Algoritmer og Datastrukturer

Stakke, køer
[CLRS, kapitel 10]

[CLRS, Del 3] : Datastrukturer

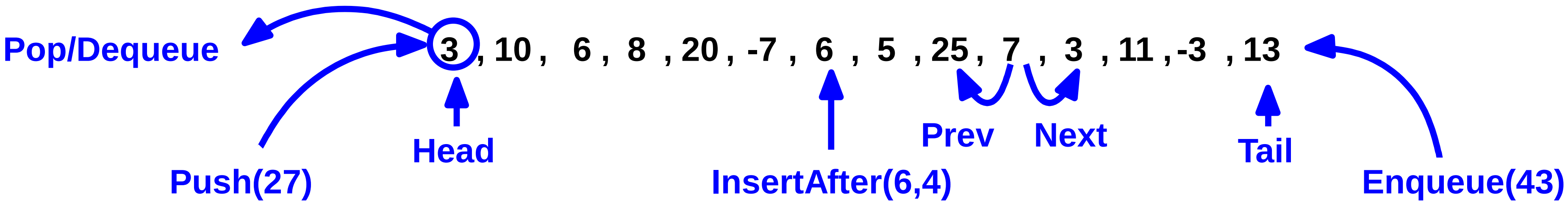
OpretHold en struktur for en **dynamisk** mængde data

Abstrakte Datastrukturer for Mængder

			-Min-prioritetskø	-Max-prioritetskø	- Ordbog
Forespørgsel	Minimum(S)	pointer til element	●		
	Maximum(S)	pointer til element		●	
	Search(S, x)	pointer til element			●
	Member(S, x)	TRUE eller FALSE			
	Successor(S, x)	pointer til element			
	Predecessor(S, x)	pointer til element			
Opdateringer	Insert(S, x)	pointer til element	●	●	●
	Delete(S, x)	-			●
	DeleteMin(S)	element	●		
	DeleteMax(S)	element		●	
	Join(S_1, S_2)	mængde S			
	Split(S, x)	mængder S_1 og S_2			

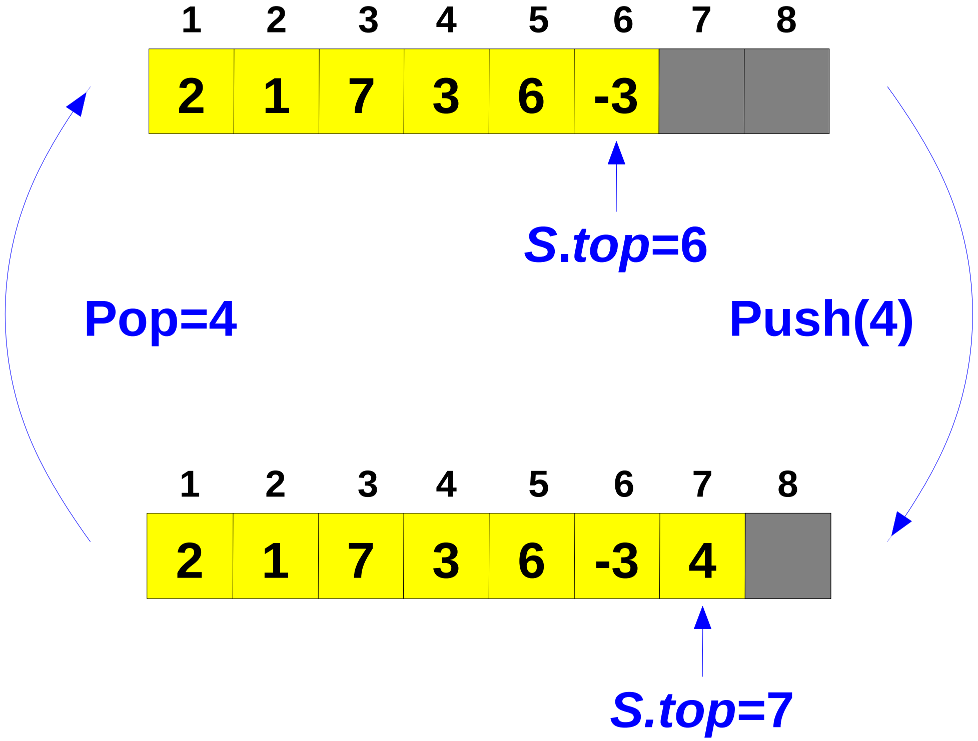
Abstrakte Datastrukturer for Lister

			-Stak	-Kø
Forespørgsel	**Empty(S)**	TRUE eller FALSE	●	●
	Head(S), Tail(S)	pointer til element		
	Next(S, x), Prev(S, x)	pointer til element		
	Search(S, x)	pointer til element		
Opdateringer	**Push(S, x)**	-	●	
	Pop/Dequeue(S)	element	●	●
	Enqueue(S, x)	-		●
	Delete(S, x)	Element		
	InsertAfter(S, x, y)	pointer til element		



Stak

Stak : Array Implementation



STACK-EMPTY(S)

```
if S.top == 0
    return TRUE
else return FALSE
```

PUSH(S, x)

```
S.top = S.top + 1
S[S.top] = x
```

POP(S)

```
if STACK-EMPTY(S)
    error "underflow"
else S.top = S.top - 1
    return S[S.top + 1]
```

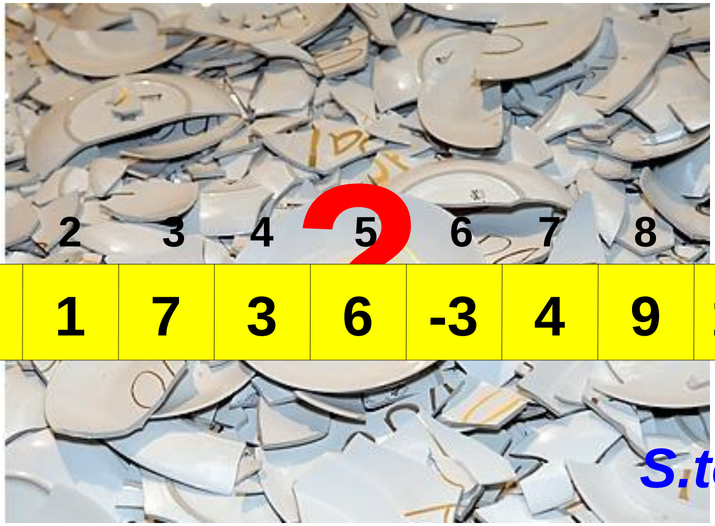
Stack-Empty, Push, Pop : $O(1)$ **tid**

Stak : Overløb

1	2	3	4	5	6	7	8
2	1	7	3	6	-3	4	9

S.top=8

Push(13)



?

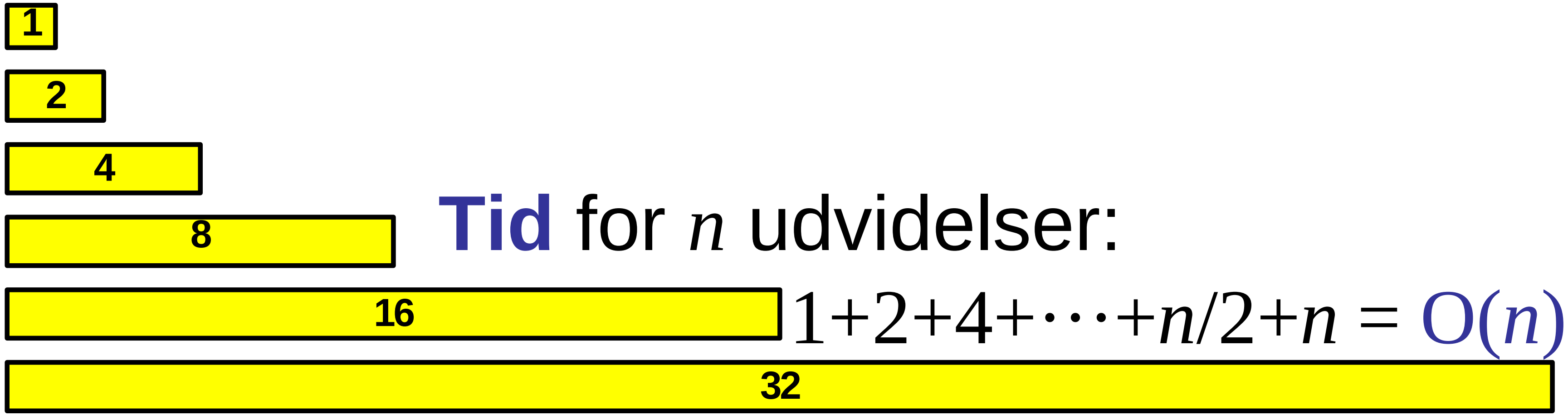
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	7	3	6	-3	4	9	13							

S.top=9

Array fordobling : $O(n)$ tid

Array Fordobling

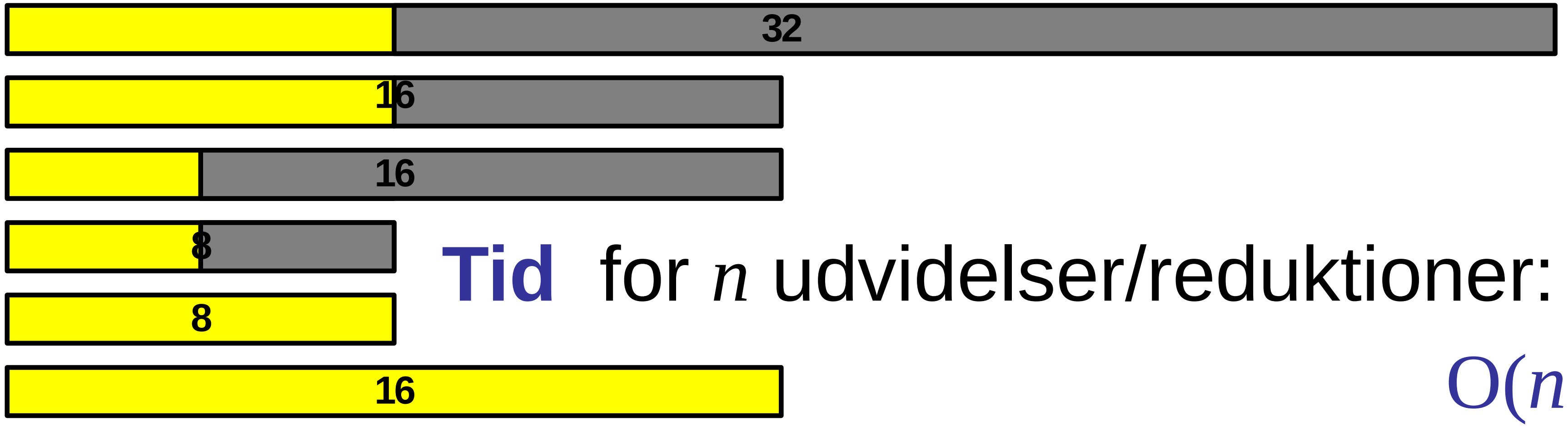
Fordoble arrayet når det er fuld



Tid for n udvidelser:

$$1+2+4+\cdots+n/2+n = O(n)$$

Halver arrayet når det er <1/4 fyldt



Tid for n udvidelser/reduktioner:

$$O(n)$$

Array Fordobling + Halvering

– en generel teknik

Tid for n udvidelser/reduktioner er $O(n)$

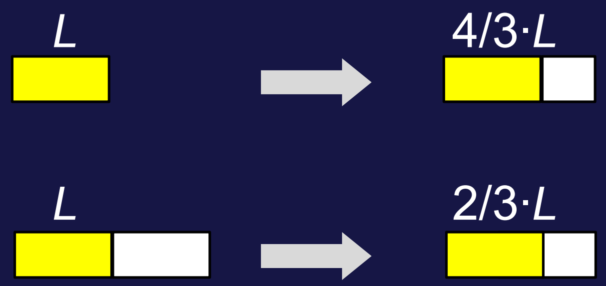
Plads $\leq$ $4 \cdot$ aktuelle antal elementer

Array implementation af Stak:

n push og pop operationer tager $O(n)$ tid

4/3 Forøgelse

Ved overløb lad den nye længde være 4/3 af den oprindelige, og når arrayet bliver halv fuldt reducer det til 2/3 længde.



Hvor langt kan arrayet maksimalt være når det gemmer *n* elementer ?

a) $3/2 \cdot n$
b) $4/3 \cdot n$

c) $2 \cdot n$
d) $8/3 \cdot n$
e) ved ikke

Ikke pensum

Reallokeringsstrategier i Java, Python og C++

Sprog	Java ArrayList (JDK SE 12.0.1)	Python list (CPython 3.7)	C++ vector (GCC 9.1)
Ved overløb	**+ 50 %**	**+ 12.5 %**	**+ 100 %**
Formindskelse	Nej (kan kalde trimToSize)	< 50 %	Nej (kan kalde shrink_to_fit)
Dokumentation	https://docs.oracle.com/javase/10/docs/api/java/util/ArrayList.html	https://docs.python.org/3/library/stdtypes.html#sequence-types-list-tuple-range	http://www.cplusplus.com/reference/vector/vector/
Kildekode	Installer JDK fra www.oracle.com/technetwork/java/javase/downloads/ Fil java.base\java\util\ArrayList.java fra C:\Program Files\Java\jdk-12.0.1\lib\src.zip	github.com/python/cpython/blob/master/Objects/listobject.c	github.com/gcc-mirror/gcc/blob/master/libstdc++-v3/include/bits/stl_vector.h

Java ArrayList

```
private int newCapacity(int minCapacity) {
    // overflow-conscious code
    int oldCapacity = elementData.length;
    int newCapacity = oldCapacity + (oldCapacity >> 1);
    if (newCapacity - minCapacity <= 0) {
        if (elementData == DEFAULTCAPACITY_EMPTY_ELEMENTDATA)
            return Math.max(DEFAULT_CAPACITY, minCapacity);
        if (minCapacity < 0) // overflow
            throw new OutOfMemoryError();
        return minCapacity;
    }
    return (newCapacity - MAX_ARRAY_SIZE <= 0)
        ? newCapacity
        : hugeCapacity(minCapacity);
}
```

Python list

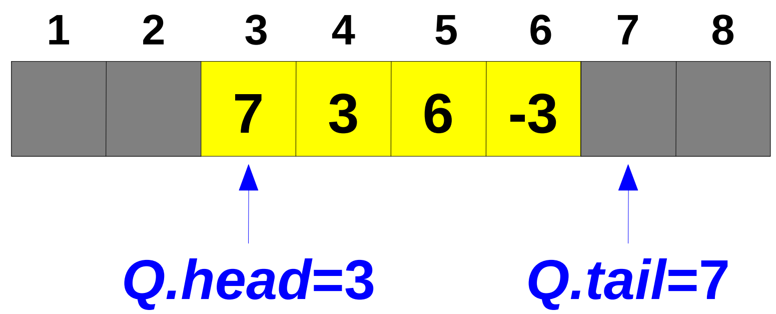
```
static int
list_resize(PyListObject *self, Py_ssize_t newsize)
{
    PyObject **items;
    size_t new_allocated, num_allocated_bytes;
    Py_ssize_t allocated = self->allocated;
    if (allocated >= newsize && newsize >= (allocated >> 1)) {
        assert(self->ob_item != NULL || newsize == 0);
        Py_SIZE(self) = newsize;
        return 0;
    }
    new_allocated =
      (size_t)newsize + (newsize >> 3) + (newsize < 9 ? 3 : 6);
    …
}
```

C++ vector

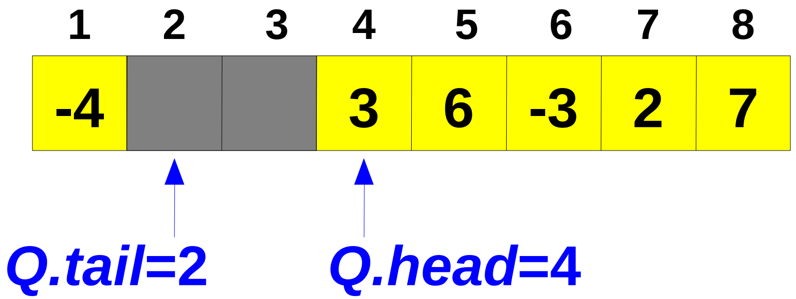
```
size_type
_M_check_len(size_type __n, const char* __s) const
{
    if (max_size() - size() < __n)
        __throw_length_error(__N(__s));

   const size_type __len = size() + (std::max)(size(), __n);
   return (__len < size() || __len > max_size())
       ? max_size()
       : __len;
}
```

Kø

Kø : Array Implementation



Enqueue(2)
Enqueue(7)
Engueue(-4)
Dequeue = 7



ENQUEUE(Q, x)

```
Q[Q.tail] = x
if Q.tail == Q.length
    Q.tail = 1
else Q.tail = Q.tail + 1
```

DEQUEUE(Q)

```
x = Q[Q.head]
if Q.head == Q.length
    Q.head = 1
else Q.head = Q.head + 1
return x
```

Enqueue, dequeue : $O(1)$ **tid**

Maximal kapacitet af en kø implementeret i et array af længde *n* ?

a) *n*


b) *n* – 1

c) *n* – 2

d) *n* / 2

e) ved ikke

Kø : Array Implementation

1	2	3	4	5	6	7	8
-4	5		3	6	-3	2	7

***Q.tail*=3** ***Q.head*=4**

Empty : *Q.tail*=*Q.head* ?

Overløb : array fordobling/ halvering

Enqueue(9)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3	6	-3	2	7	-4	5	9								

***Q.head*=1** ***Q.tail*=9**

Array implementation af Kø:

n **enqueue og dequeue operationer tager** $O(n)$ **tid**

Arrays (med Fordobling/Halvering)

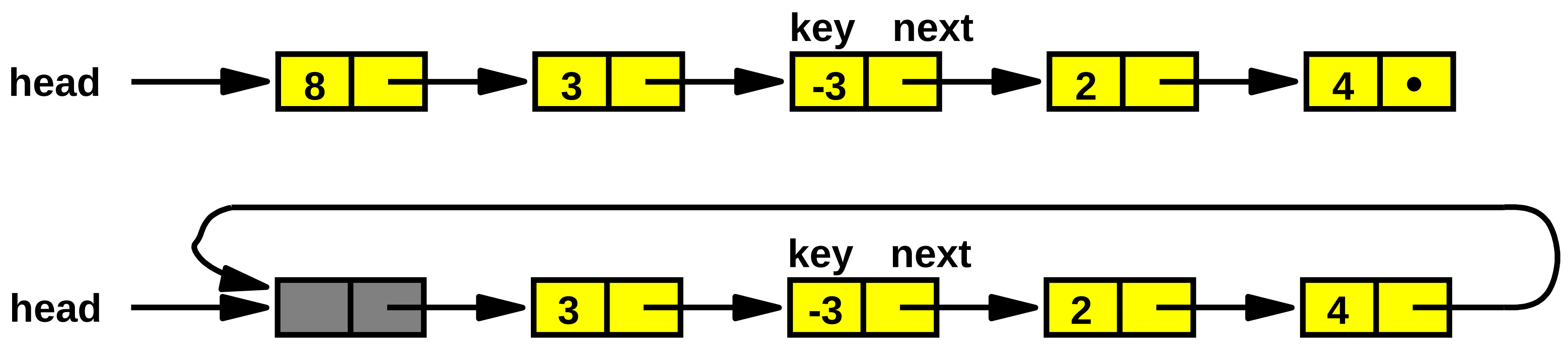
Stak	**Push(*S*, *x*)**	O(1)*
	Pop(*S*)	O(1)*
Kø	**Enqueue(*S*, *x*)**	O(1)*
	Dequeue(*S*)	O(1)*

*** Worst-case uden fordobling/halvering**
Amortiseret ([CLRS, Kap. 17]) med fordobling/halvering

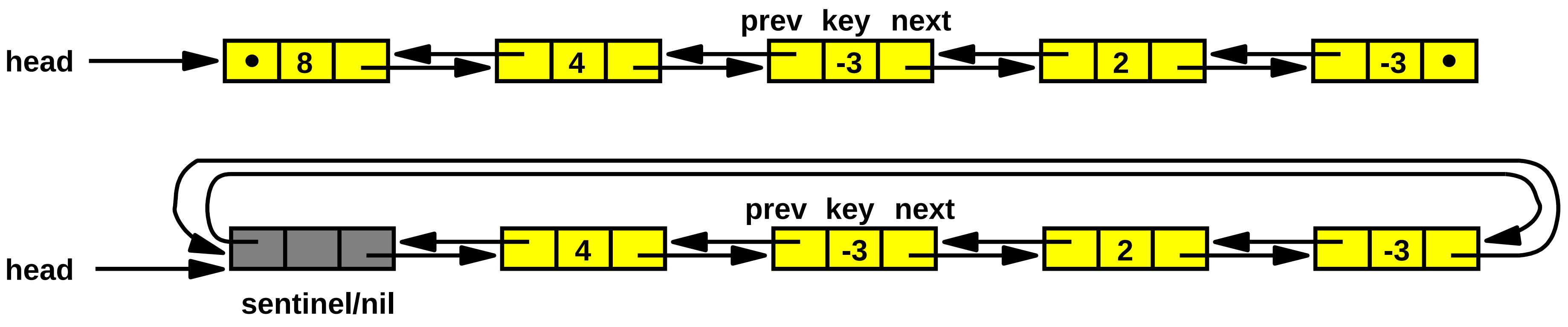
Kædede lister

Kædede Lister

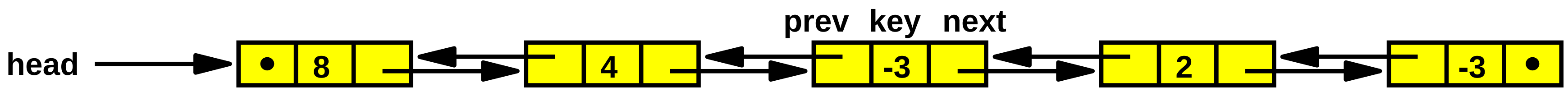
Enkelt kædede (ikke-cyklisk og cyklisk)



Dobbelt kædede (ikke-cyklisk og cyklisk)



Dobbelt Kædede Lister



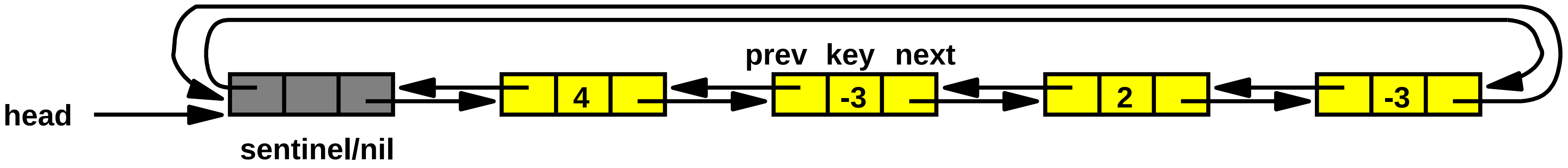

```
LIST-SEARCH(L, k)
x = L.head
while x ≠ NIL and x.key ≠ k
    x = x.next
return x
```

```
LIST-INSERT(L, x)
x.next = L.head
if L.head ≠ NIL
    L.head.prev = x
L.head = x
x.prev = NIL
```

```
LIST-DELETE(L, x)
if x.prev ≠ NIL
    x.prev.next = x.next
else L.head = x.next
if x.next ≠ NIL
    x.next.prev = x.prev
```

List-Search	$O(n)$
List-Insert	$O(1)$
List-Delete	$O(1)$

Dobbelt Kædede Cykliske Lister



LIST-SEARCH′(L, k)

```
x = L.nil.next
while x ≠ L.nil and x.key ≠ k
      x = x.next
return x
```

LIST-INSERT′(L, x)

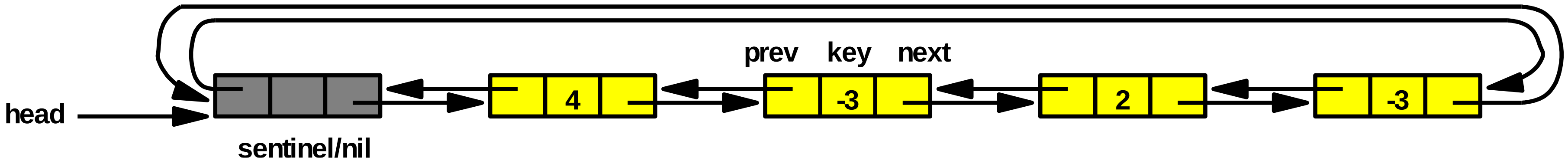
```
x.next = L.nil.next
L.nil.next.prev = x
L.nil.next = x
x.prev = L.nil
```

LIST-DELETE′(L, x)

```
x.prev.next = x.next
x.next.prev = x.prev
```

List-Search’	$O(n)$
List-Insert’	$O(1)$
List-Delete’	$O(1)$

Dobbelt Kædede Cykliske Lister



Stak	**Push(S, x)**	O(1)
	Pop(S)	O(1)
Kø	**Enqueue(S, x)**	O(1)
	Dequeue(S)	O(1)

Dancing Links

Donald E. Knuth, Stanford University

My purpose is to discuss an extremely simple technique that deserves to be better known. Suppose x points to an element of a doubly linked list; let $L[x]$ and $R[x]$ point to the predecessor and successor of that element. Then the operations

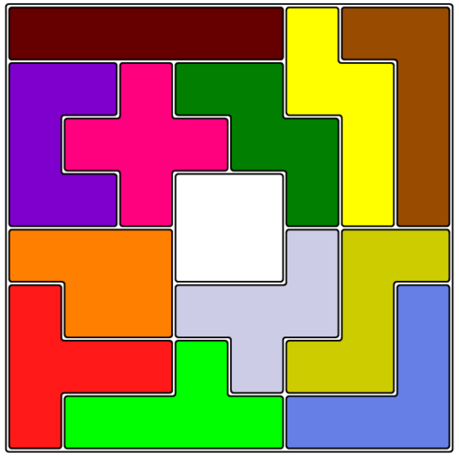
$$L\big[R[x]\big] \leftarrow L[x], \qquad R\big[L[x]\big] \leftarrow R[x] \tag{1}$$

remove x from the list; every programmer knows this. But comparatively few programmers have realized that the subsequent operations

$$L\big[R[x]\big] \leftarrow x, \qquad R\big[L[x]\big] \leftarrow x \tag{2}$$

will put x back into the list again.

Donald E. Knuth (1938-)



"The Challenge Puzzle"

?

”The Challenge Puzzle”

L := Tomt bræt
B := Alle brikker
Solve(*L*,*B*)

```
procedure Solve(Delløsning L, Brikker B)
   for alle b i B
      for alle orienteringer af b  (* max 8 forskellige *)
         if b kan placeres i nederste venstre fri then
             fjern b fra B
             indsæt b i L
             if |B|=0 then
               rapporter L er en løsning
             else
               Solve(L,B)
             fi
             slet b fra L
             genindsæt b i B
         fi
```

B

L

Nederste-venstre fri

Før

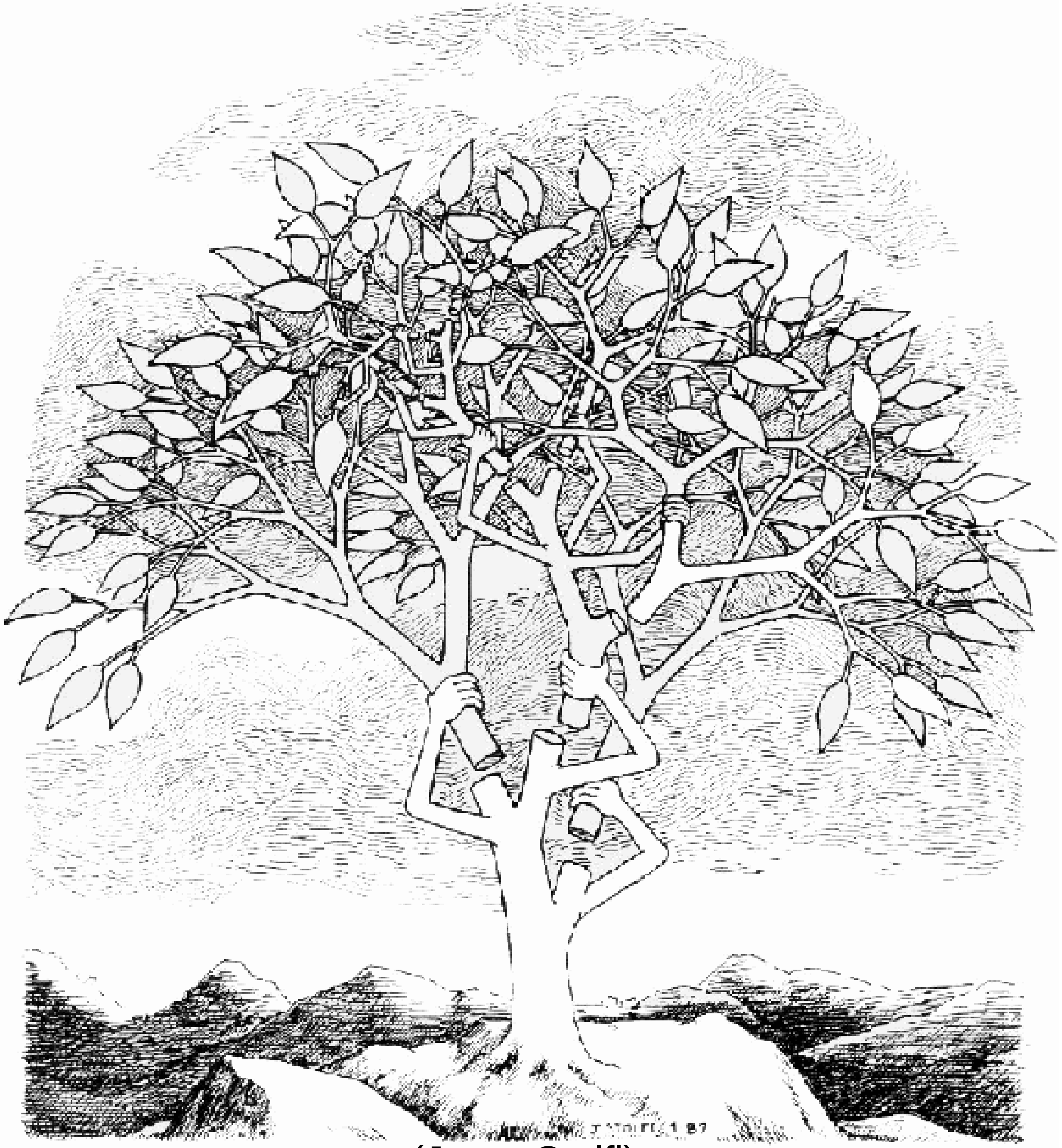
Efter

”The Challenge Puzzle”

4.040 løsninger

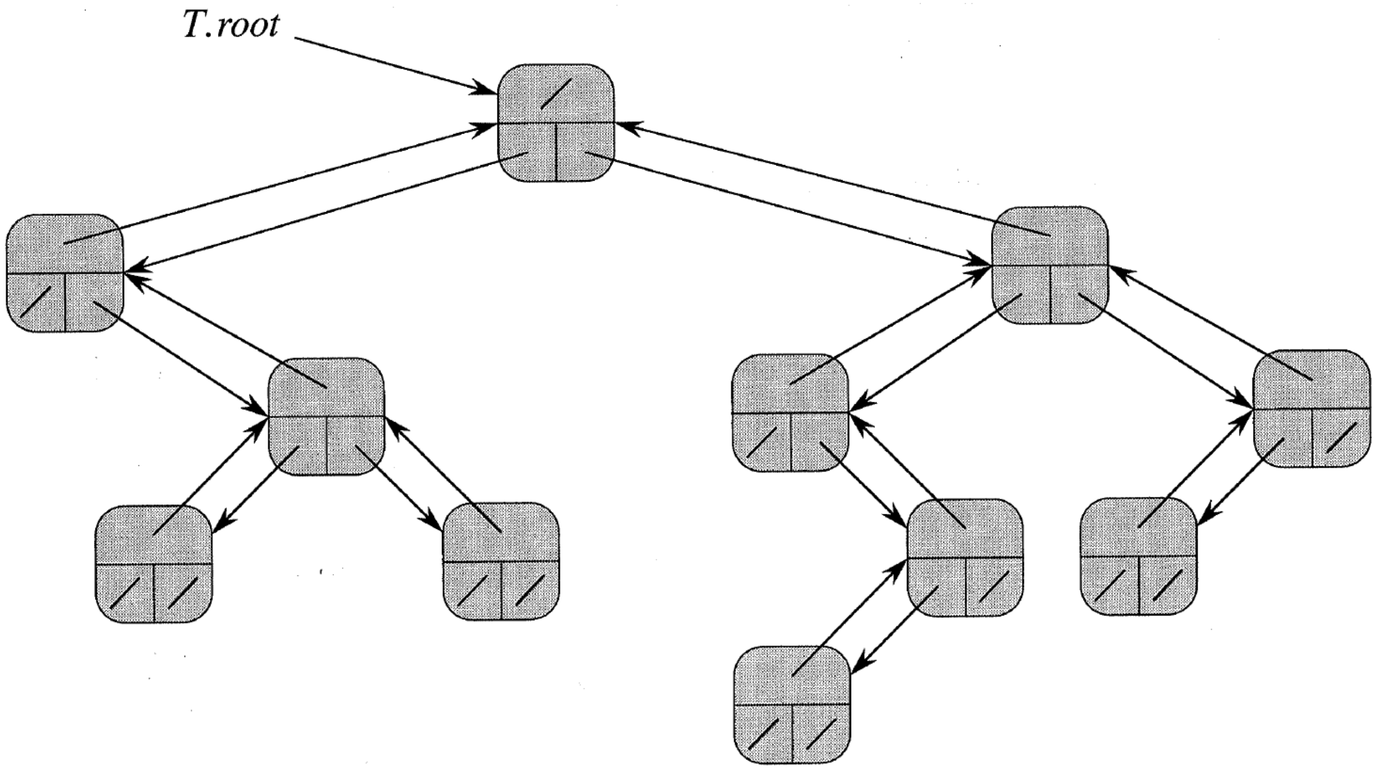
Solve placerer

8.387.259 brikker



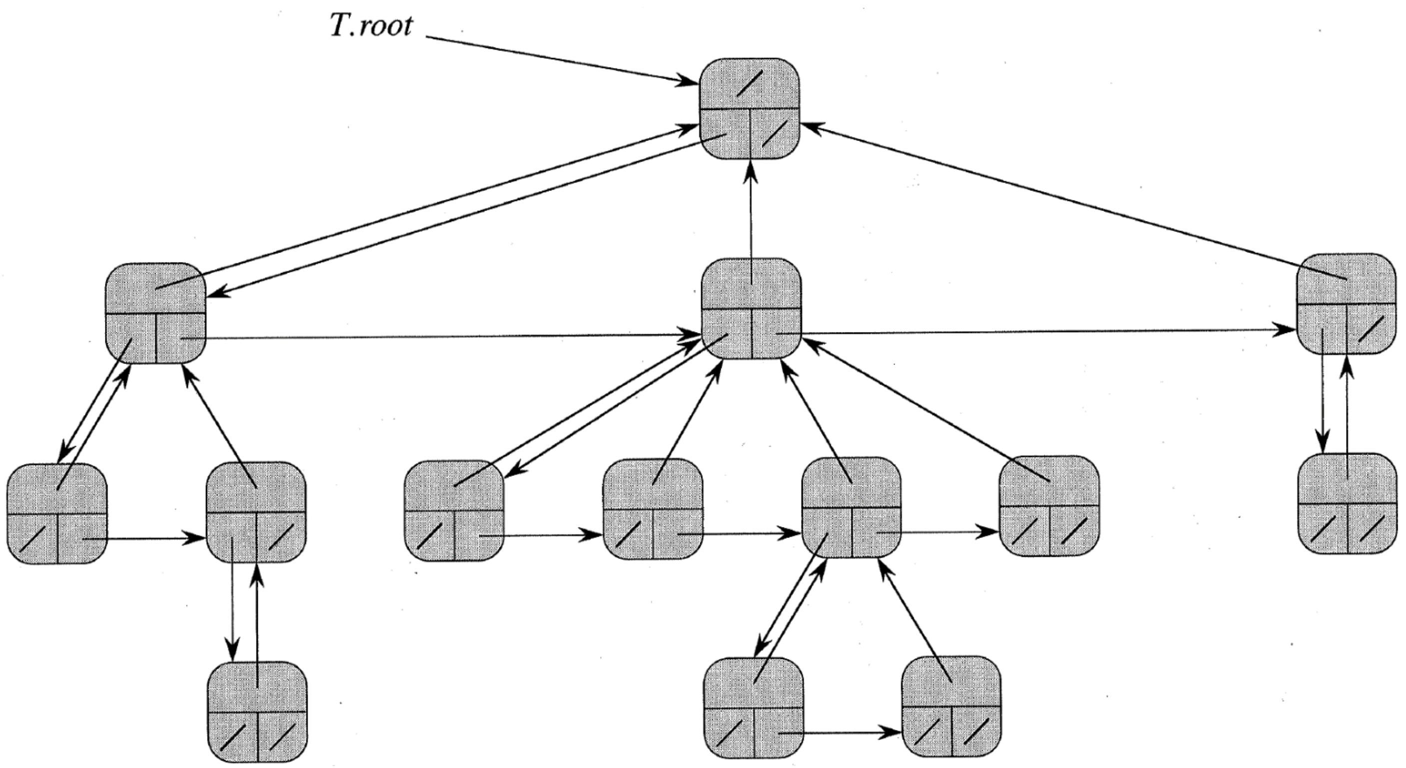
(Jorge Stolfi)

Binær Træ Repræsentation



Felter: **Left**, **right**, **parent**

Træ Repræsentation



Felter: **Left-child**, **right-sibling**, **parent**

Hvor lang tid tager det at tilgå det *i*-te barn til en knude ?

a) O(1)

b) O(i)

c) O(log i)

d) ved ikke