    |    |    |    |    |
|----|----|---|----|----|---|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4  | 5  | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 21 | 17 | 1 | 23 | 29 | 2 | 18 | 26 | 22 | 20 | 25 | 11 | 8  | 9  | 28 |
- 
- D

**Problem 64 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
17	13	3	15	4	27	30	21	22	1	20	8	12	11	25

Indicate the result of applying  $\text{PARTITION}(A, 4, 12)$  to the above array  $A$ .

- |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |
|----|----|---|---|---|---|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 17 | 13 | 3 | 4 | 1 | 8 | 30 | 21 | 22 | 15 | 20 | 27 | 12 | 11 | 25 |
- 
- A
- 
- |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |
|----|----|---|---|---|---|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 17 | 13 | 3 | 1 | 4 | 8 | 15 | 20 | 21 | 22 | 27 | 30 | 12 | 11 | 25 |
- 
- B
- 
- |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |
|----|----|---|---|---|---|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 17 | 13 | 3 | 4 | 1 | 8 | 15 | 27 | 30 | 21 | 22 | 20 | 12 | 11 | 25 |
- 
- C
- 
- |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 3 | 4 | 8 | 11 | 12 | 13 | 15 | 17 | 20 | 21 | 22 | 25 | 27 | 30 |
- 
- D

**Problem 65 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
21	24	30	25	27	3	17	29	4	15	23	2	6	5	10

Indicate the result of applying  $\text{PARTITION}(A, 4, 12)$  to the above array  $A$ .

- |    |    |    |   |   |   |    |    |    |    |    |    |    |    |    |
|----|----|----|---|---|---|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 21 | 24 | 30 | 2 | 3 | 4 | 15 | 17 | 23 | 25 | 27 | 29 | 6  | 5  | 10 |
- 
- A
- 
- |    |    |    |   |    |   |    |    |   |    |    |    |    |    |    |
|----|----|----|---|----|---|----|----|---|----|----|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6 | 7  | 8  | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 21 | 24 | 30 | 2 | 27 | 3 | 17 | 29 | 4 | 15 | 23 | 25 | 6  | 5  | 10 |
- 
- B
- 
- |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 2 | 3 | 4 | 5 | 6 | 10 | 15 | 17 | 21 | 23 | 24 | 25 | 27 | 29 | 30 |
- 
- C
- 
- |    |    |    |   |    |    |   |    |    |    |    |    |    |    |    |
|----|----|----|---|----|----|---|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4 | 5  | 6  | 7 | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 21 | 24 | 30 | 2 | 25 | 27 | 3 | 17 | 29 | 4  | 15 | 23 | 6  | 5  | 10 |
- 
- D

**Problem 66 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
30	12	26	1	5	19	8	28	29	9	22	18	2	27	21

Indicate the result of applying PARTITION( $A, 4, 13$ ) to the above array  $A$ .

- |    |    |    |   |   |    |   |    |    |    |    |    |    |    |    |
|----|----|----|---|---|----|---|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4 | 5 | 6  | 7 | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 30 | 12 | 26 | 1 | 2 | 19 | 8 | 28 | 29 | 9  | 22 | 18 | 5  | 27 | 21 |
- 
- 
- |    |    |    |   |   |   |   |   |    |    |    |    |    |    |    |
|----|----|----|---|---|---|---|---|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4 | 5 | 6 | 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 30 | 12 | 26 | 1 | 2 | 5 | 8 | 9 | 18 | 19 | 22 | 28 | 29 | 27 | 21 |
- 
- B
- 
- |    |    |    |   |   |   |    |   |    |    |    |    |    |    |    |
|----|----|----|---|---|---|----|---|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4 | 5 | 6 | 7  | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 30 | 12 | 26 | 1 | 2 | 5 | 19 | 8 | 28 | 29 | 9  | 22 | 18 | 27 | 21 |
- 
- C
- 
- |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 2 | 5 | 8 | 9 | 12 | 18 | 19 | 21 | 22 | 26 | 27 | 28 | 29 | 30 |
- 
- D

**Problem 67 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12	26	15	10	27	25	19	6	8	3	16	23	4	1	22

Indicate the result of applying PARTITION( $A, 3, 14$ ) to the above array  $A$ .

- |    |    |   |    |    |    |    |   |   |    |    |    |    |    |    |
|----|----|---|----|----|----|----|---|---|----|----|----|----|----|----|
| 1  | 2  | 3 | 4  | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 12 | 26 | 1 | 10 | 27 | 25 | 19 | 6 | 8 | 3  | 16 | 23 | 4  | 15 | 22 |
- 
- 
- |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 3 | 4 | 6 | 8 | 10 | 12 | 15 | 16 | 19 | 22 | 23 | 25 | 26 | 27 |
- 
- B
- 
- |    |    |   |    |    |    |    |    |   |    |    |    |    |    |    |
|----|----|---|----|----|----|----|----|---|----|----|----|----|----|----|
| 1  | 2  | 3 | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 12 | 26 | 1 | 15 | 10 | 27 | 25 | 19 | 6 | 8  | 3  | 16 | 23 | 4  | 22 |
- 
- C
- 
- |    |    |   |   |   |   |   |    |    |    |    |    |    |    |    |
|----|----|---|---|---|---|---|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5 | 6 | 7 | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 12 | 26 | 1 | 3 | 4 | 6 | 8 | 10 | 15 | 16 | 19 | 23 | 25 | 27 | 22 |
- 
- D

**Problem 68 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
19	24	1	11	23	6	18	21	22	10	3	2	25	13	12

Indicate the result of applying PARTITION( $A, 4, 14$ ) to the above array  $A$ .

- |    |    |   |    |   |    |   |   |    |    |    |    |    |    |    |
|----|----|---|----|---|----|---|---|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4  | 5 | 6  | 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 19 | 24 | 1 | 11 | 6 | 10 | 3 | 2 | 13 | 23 | 18 | 21 | 22 | 25 | 12 |
- 
- A
- 
- |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |
|----|----|---|---|---|---|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4 | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 19 | 24 | 1 | 2 | 3 | 6 | 10 | 11 | 13 | 18 | 21 | 22 | 23 | 25 | 12 |
- 
- B
- 
- |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 2 | 3 | 6 | 10 | 11 | 12 | 13 | 18 | 19 | 21 | 22 | 23 | 24 | 25 |
- 
- C
- 
- |    |    |   |    |   |    |   |   |    |    |    |    |    |    |    |
|----|----|---|----|---|----|---|---|----|----|----|----|----|----|----|
| 1  | 2  | 3 | 4  | 5 | 6  | 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 19 | 24 | 1 | 11 | 6 | 10 | 3 | 2 | 13 | 23 | 18 | 21 | 25 | 22 | 12 |
-

**Problem 69 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	27	11	15	21	23	10	12	1	5	16	19	13	18	28

Indicate the result of applying  $\text{PARTITION}(A, 2, 13)$  to the above array  $A$ .

- |   |    |    |    |   |   |    |    |    |    |    |    |    |    |    |
|---|----|----|----|---|---|----|----|----|----|----|----|----|----|----|
| 1 | 2  | 3  | 4  | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 11 | 10 | 12 | 1 | 5 | 13 | 15 | 21 | 23 | 16 | 19 | 27 | 18 | 28 |
- 
- 
- |   |    |    |    |   |   |    |    |    |    |    |    |    |    |    |
|---|----|----|----|---|---|----|----|----|----|----|----|----|----|----|
| 1 | 2  | 3  | 4  | 5 | 6 | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 11 | 10 | 12 | 1 | 5 | 13 | 27 | 15 | 21 | 23 | 16 | 19 | 18 | 28 |
- 
- 
- |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |
|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 5 | 7 | 10 | 11 | 12 | 13 | 15 | 16 | 18 | 19 | 21 | 23 | 27 | 28 |
- 
- 
- |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |
|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 1 | 5 | 10 | 11 | 12 | 13 | 15 | 16 | 19 | 21 | 23 | 27 | 18 | 28 |
- 

**Problem 70 (4%)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11	5	19	13	28	22	15	29	30	17	2	18	12	14	10

Indicate the result of applying  $\text{PARTITION}(A, 3, 14)$  to the above array  $A$ .

- |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |
|----|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 11 | 5 | 2 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 22 | 28 | 29 | 30 | 10 |
- 
- 
- |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 2 | 5 | 10 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 22 | 28 | 29 | 30 |
- 
- 
- |    |   |    |   |    |    |    |    |    |    |    |    |    |    |    |
|----|---|----|---|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2 | 3  | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 11 | 5 | 13 | 2 | 12 | 14 | 19 | 28 | 22 | 15 | 29 | 30 | 17 | 18 | 10 |
- 
- 
- |    |   |    |   |    |    |    |    |    |    |    |    |    |    |    |
|----|---|----|---|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2 | 3  | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 11 | 5 | 13 | 2 | 12 | 14 | 15 | 29 | 30 | 17 | 19 | 18 | 28 | 22 | 10 |
-

## Radix-sort

### Problem 71 (4 %)

2143   2102   0424   3324   1143   1224

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 2102   0424   3324   1224   2143   1143    A
- 2102   2143   1143   0424   3324   1224    B
- 0424   1143   1224   2102   2143   3324    C
- 0424   1143   1224   2143   2102   3324    D
- 2102   0424   1224   3324   1143   2143    E

### Problem 72 (4 %)

4010   3330   4410   1013   1430   1010

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 1010   1013   1430   3330   4010   4410    A
- 4010   4410   1010   1013   3330   1430    B
- 4010   4410   1010   3330   1430   1013    C
- 1013   1010   1430   3330   4010   4410    D
- 1010   4010   4410   1013   1430   3330    E

### Problem 73 (4 %)

2124   3404   2024   4324   1013   2013

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 1013   2013   3404   2124   2024   4324    A
- 1013   2024   2013   2124   3404   4324    B
- 1013   2013   2024   2124   3404   4324    C
- 3404   1013   2013   2024   2124   4324    D
- 3404   1013   2013   2124   2024   4324    E

**Problem 74 (4 %)**

1143 4432 0234 1134 1432 4034

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 4432 1432 0234 1134 4034 1143
- 0234 1143 1134 1432 4034 4432
- 0234 1134 1143 1432 4034 4432
- 1432 4432 0234 1134 4034 1143
- 4432 1432 1143 0234 1134 4034

**Problem 75 (4 %)**

2042 0041 4041 2241 4112 2212

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 4112 2212 0041 4041 2241 2042
- 0041 2042 2241 2212 4041 4112
- 0041 2042 2212 2241 4041 4112
- 0041 4041 2241 4112 2212 2042
- 2212 4112 0041 2241 4041 2042

**Problem 76 (4 %)**

4040 2123 0430 0330 0423 4123

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 0430 0330 4040 2123 0423 4123
- 0330 0423 0430 2123 4040 4123
- 0330 0430 0423 2123 4040 4123
- 0423 2123 4123 0330 0430 4040
- 2123 0423 4123 0430 0330 4040

**Problem 77 (4 %)**

3331 4143 2031 0031 4012 2012

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 4012 2012 3331 2031 0031 4143
- 0031 2031 2012 3331 4012 4143
- 0031 2012 2031 3331 4012 4143
- 2012 4012 0031 2031 3331 4143
- 3331 2031 0031 4012 2012 4143

**Problem 78 (4 %)**

0321 4000 2121 2302 2300 1102

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 4000 2300 0321 2121 2302 1102
- 4000 2300 2302 1102 0321 2121
- 2300 4000 1102 2302 0321 2121
- 0321 1102 2121 2300 2302 4000
- 0321 1102 2121 2302 2300 4000

**Problem 79 (4 %)**

4033 4313 2433 3120 4333 4320

Consider RADIX-SORT applied to the above list of numbers ( $d = 4$ ,  $k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 4313 3120 4320 4033 2433 4333
- 4313 3120 4320 2433 4033 4333
- 2433 3120 4033 4313 4333 4320
- 3120 4320 4313 4033 2433 4333
- 2433 3120 4033 4313 4320 4333

**Problem 80 (4 %)**

3241 2123 1023 0023 3233 0433

Consider RADIX-SORT applied to the above list of numbers ( $d = 4, k = 5$ ). Indicate the partially sorted list after RADIX-SORT has sorted the numbers according to the *two* least significant digits.

- 0023 0433 1023 2123 3241 3233  A
- 2123 1023 0023 3233 0433 3241  B
- 3241 2123 1023 0023 3233 0433  C
- 0023 0433 1023 2123 3233 3241  D
- 0023 1023 2123 0433 3233 3241  E

**Linear probing**

**Problem 81 (4 %)**

0	1	2	3	4	5	6	7	8	9	10
11				5	16				3	14

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 3k \bmod 11$ . Indicate the positions the five elements 0, 2, 7, 8 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 3, 5, 11, 14 and 16).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	81.1
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	81.2
INSERT(7)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	81.3
INSERT(8)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	81.4
INSERT(10)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	81.5

**Problem 82 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22	6							15	4	21

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 2k \bmod 11$ . Indicate the positions the five elements 1, 2, 3, 5 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 4, 6, 15, 21 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(1)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	82.1
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	82.2
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	82.3
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	82.4
INSERT(11)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	82.5

**Problem 83 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22	0							6	17	15

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 5k \bmod 11$ . Indicate the positions the five elements 1, 3, 4, 9 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 6, 15, 17 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(1)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	83.1
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	83.2
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	83.3
INSERT(9)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	83.4
INSERT(10)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	83.5

**Problem 84 (4%)**

0	1	2	3	4	5	6	7	8	9	10
11	22	20							4	13

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 5k \bmod 11$ . Indicate the positions the five elements 0, 2, 3, 5 and 6 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 4, 11, 13, 20 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	84.1
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	84.2
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	84.3
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	84.4
INSERT(6)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	84.5

**Problem 85 (4%)**

0	1	2	3	4	5	6	7	8	9	10
		1				3	14		21	5

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 2k \bmod 11$ . Indicate the positions the five elements 2, 4, 6, 9 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 1, 3, 5, 14 and 21).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	85.1
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	85.2
INSERT(6)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	85.3
INSERT(9)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	85.4
INSERT(10)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	85.5

**Problem 86 (4%)**

0	1	2	3	4	5	6	7	8	9	10
11	0					21			4	15

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 5k \bmod 11$ . Indicate the positions the five elements 2, 3, 5, 7 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 4, 11, 15 and 21).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	86.1
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	86.2
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	86.3
INSERT(7)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	86.4
INSERT(9)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	86.5

**Problem 87 (4%)**

0	1	2	3	4	5	6	7	8	9	10
0				2		14	3			16

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 2k \bmod 11$ . Indicate the positions the five elements 4, 5, 6, 10 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 2, 3, 14 and 16).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	87.1
INSERT(5)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	87.2
INSERT(6)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	87.3
INSERT(10)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input checked="" type="checkbox"/> J	<input type="checkbox"/> K	87.4
INSERT(11)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	87.5

**Problem 88 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22			7	18				15		16

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 2k \bmod 11$ . Indicate the positions the five elements 0, 2, 3, 5 and 6 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 7, 15, 16, 18 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	88.1
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	88.2
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	88.3
INSERT(5)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	88.4
INSERT(6)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	88.5

**Problem 89 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22	20	0							15	2

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 5k \bmod 11$ . Indicate the positions the five elements 4, 6, 7, 8 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 2, 15, 20 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	89.1
INSERT(6)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	89.2
INSERT(7)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	89.3
INSERT(8)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	89.4
INSERT(9)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	89.5

**Problem 90 (4%)**

0	1	2	3	4	5	6	7	8	9	10
		1			19			15	4	16

In the above hash table of size 11, *linear probing* is used with hash function  $h(k) = 2k \bmod 11$ . Indicate the positions the five elements 2, 6, 7, 8 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 1, 4, 15, 16 and 19).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	A	B	C	D	⊗	F	G	H	I	J	K	90.1
INSERT(6)	A	⊗	C	D	E	F	G	H	I	J	K	90.2
INSERT(7)	A	B	C	⊗	E	F	G	H	I	J	K	90.3
INSERT(8)	A	B	C	D	E	F	⊗	H	I	J	K	90.4
INSERT(10)	⊗	B	C	D	E	F	G	H	I	J	K	90.5

**Quadratic probing****Problem 91 (4%)**

0	1	2	3	4	5	6	7	8	9	10
		1	12			13			18	7

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 3k \bmod 11$  and  $h(k, i) = (h'(k) + 5i + 5i^2) \bmod 11$ . Indicate the positions the five elements 0, 2, 3, 5 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 1, 7, 12, 13 and 18).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	⊗	B	C	D	E	F	G	H	I	J	K	91.1
INSERT(2)	A	B	C	D	E	⊗	G	H	I	J	K	91.2
INSERT(3)	A	B	C	D	E	F	G	H	⊗	J	K	91.3
INSERT(5)	A	B	C	D	⊗	F	G	H	I	J	K	91.4
INSERT(9)	A	B	C	D	E	⊗	G	H	I	J	K	91.5

**Problem 92 (4 %)**

0	1	2	3	4	5	6	7	8	9	10
22					0	18	10		5	

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 4k \bmod 11$  and  $h(k, i) = (h'(k) + 2i + 3i^2) \bmod 11$ . Indicate the positions the five elements 2, 3, 7, 9 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 5, 10, 18 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	A	B	C	D	E	F	G	H	⊗	J	K	92.1
INSERT(3)	A	⊗	C	D	E	F	G	H	I	J	K	92.2
INSERT(7)	A	B	C	⊗	E	F	G	H	I	J	K	92.3
INSERT(9)	A	B	C	⊗	E	F	G	H	I	J	K	92.4
INSERT(11)	A	⊗	C	D	E	F	G	H	I	J	K	92.5

**Problem 93 (4 %)**

0	1	2	3	4	5	6	7	8	9	10
22	6					10		15	17	

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 2k \bmod 11$  and  $h(k, i) = (h'(k) + 3i + 5i^2) \bmod 11$ . Indicate the positions the five elements 0, 2, 4, 7 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 6, 10, 15, 17 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	A	B	C	D	⊗	F	G	H	I	J	K	93.1
INSERT(2)	A	B	C	D	⊗	F	G	H	I	J	K	93.2
INSERT(4)	A	B	C	D	E	⊗	G	H	I	J	K	93.3
INSERT(7)	A	B	C	⊗	E	F	G	H	I	J	K	93.4
INSERT(9)	A	B	C	D	E	F	G	⊗	I	J	K	93.5

**Problem 94 (4 %)**

0	1	2	3	4	5	6	7	8	9	10
	15	6			20		17			18

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 3k \bmod 11$  and  $h(k, i) = (h'(k) + 2i + 4i^2) \bmod 11$ . Indicate the positions the five elements 0, 4, 7, 9 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 6, 15, 17, 18 and 20).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	94.1
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	94.2
INSERT(7)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/>	<input type="checkbox"/> J	<input type="checkbox"/> K	94.3
INSERT(9)	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	94.4
INSERT(10)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/>	<input type="checkbox"/> J	<input type="checkbox"/> K	94.5

**Problem 95 (4 %)**

0	1	2	3	4	5	6	7	8	9	10
0					12	10		17	11	

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 5k \bmod 11$  and  $h(k, i) = (h'(k) + 5i^2) \bmod 11$ . Indicate the positions the five elements 1, 3, 4, 5 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 10, 11, 12 and 17).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(1)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input checked="" type="checkbox"/>	95.1
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/>	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	95.2
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	95.3
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	95.4
INSERT(9)	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	95.5

**Problem 96 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22		11				3		4	21	

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 2k \bmod 11$  and  $h(k, i) = (h'(k) + i + i^2) \bmod 11$ . Indicate the positions the five elements 1, 2, 5, 7 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 3, 4, 11, 21 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(1)	A	B	C	D	⊗	F	G	H	I	J	K	96.1
INSERT(2)	A	B	C	D	⊗	F	G	H	I	J	K	96.2
INSERT(5)	A	B	C	D	E	F	G	H	I	J	⊗	96.3
INSERT(7)	A	B	C	⊗	E	F	G	H	I	J	K	96.4
INSERT(10)	A	B	C	D	⊗	F	G	H	I	J	K	96.5

**Problem 97 (4%)**

0	1	2	3	4	5	6	7	8	9	10
3	17					14	20		21	

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 2k \bmod 11$  and  $h(k, i) = (h'(k) + 3i + 2i^2) \bmod 11$ . Indicate the positions the five elements 2, 4, 7, 9 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 3, 14, 17, 20 and 21).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	A	B	C	D	⊗	F	G	H	I	J	K	97.1
INSERT(4)	A	B	C	D	E	F	G	H	⊗	J	K	97.2
INSERT(7)	A	B	C	⊗	E	F	G	H	I	J	K	97.3
INSERT(9)	A	B	C	D	E	F	G	H	I	J	⊗	97.4
INSERT(10)	A	B	C	⊗	E	F	G	H	I	J	K	97.5

**Problem 98 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22		18					19	6	11	

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 5k \bmod 11$  and  $h(k, i) = (h'(k) + 5i + 2i^2) \bmod 11$ . Indicate the positions the five elements 0, 1, 2, 3 and 8 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 6, 11, 18, 19 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	98.1
INSERT(1)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	98.2
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input checked="" type="checkbox"/> K	98.3
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	98.4
INSERT(8)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	98.5

**Problem 99 (4%)**

0	1	2	3	4	5	6	7	8	9	10
11			18	13			20	22		

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 2k \bmod 11$  and  $h(k, i) = (h'(k) + 3i + 5i^2) \bmod 11$ . Indicate the positions the five elements 2, 5, 6, 7 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 11, 13, 18, 20 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	99.1
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input checked="" type="checkbox"/> K	99.2
INSERT(6)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	99.3
INSERT(7)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	99.4
INSERT(9)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	99.5

**Problem 100 (4%)**

0	1	2	3	4	5	6	7	8	9	10
0	22			13	8			15		

In the above hash table of size 11, *quadratic probing* is used with hash functions  $h'(k) = 2k \bmod 11$  and  $h(k, i) = (h'(k) + i^2) \bmod 11$ . Indicate the positions the five elements 2, 4, 6, 9 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 8, 13, 15 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	100.1
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input checked="" type="checkbox"/> J	<input type="checkbox"/> K	100.2
INSERT(6)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	100.3
INSERT(9)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	100.4
INSERT(11)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input checked="" type="checkbox"/> J	<input type="checkbox"/> K	100.5

**Double hashing**

**Problem 101 (4%)**

0	1	2	3	4	5	6	7	8	9	10
		19	12		14		6		3	

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 3k \bmod 11$  and  $h_2(k) = 1 + (4k \bmod 10)$ . Indicate the positions the five elements 2, 5, 8, 9 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 3, 6, 12, 14 and 19).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	101.1
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	101.2
INSERT(8)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	101.3
INSERT(9)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	101.4
INSERT(11)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	101.5

**Problem 102 (4%)**

0	1	2	3	4	5	6	7	8	9	10
		19	7		8		9	18		

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 2k \bmod 11$  and  $h_2(k) = 1 + (3k \bmod 10)$ . Indicate the positions the five elements 1, 2, 4, 6 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 7, 8, 9, 18 and 19).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(1)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	102.1
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	102.2
INSERT(4)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	102.3
INSERT(6)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	102.4
INSERT(11)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	102.5

**Problem 103 (4%)**

0	1	2	3	4	5	6	7	8	9	10
11	9			13			22			16

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 2k \bmod 11$  and  $h_2(k) = 1 + (3k \bmod 10)$ . Indicate the positions the five elements 0, 4, 5, 7 and 8 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 9, 11, 13, 16 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	103.1
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	103.2
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	103.3
INSERT(7)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	103.4
INSERT(8)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	103.5

**Problem 104 (4%)**

0	1	2	3	4	5	6	7	8	9	10
10							21	19	16	8

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 4k \bmod 11$  and  $h_2(k) = 1 + (2k \bmod 10)$ . Indicate the positions the five elements 0, 1, 2, 3 and 4 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 8, 10, 16, 19 and 21).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	A	⊗	C	D	E	F	G	H	I	J	K	104.1
INSERT(1)	A	B	C	D	⊗	F	G	H	I	J	K	104.2
INSERT(2)	A	B	⊗	D	E	F	G	H	I	J	K	104.3
INSERT(3)	A	⊗	C	D	E	F	G	H	I	J	K	104.4
INSERT(4)	A	B	C	D	E	⊗	G	H	I	J	K	104.5

**Problem 105 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22	0	6							5	19

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 4k \bmod 11$  and  $h_2(k) = 1 + (3k \bmod 10)$ . Indicate the positions the five elements 2, 3, 4, 8 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 5, 6, 19 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(2)	A	B	C	D	E	F	G	H	⊗	J	K	105.1
INSERT(3)	A	B	C	D	E	F	G	H	⊗	J	K	105.2
INSERT(4)	A	B	C	D	E	⊗	G	H	I	J	K	105.3
INSERT(8)	A	B	C	D	⊗	F	G	H	I	J	K	105.4
INSERT(9)	A	B	C	⊗	E	F	G	H	I	J	K	105.5

**Problem 106 (4%)**

0	1	2	3	4	5	6	7	8	9	10
11				1	7	18		2		

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 4k \bmod 11$  and  $h_2(k) = 1 + (2k \bmod 10)$ . Indicate the positions the five elements 0, 3, 4, 5 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 1, 2, 7, 11 and 18).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	A	<input checked="" type="checkbox"/>	C	D	E	F	G	H	I	J	K	106.1
INSERT(3)	A	<input checked="" type="checkbox"/>	C	D	E	F	G	H	I	J	K	106.2
INSERT(4)	A	B	C	<input checked="" type="checkbox"/>	E	F	G	H	I	J	K	106.3
INSERT(5)	A	B	C	D	E	F	G	H	I	<input checked="" type="checkbox"/>	K	106.4
INSERT(9)	A	B	C	<input checked="" type="checkbox"/>	E	F	G	H	I	J	K	106.5

**Problem 107 (4%)**

0	1	2	3	4	5	6	7	8	9	10
11	14			1		7				3

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 4k \bmod 11$  and  $h_2(k) = 1 + (3k \bmod 10)$ . Indicate the positions the five elements 0, 4, 6, 8 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 1, 3, 7, 11 and 14).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	A	B	<input checked="" type="checkbox"/>	D	E	F	G	H	I	J	K	107.1
INSERT(4)	A	B	C	D	E	<input checked="" type="checkbox"/>	G	H	I	J	K	107.2
INSERT(6)	A	B	<input checked="" type="checkbox"/>	D	E	F	G	H	I	J	K	107.3
INSERT(8)	A	B	C	D	E	F	G	H	I	<input checked="" type="checkbox"/>	K	107.4
INSERT(10)	A	B	C	D	E	F	G	<input checked="" type="checkbox"/>	I	J	K	107.5

**Problem 108 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22			5		12		8			11

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 5k \bmod 11$  and  $h_2(k) = 1 + (4k \bmod 10)$ . Indicate the positions the five elements 0, 1, 2, 6 and 9 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 5, 8, 11, 12 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(0)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	108.1
INSERT(1)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	108.2
INSERT(2)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	108.3
INSERT(6)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	108.4
INSERT(9)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	108.5

**Problem 109 (4%)**

0	1	2	3	4	5	6	7	8	9	10
22		1			12				21	10

In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 2k \bmod 11$  and  $h_2(k) = 1 + (3k \bmod 10)$ . Indicate the positions the five elements 3, 4, 5, 6 and 11 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 1, 10, 12, 21 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(3)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	109.1
INSERT(4)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	109.2
INSERT(5)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	109.3
INSERT(6)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	109.4
INSERT(11)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	<input type="checkbox"/> J	<input type="checkbox"/> K	109.5

**Problem 110 (4%)**

0	1	2	3	4	5	6	7	8	9	10
0		1	18		22		12			

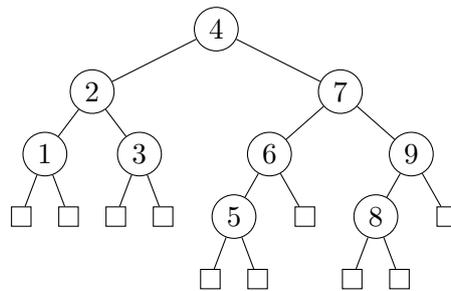
In the above hash table of size 11, *double hashing* is used with hash functions  $h_1(k) = 2k \bmod 11$  and  $h_2(k) = 1 + (2k \bmod 10)$ . Indicate the positions the five elements 3, 7, 8, 9 and 10 will be inserted at in the hash table (for each insertion, we assume the hash table only contains the elements 0, 1, 12, 18 and 22).

	0	1	2	3	4	5	6	7	8	9	10	
INSERT(3)	A	B	C	D	E	F	⊗	H	I	J	K	110.1
INSERT(7)	A	B	C	D	E	F	G	H	⊗	J	K	110.2
INSERT(8)	A	⊗	C	D	E	F	G	H	I	J	K	110.3
INSERT(9)	A	⊗	C	D	E	F	G	H	I	J	K	110.4
INSERT(10)	A	B	C	D	E	F	G	H	I	⊗	K	110.5

**Valid red-black trees**

**Problem 111 (4%)**

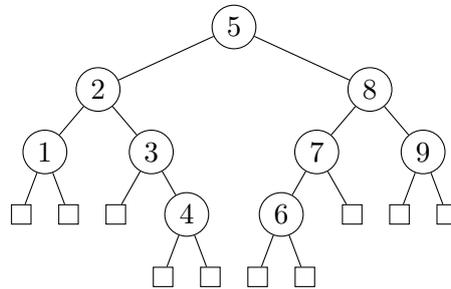
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
1, 3, 5, 7, 8	⊗	B	111.1
5, 8	⊗	B	111.2
4, 5, 8	A	⊗	111.3
2, 5, 6, 8, 9	A	⊗	111.4
2, 5, 7, 8	⊗	B	111.5

**Problem 112 (4%)**

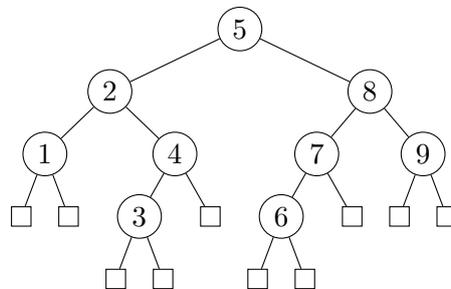
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
2, 4, 6, 7, 9	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	112.1
1, 3, 4, 6, 8	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	112.2
4, 5, 6	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	112.3
2, 4, 6, 8	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	112.4
4, 6	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	112.5

**Problem 113 (4%)**

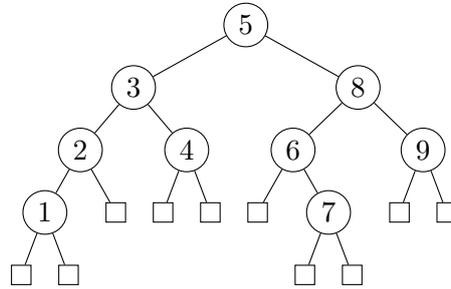
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
3, 5, 6	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	113.1
2, 3, 6, 7, 9	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	113.2
2, 3, 6, 8	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	113.3
1, 3, 4, 6, 8	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	113.4
3, 6	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	113.5

**Problem 114 (4 %)**

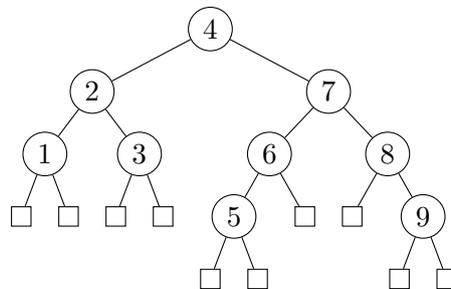
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
1, 7	<input checked="" type="checkbox"/>	<input type="checkbox"/>	114.1
1, 5, 7	<input type="checkbox"/>	<input checked="" type="checkbox"/>	114.2
1, 3, 7, 8	<input checked="" type="checkbox"/>	<input type="checkbox"/>	114.3
1, 2, 4, 7, 8	<input type="checkbox"/>	<input checked="" type="checkbox"/>	114.4
1, 3, 6, 7, 9	<input type="checkbox"/>	<input checked="" type="checkbox"/>	114.5

**Problem 115 (4 %)**

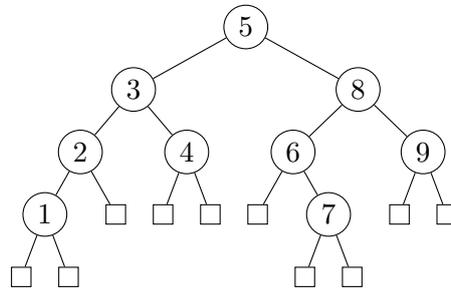
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
4, 5, 9	<input type="checkbox"/>	<input checked="" type="checkbox"/>	115.1
1, 3, 5, 6, 8, 9	<input type="checkbox"/>	<input checked="" type="checkbox"/>	115.2
1, 3, 5, 7, 9	<input checked="" type="checkbox"/>	<input type="checkbox"/>	115.3
5, 9	<input checked="" type="checkbox"/>	<input type="checkbox"/>	115.4
2, 5, 7, 9	<input checked="" type="checkbox"/>	<input type="checkbox"/>	115.5

**Problem 116 (4%)**

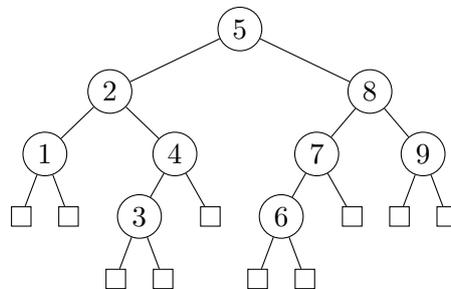
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
1, 5, 7	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	116.1
1, 3, 7, 8	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	116.2
1, 2, 4, 7, 8	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	116.3
1, 7	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	116.4
1, 3, 6, 7, 9	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	116.5

**Problem 117 (4%)**

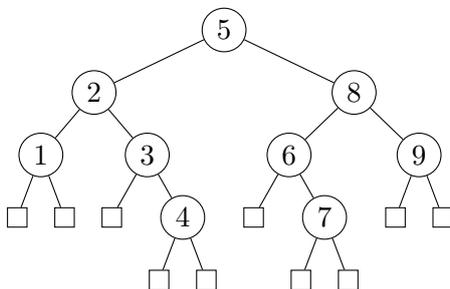
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
3, 6	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	117.1
1, 3, 4, 6, 8	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	117.2
3, 5, 6	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	117.3
2, 3, 6, 8	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	117.4
2, 3, 6, 7, 9	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	117.5

**Problem 118 (4%)**

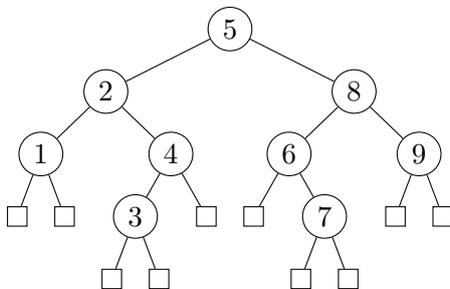
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
4, 7	<input checked="" type="checkbox"/>	<input type="checkbox"/>	118.1
1, 3, 4, 7, 8	<input type="checkbox"/>	<input checked="" type="checkbox"/>	118.2
2, 4, 7, 8	<input checked="" type="checkbox"/>	<input type="checkbox"/>	118.3
2, 4, 6, 7, 9	<input type="checkbox"/>	<input checked="" type="checkbox"/>	118.4
4, 5, 7	<input type="checkbox"/>	<input checked="" type="checkbox"/>	118.5

**Problem 119 (4%)**

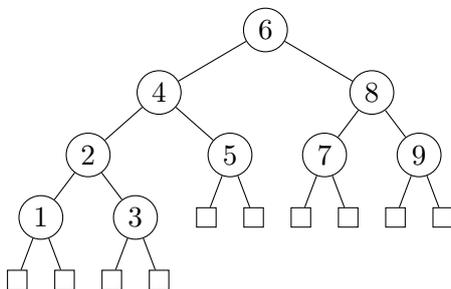
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
3, 7	<input checked="" type="checkbox"/>	<input type="checkbox"/>	119.1
1, 3, 4, 7, 8	<input type="checkbox"/>	<input checked="" type="checkbox"/>	119.2
3, 5, 7	<input type="checkbox"/>	<input checked="" type="checkbox"/>	119.3
2, 3, 7, 8	<input checked="" type="checkbox"/>	<input type="checkbox"/>	119.4
2, 3, 6, 7, 9	<input type="checkbox"/>	<input checked="" type="checkbox"/>	119.5

**Problem 120 (4 %)**

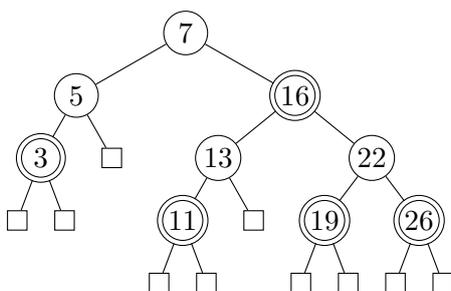
For each of the following subsets, indicate whether the binary tree below is a valid red-black tree if these nodes are colored red.



	Yes	No	
1, 3, 4, 7, 9	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	120.1
1, 3, 6	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	120.2
1, 2, 3, 5, 7, 9	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	120.3
1, 3, 4, 8	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	120.4
1, 3	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	120.5

**Red-black tree insertion**

**Problem 121 (4 %)**



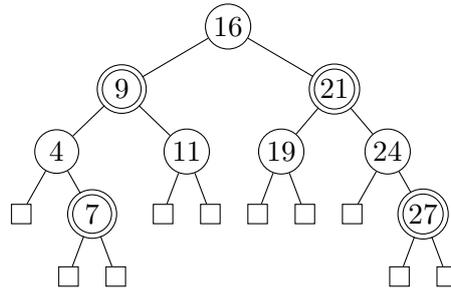
Indicate the resulting red-black tree when inserting 21 into the above red-black tree (double circles indicate red nodes).

A

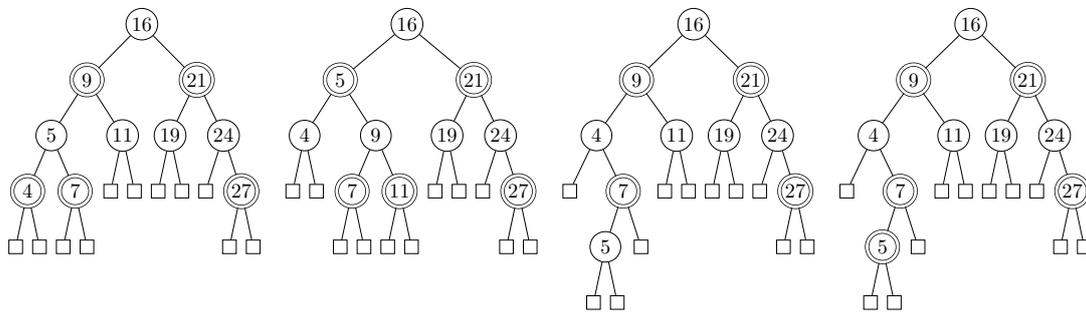
B

C

**Problem 122 (4 %)**



Indicate the resulting red-black tree when inserting 5 into the above red-black tree (double circles indicate red nodes).



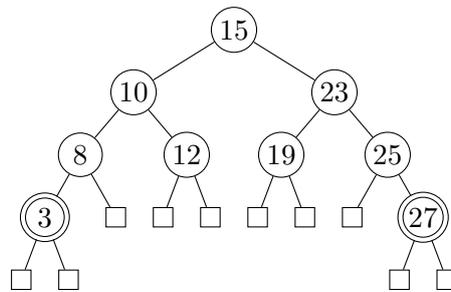
A

B

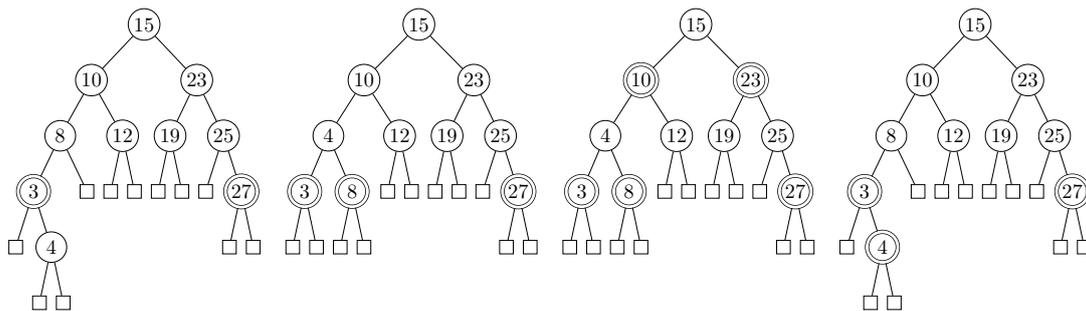
C

D

**Problem 123 (4 %)**



Indicate the resulting red-black tree when inserting 4 into the above red-black tree (double circles indicate red nodes).



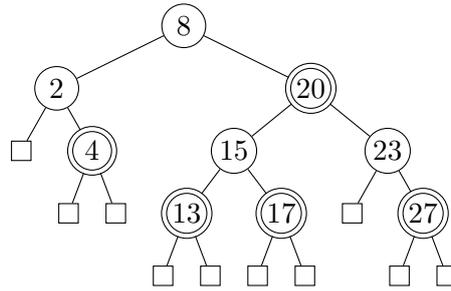
A

B

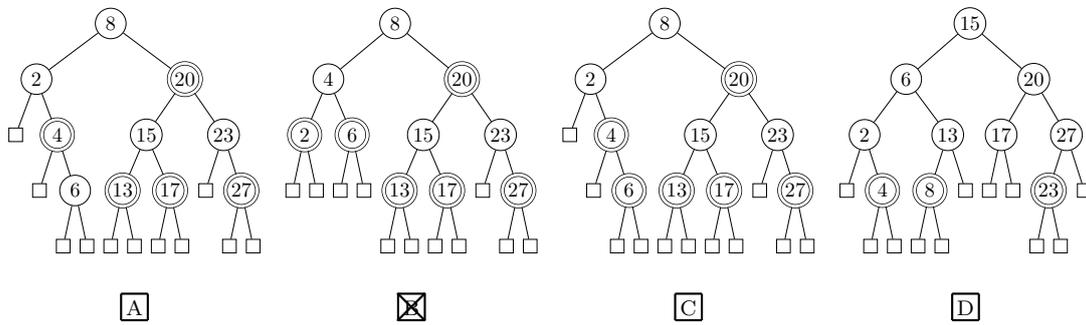
C

D

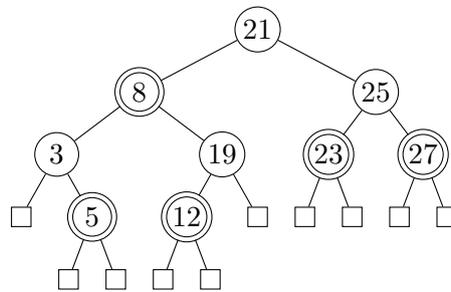
**Problem 124 (4 %)**



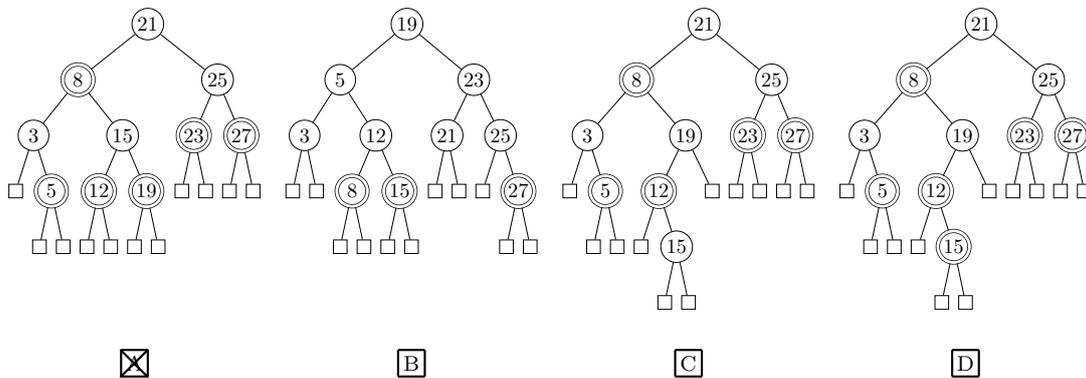
Indicate the resulting red-black tree when inserting 6 into the above red-black tree (double circles indicate red nodes).



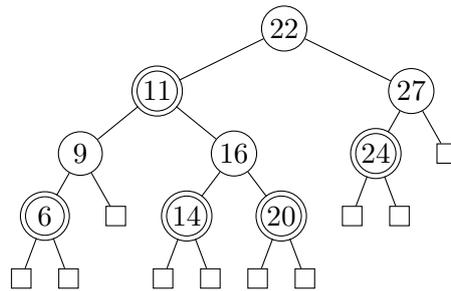
**Problem 125 (4 %)**



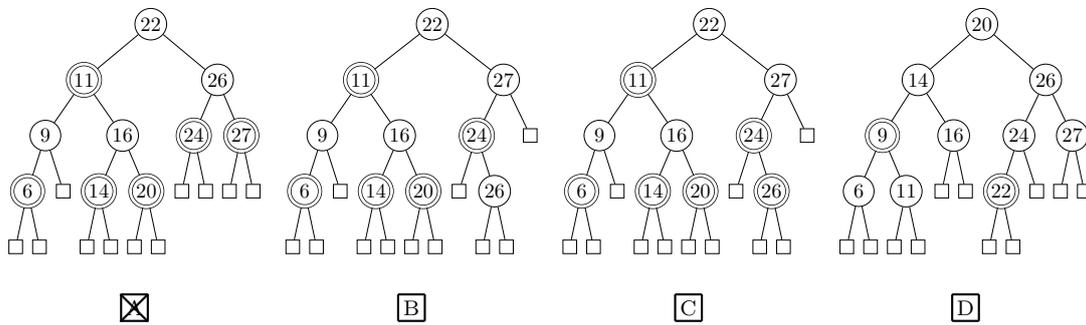
Indicate the resulting red-black tree when inserting 15 into the above red-black tree (double circles indicate red nodes).



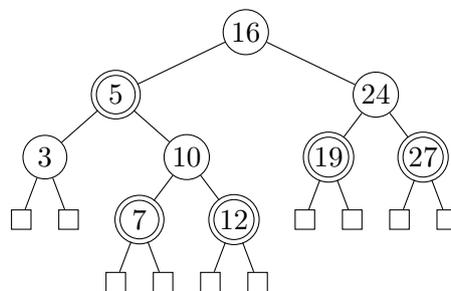
**Problem 126 (4 %)**



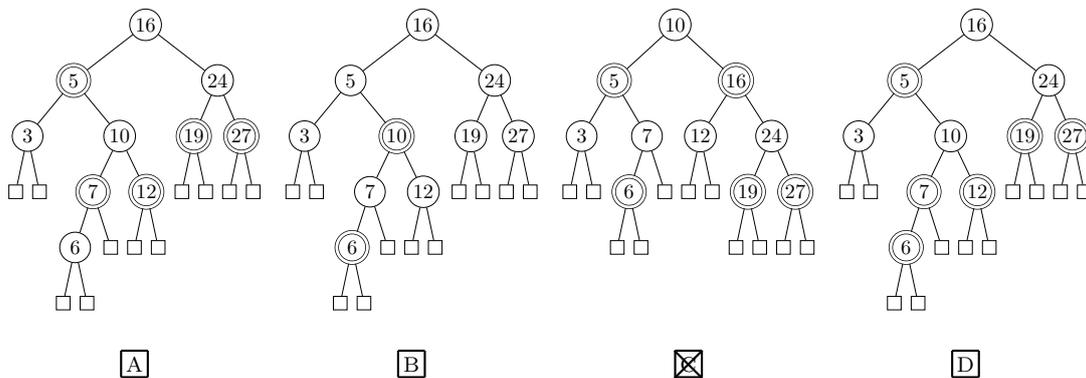
Indicate the resulting red-black tree when inserting 26 into the above red-black tree (double circles indicate red nodes).



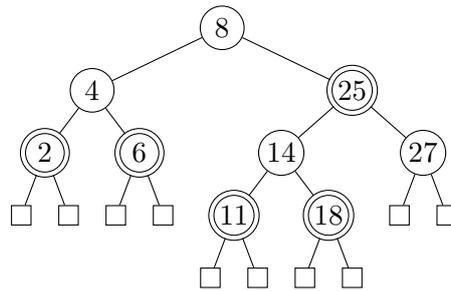
**Problem 127 (4 %)**



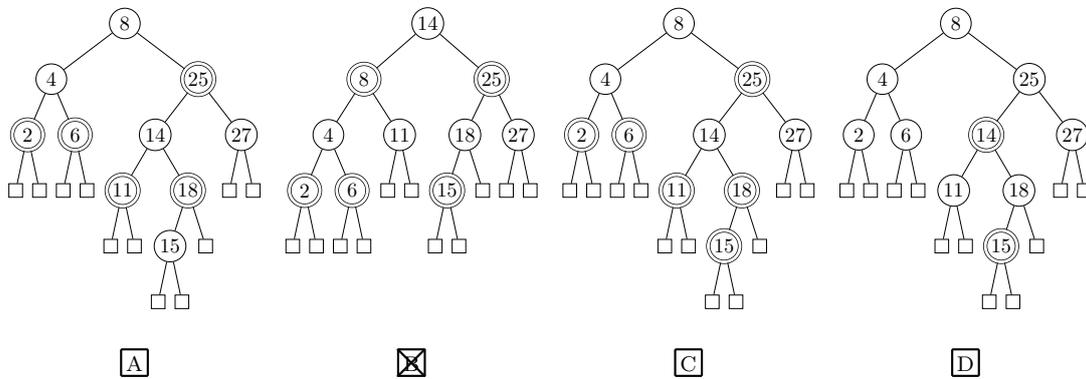
Indicate the resulting red-black tree when inserting 6 into the above red-black tree (double circles indicate red nodes).



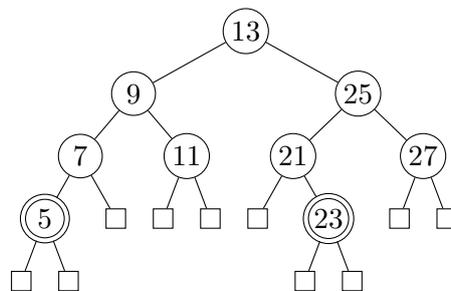
**Problem 128 (4 %)**



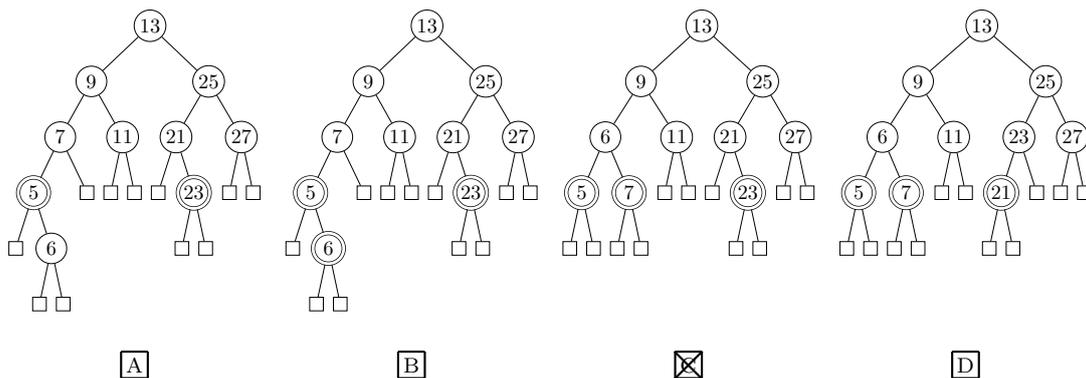
Indicate the resulting red-black tree when inserting 15 into the above red-black tree (double circles indicate red nodes).



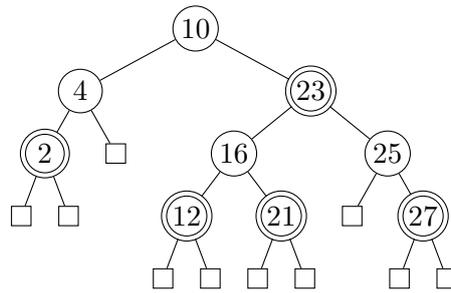
**Problem 129 (4 %)**



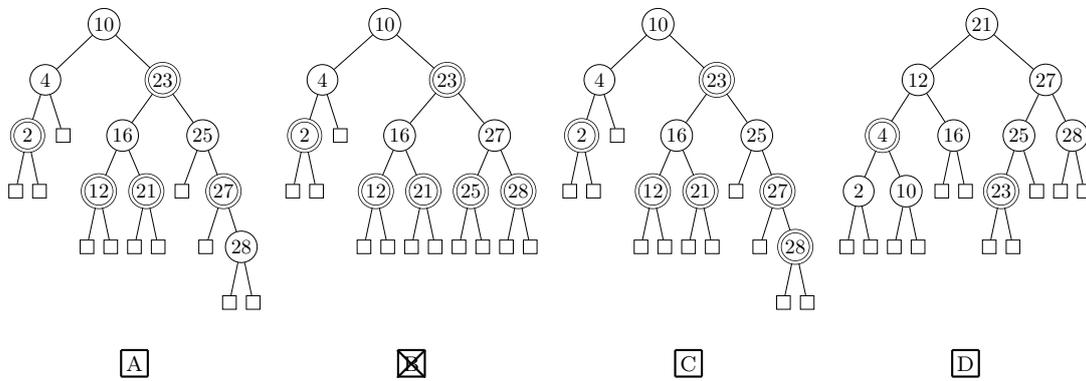
Indicate the resulting red-black tree when inserting 6 into the above red-black tree (double circles indicate red nodes).



**Problem 130 (4%)**



Indicate the resulting red-black tree when inserting 28 into the above red-black tree (double circles indicate red nodes).

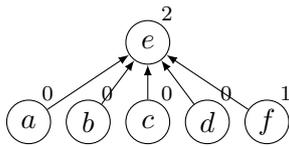


## Union-find

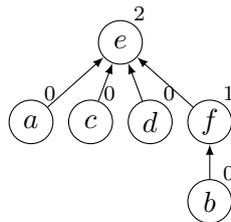
### Problem 131 (4 %)

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

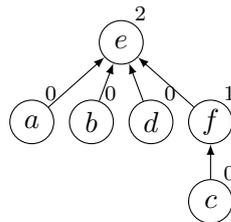
MAKESET( $a$ )  
 MAKESET( $b$ )  
 MAKESET( $c$ )  
 MAKESET( $d$ )  
 MAKESET( $e$ )  
 MAKESET( $f$ )  
 UNION( $c, f$ )  
 UNION( $c, b$ )  
 UNION( $d, e$ )  
 UNION( $c, e$ )  
 UNION( $a, c$ )  
 FIND-SET( $b$ )



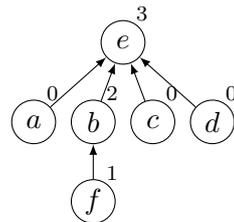
A



B



C

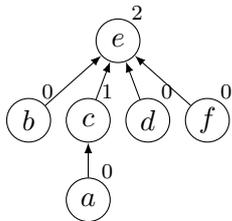


D

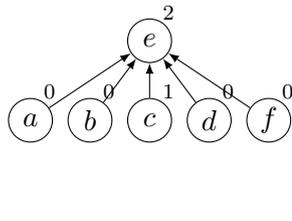
**Problem 132 (4 %)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

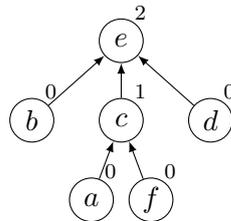
- MAKESET( $a$ )
- MAKESET( $b$ )
- MAKESET( $c$ )
- MAKESET( $d$ )
- MAKESET( $e$ )
- MAKESET( $f$ )
- UNION( $f, c$ )
- UNION( $c, a$ )
- UNION( $d, e$ )
- UNION( $a, e$ )
- UNION( $f, b$ )
- FIND-SET( $b$ )



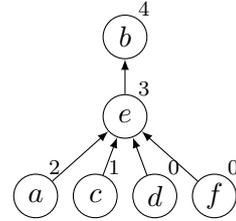
A



B



C

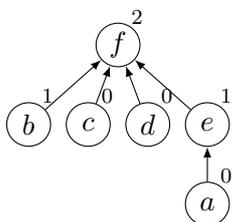


D

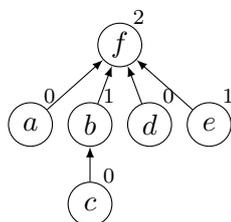
**Problem 133 (4 %)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

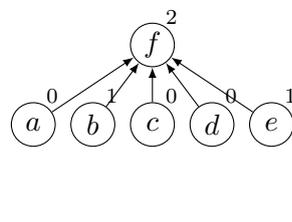
- MAKESET( $a$ )
- MAKESET( $b$ )
- MAKESET( $c$ )
- MAKESET( $d$ )
- MAKESET( $e$ )
- MAKESET( $f$ )
- UNION( $c, b$ )
- UNION( $d, f$ )
- UNION( $c, d$ )
- UNION( $a, e$ )
- UNION( $c, e$ )
- FIND-SET( $a$ )



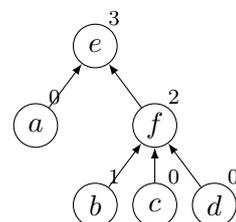
A



B



C

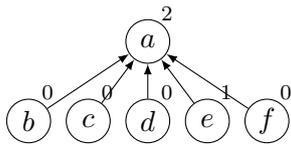


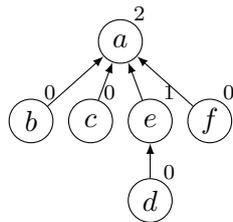
D

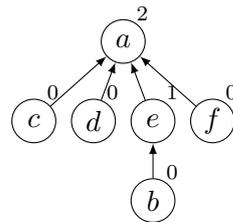
**Problem 134 (4%)**

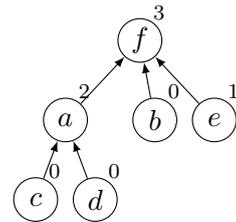
Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

MAKESET( $a$ )  
 MAKESET( $b$ )  
 MAKESET( $c$ )  
 MAKESET( $d$ )  
 MAKESET( $e$ )  
 MAKESET( $f$ )  
 UNION( $d, e$ )  
 UNION( $b, e$ )  
 UNION( $c, a$ )  
 UNION( $e, a$ )  
 UNION( $d, f$ )  
 FIND-SET( $b$ )





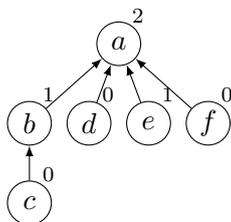




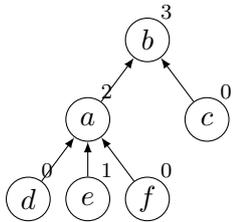
**Problem 135 (4 %)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

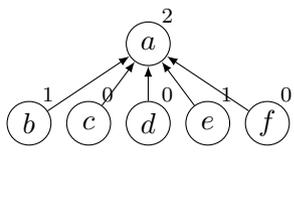
- MAKESET( $a$ )
- MAKESET( $b$ )
- MAKESET( $c$ )
- MAKESET( $d$ )
- MAKESET( $e$ )
- MAKESET( $f$ )
- UNION( $f, e$ )
- UNION( $d, a$ )
- UNION( $f, d$ )
- UNION( $c, b$ )
- UNION( $f, c$ )
- FIND-SET( $b$ )



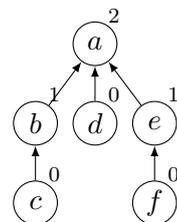
A



B



C

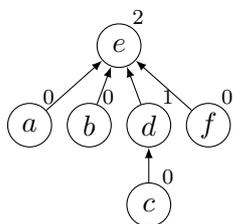


D

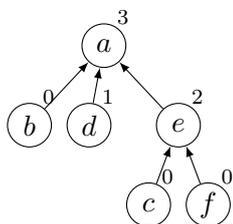
**Problem 136 (4 %)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

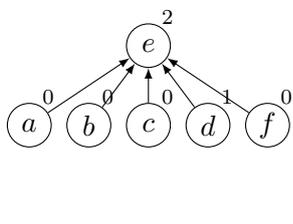
- MAKESET( $a$ )
- MAKESET( $b$ )
- MAKESET( $c$ )
- MAKESET( $d$ )
- MAKESET( $e$ )
- MAKESET( $f$ )
- UNION( $b, d$ )
- UNION( $c, b$ )
- UNION( $f, e$ )
- UNION( $c, f$ )
- UNION( $c, a$ )
- FIND-SET( $b$ )



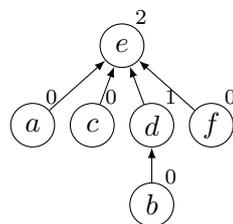
A



B



C

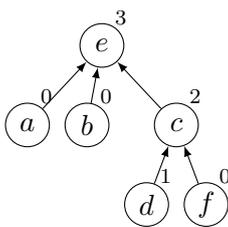


D

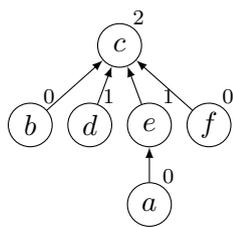
**Problem 137 (4%)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

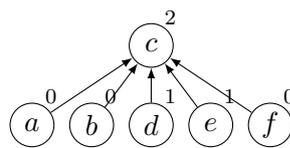
MAKESET( $a$ )  
 MAKESET( $b$ )  
 MAKESET( $c$ )  
 MAKESET( $d$ )  
 MAKESET( $e$ )  
 MAKESET( $f$ )  
 UNION( $f, d$ )  
 UNION( $b, c$ )  
 UNION( $f, c$ )  
 UNION( $a, e$ )  
 UNION( $f, a$ )  
 FIND-SET( $b$ )



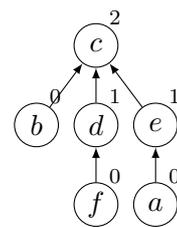
A



B



C

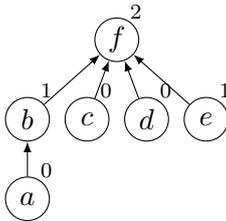


D

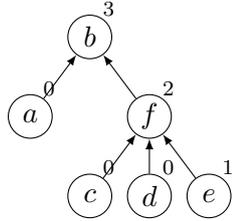
**Problem 138 (4 %)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

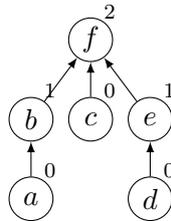
- MAKESET( $a$ )
- MAKESET( $b$ )
- MAKESET( $c$ )
- MAKESET( $d$ )
- MAKESET( $e$ )
- MAKESET( $f$ )
- UNION( $d, e$ )
- UNION( $c, f$ )
- UNION( $e, f$ )
- UNION( $a, b$ )
- UNION( $d, a$ )
- FIND-SET( $b$ )



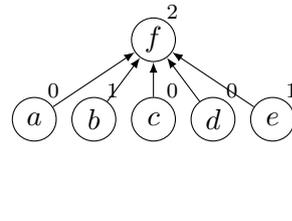
A



B



C

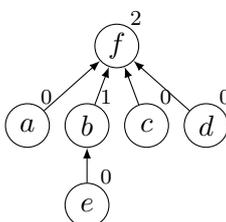


D

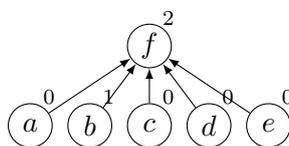
**Problem 139 (4 %)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

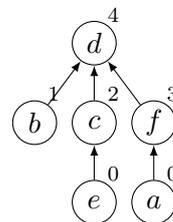
- MAKESET( $a$ )
- MAKESET( $b$ )
- MAKESET( $c$ )
- MAKESET( $d$ )
- MAKESET( $e$ )
- MAKESET( $f$ )
- UNION( $e, b$ )
- UNION( $b, c$ )
- UNION( $a, f$ )
- UNION( $e, a$ )
- UNION( $c, d$ )
- FIND-SET( $b$ )



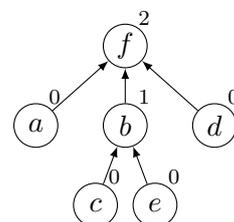
A



B



C

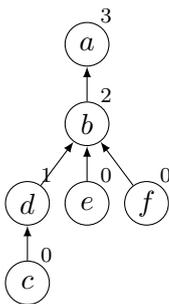


D

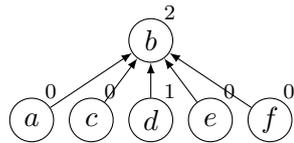
**Problem 140 (4%)**

Indicate the resulting union-find structure after the following sequence of operations, when using union-by-rank and path compression.

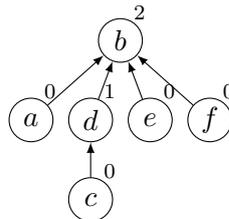
MAKESET( $a$ )  
 MAKESET( $b$ )  
 MAKESET( $c$ )  
 MAKESET( $d$ )  
 MAKESET( $e$ )  
 MAKESET( $f$ )  
 UNION( $c, d$ )  
 UNION( $e, d$ )  
 UNION( $f, b$ )  
 UNION( $c, b$ )  
 UNION( $e, a$ )  
 FIND-SET( $b$ )



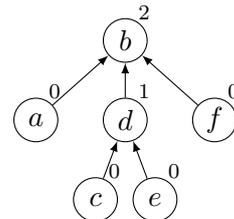
A



B



C



D

## Huffman encoding

### Problem 141 (4%)

Letter	a	b	c	d	e	f	g
Frequency	20	80	10	40	70	40	30

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **a**, **d** and **f**?

	1	2	3	4	5	6	
a	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	141.1
d	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	141.2
f	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	141.3

The file contains  $20 + 80 + 10 + 40 + 70 + 40 + 30 = 290$  letters. How many bits would a Huffman encoding of the string use?

750 790 800 810 830 840

A  B  C  D  E  F 141.4

### Problem 142 (4%)

Letter	a	b	c	d	e	f	g
Frequency	60	30	50	80	80	40	30

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **b**, **d** and **g**?

	1	2	3	4	5	6	
b	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	142.1
d	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	142.2
g	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	142.3

The file contains  $60 + 30 + 50 + 80 + 80 + 40 + 30 = 370$  letters. How many bits would a Huffman encoding of the string use?

1010 1020 1030 1040 1050 1060

A  B  C  D  E  F 142.4

**Problem 143 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	40	10	30	20	40	70	80

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for c, d and e?

	1	2	3	4	5	6	
c	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	143.1
d	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	143.2
e	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	143.3

The file contains  $40 + 10 + 30 + 20 + 40 + 70 + 80 = 290$  letters. How many bits would a Huffman encoding of the string use?

730 740 750 780 790 800

A  B  C  D  E  F 143.4

**Problem 144 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	40	20	70	60	80	50	10

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for d, e and g?

	1	2	3	4	5	6	
d	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	144.1
e	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	144.2
g	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	144.3

The file contains  $40 + 20 + 70 + 60 + 80 + 50 + 10 = 330$  letters. How many bits would a Huffman encoding of the string use?

830 850 860 870 900 910

A  B  C  D  E  F 144.4

**Problem 145 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	70	10	50	50	10	30	20

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **a**, **c** and **f**?

	1	2	3	4	5	6	
a	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	145.1
c	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	145.2
f	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	145.3

The file contains  $70 + 10 + 50 + 50 + 10 + 30 + 20 = 240$  letters. How many bits would a Huffman encoding of the string use?

590 600 610 620 630 640

<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	145.4
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**Problem 146 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	40	80	60	20	10	40	30

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **c**, **e** and **g**?

	1	2	3	4	5	6	
c	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	146.1
e	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	146.2
g	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	146.3

The file contains  $40 + 80 + 60 + 20 + 10 + 40 + 30 = 280$  letters. How many bits would a Huffman encoding of the string use?

720 730 760 770 780 790

<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	146.4
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**Problem 147 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	70	40	50	30	20	20	30

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **c**, **d** and **f**?

	1	2	3	4	5	6	
c	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	147.1
d	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	147.2
f	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	147.3

The file contains  $70 + 40 + 50 + 30 + 20 + 20 + 30 = 260$  letters. How many bits would a Huffman encoding of the string use?

660 670 680 700 710 720

A  B  C  D  E  F 147.4

**Problem 148 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	80	50	50	60	10	40	20

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **b**, **c** and **g**?

	1	2	3	4	5	6	
b	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	148.1
c	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	148.2
g	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	148.3

The file contains  $80 + 50 + 50 + 60 + 10 + 40 + 20 = 310$  letters. How many bits would a Huffman encoding of the string use?

780 800 810 820 830 850

A  B  C  D  E  F 148.4

**Problem 149 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	50	20	70	30	30	40	20

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **b**, **d** and **e**?

	1	2	3	4	5	6	
b	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	149.1
d	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	149.2
e	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	149.3

The file contains  $50 + 20 + 70 + 30 + 30 + 40 + 20 = 260$  letters. How many bits would a Huffman encoding of the string use?

700 710 720 730 740 750

B  C  D  E  F 149.4

**Problem 150 (4%)**

Letter	a	b	c	d	e	f	g
Frequency	40	60	60	10	50	50	20

Assume we have a file with the above letters and frequencies. If one constructs a Huffman tree for these frequencies, what are the lengths of the codes for **b**, **e** and **f**?

	1	2	3	4	5	6	
b	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	150.1
e	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	150.2
f	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	150.3

The file contains  $40 + 60 + 60 + 10 + 50 + 50 + 20 = 290$  letters. How many bits would a Huffman encoding of the string use?

720 730 740 760 770 780

A  B  C  D  E  150.4

## Recurrence relations

### Problem 151 (4 %)

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = T(n-1) + \log n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	151.1
$T(n) = 4 \cdot T(n/5) + n^3$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	151.2
$T(n) = 4 \cdot T(n/2) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	151.3
$T(n) = 3 \cdot T(n/5) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	151.4
$T(n) = T(n-1) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	151.5

### Problem 152 (4 %)

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = T(n-1) + \log n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	152.1
$T(n) = 4 \cdot T(n/5) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	152.2
$T(n) = T(n/3) + 3$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	152.3
$T(n) = 2 \cdot T(n/2) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	152.4
$T(n) = 3 \cdot T(n/9) + 3$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	152.5

### Problem 153 (4 %)

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 2 \cdot T(n/4) + 2$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	153.1
$T(n) = 5 \cdot T(n/5) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	153.2
$T(n) = 3 \cdot T(n/9) + 1$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	153.3
$T(n) = 8 \cdot T(n/2) + 2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	153.4
$T(n) = 4 \cdot T(n/2) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	153.5

### Problem 154 (4 %)

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 2 \cdot T(n/5) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	154.1
$T(n) = 4 \cdot T(n/2) + 1$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	154.2
$T(n) = 4 \cdot T(n/4) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	154.3
$T(n) = T(n/4) + 3$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	154.4
$T(n) = T(n-1) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	154.5

**Problem 155 (4 %)**

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 2 \cdot T(n/4) + 1$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	155.1
$T(n) = 5 \cdot T(n/5) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	155.2
$T(n) = 8 \cdot T(n/2) + 1$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	155.3
$T(n) = T(n-1) + 2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	155.4
$T(n) = T(n-1) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	155.5

**Problem 156 (4 %)**

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 4 \cdot T(n/2) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	156.1
$T(n) = 9 \cdot T(n/3) + 3$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	156.2
$T(n) = 4 \cdot T(n/2) + 2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	156.3
$T(n) = 2 \cdot T(n/4) + 2$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	156.4
$T(n) = 8 \cdot T(n/2) + 1$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	156.5

**Problem 157 (4 %)**

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 8 \cdot T(n/2) + 2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	157.1
$T(n) = T(n/3) + 5$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	157.2
$T(n) = T(n-1) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	157.3
$T(n) = 2 \cdot T(n/4) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	157.4
$T(n) = 3 \cdot T(n/4) + n^3$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	157.5

**Problem 158 (4 %)**

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 9 \cdot T(n/3) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	158.1
$T(n) = 4 \cdot T(n/2) + 2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	158.2
$T(n) = T(n/4) + 3$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	158.3
$T(n) = 3 \cdot T(n/9) + 1$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	158.4
$T(n) = 3 \cdot T(n/5) + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	158.5

**Problem 159 (4 %)**

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 3 \cdot T(n/9) + 3$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	159.1
$T(n) = T(n - 1) + \log n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	159.2
$T(n) = 2 \cdot T(n/4) + 2$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	159.3
$T(n) = T(n - 1) + 3$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	159.4
$T(n) = T(n/3) + 5$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	159.5

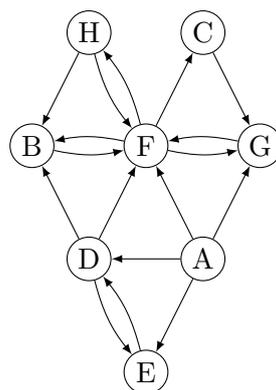
**Problem 160 (4 %)**

State the solution for each of the below recurrence relations, where  $T(n) = 1$  for  $n \leq 1$ .

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$	
$T(n) = 9 \cdot T(n/3) + 3$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	160.1
$T(n) = 4 \cdot T(n/5) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	160.2
$T(n) = 4 \cdot T(n/2) + n^2$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	160.3
$T(n) = 3 \cdot T(n/9) + 2$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	160.4
$T(n) = 2 \cdot T(n/4) + n^3$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	160.5

**BFS**

**Problem 161 (4 %)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

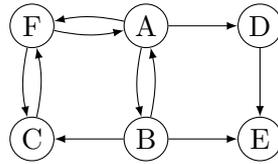
ADEFGBCH   ADEFGBHC   AGDEFBCH   ADBFCGHE

B

C

D

**Problem 162 (4 %)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

ABCFED    ABDFCE    ABDFEC    AFDBCE

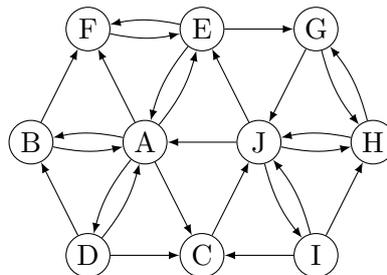
A

B

C

D

**Problem 163 (4 %)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

ABFEGHJICD    ABCDEFJGHI    AEBFDCGJHI    ABCDEFJGHI

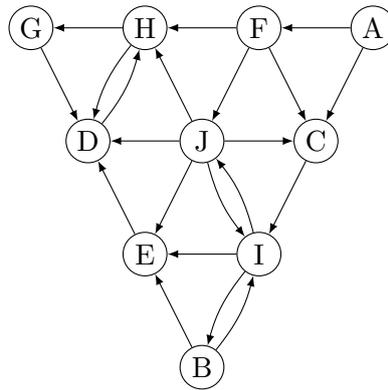
A

B

C

D

**Problem 164 (4%)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

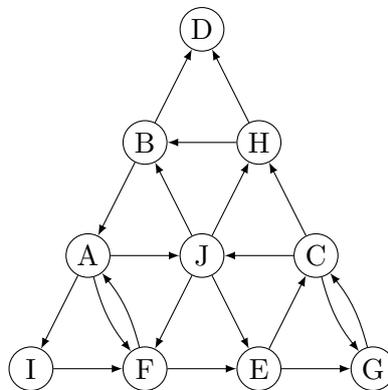
ACIBEDHGFJ    ACFIHJBEGD    ACFIHJEBGD    ACFIHJBEDG

A

B

C

**Problem 165 (4%)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

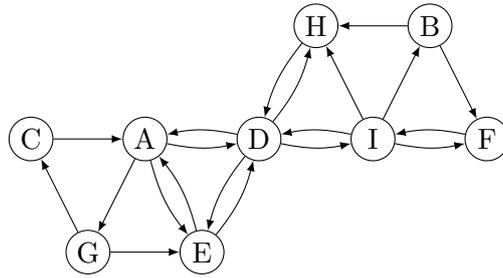
AFIJE BHCGD    AJFIH BEDCG    AFIJE BHGCD    AFECGHBDJI

B

C

D

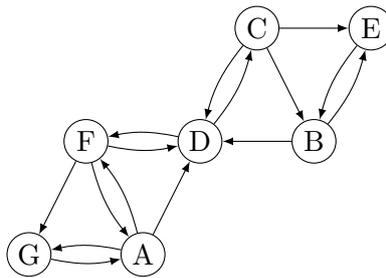
**Problem 166 (4%)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

ADEGHICBF    ADEHIBFGC    ADEGHICFB    ADEGIHCFB

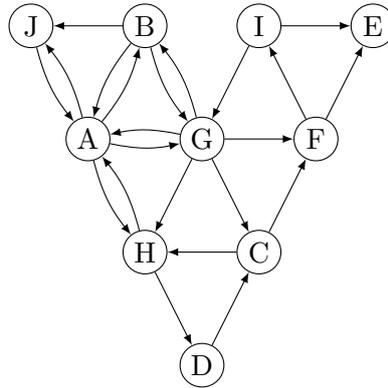
**Problem 167 (4%)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are removed from the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

ADFGCBE    ADGFCEB    ADFGCEB    ADCBEFG

**Problem 168 (4 %)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are removed from the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

ABGCFEIHDJ    ABGHJCFDIE    ABGHJCFDEI    ABHJGDCFEI

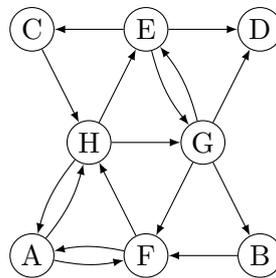
A

B

C

D

**Problem 169 (4 %)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

AFHECDGB    AHFGEBDC    AFHEGDCB    AFHEGCDB

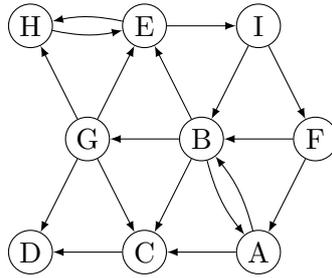
A

B

C

D

**Problem 170 (4%)**



For a breadth first traversal (BFS) of the graph above **starting in node A**, indicate the order in which the nodes are inserted into the queue  $Q$  in the BFS algorithm. It is assumed that the graph is given by alphabetically sorted adjacency lists.

ABCDEHIFG   ACBDGEHIF   ABCEGDHIF   ABCEGDIHF

A

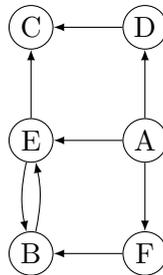
B

C

D

**Valid BFS trees**

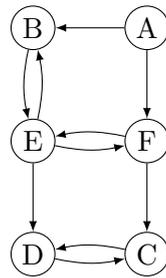
**Problem 171 (4%)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,D) (A,F) (B,E) (E,C) (F,B)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	171.1
(A,D) (A,E) (A,F) (E,B) (E,C)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	171.2
(A,D) (A,E) (A,F) (D,C) (E,B)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	171.3
(A,D) (A,F) (B,E) (D,C) (F,B)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	171.4
(A,D) (A,E) (A,F) (E,C) (F,B)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	171.5

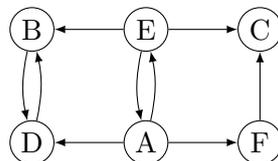
**Problem 172 (4 %)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,B) (A,F) (E,D) (F,C) (F,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	172.1
(A,B) (B,E) (D,C) (E,D) (E,F)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	172.2
(A,B) (A,F) (D,C) (E,D) (F,E)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	172.3
(A,F) (E,B) (E,D) (F,C) (F,E)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	172.4
(A,B) (A,F) (C,D) (F,C) (F,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	172.5

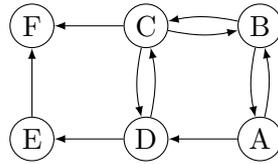
**Problem 173 (4 %)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,D) (A,E) (A,F) (D,B) (F,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	173.1
(A,E) (A,F) (B,D) (E,B) (E,C)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	173.2
(A,D) (A,E) (A,F) (E,B) (F,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	173.3
(A,D) (A,E) (A,F) (E,B) (E,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	173.4
(A,D) (A,E) (A,F) (D,B) (E,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	173.5

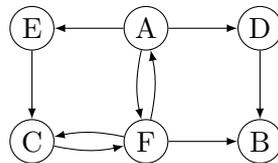
**Problem 174 (4%)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,B) (A,D) (C,F) (D,C) (D,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	174.1
(A,D) (C,B) (D,C) (D,E) (E,F)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	174.2
(A,B) (A,D) (D,C) (D,E) (E,F)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	174.3
(A,B) (A,D) (B,C) (C,F) (D,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	174.4
(A,B) (A,D) (B,C) (D,E) (E,F)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	174.5

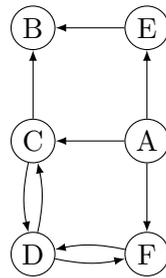
**Problem 175 (4%)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,D) (A,E) (A,F) (D,B) (F,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	175.1
(A,D) (A,E) (A,F) (D,B) (E,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	175.2
(A,D) (A,E) (A,F) (E,C) (F,B)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	175.3
(A,D) (A,E) (C,F) (E,C) (F,B)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	175.4
(A,D) (A,E) (A,F) (F,B) (F,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	175.5

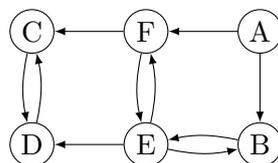
**Problem 176 (4 %)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,C) (A,E) (A,F) (C,D) (E,B)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	176.1
(A,C) (A,E) (A,F) (E,B) (F,D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	176.2
(A,E) (A,F) (C,B) (D,C) (F,D)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	176.3
(A,C) (A,E) (A,F) (C,B) (C,D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	176.4
(A,C) (A,E) (A,F) (C,B) (F,D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	176.5

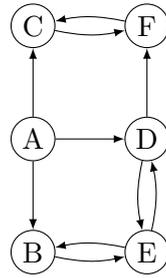
**Problem 177 (4 %)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,B) (A,F) (E,D) (F,C) (F,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	177.1
(A,B) (B,E) (D,C) (E,D) (E,F)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	177.2
(A,B) (A,F) (B,E) (E,D) (F,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	177.3
(A,F) (C,D) (E,B) (F,C) (F,E)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	177.4
(A,B) (A,F) (B,E) (D,C) (E,D)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	177.5

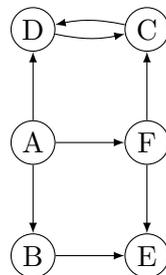
**Problem 178 (4 %)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,D) (D,E) (D,F) (E,B) (F,C)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	178.1
(A,B) (A,C) (A,D) (C,F) (D,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	178.2
(A,B) (A,C) (A,D) (B,E) (C,F)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	178.3
(A,B) (A,C) (B,E) (C,F) (E,D)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	178.4
(A,B) (A,C) (A,D) (D,E) (D,F)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	178.5

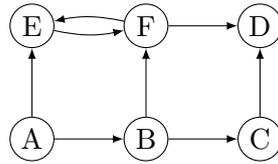
**Problem 179 (4 %)**



Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,B) (A,F) (C,D) (F,C) (F,E)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	179.1
(A,B) (A,D) (A,F) (B,E) (F,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	179.2
(A,B) (A,D) (A,F) (F,C) (F,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	179.3
(A,B) (A,D) (A,F) (B,E) (D,C)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	179.4
(A,B) (A,D) (A,F) (D,C) (F,E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	179.5

**Problem 180 (4 %)**

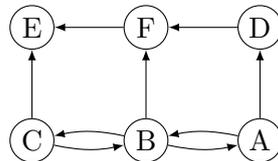


Indicate for each of the sets of edges below whether they form a valid BFS tree for a breadth first traversal of the graph above **starting in node A** and for a suitable ordering of the graph's adjacency lists.

	Yes	No	
(A,B) (B,C) (B,F) (F,D) (F,E)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	180.1
(A,B) (A,E) (B,C) (C,D) (E,F)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	180.2
(A,B) (A,E) (B,C) (B,F) (C,D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	180.3
(A,B) (A,E) (B,C) (E,F) (F,D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	180.4
(A,B) (A,E) (B,C) (B,F) (F,D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	180.5

**DFS**

**Problem 181 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **discovery time**.

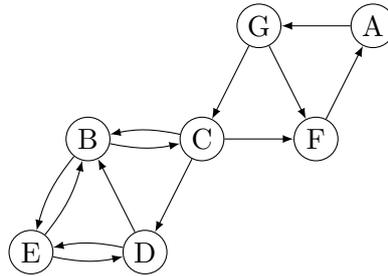
ABCEFD    ADBFCE    ABDCFE    ADFEBC

<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	181.1
-------------------------------------	--------------------------	--------------------------	--------------------------	-------

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(A, D)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	181.2
(C, B)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	181.3
(F, E)	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	181.4
(B, A)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	181.5

**Problem 182 (4%)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **discovery time**.

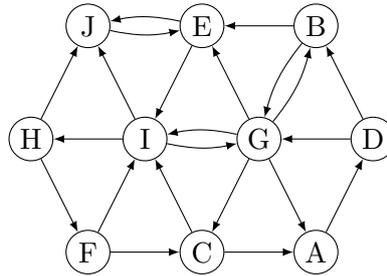
AGCFBED    AGCBEDF    AGCFBDE    AGFCBED

A                       B                       C                       D                      182.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(C, D)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	182.2
(D, E)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	182.3
(D, B)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	182.4
(C, B)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	182.5

**Problem 183 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **discovery time**.

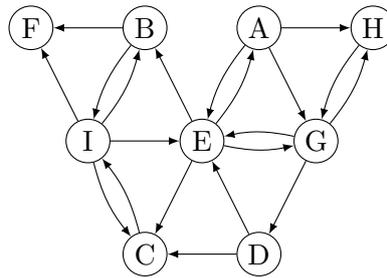
ADBGECIJHF    ADBEIHJFGC    ADBEIGCHFJ    ADGBEJIHFC

A                       B                       C                       D                      183.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(F, C)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	183.2
(E, J)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	183.3
(F, I)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	183.4
(B, E)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	183.5

**Problem 184 (4%)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **finishing time**.

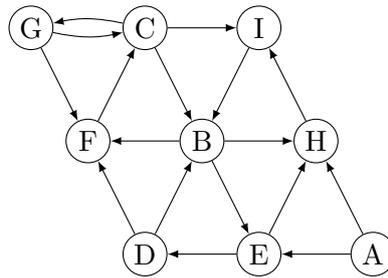
FCIBDHGEA    HDGCIFBEA    FBICDHGEA    IFDCBHGEA

                                                                                       184.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(D, E)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	184.2
(A, H)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	184.3
(A, E)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	184.4
(I, F)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	184.5

**Problem 185 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **discovery time**.

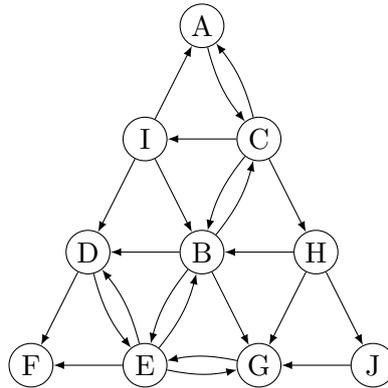
AEDBHF CIG    AEDBHIFCG    AEDBF C G I H    AEHDIBFCG

A                       B                       C                       D                      185.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(D, F)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	185.2
(I, B)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	185.3
(B, H)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	185.4
(H, I)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	185.5

**Problem 186 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **finishing time**.

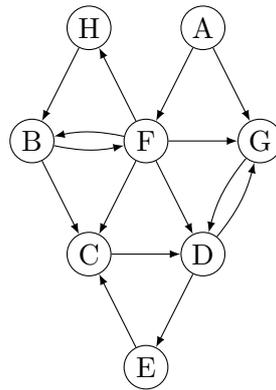
I J H G F E D B C A    F G B E D I J H C A    F J G E D I H B C A    F G E D B J H I C A

A                                       B                                       C                                       186.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(D, F)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	186.2
(D, E)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	186.3
(I, A)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	186.4
(I, D)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	186.5

**Problem 187 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **finishing time**.

EGDCBHFA    EHDCBGFA    HGEDCBFA    GEDCBHFA

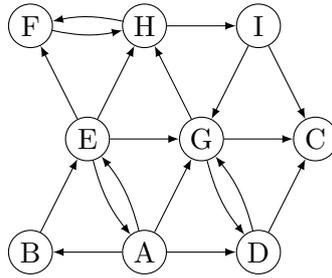
                                                                                       187.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(G, D)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	187.2
(H, B)	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	187.3
(D, G)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	187.4
(A, G)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	187.5



**Problem 189 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **discovery time**.

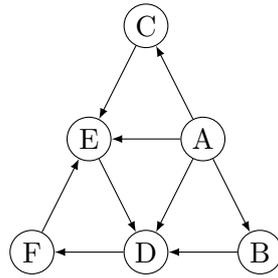
ABEFHICGD    ABDEGCFHI    ABEFHIGDC    AEFHIGCDB

                                                                                       189.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(E, H)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	189.2
(G, C)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	189.3
(E, F)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	189.4
(H, F)	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	189.5

**Problem 190 (4 %)**



Consider a depth first traversal (DFS) of the graph above, where the DFS traversal starts in **node A**, and where the outgoing edges of a node are visited in alphabetical order. Indicate the order in which the nodes receive their **finishing time**.

CEFDBA   EFDBCA   FDEBCA   FEDCBA

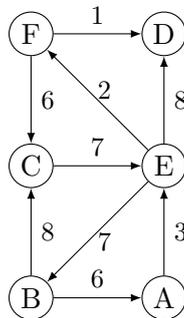
A             B             C             D            190.1

Indicate for each of the edges below which type the edge is in the DFS traversal.

	Tree edge	Back edge	Cross edge	Forward edge	
(F, E)	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	190.2
(E, D)	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	190.3
(C, E)	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	190.4
(A, E)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	190.5

**Dijkstra’s algorithm**

**Problem 191 (4 %)**

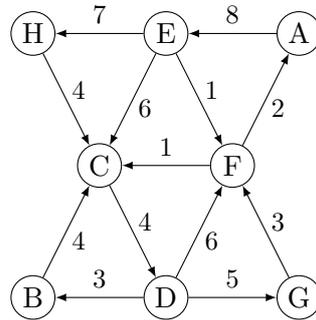


Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

AEBCDF   AEBDFC   AEFDCB   AEFDBC

A             B             C             D

**Problem 192 (4%)**



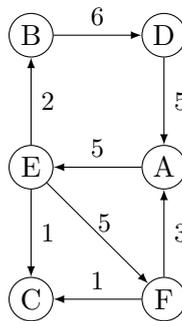
Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- AECD BFGH    AEFCD BFGH    AECFHD BFG    AEFCDH BFG





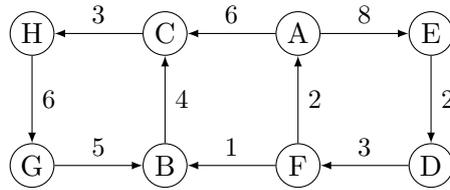
**Problem 193 (4%)**



Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- AEBCFD    AECBFD    AECBDF    AEBDCF

**Problem 194 (4 %)**

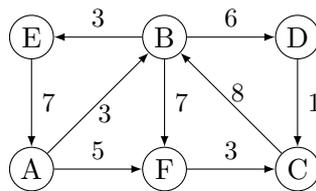


Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- ACEHGBDF    ACEHDGFB    ACEHDFBG    ACHGBEDF

- A                       B                       C                       D

**Problem 195 (4 %)**

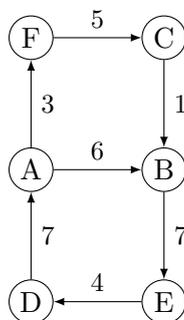


Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- ABFCED    ABDCEF    ABFECD    ABFDEC

- A                       B                       C                       D

**Problem 196 (4 %)**

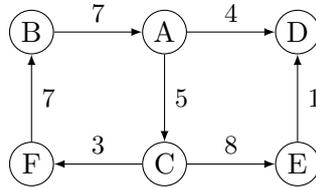


Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- AFBCED    AFBEDC    ABFECD    ABEDFC

- A                       B                       C                       D

**Problem 197 (4 %)**



Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- ADCFBE   ADCFEB   ACDEFB   ACEDFB

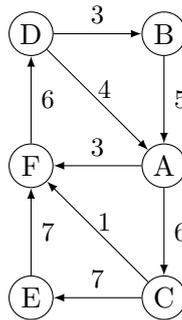
A

B

C

D

**Problem 198 (4 %)**



Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

- ACEFDB   AFCEDB   ACFEDB   AFCDBE

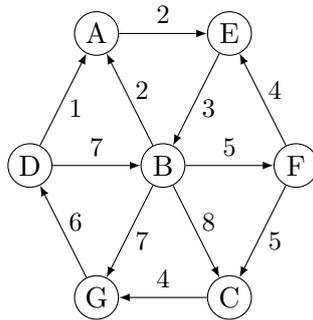
A

B

C

D

**Problem 199 (4 %)**



Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

AEBCFGD    AEBGDF    AEBFCGD    AEBFGCD

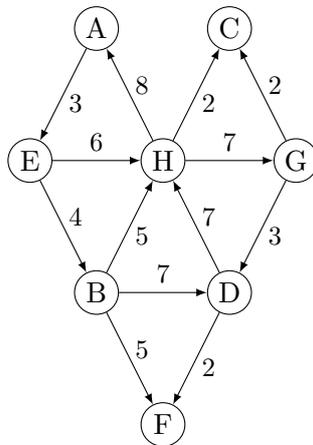
A

B

C

D

**Problem 200 (4 %)**



Assume Dijkstra’s algorithm is used to find shortest distances from **node A** to all nodes in the graph above. Indicate the order in which the nodes are removed from the priority queue in Dijkstra’s algorithm.

AEBHCGDF    AEBHCFDG    AEBDFHCG    AEBHDFCG

A

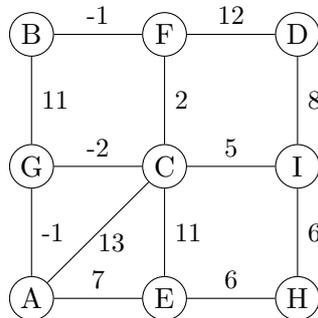
B

C

D

### Prim's algorithm

Problem 201 (4%)

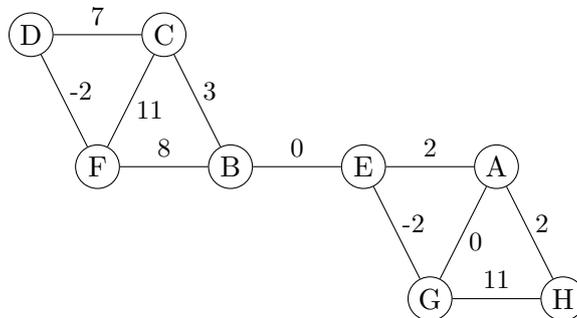


Assume Prim's algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim's algorithm).

AGCFBIHED    AGCFBDIHE    AGCFBIHDE    AGCFBIEHD



Problem 202 (4%)

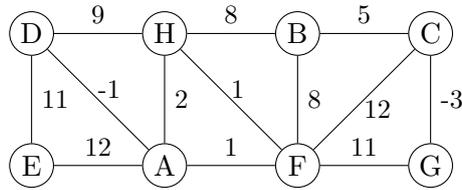


Assume Prim's algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim's algorithm).

AGEBHCDF    AGEBCHFD    AGEBCDFH    AGEBCHDF



**Problem 203 (4 %)**



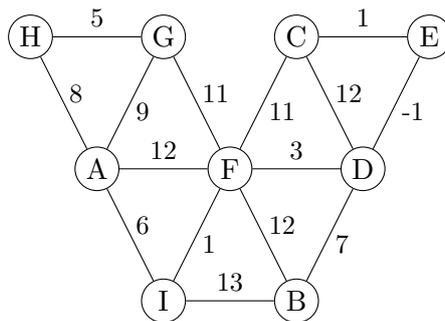
Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

ADFBEGC    ADFHBCGE    ADFHBECG    ADHFBCGE





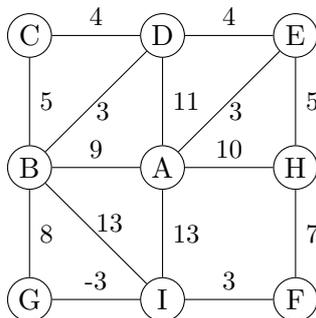
**Problem 204 (4 %)**



Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

AIFDECBHG    AIFHGDECB    AIFDECBGH    AIFHDECGB

**Problem 205 (4 %)**



Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

- AEDBCGIFH    AEDHBCIGF    AEDBCHFIG    AEDHBCFIG

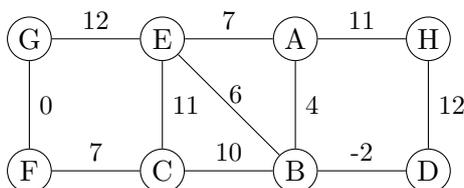
A

B

C

D

**Problem 206 (4 %)**



Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

- ABDEHCGF    ABDHECFG    ABDEHCFG    ABDECFGH

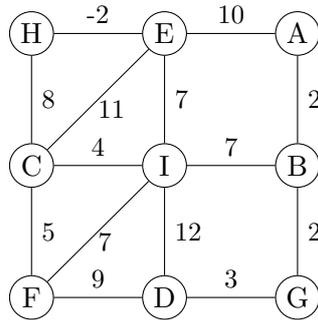
A

B

C

D

**Problem 207 (4 %)**



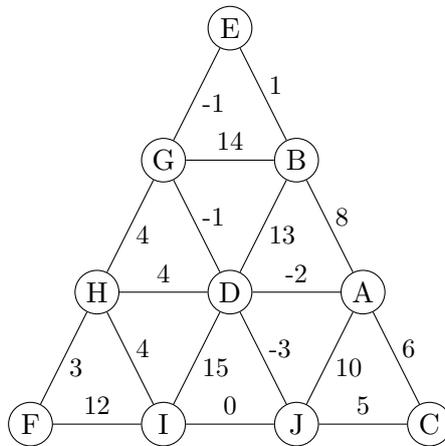
Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

- ABGDFCIEH    ABGDIEHCF    ABGDICFEH    ABGDICEHF





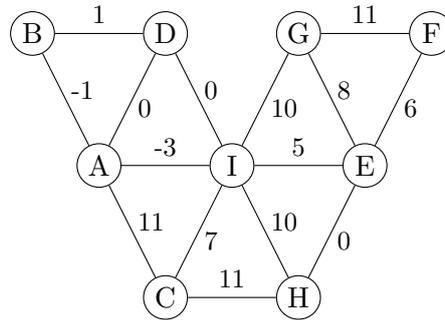
**Problem 208 (4 %)**



Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

- ADJIHFGEB    ADJGEIBHFC    ADJIGEBCHF    ADJIGEBHCF

**Problem 209 (4%)**



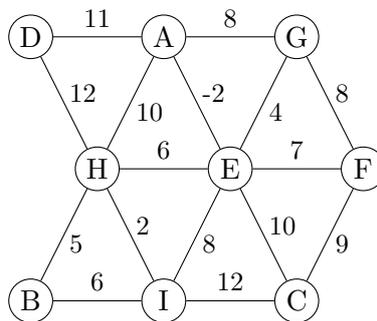
Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

AIBDEHCFG    AIDBEHCGF    AIDBEHCFG    AIBDEHFCG





**Problem 210 (4%)**

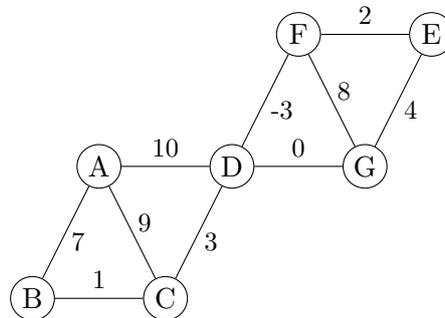


Assume Prim’s algorithm is used to find a minimum spanning tree for the graph above, and the algorithm starts in **node A**. Indicate the order in which the nodes are included in the minimum spanning tree (taken out of the priority queue in Prim’s algorithm).

AEGHFIBDC    AEGFCIHBD    AEGHIBFCD    AEGHFICBD

## Kruskal's algorithm

### Problem 211 (4%)

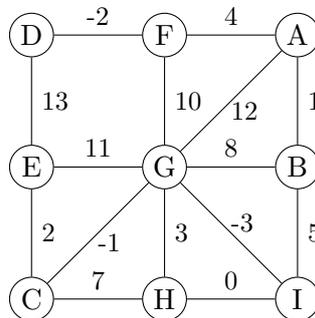


Assume Kruskal's algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

(B, C) (A, B) (E, G) (E, F) (D, G)

A     B     C     D     E

### Problem 212 (4%)

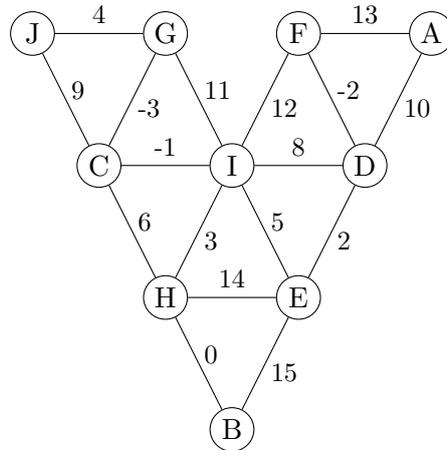


Assume Kruskal's algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

(B, I) (C, H) (C, E) (H, I) (G, I)

A     B     C     D     E

**Problem 213 (4%)**

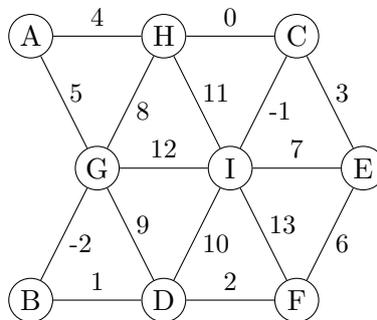


Assume Kruskal’s algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

- (B, H) (B, E) (A, D) (C, I) (G, J)

- A     B     C     D     E

**Problem 214 (4%)**

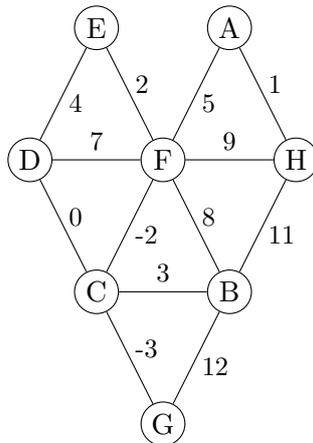


Assume Kruskal’s algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

- (B, G) (A, G) (C, E) (A, H) (D, F)

- A     B     C     D     E

**Problem 215 (4 %)**

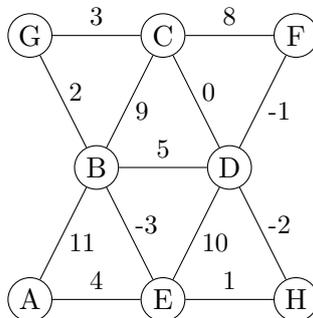


Assume Kruskal’s algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

- (D, E) (A, F) (E, F) (C, F) (B, C)

A     B     C     D     E

**Problem 216 (4 %)**

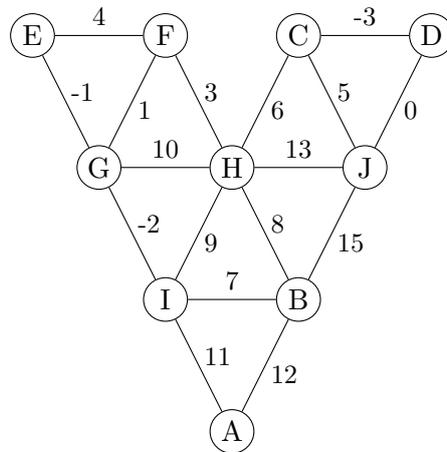


Assume Kruskal’s algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

- (D, F) (C, D) (A, E) (B, G) (B, E)

A     B     C     D     E

**Problem 217 (4%)**

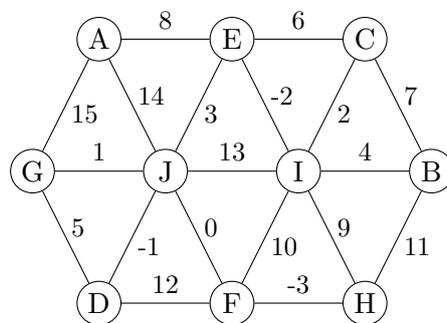


Assume Kruskal's algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

- (B, I) (C, H) (D, J) (B, J) (A, I)

- A     B     C     D

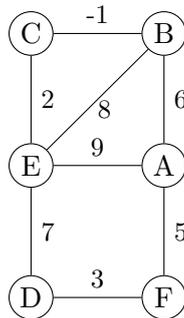
**Problem 218 (4%)**



Assume Kruskal's algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

- (D, G) (B, I) (C, I) (A, E) (G, J)

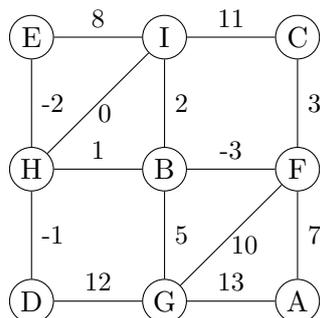
- A     B     C     D     E

**Problem 219 (4%)**

Assume Kruskal's algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

(A, F) (B, C) (D, F) (C, E) (A, B)

A     B     C     D   

**Problem 220 (4%)**

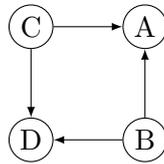
Assume Kruskal's algorithm is used to find a minimum spanning tree for the graph above. Indicate which edge is **last** included in the minimum spanning tree.

(C, F) (B, G) (B, H) (D, G) (A, F)

A     B     C     D

## Topological sorting

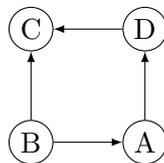
### Problem 221 (4%)



For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

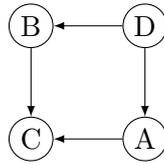
	Yes	No	
B C D A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	221.1
C B A D	<input checked="" type="checkbox"/>	<input type="checkbox"/>	221.2
C D A B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	221.3
B C A D	<input checked="" type="checkbox"/>	<input type="checkbox"/>	221.4
D B A C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	221.5

### Problem 222 (4%)



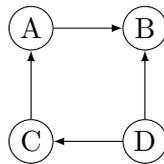
For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
A B D C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	222.1
C A D B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	222.2
B D A C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	222.3
B A D C	<input checked="" type="checkbox"/>	<input type="checkbox"/>	222.4
B A C D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	222.5

**Problem 223 (4 %)**

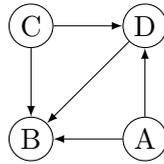
For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
D C A B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	223.1
D A B C	<input checked="" type="checkbox"/>	<input type="checkbox"/>	223.2
D B A C	<input checked="" type="checkbox"/>	<input type="checkbox"/>	223.3
C A B D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	223.4
A D B C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	223.5

**Problem 224 (4 %)**

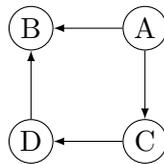
For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
D C B A	<input type="checkbox"/>	<input checked="" type="checkbox"/>	224.1
D C A B	<input checked="" type="checkbox"/>	<input type="checkbox"/>	224.2
D A C B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	224.3
B C A D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	224.4
C D A B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	224.5

**Problem 225 (4 %)**

For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

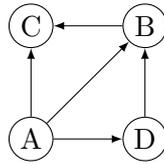
	Yes	No	
A C D B	<input checked="" type="checkbox"/>	<input type="checkbox"/>	225.1
B C D A	<input type="checkbox"/>	<input checked="" type="checkbox"/>	225.2
C A B D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	225.3
C A D B	<input checked="" type="checkbox"/>	<input type="checkbox"/>	225.4
C D A B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	225.5

**Problem 226 (4 %)**

For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
A C D B	<input checked="" type="checkbox"/>	<input type="checkbox"/>	226.1
A D C B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	226.2
A B D C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	226.3
D C A B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	226.4
C A D B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	226.5

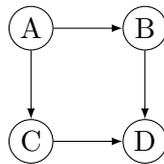
**Problem 227 (4 %)**



For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

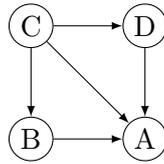
	Yes	No	
A D C B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	227.1
A D B C	<input checked="" type="checkbox"/>	<input type="checkbox"/>	227.2
C D B A	<input type="checkbox"/>	<input checked="" type="checkbox"/>	227.3
A C B D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	227.4
D A B C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	227.5

**Problem 228 (4 %)**



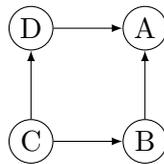
For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
B A C D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	228.1
C B A D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	228.2
A C B D	<input checked="" type="checkbox"/>	<input type="checkbox"/>	228.3
A D B C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	228.4
A B C D	<input checked="" type="checkbox"/>	<input type="checkbox"/>	228.5

**Problem 229 (4 %)**

For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
D B C A	<input type="checkbox"/>	<input checked="" type="checkbox"/>	229.1
C B A D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	229.2
A B D C	<input type="checkbox"/>	<input checked="" type="checkbox"/>	229.3
C B D A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	229.4
C D B A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	229.5

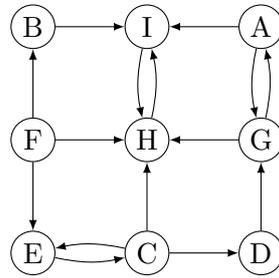
**Problem 230 (4 %)**

For each of the following orderings of the nodes in the graph above, indicate whether it is a valid topological sort.

	Yes	No	
C D B A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	230.1
C B D A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	230.2
C D A B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	230.3
C B A D	<input type="checkbox"/>	<input checked="" type="checkbox"/>	230.4
D B C A	<input type="checkbox"/>	<input checked="" type="checkbox"/>	230.5

### Strongly connected components

**Problem 231 (4%)**

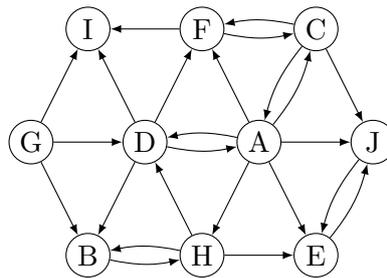


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9

- A  B  C  D  E  F  G  H  I

**Problem 232 (4%)**

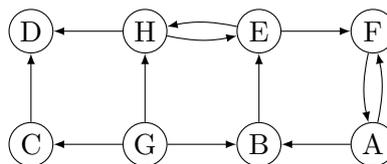


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9 10

- A  B  C  D  E  F  G  H  I  J

**Problem 233 (4%)**

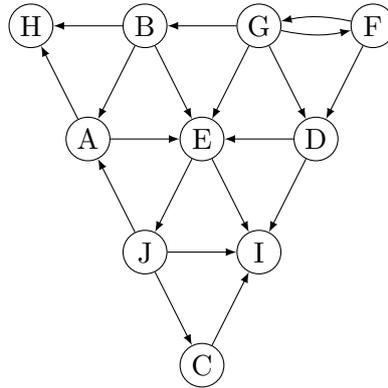


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8

- A  B  C  D  E  F  G  H

**Problem 234 (4 %)**

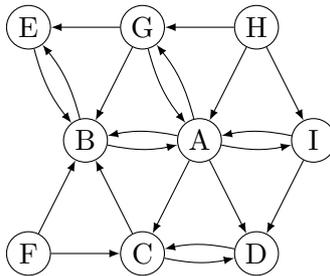


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9 10

- A  B  C  D  E  F  H  I  J

**Problem 235 (4 %)**

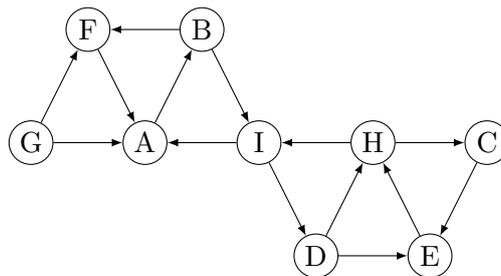


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9

- A  B  D  E  F  G  H  I

**Problem 236 (4 %)**

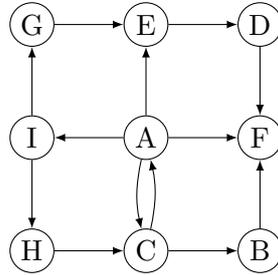


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9

- A  B  C  D  E  F  G  H  I

**Problem 237 (4 %)**

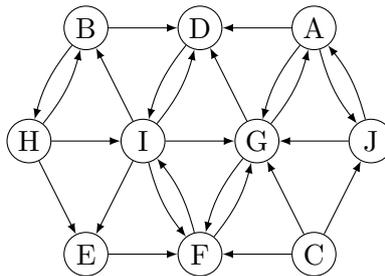


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9

- A  B  C  D  E  F  G  H  I

**Problem 238 (4 %)**

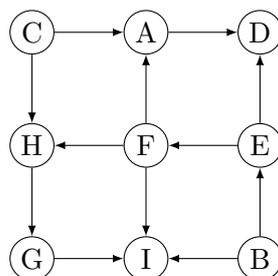


What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9 10

- A  B  C  D  E  F  G  H  I  J

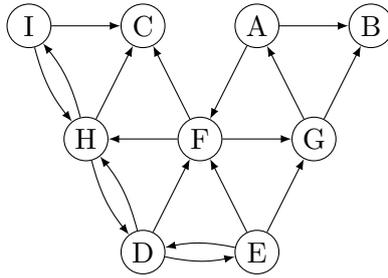
**Problem 239 (4 %)**



What is the number of strongly connected components in the graph above?

- 1 2 3 4 5 6 7 8 9

- A  B  C  D  E  F  G  H  I

**Problem 240 (4%)**

What is the number of strongly connected components in the graph above?

1 2 3 4 5 6 7 8 9

A  B  3  D  E  F  G  H  I

## Loop problems

### Problem 241

<p><b>Algorithm loop1(<math>n</math>)</b>  <math>i = 1</math>  <b>while</b> <math>i \leq n</math>          <math>j = 1</math>          <b>while</b> <math>j \leq i</math>              <math>j = 2 * j</math>          <math>i = 2 * i</math></p>	<p><b>Algorithm loop2(<math>n</math>)</b>  <math>i = 1</math>  <b>while</b> <math>i \leq n</math>          <math>j = 1</math>          <b>while</b> <math>j \leq n</math>              <math>j = 2 * j</math>          <math>i = 2 * i</math></p>	<p><b>Algorithm loop3(<math>n</math>)</b>  <math>i = 1</math>  <b>while</b> <math>i \leq n</math>          <math>j = i</math>          <b>while</b> <math>j \leq n</math>              <math>j = 2 * j</math>          <math>i = 2 * i</math></p>
<p><b>Algorithm loop4(<math>n</math>)</b>  <math>i = 1</math>  <b>while</b> <math>i \leq n</math>          <math>j = i</math>          <b>while</b> <math>j &gt; 0</math>              <math>j = \lfloor j/2 \rfloor</math>          <math>i = i + i</math></p>	<p><b>Algorithm loop5(<math>n</math>)</b>  <math>i = n</math>  <b>while</b> <math>i &gt; 0</math>          <math>j = i</math>          <b>while</b> <math>j &gt; 0</math>              <math>j = \lfloor j/2 \rfloor</math>          <math>i = \lfloor i/2 \rfloor</math></p>	<p><b>Algorithm loop6(<math>n</math>)</b>  <math>s = 1</math>  <b>for</b> <math>i = 1</math> <b>to</b> <math>n</math>          <math>j = 1</math>          <b>while</b> <math>j \leq s</math>              <math>j = j + 1</math>          <math>s = 2 * s</math></p>
<p><b>Algorithm loop7(<math>n</math>)</b>  <math>s = 1</math>  <b>for</b> <math>i = 1</math> <b>to</b> <math>n</math>          <math>j = s</math>          <b>while</b> <math>j &gt; 0</math>              <math>s = s + 1</math>              <math>j = j - 1</math></p>	<p><b>Algorithm loop8(<math>n</math>)</b>  <math>i = 1</math>  <math>p = 1</math>  <b>while</b> <math>p \leq n</math>          <math>i = i + 1</math>          <math>p = p * i</math></p>	<p><b>Algorithm loop9(<math>n</math>)</b>  <math>i = 1</math>  <math>j = n</math>  <b>while</b> <math>i &lt; j</math>          <math>i = 2 * i</math>          <math>j = j + n</math></p>

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(2^n)$	$\Theta(n^2)$	$\Theta(n^3)$	$\Theta(\log n)$	$\Theta(\frac{\log n}{\log \log n})$	$\Theta(n \log n)$	$\Theta(n)$	$\Theta((\log n)^2)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	241.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	241.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	241.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	241.4
loop5	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/> H	241.5
loop6	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	241.6
loop7	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	241.7
loop8	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	241.8
loop9	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	241.9

## Problem 242

<b>Algorithm</b> loop1( $n$ )	<b>Algorithm</b> loop2( $n$ )	<b>Algorithm</b> loop3( $n$ )
$i = 1$	$i = 1$	$i = 1$
$j = n$	$j = n$	<b>while</b> $i * i \leq n$
<b>while</b> $i \leq j$	<b>while</b> $i \leq j$	$i = i + i$
$i = 4 * i$	$i = i * 2$	
$j = 2 * j$	$j = \lfloor j/2 \rfloor$	
<b>Algorithm</b> loop4( $n$ )	<b>Algorithm</b> loop5( $n$ )	<b>Algorithm</b> loop6( $n$ )
$i = 1$	$i = 1$	$i = 1$
<b>while</b> $i \leq n$	<b>while</b> $i \leq n$	<b>while</b> $i \leq n$
$i = 2 * i$	$i = 3 * i$	$i = i + i$
<b>Algorithm</b> loop7( $n$ )	<b>Algorithm</b> loop8( $n$ )	<b>Algorithm</b> loop9( $n$ )
$i = 1$	$i = 1$	$i = 1$
<b>while</b> $i \leq n * n$	<b>while</b> $i \leq n * n$	$j = n * n$
$i = 2 * i$	$i = 3 * i$	<b>while</b> $i \leq j$
		$i = 2 * i$
		$j = j - 1$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n \log n)$	$\Theta(n^3)$	$\Theta(\log n)$	$\Theta(\sqrt{n} \log n)$	$\Theta(n^2)$	$\Theta(\frac{\log n}{\log \log n})$	$\Theta((\log n)^2)$	$\Theta(n)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.4
loop5	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.5
loop6	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.6
loop7	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.7
loop8	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.8
loop9	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	242.9

## Problem 243

<b>Algorithm loop1(<math>n</math>)</b> $i = n$ <b>while</b> $i > 0$ <b>if</b> $i$ ulige <b>then</b> $i = i - 1$ <b>else</b> $i = i/2$	<b>Algorithm loop2(<math>n</math>)</b> $i = n$ <b>while</b> $i \leq n * n$ $i = 2 * i$	<b>Algorithm loop3(<math>n</math>)</b> $s = n$ <b>while</b> $s > 0$ $s = \lfloor s/2 \rfloor$
<b>Algorithm loop4(<math>n</math>)</b> $i = 2$ <b>while</b> $i \leq n$ $i = i * i$	<b>Algorithm loop5(<math>n</math>)</b> $s = 2$ <b>while</b> $s \leq n$ $s = s * s$	<b>Algorithm loop6(<math>n</math>)</b> $i = 0$ $s = 0$ $q = 0$ <b>while</b> $q \leq n$ $i = i + 1$ $s = s + i$ $q = q + s$
<b>Algorithm loop7(<math>n</math>)</b> $i = 0$ $s = 0$ <b>while</b> $s \leq n$ $i = i + 1$ $s = s + i$	<b>Algorithm loop8(<math>n</math>)</b> $i = 1$ $j = 1$ $s = 0$ <b>while</b> $s \leq n$ <b>while</b> $j \leq s$ $j = 2 * j$ $s = s + i$ $i = i + 1$	<b>Algorithm loop9(<math>n</math>)</b> $j = n$ $i = 1$ <b>while</b> $j \geq 0$ $j = j - i$ $i = i + 1$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(\sqrt[3]{n})$	$\Theta(\log \log n)$	$\Theta(n)$	$\Theta((\log n)^2)$	$\Theta(\log n)$	$\Theta(n^3)$	$\Theta(\sqrt{n})$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/>	<input type="checkbox"/> H	<input type="checkbox"/> I	243.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/>	<input type="checkbox"/> H	<input type="checkbox"/> I	243.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/>	<input type="checkbox"/> H	<input type="checkbox"/> I	243.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	243.4
loop5	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/>	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	243.5
loop6	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/>	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input type="checkbox"/> I	243.6
loop7	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/>	243.7
loop8	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/>	243.8
loop9	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	<input checked="" type="checkbox"/>	243.9

## Problem 244

<b>Algorithm loop1(<math>n</math>)</b> $s = 0$ $i = 1$ <b>while</b> $s \leq n$ $s = s + i$ $i = i + 1$	<b>Algorithm loop2(<math>n</math>)</b> $i = 1$ <b>while</b> $i \leq n$ $j = 1$ $k = 1$ <b>while</b> $k \leq n$ $j = j + 1$ $k = k + j$ $i = 2 * i$	<b>Algorithm loop3(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = i$ <b>while</b> $j \leq n$ $j = 2 * j$
<b>Algorithm loop4(<math>n</math>)</b> $i = 0$ $j = n$ <b>while</b> $i \leq j$ $i = i + 1$ $j = j - 1$	<b>Algorithm loop5(<math>n</math>)</b> $i = 0$ $j = n$ <b>while</b> $i < j$ $i = i + 2$ $j = j + 1$	<b>Algorithm loop6(<math>n</math>)</b> $i = 1$ $j = 0$ <b>while</b> $i \leq n$ $i = i + i$ <b>while</b> $j < i$ $j = j + 1$
<b>Algorithm loop7(<math>n</math>)</b> $i = 1$ $j = 1$ <b>while</b> $i \leq n$ <b>while</b> $j \leq i$ $j = j + 1$ $i = 2 * i$	<b>Algorithm loop8(<math>n</math>)</b> $i = 1$ $s = 0$ <b>while</b> $s \leq n$ $j = 1$ <b>while</b> $j \leq i$ $j = j + 1$ $s = s + i$ $i = i + 1$	<b>Algorithm loop9(<math>n</math>)</b> $i = 1$ $s = 1$ <b>while</b> $s \leq n * n$ $i = i + 1$ $s = s + i$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta((\log n)^2)$	$\Theta(n^2)$	$\Theta(n^3)$	$\Theta(\sqrt{n} \log n)$	$\Theta(n)$	$\Theta(n \log n)$	
loop1	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	244.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	244.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.4
loop5	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.5
loop6	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.6
loop7	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.7
loop8	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.8
loop9	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input checked="" type="checkbox"/> G	<input type="checkbox"/> H	244.9



## Problem 246

<b>Algorithm loop1(<math>n</math>)</b> $s = 1$ $i = 1$ <b>while</b> $i \leq n$ <b>for</b> $j = 1$ <b>to</b> $i$ $s = s + 1$ $i = 2 * i$	<b>Algorithm loop2(<math>n</math>)</b> $s = 1$ <b>while</b> $s \leq n$ $s = s + 1$	<b>Algorithm loop3(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = 0$ <b>while</b> $j \leq n$ $j = j + i$
<b>Algorithm loop4(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = 1$ <b>while</b> $j \leq i$ $j = 2 * j$	<b>Algorithm loop5(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = 1$ <b>while</b> $j \leq n$ $j = 2 * j$	<b>Algorithm loop6(<math>n</math>)</b> <b>for</b> $i = 1$ <b>to</b> $n$ $j = i$ <b>while</b> $j > 1$ $j = \lfloor j/2 \rfloor$
<b>Algorithm loop7(<math>n</math>)</b> $i = 0$ <b>while</b> $i \leq n$ $j = i$ <b>while</b> $j > 0$ $j = \lfloor j/2 \rfloor$ $i = i + 1$	<b>Algorithm loop8(<math>n</math>)</b> $i = 1$ $j = 1$ $s = 0$ <b>while</b> $i \leq n$ <b>if</b> $i = j$ <b>then</b> <b>for</b> $k = 1$ <b>to</b> $n$ $s = s + 1$ $j = 2 * j$ $i = i + 1$	<b>Algorithm loop9(<math>n</math>)</b> $i = 1$ $s = 0$ <b>while</b> $i \leq n$ <b>for</b> $j = i$ <b>to</b> $n$ $s = s + 1$ $i = i + 1$

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(\log n)$	$\Theta(\frac{\log n}{\log \log n})$	$\Theta(n)$	$\Theta(n^3)$	$\Theta(n\sqrt{n})$	$\Theta((\log n)^2)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.2
loop3	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.3
loop4	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.4
loop5	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.5
loop6	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.6
loop7	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.7
loop8	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.8
loop9	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input type="checkbox"/> H	246.9

**Problem 247**

```

Algorithm loop1(n)   Algorithm loop2(n)   Algorithm loop3(n)
i = 1                 i = 1                 i = 1
while i ≤ n         while i ≤ n         while i ≤ n
  j = 0               j = 1                 j = i
  while j ≤ n         while j ≤ i         while j ≤ n
    j = j + 1         j = 2 * j           j = j + 1
  i = 2 * i           i = i + 1             i = 2 * i

Algorithm loop4(n) Algorithm loop5(n) Algorithm loop6(n)
i = 1                 s = 0                 for i = 0 to n
while i ≤ n         i = n                 j = 0
  j = n               while i > 1           s = 0
  while j > 1         for j = 1 to n         while s ≤ i
    j = j - 1         s = s + 1           j = j + 1
  i = 2 * i           i = ⌊i/2⌋           s = s + j

Algorithm loop7(n) Algorithm loop8(n) Algorithm loop9(n)
for i = 1 to n     for i = 1 to n     i = 0
  j = 1               j = i                 j = 0
  while j ≤ i         while j > 0         while i ≤ n
    j = j + 1         j = j - 1           if i < j then
                                     i = i + 1
                                     else
                                       j = j + 1
                                       i = 0

```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n \log n)$	$\Theta(2^n)$	$\Theta(n^3)$	$\Theta(n^2)$	$\Theta((\log n)^2)$	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n\sqrt{n})$	
loop1	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.1
loop2	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.2
loop3	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.3
loop4	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.4
loop5	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.5
loop6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	247.6
loop7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.7
loop8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.8
loop9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	247.9

**Problem 248**

**Algorithm loop1( $n$ )**

```

i = 1
while i ≤ n
    j = 1
    while j ≤ i
        j = j + 1
    i = i + 1
    
```

**Algorithm loop2( $n$ )**

```

i = 1
while i ≤ n
    j = 1
    while j ≤ n
        j = j + 1
    i = i + 1
    
```

**Algorithm loop3( $n$ )**

```

i = n
j = 0
while i > 0
    if j < i
        j = j + 1
    else
        j = 0
        i = i - 1
    
```

**Algorithm loop4( $n$ )**

```

s = 0
for i = 1 to n
    for j = 1 to n
        if i = j then
            for k = 1 to n
                s = s + 1
    
```

**Algorithm loop5( $n$ )**

```

s = 0
i = n
while i > 0
    for j = 1 to i
        s = s + 1
    i = i - 1
    
```

**Algorithm loop6( $n$ )**

```

s = 1
for i = 1 to n
    for j = 1 to n
        s = s + 1
    
```

**Algorithm loop7( $n$ )**

```

s = 1
for i = 1 to n
    for j = 1 to n
        s = s + 1
    for k = 1 to n
        s = s + 1
    
```

**Algorithm loop8( $n$ )**

```

s = 1
for i = 1 to n
    for j = i to n
        s = s + 1
    
```

**Algorithm loop9( $n$ )**

```

s = 1
for i = n to 1 step -1
    for j = n to 1 step -1
        s = s + 1
    
```

Indicate for each of the above algorithms the running time as a function of  $n$  in  $\Theta$ -notation.

	$\Theta(n \log n)$	$\Theta(n)$	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n^3)$	$\Theta((\log n)^2)$	$\Theta(\sqrt{n} \log n)$	$\Theta(n^2)$	
loop1	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.1
loop2	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.2
loop3	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.3
loop4	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.4
loop5	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.5
loop6	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.6
loop7	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.7
loop8	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.8
loop9	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	<input checked="" type="checkbox"/>	248.9



## Amortized analysis

### Problem 250 (4%)

Assume a list  $L$  is used to store a set of distinct integers. The following two operations are to be supported:  $\text{ADD}(x)$  adds  $x$  to the list (where  $x$  is assumed to be distinct from all integers already in the list), and  $\text{REMOVELARGERHALF}$  removes and returns the  $\lfloor |L|/2 \rfloor$  largest elements from the list.  $\text{ADD}(x)$  simply appends the new element to the end of the list  $L$  in worst-case  $O(1)$  time.  $\text{REMOVELARGERHALF}$  first uses deterministic selection to find the  $\lfloor |L|/2 \rfloor + 1$  smallest element  $e$  in  $L$  in worst-case  $O(|L|)$  time, after which  $L$  is scanned and all elements greater than or equal to  $e$  are removed.

For each of the following functions, indicate whether it is a potential function with which one can argue that both operations take amortized  $O(1)$  time.

	Yes	No	
$ L $	<input checked="" type="checkbox"/>	<input type="checkbox"/>	250.1
$\log  L $	<input type="checkbox"/>	<input checked="" type="checkbox"/>	250.2
$2 L $	<input checked="" type="checkbox"/>	<input type="checkbox"/>	250.3
$ L /2$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	250.4
$ L ^2$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	250.5
$ L  \cdot \log  L $	<input type="checkbox"/>	<input checked="" type="checkbox"/>	250.6

### Problem 251 (4%)

Assume an array  $X$  of size  $n$  contains two stacks  $S$  and  $T$  of sizes  $s$  and  $t$ , respectively, such that  $X[1..s] = S$  and  $X[n+1-t..n] = T$ , where the tops of the two stacks are  $X[s]$  and  $X[n+1-t]$ , respectively. When  $X$  becomes full, i.e.,  $s+t = n$ , a new array of double the size is allocated for  $X$ , and  $S$  and  $T$  are copied into this array.

For each of the following functions, indicate whether it is a potential function with which one can argue that the stack operations  $\text{PUSH}$  and  $\text{POP}$  on the two stacks take amortized  $O(1)$  time.

	Yes	No	
$s + t$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	251.1
$t - s$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	251.2
$s + n - t$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	251.3
$\max\{0, 2(s+t) - n\}$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	251.4
$n - s - t$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	251.5
$\max\{0, 2s + 3t - n\}$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	251.6

**Problem 252 (4 %)**

Given a sorted list  $L$  with  $N = 2^k - 1$  elements, for a positive integer  $k$ , one can in  $O(N)$  time construct a perfectly balanced binary search tree containing  $L$ . In the following, we assume that we only perform deletions in the search tree, as described in [CLRS, Chapter 12.3], i.e., deletions do not attempt to keep the tree balanced. Let  $n$  denote the current number of elements in the tree. To keep the height of the tree logarithmic in  $n$ , the tree is reconstructed as a perfectly balanced binary search tree in  $O(n)$  time when half of the elements have been deleted, i.e., when  $n < N/2$ , where  $N$  denotes the number of elements in the tree the last time it was reconstructed.

For each of the following functions, indicate whether it is a potential function with which one can argue that deletions take amortized  $O(\log n)$  time.

	Yes	No	
$N$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	252.1
$n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	252.2
$N - n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	252.3
$(N - n) \cdot \log n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	252.4
$\log n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	252.5
$(N - n) \cdot \log N$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	252.6

**Problem 253 (4 %)**

Consider a list  $L = (x_1, \dots, x_N)$  of  $N$  integers, on which we can perform the following two operations: APPEND( $x$ ) appends the integer  $x$  to the end of the list, and ADDEPAIRS replaces for all  $i = 1.. \lfloor |L|/2 \rfloor$  the  $2i - 1$ 'th and  $2i$ 'th integers with their sum, such that the new list has length  $\lfloor |L|/2 \rfloor$ . For example, ADDEPAIRS on the list 3, 5, 7, 4, 11, 2, 6 results in the new list 8, 11, 13, 6. The worst-case time for APPEND and ADDEPAIRS is  $O(1)$  and  $O(|L|)$ , respectively.

For each of the following functions, indicate whether it is a potential function with which one can argue that both operations take amortized  $O(1)$  time.

	Yes	No	
$N$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	253.1
$\log N$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	253.2
$N + \log N$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	253.3
$N/2$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	253.4
$N \cdot \log N$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	253.5
$\sqrt{N}$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	253.6

**Problem 254 (4 %)**

Consider an unordered list of  $n$  integers, where we can in  $O(1)$  time insert a new integer, and in  $O(n)$  time perform the operation `NEGATEREMOVE`, which removes all non-positive integers ( $x \leq 0$ ) from the list and replaces each positive integer  $y > 0$  with the corresponding negative integer  $-y$ . `NEGATEREMOVE(3, -4, -2, 7, 6, -2) = (-3, -7, -6)`.

For each of the following functions, indicate whether it is a potential function with which one can argue that both insertions and `NEGATEREMOVE` take amortized  $O(1)$  time, where  $P$  is the number of positive integers in the list and  $N$  is the number of non-positive integers in the list.

	Yes	No	
$N$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	254.1
$P$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	254.2
$2N + P$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	254.3
$N + P$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	254.4
$N + 2P$	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	254.5
$P + N/2$	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	254.6

**Problem 255 (4 %)**

Consider a red-black search tree extended with the operation `INSERTCUT`( $x$ ), which inserts the element  $x$  into the search tree and deletes all elements less than  $x$  from the search tree. If the insertion deletes  $k$  elements, this takes worst-case  $O((k + 1) \log n)$  time, where  $n$  is the number of elements in the tree before `INSERTCUT` is performed.

For each of the following functions, indicate whether it is a potential function with which one can argue that `INSERTCUT` takes amortized  $O(\log n)$ .

	Yes	No	
$\log n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	255.1
$n \cdot \log n$	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	255.2
$n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	255.3
$\sum_{i=1}^n \log i$	<input checked="" type="checkbox"/>	<input type="checkbox"/> B	255.4
$k \cdot \log n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/>	255.5

**Problem 256 (4 %)**

Assume we want to store a set of  $n$  integers using hashing with linear probing in an array of size  $N$ . We guarantee that the array is always between  $1/4$  and  $3/4$  full. If there are fewer than  $N/4$  or more than  $3N/4$  integers in the set, we reinsert all integers into a new array of size  $2n$ , i.e., the new array is  $1/2$  full.

For each of the following functions, indicate whether it is a potential function with which one can argue that the total number of reinsertions in the hash tables is amortized  $O(1)$  per insertion and deletion in the set.

	Yes	No	
$ 2n - N $	<input checked="" type="checkbox"/>	<input type="checkbox"/>	256.1
$\min(n, N - n)$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	256.2
$N - n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	256.3
$4 N/2 - n $	<input checked="" type="checkbox"/>	<input type="checkbox"/>	256.4
$N$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	256.5
$N/2 - \min(n, N - n)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	256.6

**Problem 257 (4 %)**

A binary max-heap supports INSERT and HEAP-EXTRACT-MAX on a max-heap with  $n$  elements in worst-case  $O(\log n)$  time.

For each of the following functions, indicate whether it is a potential function with which one can argue that INSERT takes amortized  $O(\log n)$  time and HEAP-EXTRACT-MAX takes amortized  $O(1)$  time.

	Yes	No	
$n \cdot \log n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	257.1
$\sum_{i=1}^n \log i$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	257.2
$n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	257.3
$\log n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	257.4
$(\log n)^2$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	257.5

**Problem 258 (4 %)**

Consider a binary max-heap implemented in an array. Overflow is handled by allocating a new array of double size and copying the contents of the old array to the new array. Let the current size of the array be  $N$  and the number of elements in the heap be  $n$ .

For each of the following functions, indicate whether it is a potential function with which one can argue that INSERT and HEAP-EXTRACT-MAX require amortized  $O(\log n)$  time.

	Yes	No	
$n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	258.1
$ 2n - N $	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	258.2
$\max(0, 2n - N)$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	258.3
$N - 2n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	258.4
$\log n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	258.5
$\log(N/n)$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	258.6

**Problem 259 (4 %)**

Consider a stack implemented in an array, where overflow is handled by allocating a new larger array and copying the contents of the old array to the new array. Indicate the amortized time for a PUSH operation, when the new array has the following size  $N$  and  $n$  is the number of elements on the stack before the PUSH operation.

	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	
$N = n + 1$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	259.1
$N = n + \lceil \sqrt{n} \rceil$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	259.2
$N = \lceil \frac{3}{2}n \rceil$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	259.3
$N = 2n$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	259.4
$N = 3n$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	259.5

**Problem 260 (4 %)**

A binary max-heap supports MAX-HEAP-INSERT and HEAP-EXTRACT-MAX in worst-case time  $O(\log n)$ , where  $n$  is the number of elements in the heap. We now want to support the operation DELETE, which given a pointer to an element in the heap, deletes the element from the heap. We implement DELETE by simply marking the element as deleted in worst-case  $O(1)$  time. When we perform HEAP-EXTRACT-MAX, we repeat this until the first unmarked element is returned. I.e., if HEAP-EXTRACT-MAX deletes  $D$  marked elements, the worst-case time is  $O((D + 1) \log N)$ , where  $N$  is the number of marked and unmarked elements in the heap.

For each of the following functions, indicate whether it is a potential function with which one can argue that the operations MAX-HEAP-INSERT, DELETE, and HEAP-EXTRACT-MAX take amortized  $O(\log N)$  time, where  $M$  denotes the number of marked elements in the heap.

	Yes	No	
$N$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	260.1
$M$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	260.2
$N \cdot \log N$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	260.3
$M \cdot \log N$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	260.4
$N \cdot \log M$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	260.5
$M \cdot \log M$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	260.6

**Problem 261 (4 %)**

A binary max-heap supports INSERT and HEAP-EXTRACT-MAX on a heap with  $n$  elements in worst-case  $O(\log n)$  time. Note that the same value can be inserted multiple times in a max-heap. We now want to change HEAP-EXTRACT-MAX, such that it removes all occurrences of the maximum value from the heap, i.e., the original HEAP-EXTRACT-MAX operation is repeated until the root contains a smaller value or the heap is empty. If the maximum occurs  $m$  times in the heap, then the HEAP-EXTRACT-MAX operation will take worst-case  $O(m \log n)$  time.

For each of the following functions, indicate whether it is a potential function with which one can argue that the operations INSERT and HEAP-EXTRACT-MAX take amortized  $O(\log n)$  time. The number of elements in the heap is denoted  $n$  and the number of distinct elements in the heap  $N$ , where  $N \leq n$ .

	Yes	No	
$n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	261.1
$N$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	261.2
$n - N$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	261.3
$n \cdot \log n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	261.4
$N \cdot \log n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	261.5
$(n - N) \cdot \log n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	261.6

## Invariants

### Problem 262 (4 %)

Given two non-negative integers  $n$  and  $m$ , the below algorithm computes  $n \cdot m$ .

```

Algorithm Multiplication( $n, m$ )
Input   :  $n \geq 0 \wedge m \geq 0$ 
Output  :  $r = n_0 \cdot m_0$ 
Method  :  $r \leftarrow 0$ 
         { $I$ } while  $n > 0$  do
           if  $n$  is odd then
              $r \leftarrow r + m$ 
              $n \leftarrow n - 1$ 
           else
              $m \leftarrow m * 2$ 
              $n \leftarrow n / 2$ 

```

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Multiplication, where  $n_0$  and  $m_0$  denote the initial values of  $n$  and  $m$ , respectively.

	Yes	No	
$0 \leq n \leq n_0$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	262.1
$0 \leq m \leq m_0$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	262.2
$r = m \cdot n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	262.3
$n_0 \cdot m_0 = r + n \cdot m$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	262.4
$n_0 \cdot m_0 + r = n \cdot m$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	262.5

**Problem 263 (4 %)**

Given positive integers  $x$  and  $y$ , the below algorithm computes  $x^y$ .

**Algorithm** Power( $x, y$ )  
 Input :  $x \geq 1 \wedge y \geq 1$   
 Output :  $r = x^y$   
 Method :  $r \leftarrow 1$   
     { $I$ } **while**  $y \geq 1$  **do**  
         **if**  $y$  is odd **then**  
              $y \leftarrow y - 1$   
              $r \leftarrow r * x$   
         **else**  
              $y \leftarrow y/2$   
              $x \leftarrow x * x$

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Power, where  $x_0$  and  $y_0$  denote the initial values of  $x$  and  $y$ .

	Yes	No	
$r = x^y$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	263.1
$r = x_0^{y_0}$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	263.2
$r = x_0^{y_0 - y}$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	263.3
$x_0^{y_0} = r \cdot x^y$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	263.4
$x^y = r \cdot x_0^{y_0}$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	263.5

**Problem 264 (4 %)**

Given a positive integer  $n$ , the below algorithm computes  $n^2$ .

**Algorithm** Square( $n$ )  
 Input :  $n \geq 1$   
 Output :  $r = n^2$   
 Method :  $r \leftarrow 0$   
      $i \leftarrow 0$   
     { $I$ } **while**  $i < n$  **do**  
          $i \leftarrow i + 1$   
          $r \leftarrow r + i$   
      $r \leftarrow 2 * r - n$

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Square.

	Yes	No	
$i \geq 0 \wedge r = i^2$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	264.1
$0 \leq i \leq n$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	264.2
$i \geq 0 \wedge r = i(i + 1)/2$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	264.3
$r \geq i \geq 0$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	264.4
$0 \leq r \leq n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	264.5

**Problem 265 (4%)**

Given an array  $A[1..n]$  containing  $n \geq 1$  integers and an integer  $x$ , the below algorithm computes the number of occurrences of  $x$  in  $A$ , i.e.,  $count(x, A) = |\{i \mid 1 \leq i \leq n \wedge A[i] = x\}|$ .

```

Algorithm Count( $x, A$ )
Input   :  $x \wedge A[1..n] \wedge n \geq 1$ 
Output  :  $r = count(x, A)$ 
Method  :  $i \leftarrow 0$ 
           $r \leftarrow 0$ 
          {I} while  $i < n$  do
               $i \leftarrow i + 1$ 
              if  $x = A[i]$  then
                   $r \leftarrow r + 1$ 

```

For each of the following statements, indicate whether it is an invariant  $I$  for the above algorithm Count.

	Yes	No	
$i \leq n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	265.1
$r = count(x, A[1..i])$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	265.2
$r = count(x, A[1..i + 1])$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	265.3
$i < n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	265.4
$r = 0$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	265.5

**Problem 266 (4 %)**

Assume that a sorted array  $A[1..n]$  contains  $n \geq 2$  different integers, i.e.,  $A[1] < A[2] < \dots < A[n-1] < A[n]$ . Given a positive integer  $x > 0$ , the below algorithm identifies whether there exist  $1 \leq i < j \leq n$  such that  $x = A[j] - A[i]$ .

**Algorithm** FindDiff( $A[1..n], x$ )  
**Input** :  $x > 0 \wedge A[1..n] \wedge n \geq 2 \wedge A[1] < \dots < A[n]$   
**Output** :  $1 \leq i < j \leq n$ , where  $x = A[j] - A[i]$ ;  
           or  $j = n + 1$  if no such pair exists  
**Method** :  $i \leftarrow 1$ ;  
            $j \leftarrow 1$ ;  
           {I} **while**  $j \leq n$  **and**  $x \neq A[j] - A[i]$  **do**  
               **if**  $x > A[j] - A[i]$  **then**  
                    $j \leftarrow j + 1$   
               **else**  
                    $i \leftarrow i + 1$

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm FindDiff.

	Yes	No	
$i \leq j$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	266.1
$1 \leq i \leq j \leq n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	266.2
$x \neq A[j] - A[i]$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	266.3
$\forall 1 \leq i' < j' \leq n : (i' \geq i \vee j' \geq j) \Rightarrow x \neq A[j'] - A[i']$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	266.4
$\forall 1 \leq i' < j' \leq n : (i' < i \vee j' < j) \Rightarrow x \neq A[j'] - A[i']$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	266.5

**Problem 267 (4 %)**

Let  $\|x\|$  denote the number of bits with value 1 in the binary representation of a non-negative integer  $x$ , e.g.,  $\|14_{10}\| = \|1110_2\| = 3$ . The below algorithm computes  $\|x\|$ .

**Algorithm** Bits( $x$ )  
 Input :  $x \geq 0$   
 Output :  $r = \|x\|$   
 Method :  $r \leftarrow 0$ ;  
           { $I$ } **while**  $x > 0$  **do**  
                   **if**  $x$  is odd **then**  
                            $x \leftarrow x - 1$ ;  
                            $r \leftarrow r + 1$   
                            $x \leftarrow x/2$

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Bits, where  $x_0$  denotes the initial value of  $x$ .

	Yes	No	
$r = \ x\ $	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	267.1
$r + \ x\  = \ x_0\ $	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	267.2
$r + \ x_0\  = \ x\ $	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	267.3
$r + \ x\  \leq x_0$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	267.4
$\ x\  + 2^r = x_0$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	267.5

**Problem 268 (4 %)**

The below algorithm computes the integer base-2 logarithm of  $n$ , i.e.,  $\text{intlog}(n) = \lfloor \log_2 n \rfloor$ .

```

Algorithm Log2( $n$ )
Input   :  $n \geq 2$ 
Output  :  $r = \text{intlog}(n) = \lfloor \log_2 n \rfloor$ 
Method  :  $i \leftarrow 1$ ;
           $r \leftarrow 1$ ;
           $p \leftarrow 2$ ;
          {I} while  $2p \leq n$  do
              if  $p * p \leq n$  then
                   $p \leftarrow p * p$ ;
                   $r \leftarrow 2 * r$ 
              else
                   $p \leftarrow 2 * p$ ;
                   $r \leftarrow r + 1$ 

```

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Log2.

	Yes	No	
$1 \leq r < p$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	268.1
$2p \leq n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	268.2
$p = 2^r$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	268.3
$p = 2r$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	268.4
$p = 2^{\text{intlog}(p)}$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	268.5

**Problem 269 (4 %)**

Assume that an array  $A[1..n - 1]$  contains  $n - 1$  different numbers from the set  $\{1, 2, 3, \dots, n\}$ , where  $n \geq 2$ . The below algorithm Missing identifies the element  $r \in \{1, 2, 3, \dots, n\}$  that is not in  $A$ .

**Algorithm** Missing( $A[1..n - 1]$ )

Input :  $A[1..n - 1] \wedge n \geq 2 \wedge A[i] \in \{1, 2, \dots, n\} \wedge A[i] \neq A[j]$  for  $1 \leq i < j < n$

Output :  $r \in \{1, 2, 3, \dots, n\} \setminus A$

Method :  $i \leftarrow 1$ ;

$x \leftarrow 1$ ;

$y \leftarrow A[1]$ ;

{ $I$ } **while**  $i < n - 1$  **do**

$i \leftarrow i + 1$ ;

$x \leftarrow x + i$ ;

$y \leftarrow y + A[i]$

$r \leftarrow n + x - y$

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Missing.

	Yes	No	
$1 \leq i \leq n - 1$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	269.1
$y = x - i + A[i]$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	269.2
$x = i(i + 1)/2$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	269.3
$y = \sum_{j=1}^i A[j]$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	269.4
$i = i + 1$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	269.5

**Problem 270 (4%)**

Assume that an array  $A[1..n]$  contains  $n \geq 1$  integers. Position  $i$  in  $A$  is said to be *dominating* if  $A[i] > A[j]$  for all  $1 \leq j < i$ . Let  $\text{dom}(A)$  denote the number of dominating positions  $i$  in  $A$ , where  $1 \leq i \leq n$ . The below algorithm computes  $\text{dom}(A)$ .

```

Algorithm Domination( $A$ )
Input   : Array  $A[1..n] \wedge n \geq 1$ 
Output  :  $r = \text{dom}(A)$ 
Method  :  $i \leftarrow 1$ ;
           $x \leftarrow A[1]$ ;
           $r \leftarrow 1$ ;
          { $I$ } while  $i < n$  do
               $i \leftarrow i + 1$ ;
              if  $A[i] > x$  then
                   $x \leftarrow A[i]$ ;
                   $r \leftarrow r + 1$ 

```

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm.

	Yes	No	
$1 \leq i < n$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	270.1
$x = A[i]$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	270.2
$x \geq A[i]$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	270.3
$r = \text{dom}(A[1..n])$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	270.4
$r = \text{dom}(A[1..i])$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	270.5

**Problem 271 (4%)**

Assume  $A[1..n]$  is a sorted array with  $n$  different positive integers. Let  $\text{squares}(A)$  denote the number of  $A[i]$  such that  $A[i]^2$  also occurs in  $A$ . For example,  $\text{squares}(1, 3, 4, 7, 9, 16) = 3$ , since  $1^2 = 1$ ,  $3^2 = 9$  and  $4^2 = 16$ . The below algorithm Squares computes  $\text{squares}(A)$ .

```

Algorithm Squares( $A[1..n]$ )
Input   : Array  $A[1..n] \wedge 0 < A[1] < A[2] < \dots < A[n]$ 
Output  :  $r = \text{squares}(A)$ 
Method  :  $i \leftarrow 1$ ;
           $j \leftarrow 1$ ;
           $r \leftarrow 0$ ;
          { $I$ } while  $j \leq n$  do
              if  $A[i]^2 < A[j]$  then
                   $i \leftarrow i + 1$ 
              else if  $A[i]^2 = A[j]$  then
                   $i \leftarrow i + 1$ ;  $j \leftarrow j + 1$ ;  $r \leftarrow r + 1$ 
              else if  $A[i]^2 > A[j]$  then
                   $j \leftarrow j + 1$ 

```

For each of the following statements, indicate whether it is an invariant  $I$  for the above algorithm Squares. It is assumed that  $A[0] = 0$  and  $A[n + 1] = +\infty$ .

	Yes	No	
$1 \leq i \leq j \leq n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	271.1
$i \leq j$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	271.2
$A[i - 1]^2 < A[j] \wedge r = \text{squares}(A[1..j])$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	271.3
$A[i - 1]^2 < A[j] \wedge r = \text{squares}(A[1..j - 1])$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	271.4
$r = j - 1$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	271.5

**Problem 272 (4 %)**

Given a positive integer  $n$ , the below algorithm computes  $n^3$ .

**Algorithm** Power3( $n$ )  
 Input :  $n \geq 1$   
 Output :  $r = n^3$   
 Method :  $i \leftarrow 1$   
            $s \leftarrow 1$   
            $r \leftarrow 1$   
           { $I$ } **while**  $i < n$  **do**  
                $i \leftarrow i + 1$   
                $s \leftarrow s + 2i - 1$   
                $r \leftarrow r + 3s - 3i + 1$

For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Power3.

	Yes	No	
$1 \leq i < n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	272.1
$1 \leq i \leq s \leq r$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	272.2
$r = i^3$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	272.3
$r = n^3$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	272.4
$s = s + 2i - 1$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	272.5

**Problem 273 (4 %)**

Given a non-negative integer  $x$  and a positive integer  $y$ , the below algorithm computes  $\lfloor x/y \rfloor$ .

**Algorithm** Division( $x, y$ )  
 Input :  $x \geq 0 \wedge y \geq 1$   
 Output :  $r = \lfloor x/y \rfloor$   
 Method :  $r \leftarrow 0$   
           { $I$ } **while**  $x \geq y$  **do**  
                $x \leftarrow x - y$   
                $r \leftarrow r + 1$

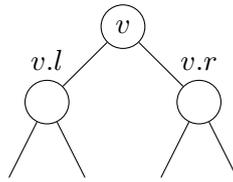
For each of the following statements, indicate whether it is an invariant  $I$  for the algorithm Division, where  $x_0$  and  $y_0$  denote the initial values of  $x$  and  $y$ , respectively.

	Yes	No	
$r = \lfloor x/y \rfloor$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	273.1
$r = \lfloor x_0/y_0 \rfloor$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	273.2
$r = \lfloor (x_0 - x)/y \rfloor$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	273.3
$x + ry = x_0$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	273.4
$r(x - x_0) = y$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	273.5

## Augmented search trees

### Problem 274 (4%)

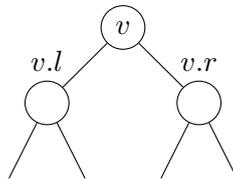
Consider a red-black tree where each node stores a pair of integers (*element*, *weight*), and the pairs are sorted from left-to-right according to increasing *element* value. For a node  $v$  in the tree we let  $v.e$  and  $v.w$  denote the pair  $(e, w)$  stored in the node. Furthermore,  $v$  stores the value  $v.W$  which is the sum of the weights in all nodes in  $v$ 's subtree, and  $v.prefix$  which is the maximum sum of the weights a *prefix* of the pairs in  $v$ 's subtree can have (when the pairs are sorted according to element value). Indicate how  $v.prefix$  can be computed when the corresponding information is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



- $v.prefix = \max(v.l.prefix, v.l.W + v.w, v.l.prefix + v.w + v.r.prefix)$   A  
 $v.prefix = \max(v.l.prefix, v.l.W + v.w, v.l.W + v.w + v.r.prefix)$   B  
 $v.prefix = \max(v.l.prefix, v.W + v.r.prefix)$   C  
 $v.prefix = \max(v.l.W, v.l.W + v.w, v.l.W + v.w + v.r.W)$   D

### Problem 275 (4%)

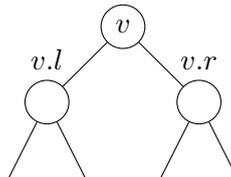
An interval tree is a red-black tree where each node stores exactly one interval (*low*, *high*), and the intervals are sorted from left-to-right according to increasing *low* value. For a node  $v$  in the tree we let  $v.low$  and  $v.high$  denote the endpoints of the interval stored in the node. Furthermore,  $v$  stores the value  $v.max$  which is the maximum value in an interval stored in  $v$ 's subtree. Indicate how  $v.max$  can be computed when the corresponding information is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



- $v.max = v.r.max$   A  
 $v.max = \max(v.r.max, v.high)$   B  
 $v.max = \max(v.r.max, v.l.max, v.high)$   C  
 $v.max = \max(v.l.high, v.high, v.r.high)$   D

**Problem 276 (4%)**

Consider a search tree where each node  $v$  stores a number  $v.x$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore, a node  $v$  stores two values  $v.size$  and  $v.avg$ , which are respectively the number of elements in  $v$ 's subtree and the average of all numbers in  $v$ 's subtree. Indicate how  $v.size$  and  $v.avg$  can be computed when the corresponding information is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



$$v.size = v.size + v.l.size + v.r.size \quad \boxed{A}$$

$$v.size = 1 + v.l.size + v.r.size \quad \boxed{\times}$$

$$v.size = v.l.size + v.r.size \quad \boxed{C}$$

$$v.size = 1 + (v.r.size - v.l.size) \quad \boxed{D}$$

$$v.avg = (v.x + v.l.avg + v.r.avg)/3 \quad \boxed{A}$$

$$v.avg = (v.x + v.l.avg * v.l.size + v.r.avg * v.r.size)/v.size \quad \boxed{\times}$$

$$v.avg = (v.x + v.l.avg + v.r.avg)/v.size \quad \boxed{C}$$

$$v.avg = v.x + (v.l.avg + v.r.avg)/2 \quad \boxed{D}$$

**Problem 277 (4 %)**

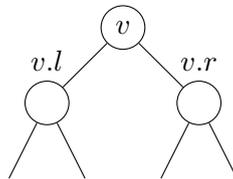
Consider a list of  $n$  points  $(x_1, y_1), \dots, (x_n, y_n)$ , where  $x_i$  and  $y_i$  are real numbers and  $x_1 < x_2 < \dots < x_n$ . We want to find a contiguous sequence of points

$$(x_i, y_i), (x_{i+1}, y_{i+1}), \dots, (x_{j-1}, y_{j-1}), (x_j, y_j),$$

where  $i \leq j$ , such that  $Y_{i,j} = \sum_{k=i}^j y_k = y_i + y_{i+1} + \dots + y_{j-1} + y_j$  is as large as possible. We want for a dynamic list of points to maintain this maximum contiguous  $y$ -sum, *maxysum*.

Consider a search tree where each node  $v$  stores a point  $(v.x, v.y)$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore, a node  $v$ , which in its subtree contains the points  $(x_k, y_k), \dots, (x_\ell, y_\ell)$ , stores four values  $v.sum = Y_{k,\ell}$ ,  $v.maxysum = \max_{k \leq i \leq j \leq \ell} Y_{i,j}$ ,  $v.pre = \max\{0, \max_{k \leq j \leq \ell} Y_{k,j}\}$ , and  $v.suf = \max\{0, \max_{k \leq i \leq \ell} Y_{i,\ell}\}$ .

Indicate how  $v.sum$  and  $v.maxysum$  can be computed when the corresponding information (incl. *pre* and *suf*) is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



$$v.sum = v.l.sum + v.r.sum \quad \boxed{\text{A}}$$

$$v.sum = v.l.sum + v.y + v.r.sum \quad \boxed{\text{X}}$$

$$v.sum = v.l.suf + v.r.pre \quad \boxed{\text{C}}$$

$$v.sum = v.l.suf + v.y + v.r.pre \quad \boxed{\text{D}}$$

$$v.maxysum = \max\{v.l.maxysum, v.r.maxysum\} \quad \boxed{\text{A}}$$

$$v.maxysum = \max\{v.l.maxysum, v.y, v.r.maxysum\} \quad \boxed{\text{B}}$$

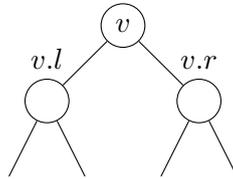
$$v.maxysum = \max\{v.l.maxysum, v.l.suf + v.y + v.r.pre, v.r.maxysum\} \quad \boxed{\text{X}}$$

$$v.maxysum = \max\{v.l.maxysum, v.l.suf + v.r.pre, v.r.maxysum\} \quad \boxed{\text{D}}$$

**Problem 278 (4 %)**

Consider a list of  $n$  points  $(x_1, y_1), \dots, (x_n, y_n)$ , where  $x_i$  and  $y_i$  are real numbers and  $x_1 < x_2 < \dots < x_n$ . A point  $(x_i, y_i)$  is dominated by a point  $(x_j, y_j)$  if  $x_i \leq x_j$  and  $y_i \leq y_j$ . A list of points is *dominance-free* if and only if no point in the list is dominated by another point. For example, the list  $(3, 7), (5, 3), (7, 6), (13, 4)$  is *not* dominance-free, since  $(5, 3)$  is dominated by  $(13, 4)$ . We want for a dynamic list of points to maintain the statement of whether the list is dominance-free.

Consider a search tree where each node  $v$  stores a point  $(v.x, v.y)$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore,  $v$  also stores  $v.DF$  and  $v.miny$  and  $v.maxy$ , where  $v.DF$  is true if and only if the set of points in  $v$ 's subtree is dominance-free, and  $v.miny$  and  $v.maxy$  are the smallest and largest  $y$  value stored in  $v$ 's subtree. Indicate how  $v.miny$  and  $v.DF$  can be computed when the corresponding information (incl.  $maxy$ ) is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



$$v.miny = v.l.miny \quad \boxed{\text{A}}$$

$$v.miny = \min\{v.l.miny, v.r.miny\} \quad \boxed{\text{B}}$$

$$v.miny = \min\{v.l.miny, v.y, v.r.miny\} \quad \boxed{\otimes}$$

$$v.DF = v.l.DF \wedge v.r.DF \quad \boxed{\text{A}}$$

$$v.DF = v.l.DF \wedge v.r.DF \wedge v.y < v.l.miny \wedge v.y > v.r.maxy \quad \boxed{\otimes}$$

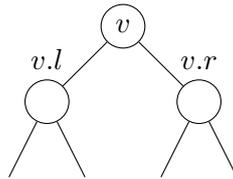
$$v.DF = v.l.DF \wedge v.r.DF \wedge v.y > v.l.miny \wedge v.y < v.r.maxy \quad \boxed{\text{C}}$$

$$v.DF = v.l.DF \wedge v.r.DF \wedge v.y < v.l.miny \quad \boxed{\text{D}}$$

**Problem 279 (4%)**

Consider a list of  $n$  pairs  $(x_1, c_1), \dots, (x_n, c_n)$ , where  $x_1 < x_2 < \dots < x_n$  are real numbers and each  $c_i \in \{\text{Red, Green, Blue}\}$  is a color.

Consider a search tree where each node  $v$  stores a pair  $(v.x, v.c)$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore, a node  $v$  stores a boolean value  $v.mono$ , which indicates whether all elements in  $v$ 's subtree have the same color, and the set  $v.missing$  of the colors that *do not* occur in  $v$ 's subtree. Indicate how  $v.mono$  and  $v.missing$  can be computed when the corresponding information is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



$$v.mono = v.l.mono \wedge v.r.mono \quad \boxed{\text{A}}$$

$$v.mono = v.l.mono \wedge v.r.mono \wedge v.l.c = v.r.c \wedge v.c = v.r.c \quad \boxed{\times}$$

$$v.mono = v.l.mono \wedge v.r.mono \wedge v.l.c = v.r.c \quad \boxed{\text{C}}$$

$$v.mono = v.l.mono \wedge v.r.mono \wedge v.mono \quad \boxed{\text{D}}$$

$$v.missing = v.l.missing \cup v.r.missing \quad \boxed{\text{A}}$$

$$v.missing = v.l.missing \cap v.r.missing \quad \boxed{\text{B}}$$

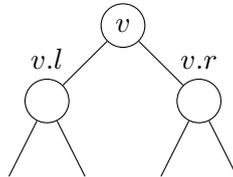
$$v.missing = \{v.c\} \cup v.l.missing \cup v.r.missing \quad \boxed{\text{C}}$$

$$v.missing = (v.l.missing \cap v.r.missing) \setminus \{v.c\} \quad \boxed{\times}$$

**Problem 280 (4 %)**

Given a string  $T$  containing letters and opening and closing parentheses ( and ), we assume that all positions with parentheses are stored in a search tree, sorted from left-to-right according to increasing position. A node  $v$  stores a position  $v.p$  and the corresponding parenthesis  $v.c = T[v.p]$  from  $T$ . For  $T = \text{"a)b(cd(x)dc(a"$  the  $\langle v.p, v.c \rangle$  pairs  $\langle 2, \rangle \rangle, \langle 4, ( \rangle, \langle 7, ( \rangle, \langle 9, \rangle \rangle$  and  $\langle 12, ( \rangle$  are stored in the tree.

We want to maintain information about whether the parentheses are balanced. In the above example the parentheses “) ( ( ) (” are not balanced, since only the marked parentheses match each other. The remaining parentheses “) ( (” will always be  $R$ )-parentheses followed by  $L$  (-parentheses, where  $R \geq 0$  and  $L \geq 0$ . In the example we have  $R = 1$  and  $L = 2$ . In a node  $v$  in the tree these values  $v.R$  and  $v.L$  are stored for the subsequence of parentheses in  $v$ 's subtree. Indicate how  $v.R$  can be computed when  $v.c = )$  and the  $R$  and  $L$  values are known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



$$v.R = v.l.R + 1 + v.r.R \quad \boxed{\text{A}}$$

$$v.R = v.l.R + v.r.R + 1 - v.l.L \quad \boxed{\text{B}}$$

$$v.R = v.l.R + \max\{0, v.r.R + 1 - v.l.L\} \quad \boxed{\otimes}$$

$$v.R = v.l.R + 1 + \max\{0, v.r.R - v.l.L\} \quad \boxed{\text{D}}$$

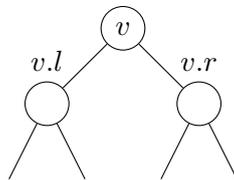
**Problem 281 (4%)**

For  $n$  numbers  $x_1, \dots, x_n$  we want to compute the coefficients  $a$  and  $b$  for the polynomial

$$P(y) = \sum_{i=1}^n (x_i - y)^2 = n \cdot y^2 + a \cdot y + b.$$

For example, for  $x_1 = 2$ ,  $x_2 = 3$ , and  $x_3 = 5$ , we have the polynomial  $P(y) = (2 - y)^2 + (3 - y)^2 + (5 - y)^2 = 3y^2 - 20y + 38$ , i.e.,  $a = -20$  and  $b = 38$ .

Consider a search tree where each node  $v$  stores a number  $v.x$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore, a node  $v$  stores two values  $v.a$  and  $v.b$ , which are the  $a$  and  $b$  coefficients for the  $P(y)$  polynomial defined by all  $x$  values in  $v$ 's subtree. Indicate how  $v.a$  and  $v.b$  can be computed when the corresponding information is known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



$$v.a = v.l.a + v.r.a + v.x \quad \boxed{\text{A}}$$

$$v.a = v.l.a + v.r.a - 2 * v.x \quad \boxed{\text{X}}$$

$$v.a = v.l.a * v.r.a * (v.x)^2 \quad \boxed{\text{C}}$$

$$v.a = v.l.a + v.r.a + (v.x)^2 \quad \boxed{\text{D}}$$

$$v.b = v.l.b + v.r.b + v.x \quad \boxed{\text{A}}$$

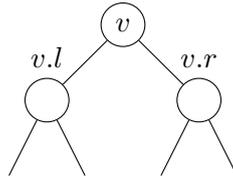
$$v.b = v.l.b + v.r.b + (v.x)^2 \quad \boxed{\text{X}}$$

$$v.b = v.l.b + v.r.b - 2 * v.x \quad \boxed{\text{C}}$$

$$v.b = v.l.b + v.r.b + 2 * v.x \quad \boxed{\text{D}}$$

**Problem 282 (4 %)**

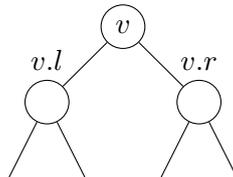
Consider a search tree where each node  $v$  stores a number  $v.x$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore, a node  $v$  stores three values  $v.min$ ,  $v.max$ , and  $v.closest$ . The values  $v.min$  and  $v.max$  are respectively the smallest and largest number in  $v$ 's subtree, and  $v.closest$  is the smallest difference between two numbers in  $v$ 's subtree. If  $v$ 's subtree contains only one number then  $v.closest = +\infty$ . Indicate how  $v.closest$  can be computed when the  $min$ ,  $max$ , and  $closest$  values are known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



- $v.closest = \min(v.l.closest, v.r.x - v.l.x, v.r.closest)$   A  
 $v.closest = \min(v.l.closest, v.x - v.l.min, v.r.max - v.x, v.r.closest)$   B  
 $v.closest = \min(v.l.closest, v.x - v.l.max, v.r.min - v.x, v.r.closest)$   C  
 $v.closest = \min(v.l.closest, v.r.min - v.l.max, v.r.closest)$   D  
 $v.closest = v.r.closest - v.l.closest$   E

**Problem 283 (4 %)**

For a sorted list of numbers  $x_1 \leq x_2 \leq \dots \leq x_n$  we define the *sum of square gaps* ( $ssg$ ) as  $\sum_{i=2..n} (x_i - x_{i-1})^2$ . Consider a red-black tree where each node  $v$  stores an integer  $v.x$ , and the nodes are ordered left-to-right according to increasing  $v.x$ . Furthermore,  $v$  stores the values  $v.min$ ,  $v.max$ , and  $v.ssg$ , which are respectively the smallest, largest, and *sum of square gaps* of the elements in the subtree rooted at  $v$ . Indicate how  $v.ssg$  can be computed when  $v.min$ ,  $v.max$ , and  $v.ssg$  are known for the two children  $v.l$  and  $v.r$  (it can be assumed that both exist).



- $v.ssg = v.l.ssg + v.r.ssg$   A  
 $v.ssg = v.l.ssg + (v.r.min - v.l.max)^2 + v.r.ssg$   B  
 $v.ssg = v.l.ssg + (v.x - v.l.max)^2 + (v.x - v.r.min)^2 + v.r.ssg$   C  
 $v.ssg = v.l.ssg + (v.r.x - v.l.x)^2 + v.r.ssg$   D  
 $v.ssg = v.l.ssg + (v.x - v.l.x)^2 + (v.x - v.r.x)^2 + v.r.ssg$   E

## Miscellaneous questions

### Problem 284 (4 %)

For each of the following algorithms, give the best-case, worst-case, and expected running time on input of size  $n$ , where the input may contain identical elements.

	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(2^n)$	
Worst-case time for INSERTION-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	284.1
Worst-case time for MERGE-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.2
Worst-case time for HEAPSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.3
Worst-case time for QUICKSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	284.4
Best-case time for INSERTION-SORT	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.5
Best-case time for MERGE-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.6
Best-case time for HEAPSORT	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.7
Best-case time for QUICKSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.8
Expected time for INSERTION-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	284.9
Expected time for MERGE-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.10
Expected time for HEAPSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.11
Expected time for QUICKSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	284.12

### Problem 285 (4 %)

Give the worst-case time for HEAPSORT on an array of  $n$  identical elements.

$\Theta(\sqrt{n})$     $\Theta(n)$     $\Theta(n \log n)$     $\Theta(n\sqrt{n})$     $\Theta(n^2)$

A    B    C    D    E

### Problem 286 (4 %)

Give the number of times the *largest element* in an array of  $n$  elements can be compared with other elements in the worst case during the execution of MERGE-SORT.

$\Theta(1)$     $\Theta(\log n)$     $\Theta(\sqrt{n})$     $\Theta(n)$     $\Theta(n \log n)$

A    B    C    D    E

**Problem 287 (4 %)**

Given a sorted array  $A[1..n]$  ( $A[1] < A[2] < \dots < A[n]$ ) and an element  $x$ , we want to find the index  $\ell$  such that  $A[\ell] \leq x < A[\ell + 1]$ . It is assumed that  $A[1] \leq x < A[n]$ . Which of the following algorithms is correct (only lines 2 and 4 vary in the algorithms).

$\ell = 1, h = n + 1$ <b>while</b> $\ell < h$ $m = \lfloor (h + \ell) / 2 \rfloor$ <b>if</b> $A[m] > x$ $\ell = m$ <b>else</b> $h = m$	$\ell = 1, h = n + 1$ <b>while</b> $\ell + 1 < h$ $m = \lfloor (h + \ell) / 2 \rfloor$ <b>if</b> $A[m] > x$ $\ell = m$ <b>else</b> $h = m$	$\ell = 1, h = n + 1$ <b>while</b> $\ell < h$ $m = \lfloor (h + \ell) / 2 \rfloor$ <b>if</b> $A[m] \leq x$ $\ell = m$ <b>else</b> $h = m$	$\ell = 1, h = n + 1$ <b>while</b> $\ell + 1 < h$ $m = \lfloor (h + \ell) / 2 \rfloor$ <b>if</b> $A[m] \leq x$ $\ell = m$ <b>else</b> $h = m$
<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D

**Problem 288 (4 %)**

Strassen's algorithm for multiplication of square  $n \times n$  matrices is a divide-and-conquer algorithm. Indicate which recurrence relation describes the execution time of Strassen's algorithm.

$$T(n) \leq 7 \cdot T(n/2) + c \cdot n \quad \text{A}$$

$$T(n) \leq 7 \cdot T(n/4) + c \cdot n^2 \quad \text{B}$$

$$T(n) \leq 7 \cdot T(n/2) + c \cdot n^2 \quad \text{C}$$

$$T(n) \leq 7 \cdot T(n/2) + c \cdot n^3 \quad \text{D}$$

**Problem 289 (4 %)**

Give the worst-case execution time for each of the following algorithms when the input is an array of size  $n$ .

	$\Theta(1)$	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^3)$	
BINARY-SEARCH	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	289.1
INSERTION-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	289.2
MERGE-SORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	289.3
HEAPSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	289.4
QUICKSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G	289.5
PARTITION	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	<input type="checkbox"/> F	<input type="checkbox"/> G	289.6

**Problem 290 (4 %)**

Given an array of size  $n$  containing the numbers  $1, 2, \dots, n$  in ascending order, what is the worst-case time for the following sorting algorithms when applied to the array?

	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	
INSERTION-SORT	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	290.1
HEAPSORT	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	290.2
MERGE-SORT	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	290.3
QUICKSORT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	290.4

**Problem 291 (4 %)**

The selection algorithm to find the  $i$ th smallest element in an unsorted list in worst-case time  $O(n)$  [CLRS, Chapter 9.3], divides the input into groups of 5 elements, recursively finds the median of the groups' medians, uses the found element as a pivot for a partition, and recursively calls on one of the two parts. The execution time can be described by the following recurrence relation (ignoring rounding):

$$T(n) = T(n/5) + T(7n/10) + c \cdot n$$

What will the execution time be if you change the groups in the algorithm to have size 3 instead of 5?

$$\Theta(\log n) \quad \Theta(n) \quad \Theta(n \log n) \quad \Theta(n^2)$$

A       B       C       D

**Problem 292 (4 %)**

For a binary max-heap of size  $n$ , give the best-case and worst-case execution time for the following operations.

	$\Theta(1)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n \log n)$	
INSERT, worst-case	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	292.1
INSERT, best-case	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	292.2
HEAP-EXTRACT-MAX, worst-case	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	292.3
HEAP-EXTRACT-MAX, best-case	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	292.4
BUILD-MAX-HEAP, worst-case	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	292.5
BUILD-MAX-HEAP, best-case	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	292.6

**Problem 293 (4 %)**

For QUICKSORT on input of size  $n$ , and a given element  $e$  in the input, how many comparisons will this element  $e$  be involved in during the execution of QuickSort? The expected number of comparisons here is the expected number of comparisons for a random permutation of the input.

	$\Theta(1)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	
Worst-case number of comparisons	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	293.1
Best-case number of comparisons	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	293.2
Expected number of comparisons	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	293.3

**Problem 294 (4 %)**

Which of the following statements are true for all binary search trees with  $n$  elements.

	Yes	No	
The element in a leaf is always $\leq$ the element in the root	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	294.1
Elements at the same depth in the tree are sorted from left to right	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	294.2
Longest root-to-leaf path contains $O(\log n)$ nodes	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	294.3
The right child of a node with rank $r$ has rank $r + 1$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	294.4
Along the path from the root to the largest element, the elements are increasing	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	294.5

**Problem 295 (4 %)**

Consider an  $n \times n$  matrix  $M$  of integers, where all rows and columns are sorted, i.e.,  $M[i, j] \leq M[i', j']$  for all  $1 \leq i \leq i' \leq n$  and  $1 \leq j \leq j' \leq n$ .

What is the best worst-case time one can achieve to search for an integer in  $M$ ?

$O(\log n)$     $O(n)$     $O(n \log n)$     $O(n^2)$     $O(n\sqrt{n})$     $O(\sqrt{n})$     $O((\log n)^2)$

A    B    C    D    E    F    G

**Problem 296 (4 %)**

Consider a queue implemented in a circular array  $Q$ , where  $Q.head$  indicates the head of the queue and  $Q.tail$  is the next free space in the queue (as described in [CLRS]).

Which of the following expressions correctly computes the size of the queue?

	Yes	No	
$Q.tail - Q.head$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	296.1
$ Q.tail - Q.head $	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	296.2
$ Q.tail - Q.head  \bmod Q.length$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	296.3
$(Q.tail - Q.head + Q.length - 1) \bmod Q.length$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	296.4
$(Q.tail - Q.head + Q.length) \bmod Q.length$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	296.5

**Problem 297 (4 %)**

Consider an arbitrary binary search tree, and let  $k$  be an element in a leaf of the search tree. Let  $B$  be all elements on the path from the root down to  $k$ , and let  $A$  be all elements to the left of the path, and  $C$  all elements to the right of the path. Does the following statement always hold for all  $a \in A$ ,  $b \in B$  and  $c \in C$ ?

	Yes	No	
$a \leq b$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	297.1
$b \leq c$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	297.2
$a \leq c$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	297.3

**Problem 298 (4 %)**

Given two search trees  $T_1$  and  $T_2$  containing the same  $n$  elements,  $T_1$  can be transformed into  $T_2$  by a series of rotations. What is the worst-case number of rotations required to transform a search tree with  $n$  elements into another search tree  $T_2$  containing the same  $n$  elements?

- $\Theta(\log n)$     $\Theta(n)$     $\Theta(n \log n)$     $\Theta(n^2)$   
 A    B    C    D

**Problem 299 (4 %)**

For each of the sums below, give their value in  $\Theta$ -notation. It is assumed that  $n$  is a power of two.

	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	
$\sum_{i=1}^n i = 1 + 2 + 3 + \dots + n$	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	299.1
$\sum_{i=0}^{\log n} 2^i = 1 + 2 + 4 + 8 + \dots + n$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	299.2
$\sum_{i=1}^{\log n} i \frac{n}{2^i} = 1 \frac{n}{2^1} + 2 \frac{n}{2^2} + 3 \frac{n}{2^3} + \dots + \log n \frac{n}{2^{\log n}}$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	299.3
$\sum_{i=1}^n \log i = \log 1 + \log 2 + \log 3 + \dots + \log n$	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	299.4
$\sum_{i=0}^{\log n} \frac{n}{2^i} = n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \dots + \frac{n}{n}$	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	299.5

**Problem 300 (4 %)**

Consider the variant of the binary max-heap where each node has  $d$  children instead of two, where  $d \geq 2$  is a parameter. What is the time for INSERT and EXTRACT-MAX expressed as a function of  $n$  and  $d$ .

	$\Theta(d)$	$\Theta(\log_2 n)$	$\Theta(\log_d n)$	$\Theta(d \log_2 n)$	$\Theta(d \log_d n)$	
INSERT	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	300.1
EXTRACT-MAX	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input checked="" type="checkbox"/> E	300.2

**Problem 301 (4 %)**

Consider a binary max-heap with  $n = 2^k - 1$  different elements, for an integer  $k \geq 1$ . How many different nodes in the max-heap can the smallest element be located in?

- 1    $n$     $k$     $k - 1$     $2^{k-1}$     $2^k - 1$   
 A    B    C    D    E    F

**Problem 302 (4 %)**

How many times is RANDOMIZED-PARTITION called in RANDOMIZED-SELECT on an array of size  $n$ ?

	$\Theta(1)$	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	
Expected number of calls	<input type="checkbox"/> A	<input checked="" type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	302.1
Fewest number of calls (best case)	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	302.2
Most number of calls (worst case)	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E	302.3

**Problem 303 (4 %)**

Let  $v$  be a node in a red-black search tree, and assume there are  $n$  elements in  $v$ 's left subtree. How many elements can there be at most in  $v$ 's right subtree, i.e., how unbalanced can a node  $v$  be in a red-black search tree?

	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n\sqrt{n})$	$\Theta(n^2)$	$\Theta(2^n)$
	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input checked="" type="checkbox"/> D	<input type="checkbox"/> E

**Problem 304 (4 %)**

Assume an array  $A[1..n]$  represents a binary max-heap containing  $n$  elements. How quickly can one construct a search tree (not necessarily balanced) containing the elements  $A[1..n]$ ?

	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$
	<input type="checkbox"/> A	<input type="checkbox"/> B	<input checked="" type="checkbox"/> C	<input type="checkbox"/> D

**Problem 305 (4 %)**

Assume MERGE-SORT is performed on an input of size  $n$  and containing two elements  $x$  and  $y$ . What is the worst-case number of comparisons of  $x$  with  $y$  during the execution of MERGE-SORT?

	$\Theta(1)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$
	<input checked="" type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E

**Problem 306 (4 %)**

Assume we have a directed graph with  $n$  nodes and positively weighted edges, where we continuously add additional edges. We want to maintain a distance table of the shortest distances between all pairs of nodes.

What is the best worst-case time we can achieve to update the distance table, when adding a new edge with positive weight to the graph?

	$\Theta(1)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \cdot \log n)$	$\Theta(n \cdot \sqrt{n})$	$\Theta(n^2)$	$\Theta(n^3)$
	<input type="checkbox"/> A	<input type="checkbox"/> B	<input type="checkbox"/> C	<input type="checkbox"/> D	<input type="checkbox"/> E	<input checked="" type="checkbox"/> F	<input type="checkbox"/> G