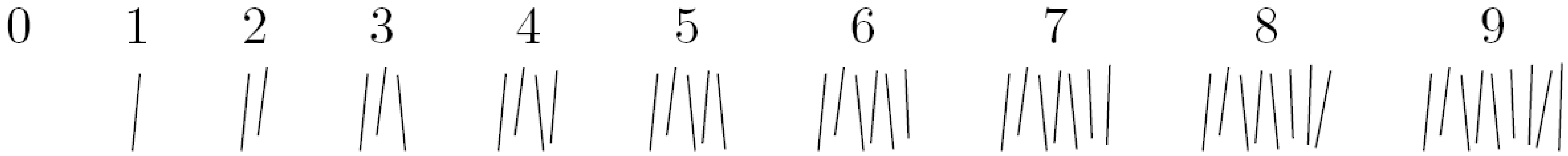
Algoritmer og Datastrukturer

Heltalsaritmetik: Binære og decimal tal, addition, subtraktion, multiplikation, division

Cifre



For hvert lille tal har man et specielt symbol

10-tals systemet (10 = 9 + 1)

Værdien af $d_{n-1}d_{n-2}\cdots d_1d_0$

$$\sum_{i=0}^{n-1} d_i \cdot 10^i = d_{n-1} \cdot 10^{d_{n-1}} + d_{n-2} \cdot 10^{d_{n-2}} + \cdots + d_1 \cdot 10^1 + d_0 \cdot 10^0$$

Eksempel: $1849_{10} = 1 \cdot 10^3 + 8 \cdot 10^2 + 4 \cdot 10^1 + 9 \cdot 10^0$

Repræsentation base *b*

Værdien af $d_{n-1} d_{n-2} \cdots d_1 d_0$

$$\sum_{i=0}^{n-1} d_i \cdot b^i$$

Eksempel: $1849_{10} = 11100111001_2 = 3471_8 = 739_{16}$

Decimal value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary symbol	0	1														
Octal symbol	0	1	2	3	4	5	6	7								
Hexadecimal symbol	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Addition

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Addition - skolemetoden

$$
\begin{array}{rcccc}
 & 1 & 1 & & \\
 & & 8 & 4 & 3 \\
+ & & 5 & 7 & 2 \\
\hline
 & 1 & 4 & 1 & 5
\end{array}
$$

$$
\begin{aligned}
843 + 572 &= (\mathbf{8} \cdot 10^2 + \mathbf{4} \cdot 10^1 + \mathbf{3} \cdot 10^0) + (\mathbf{5} \cdot 10^2 + \mathbf{7} \cdot 10^1 + \mathbf{2} \cdot 10^0) \\
&= (\mathbf{8} + \mathbf{5}) \cdot 10^2 + (\mathbf{4} + \mathbf{7}) \cdot 10^1 + (\mathbf{3} + \mathbf{2}) \cdot 10^0 \\
&= (8 + 5) \cdot 10^2 + (4 + 7) \cdot 10^1 + \mathbf{5} \cdot 10^0 \\
&= (8 + 5) \cdot 10^2 + \mathbf{11} \cdot 10^1 + 5 \cdot 10^0 \\
&= (8 + 5) \cdot 10^2 + (\mathbf{1} \cdot 10 + \mathbf{1}) \cdot 10^1 + 5 \cdot 10^0 \\
&= (8 + 5) \cdot 10^2 + (\mathbf{1} \cdot 10^2 + \mathbf{1} \cdot 10^1) + 5 \cdot 10^0 \\
&= ((8 + 5) + \mathbf{1}) \cdot 10^2 + \mathbf{1} \cdot 10^1 + 5 \cdot 10^0 \\
&= (\mathbf{13} + 1) \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\
&= \mathbf{14} \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\
&= (\mathbf{1} \cdot 10 + \mathbf{4}) \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\
&= \mathbf{1} \cdot 10^3 + \mathbf{4} \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0 \\
&= 1415
\end{aligned}
$$

Binær addition

$$\begin{array}{c|cc} + & 0 & 1 \\ \hline 0 & 0 & 1 \\ 1 & 1 & 10 \end{array}$$

$$\begin{array}{rccccc} & & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & \\ & & 1 & 1 & 1 & 0_2 \\ + & & 1 & 0 & 1 & 1_2 \\ \hline & 1 & 1 & 0 & 0 & 1_2 \end{array}$$

Skolemetoden

Addition af flere tal

$$
\begin{array}{rcccc}
 & {\scriptstyle 3} & {\scriptstyle 3} & {\scriptstyle 2} & \\
 & & 2 & 7 & 7 \\
+ & & 9 & 5 & 5 \\
+ & & 7 & 7 & 6 \\
+ & & 4 & 9 & 1 \\
+ & 1 & 5 & 2 & 4 \\
+ & & 3 & 3 & 6 \\
\hline
 & 4 & 3 & 5 & 9
\end{array}
$$

Generaliseret skolemetode

$$
\begin{array}{rcccc}
 & {\scriptstyle 1} & {\scriptstyle 10} & {\scriptstyle 10} & \\
 & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
+ & & & 9 & 9 \\
\hline
 & 1 & 0 & 9 & 8
\end{array}
$$

...menterne kan have flere cifre

$$
\begin{array}{rcccc}
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \\
 & & 2 & 7 & 7 \\
+ & & 9 & 5 & 5 \\
\hline
 & {\scriptstyle 1} & {\scriptstyle 1} & & \\
 & 1 & 2 & 3 & 2 \\
+ & & 7 & 7 & 6 \\
\hline
 & 2 & 0 & 0 & 8 \\
+ & & 4 & 9 & 1 \\
\hline
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \\
 & 2 & 4 & 9 & 9 \\
+ & 1 & 5 & 2 & 4 \\
\hline
 & 4 & 0 & 2 & 3 \\
+ & & 3 & 3 & 6 \\
\hline
 & 4 & 3 & 5 & 9
\end{array}
$$

Gentagen addition af to tal

Subtraktion - skolemetoden

$$
\begin{array}{ccccc}
 & -1 & -1 & & \\
 & 5 & 3 & 4 & 7 \\
- & 1 & 8 & 6 & 3 \\
\hline
 & 3 & 4 & 8 & 4
\end{array}
\qquad\qquad
\begin{array}{ccccccc}
 & -1 & -1 & & -1 & & \\
 & 1 & 0 & 0 & 1 & 0 & 1_2 \\
- & & 1 & 1 & 0 & 1 & 0_2 \\
\hline
 & 0 & 0 & 1 & 0 & 1 & 1_2
\end{array}
$$

4 – 6 = -2 = -10 + 8

Multiplikation

$\cdot$	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Multiplikation

$$\begin{array}{r} {\scriptstyle 2\;\;1\;\;4} \\ 6 \cdot 427 \\ \hline 2562 \end{array}$$

Multiplikation med enkelt ciffer

$$\begin{array}{r} 365 \cdot 427 \\ \hline 1281 \\ 2562 \\ 2135 \\ \hline 155855 \end{array}$$

$$\begin{aligned} 365 \cdot 427 &= (3 \cdot 10^2 + 6 \cdot 10^1 + 5 \cdot 10^0) \cdot 427 \\ &= 3 \cdot 427 \cdot 10^2 + 6 \cdot 427 \cdot 10^1 + 5 \cdot 427 \cdot 10^0 \\ &= 1281 \cdot 10^2 + 2562 \cdot 10^1 + 2135 \cdot 10^0 \\ &= 128100 + 25620 + 2135 \\ &= 155855 \end{aligned}$$

Binær multiplikation

$$\begin{array}{c|cc} \cdot & 0 & 1 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 1 \end{array}$$

Binær multiplikation

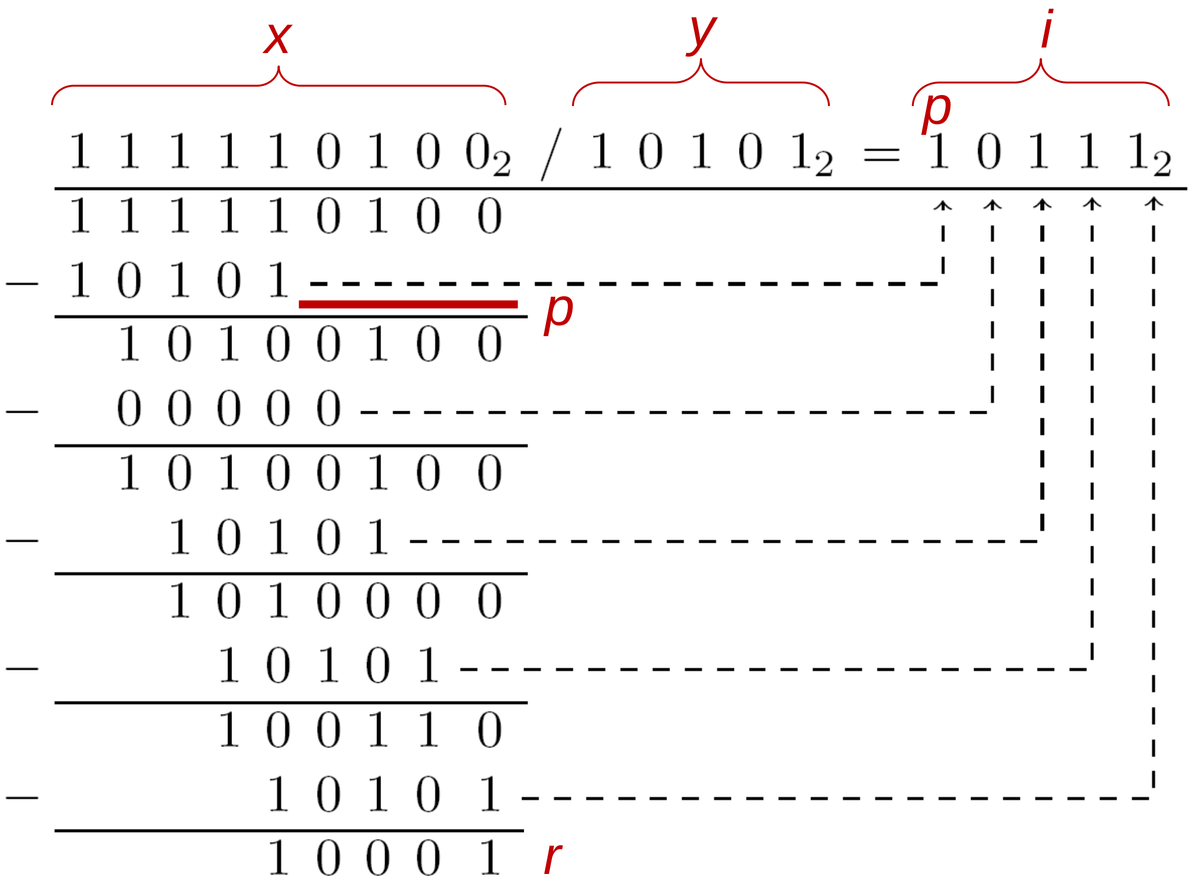
$$
\begin{array}{ccccccccccc}
1 & 0 & 1 & 1 & 1_2 & \cdot & 1 & 0 & 1 & 0 & 1_2 \\
\hline
 & & 1 & 0 & 1 & 0 & 1_2 & & & & \\
 & & & 0 & 0 & 0 & 0 & 0_2 & & & \\
 & & & & 1 & 0 & 1 & 0 & 1_2 & & \\
 & & & & & 1 & 0 & 1 & 0 & 1_2 & \\
 & & & & & & 1 & 0 & 1 & 0 & 1_2 \\
\hline
 & & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1_2
\end{array}
$$

Binær multiplikation af blokke med 1

$$\overbrace{11111}^{j}\,\overbrace{00000}^{i}{}_2 = 1\,\overbrace{0000000000}^{i+j}{}_2 - 1\,\overbrace{00000}^{i}{}_2$$

$$\begin{array}{lr}
 & 1\;0\;\overbrace{1\;1\;1}{}_2 \cdot 1\;0\;1\;0\;1_2 \\ \hline
 & 1\;0\;1\;0\;1_2 \\
\left\{\begin{array}{l} + \\ - \end{array}\right. & \begin{array}{r} 1\;0\;1\;0\;1_2 \\ 1\;0\;1\;0\;1_2 \end{array} \\ \hline
 & 1\;1\;1\;1\;0\;0\;0\;1\;1_2
\end{array}$$

Division



Algorithm $\textsc{IntegerDivision}(x, y)$

Input Integers $x \geq 0$ and $y \geq 1$

Output Integer $i = \lfloor x/y \rfloor$, i.e. $0 \leq x - i \cdot y < y$

```
p = 0
while y · 2^(p+1) ≤ x
    p = p + 1
i = 0
r = x
# Invariant: x = i · y + r and r < y · 2^(p+1)
while y ≤ r
    if y · 2^p ≤ r
        i = i + 2^p        # set position p in i = 1
        r = r − y · 2^p
    p = p − 1
return i
```

Konvertering til base *b*

Algorithm $\textsc{BaseRepresentation}(x, b)$

Input Integers $x \geq 0$ and base $b \geq 2$

Output Digits $d_0, d_1, \ldots$ of the b-ary representation of x

$p = 0$

while $x > 0$ **do**

  $i = \lfloor x/b \rfloor$ $\textsc{IntegerDivision}(x, b)$

  $d_p = x - i \cdot b$

  $x = i$

  $p = p + 1$