

Algoritmer og Datastrukturer

Heltalsaritmetik: Binære og decimal tal, addition, subtraktion, multiplikation, division

Cifre

0	1	2	3	4	5	6	7	8	9
			\	\		\	\	\	\

For hvert lille tal har man et specielt symbol

10-tals systemet (10 = 9 + 1)

Værdien af $d_{n-1}d_{n-2} \cdots d_1d_0$

$$\sum_{i=0}^{n-1} d_i \cdot 10^i = d_{n-1} \cdot 10^{d_{n-1}} + d_{n-2} \cdot 10^{d_{n-2}} + \cdots + d_1 \cdot 10^1 + d_0 \cdot 10^0$$

Eksempel: $1849_{10} = 1 \cdot 10^3 + 8 \cdot 10^2 + 4 \cdot 10^1 + 9 \cdot 10^0$

Repræsentation base b

Værdien af $d_{n-1}d_{n-2} \cdots d_1d_0$

$$\sum_{i=0}^{n-1} d_i \cdot b^i$$

Eksempel: $1849_{10} = \overset{\underline{a}}{\underline{1}}\overset{\underline{b}}{\underline{1}}\overset{\underline{c}}{\underline{1}}\overset{\underline{d}}{\underline{1}}\overset{\underline{x}}{\underline{0}}\overset{\underline{y}}{\underline{0}}\overset{\underline{z}}{\underline{0}}\overset{\underline{0}}{\underline{1}}_2 = \overset{a}{\downarrow}\overset{b}{\downarrow}\overset{c}{\downarrow}\overset{d}{\downarrow}3471_8 = \overset{x}{\downarrow}\overset{y}{\downarrow}\overset{z}{\downarrow}739_{16}$

Decimal value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary symbol	0	1														
Octal symbol	0	1	2	3	4	5	6	7								
Hexadecimal symbol	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Addition

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Addition - skolemetoden

$$\begin{array}{r} 1 \ 1 \\ 8 \ 4 \ 3 \\ + \ 5 \ 7 \ 2 \\ \hline 1 \ 4 \ 1 \ 5 \end{array}$$

$$\begin{aligned} 843 + 572 &= (8 \cdot 10^2 + 4 \cdot 10^1 + 3 \cdot 10^0) + (5 \cdot 10^2 + 7 \cdot 10^1 + 2 \cdot 10^0) \\ &= (8 + 5) \cdot 10^2 + (4 + 7) \cdot 10^1 + (3 + 2) \cdot 10^0 \\ &= (8 + 5) \cdot 10^2 + (4 + 7) \cdot 10^1 + 5 \cdot 10^0 \\ &= (8 + 5) \cdot 10^2 + 11 \cdot 10^1 + 5 \cdot 10^0 \\ &= (8 + 5) \cdot 10^2 + (1 \cdot 10 + 1) \cdot 10^1 + 5 \cdot 10^0 \\ &= (8 + 5) \cdot 10^2 + (1 \cdot 10^2 + 1 \cdot 10^1) + 5 \cdot 10^0 \\ &= ((8 + 5) + 1) \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\ &= (13 + 1) \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\ &= 14 \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\ &= (1 \cdot 10 + 4) \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\ &= 1 \cdot 10^3 + 4 \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\ &= 1415 \end{aligned}$$

Binær addition

+	0	1
0	0	1
1	1	10

$$\begin{array}{r} \\ \\ \\ + \\ \hline 1 \end{array}$$

Skolemetoden

Subtraktion - skolemetoden

$$\begin{array}{r} \\ \\ \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \\ \\ \\ \hline \end{array}$$

$4 - 6 = -2 = -10 + 8$

Multiplikation

.	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Multiplikation

$$\begin{array}{r} 2 \ 1 \ 4 \\ 6 \cdot 4 \ 2 \ 7 \\ \hline 2 \ 5 \ 6 \ 2 \end{array}$$

Multiplikation med
enkelt ciffer

$$\begin{array}{r} 3 \ 6 \ 5 \cdot 4 \ 2 \ 7 \\ \hline 1 \ 2 \ 8 \ 1 \\ 2 \ 5 \ 6 \ 2 \\ 2 \ 1 \ 3 \ 5 \\ \hline 1 \ 5 \ 5 \ 8 \ 5 \ 5 \end{array}$$

$$\begin{aligned} 365 \cdot 427 &= (3 \cdot 10^2 + 6 \cdot 10^1 + 5 \cdot 10^0) \cdot 427 \\ &= 3 \cdot 427 \cdot 10^2 + 6 \cdot 427 \cdot 10^1 + 5 \cdot 427 \cdot 10^0 \\ &= 1281 \cdot 10^2 + 2562 \cdot 10^1 + 2135 \cdot 10^0 \\ &= 128100 + 25620 + 2135 \\ &= 155855 \end{aligned}$$

Binær multiplikation

.	0	1
0	0	0
1	0	1

Binær multiplikation

$$\begin{array}{r} 1011_2 \cdot 1010_2 \\ \hline 1010_2 \\ 0000_2 \\ 1010_2 \\ 1010_2 \\ 1010_2 \\ \hline 1111001_2 \end{array}$$

The diagram illustrates the binary multiplication of 1011_2 and 1010_2 . The multiplicand 1011_2 is aligned under the multiplier 1010_2 . A horizontal line is drawn under the multiplier. The partial products are shown as follows:

- The first partial product is 1010_2 , obtained by multiplying 1011_2 by the least significant bit of the multiplier (0).
- The second partial product is 0000_2 , obtained by multiplying 1011_2 by the second bit of the multiplier (1).
- The third partial product is 1010_2 , obtained by multiplying 1011_2 by the third bit of the multiplier (0).
- The fourth partial product is 1010_2 , obtained by multiplying 1011_2 by the fourth bit of the multiplier (1).
- The fifth partial product is 1010_2 , obtained by multiplying 1011_2 by the fifth bit of the multiplier (1).

The final result is 1111001_2 , obtained by summing all the partial products.

Binær multiplikation af blokke med 1

$$\overbrace{11111}^j \overbrace{00000}^i {}_2 = 1 \overbrace{0000000000}^{i+j} {}_2 - 1 \overbrace{00000}^i {}_2$$

$$\begin{array}{r}
 1 \ 0 \ \overbrace{1 \ 1 \ 1}_2 \cdot 1 \ 0 \ 1 \ 0 \ 1_2 \\
 \hline
 \ 1 \ 0 \ 1 \ 0 \ 1_2 \\
 \left\{ \begin{array}{l} + \\ - \end{array} \right. \ 1 \ 0 \ 1 \ 0 \ 1_2 \\
 \ 1 \ 0 \ 1 \ 0 \ 1_2 \\
 \hline
 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1_2
 \end{array}$$

Division

$$\begin{array}{r} \overbrace{111110100}_x \text{ }_2 / \overbrace{10101}_y \text{ }_2 = \overbrace{10111}_i \text{ }_2 \\ \underline{111110100} \\ - 10101 \text{ }_2 \\ \underline{10100100} \text{ }_2 \\ - 00000 \text{ }_2 \\ \underline{10100100} \\ - 10101 \text{ }_2 \\ \underline{10100000} \\ - 10101 \text{ }_2 \\ \underline{100110} \\ - 10101 \text{ }_2 \\ \underline{10001} \text{ }_2 \end{array}$$

The diagram illustrates the long division of $x = 111110100_2$ by $y = 10101_2$. The quotient is $i = 10111_2$ and the remainder is $r = 10001_2$. Red annotations highlight the current step: a red bracket labeled x is above the dividend, a red bracket labeled y is above the divisor, and a red bracket labeled i is above the quotient. A red p is placed below the first subtraction step, and a red r is placed below the final remainder. Dashed lines and arrows connect the p and r to the corresponding bits in the quotient and remainder.

Algorithm INTEGERDIVISION(x, y)

Input Integers $x \geq 0$ and $y \geq 1$

Output Integer $i = \lfloor x/y \rfloor$, i.e. $0 \leq x - i \cdot y < y$

```
1   $p = 0$ 
2  while  $y \cdot 2^{p+1} \leq x$ 
3       $p = p + 1$ 
4   $i = 0$ 
5   $r = x$ 
6  # Invariant:  $x = i \cdot y + r$  and  $r < y \cdot 2^{p+1}$ 
7  while  $y \leq r$ 
8      if  $y \cdot 2^p \leq r$ 
9           $i = i + 2^p$  # set position  $p$  in  $i = 1$ 
10          $r = r - y \cdot 2^p$ 
11      $p = p - 1$ 
12 return  $i$ 
```

Konvertering til base b

Algorithm BASEREPRESENTATION(x, b)

Input Integers $x \geq 0$ and base $b \geq 2$

Output Digits d_0, d_1, \dots of the b -ary representation of x

1 $p = 0$

2 **while** $x > 0$ **do**

3 $i = \lfloor x/b \rfloor$ 

4 $d_p = x - i \cdot b$

5 $x = i$

6 $p = p + 1$