

Algoritmer og Datastrukturer

Rød-Sorte Søgetræer [CLRS, kapitel 13]

Rød-Sorte Søgetræer

Mål

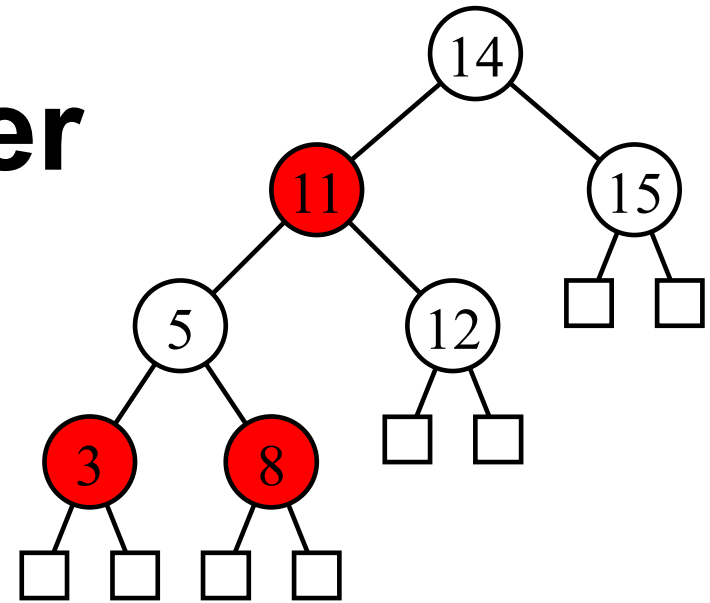
Søgetræer med dybde $O(\log n)$

Invarianter

- Hver knude er enten **RØD** eller **SORT**
- Hver **RØD** knude har en **SORT** far
- Alle stier fra roden til et eksternt blad har **samme antal sorte knuder**

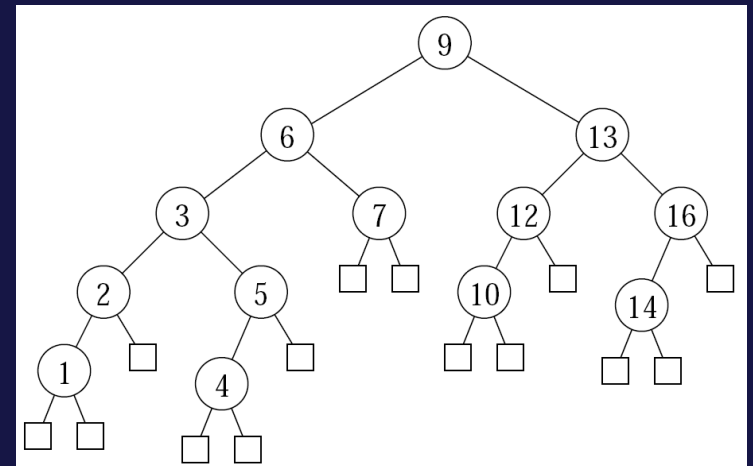
Sætning

Et rød-sort træ har højde $\leq 2 \cdot \log(n+1)$

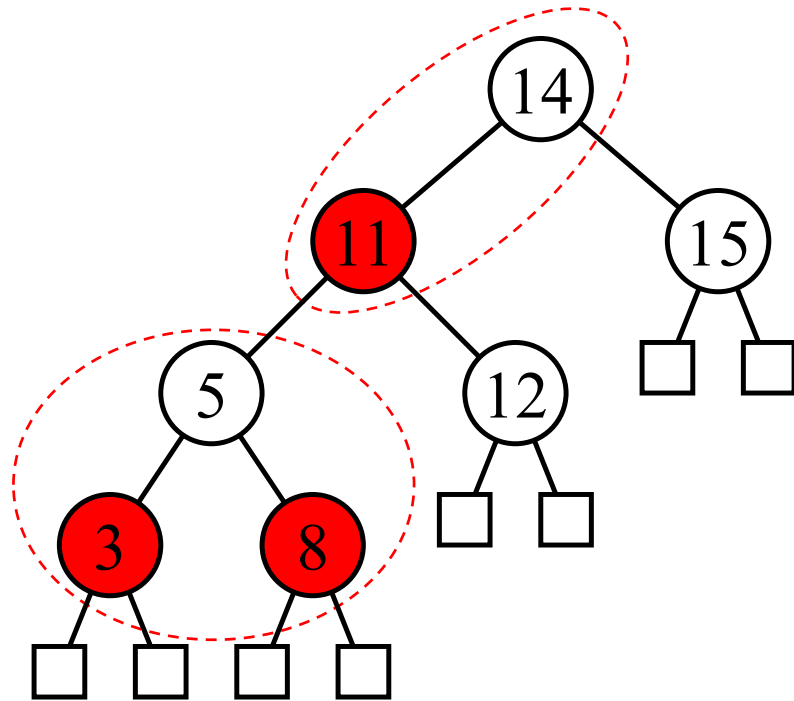


Hvilke knuder skal farves røde for at træet bliver et rød-sort søgetræ?

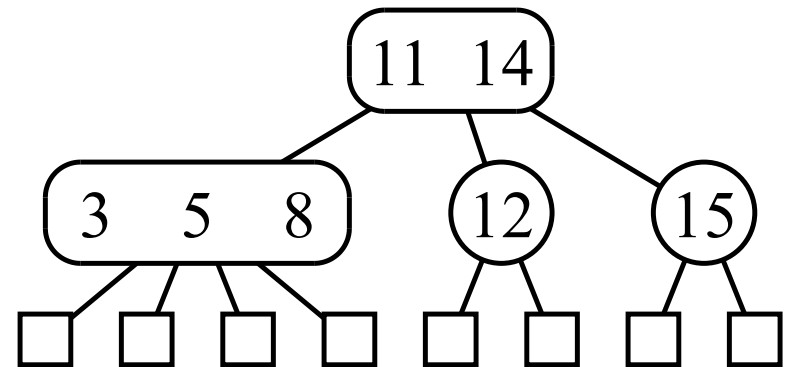
- a) 1, 4, 10, 14
- b) 2, 5, 10, 14
- c) 1, 3, 4, 7, 12, 16
- d) 1, 3, 4, 10, 14
- e) ingen
- f) ved ikke



Rød-Sorte vs (2-4)-træer



Rød-sort træ

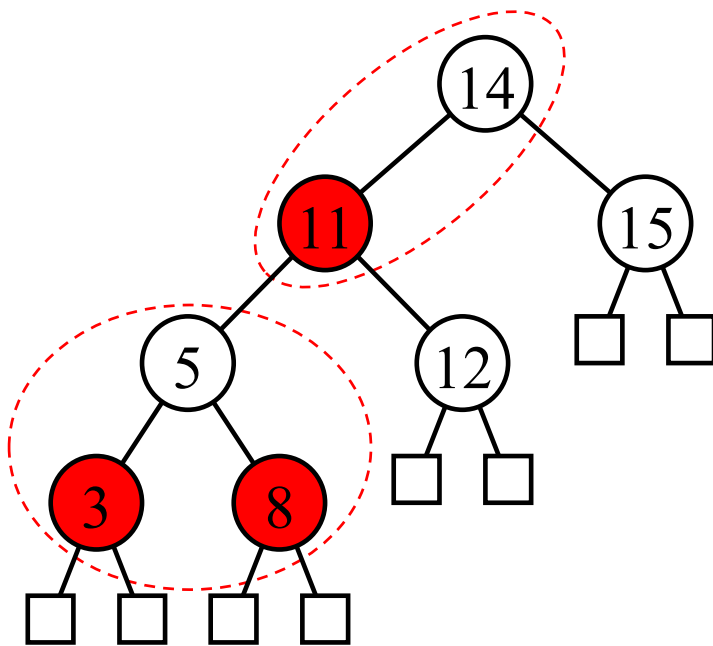


(2,4)-træ

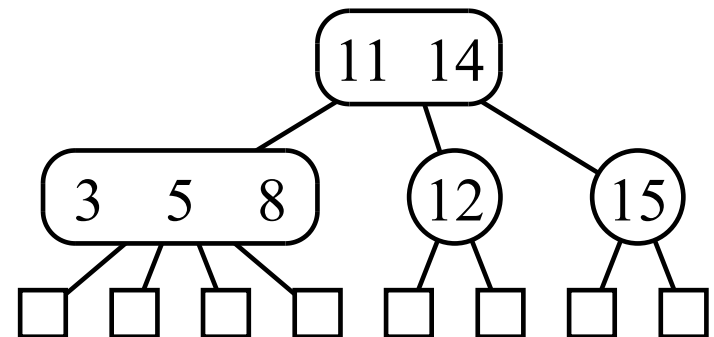
(2,4)-træ $\equiv$ **rød-sort træ** hvor alle røde knuder er slået sammen med faderen

Egenskaber ved (2,4)-træer

- Alle interne knuder har mellem **2 og 4 børn**
- Knude med i **børn** gemmer $i-1$ **elementer**
- Stier fra roden til et eksternt blad er lige lange

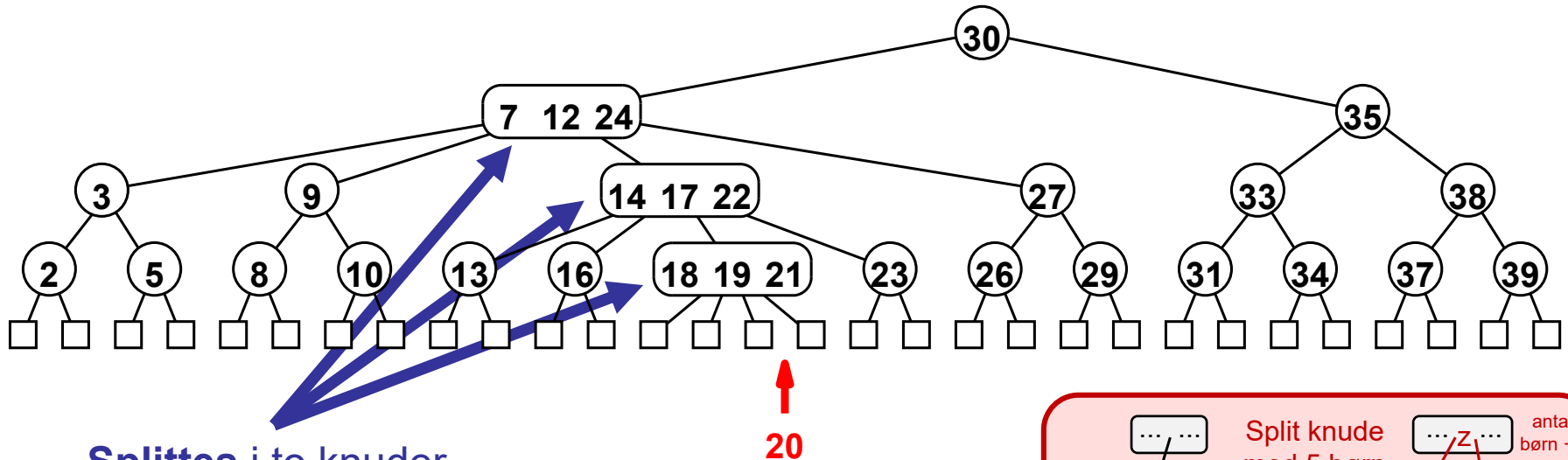


Rød-sort træ

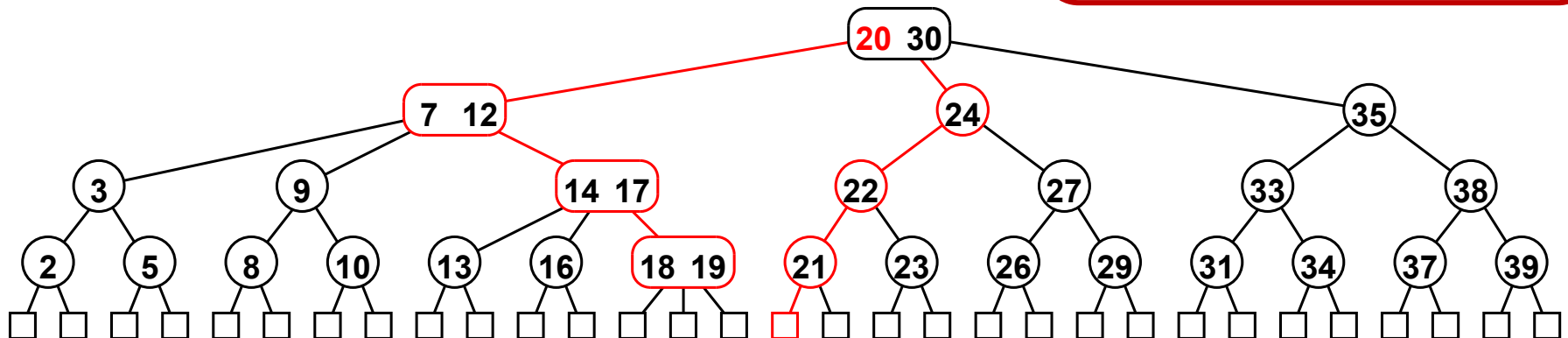
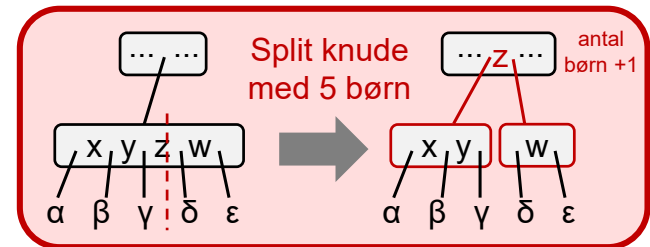


(2,4)-træ

Indsættelse i et (2,4)-træ

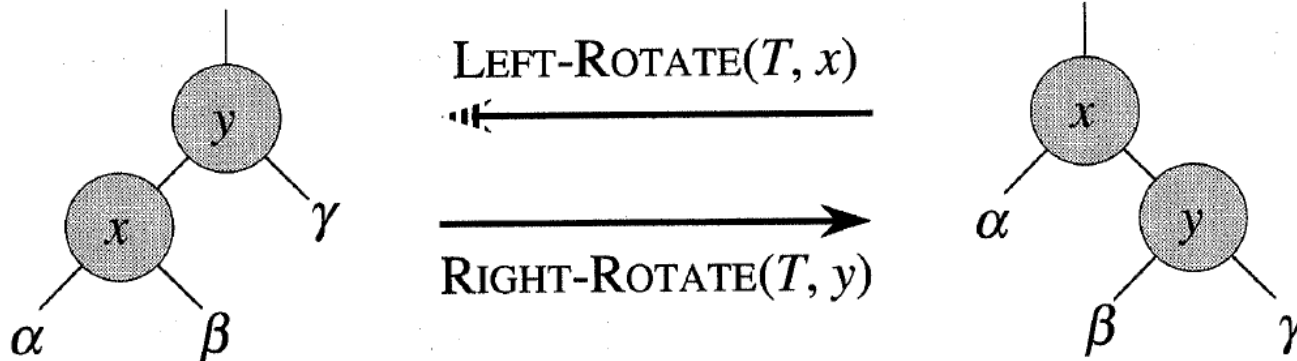


Splittes i to knuder
og midterste nøgle
flyttes et niveau op



Opdateringer af Rød-Sorte Træer

Rotationer



$\text{LEFT-ROTATE}(T, x)$

```

1   $y = x.\text{right}$            // set y
2   $x.\text{right} = y.\text{left}$        // turn y's left subtree into x's right subtree
3  if  $y.\text{left} \neq T.\text{nil}$ 
4       $y.\text{left}.p = x$ 
5   $y.p = x.p$                // link x's parent to y
6  if  $x.p == T.\text{nil}$ 
7       $T.\text{root} = y$ 
8  elseif  $x == x.p.\text{left}$ 
9       $x.p.\text{left} = y$ 
10 else  $x.p.\text{right} = y$ 
11  $y.\text{left} = x$            // put x on y's left
12  $x.p = y$ 
    
```


Insert

Ubalanceret
indsættelse

RB-INSERT(T, z)

søg

```

1   $y = T.nil$ 
2   $x = T.root$ 
3  while  $x \neq T.nil$ 
4       $y = x$ 
5      if  $z.key < x.key$ 
6           $x = x.left$ 
7      else  $x = x.right$ 

```

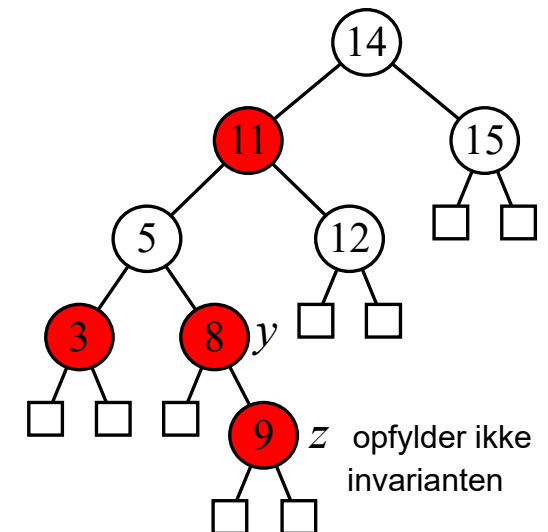
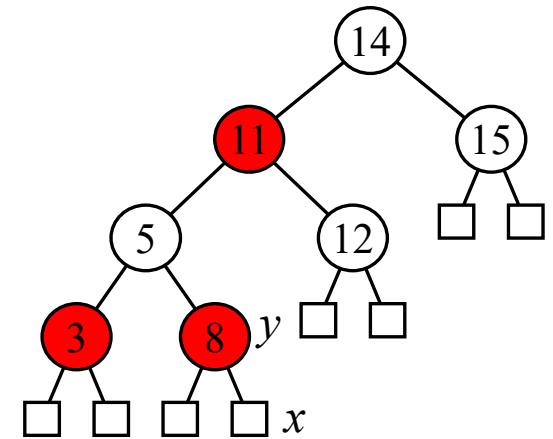
opret z

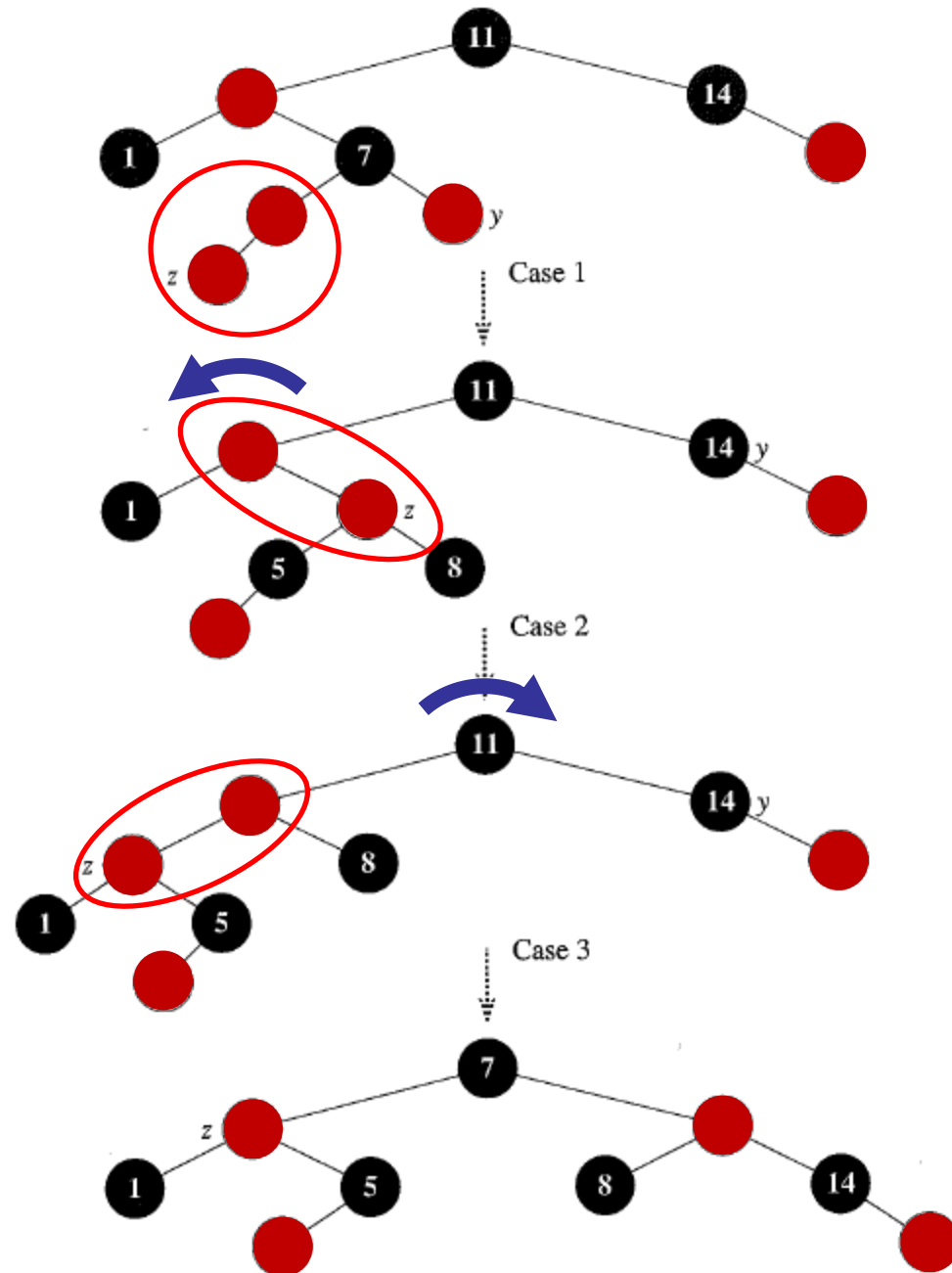
```

8   $z.p = y$ 
9  if  $y == T.nil$ 
10      $T.root = z$ 
11  elseif  $z.key < y.key$ 
12      $y.left = z$ 
13  else  $y.right = z$ 
14   $z.left = T.nil$ 
15   $z.right = T.nil$ 
16   $z.color = RED$ 
17  RB-INSERT-FIXUP( $T, z$ )

```

Insert(9)

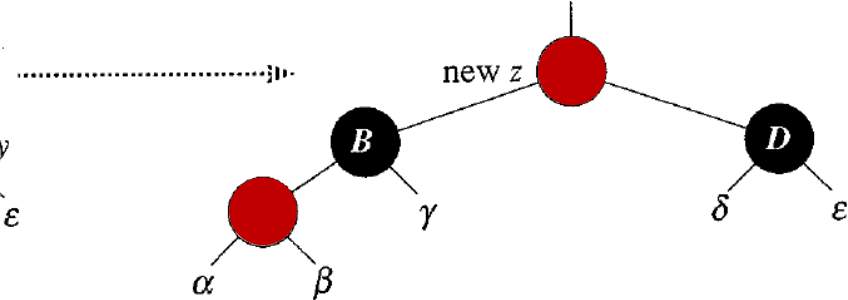
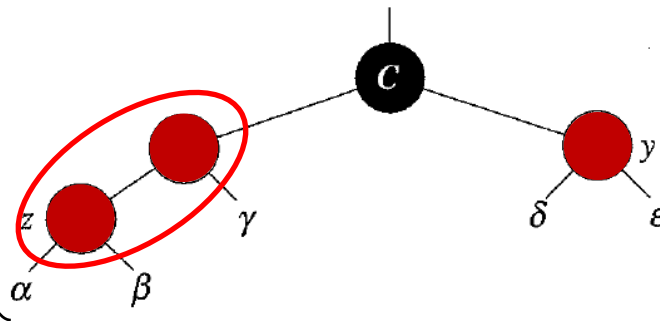
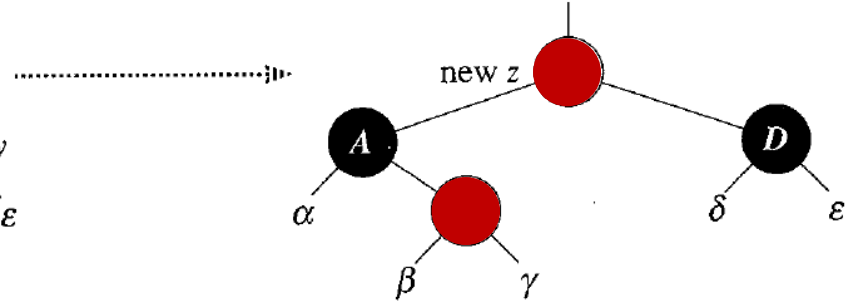
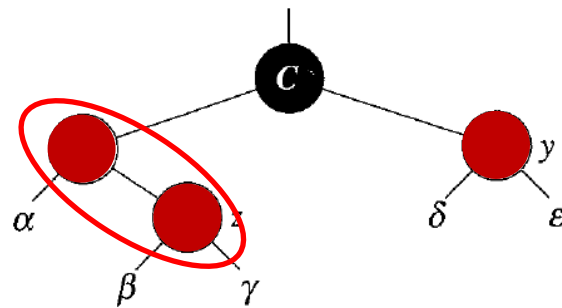




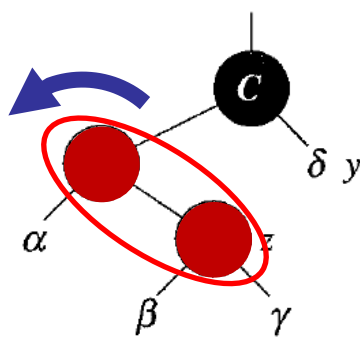
Indsættelse i rød-sort træer: rebalancering

Omfarv C
og dens to
røde børn

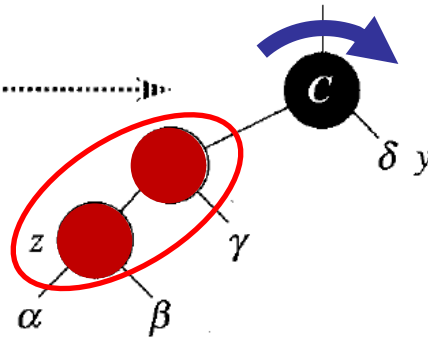
Case 1



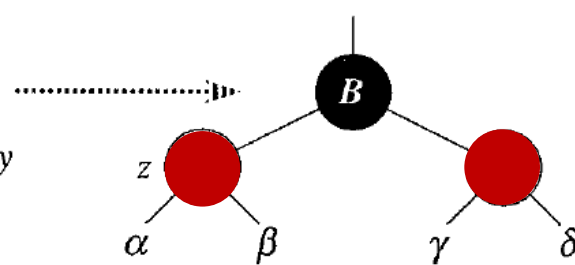
Afslut



Case 2



Case 3

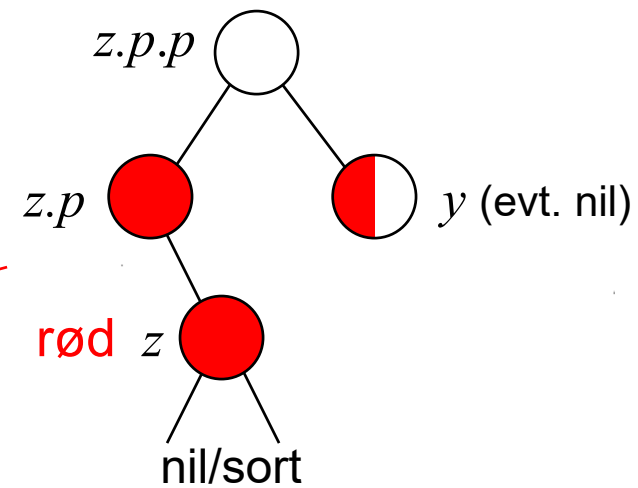


RB-INSERT-FIXUP(T, z)

```

1  while  $z.p.color == RED$ 
2      if  $z.p == z.p.p.left$ 
3           $y = z.p.p.right$ 
4          if  $y.color == RED$ 
5               $z.p.color = BLACK$  // case 1
6               $y.color = BLACK$  // case 1
7               $z.p.p.color = RED$  // case 1
8               $z = z.p.p$  // case 1
9          else if  $z == z.p.right$ 
10              $z = z.p$  // case 2
11             LEFT-ROTATE( $T, z$ ) // case 2
12              $z.p.color = BLACK$  // case 3
13              $z.p.p.color = RED$  // case 3
14             RIGHT-ROTATE( $T, z.p.p$ ) // case 3
15         else (same as then clause
                with “right” and “left” exchanged)
16   $T.root.color = BLACK$ 

```



Worst-case antal rotationer ved indsættelse i et rød-sort træ ?

- a) 1
- b) 2
- c) $\log n$
- d) $2 \cdot \log n$
- e) Ved ikke

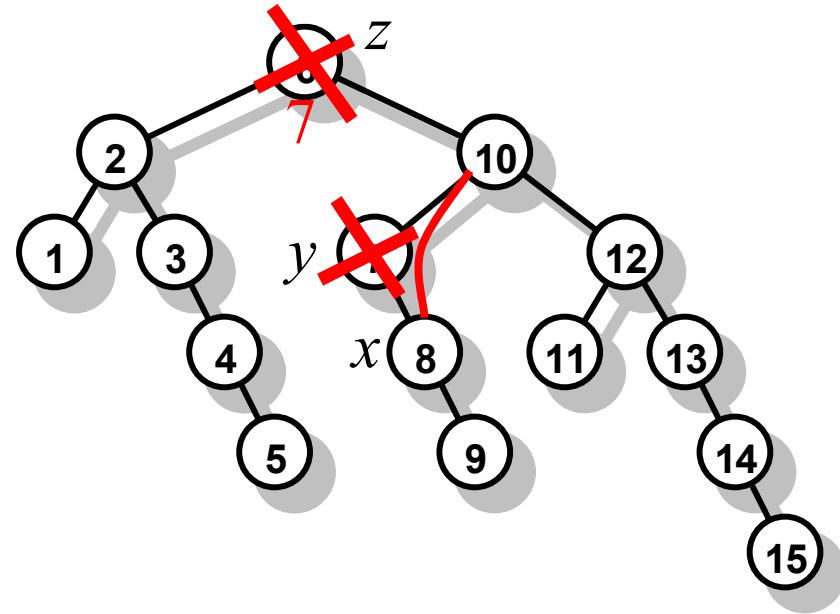
Delete

RB-DELETE(T, z)

Ubalanceret slettelse

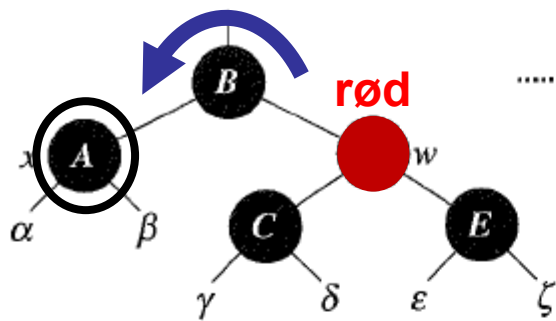
```

1   $y = z$ 
2   $y\text{-original-color} = y.\text{color}$ 
3  if  $z.\text{left} == T.\text{nil}$ 
4       $x = z.\text{right}$ 
5      RB-TRANSPLANT( $T, z, z.\text{right}$ )
6  elseif  $z.\text{right} == T.\text{nil}$ 
7       $x = z.\text{left}$ 
8      RB-TRANSPLANT( $T, z, z.\text{left}$ )
9  else  $y = \text{TREE-MINIMUM}(z.\text{right})$ 
10      $y\text{-original-color} = y.\text{color}$ 
11      $x = y.\text{right}$ 
12     if  $y.p == z$ 
13          $x.p = y$ 
14     else RB-TRANSPLANT( $T, y, y.\text{right}$ )
15          $y.\text{right} = z.\text{right}$ 
16          $y.\text{right}.p = y$ 
17     RB-TRANSPLANT( $T, z, y$ )
18      $y.\text{left} = z.\text{left}$ 
19      $y.\text{left}.p = y$ 
20      $y.\text{color} = z.\text{color}$ 
21 if  $y\text{-original-color} == \text{BLACK}$ 
22     RB-DELETE-FIXUP( $T, x$ )
  
```

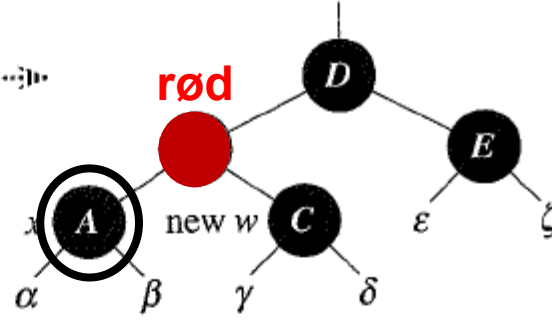
RB-TRANSPLANT(T, u, v)

```

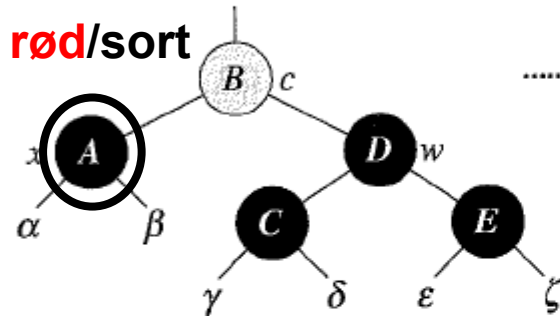
1  if  $u.p == T.\text{nil}$ 
2       $T.\text{root} = v$ 
3  elseif  $u == u.p.\text{left}$ 
4       $u.p.\text{left} = v$ 
5  else  $u.p.\text{right} = v$ 
6   $v.p = u.p$ 
  
```

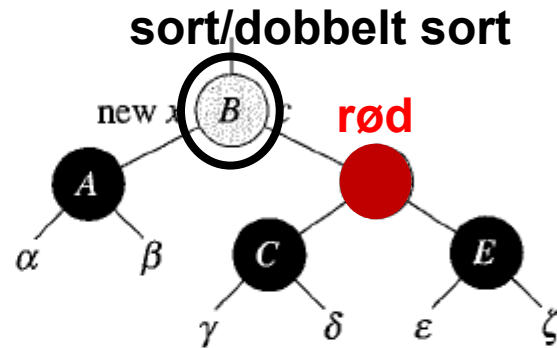
Case 1



Case 1 $\rightarrow$ 2,3,4

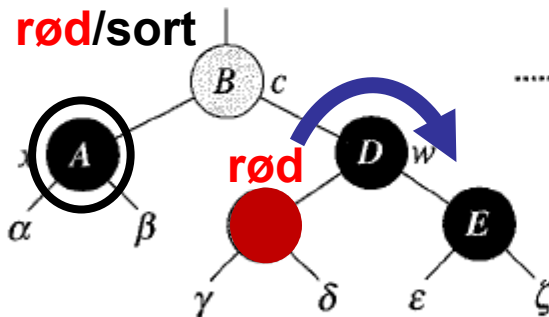


Case 2

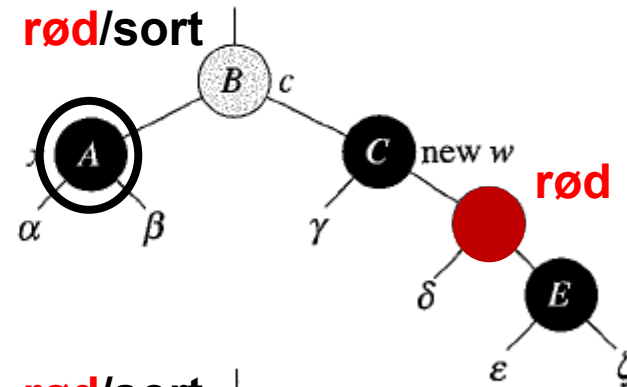


sort/dobbelt sort

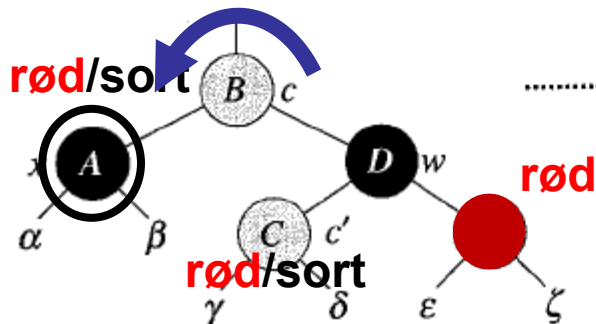
Case 2 $\rightarrow$
Problemet x flyttet
tættere på roden
eller færdig



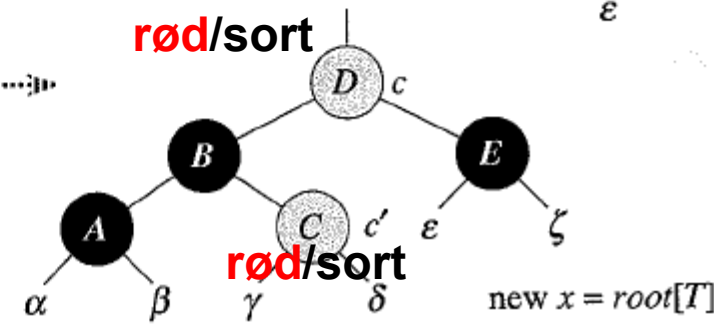
Case 3



Case 3 $\rightarrow$ 4
(og færdig)



Case 4



Case 4 $\rightarrow$ Færdig

RB-DELETE-FIXUP(T, x)

```

1  while  $x \neq T.root$  and  $x.color == BLACK$ 
2      if  $x == x.p.left$ 
3           $w = x.p.right$ 
4          if  $w.color == RED$ 
5               $w.color = BLACK$  // case 1
6               $x.p.color = RED$  // case 1
7              LEFT-ROTATE( $T, x.p$ ) // case 1
8               $w = x.p.right$  // case 1
9          if  $w.left.color == BLACK$  and  $w.right.color == BLACK$ 
10              $w.color = RED$  // case 2
11              $x = x.p$  // case 2
12         else if  $w.right.color == BLACK$ 
13              $w.left.color = BLACK$  // case 3
14              $w.color = RED$  // case 3
15             RIGHT-ROTATE( $T, w$ ) // case 3
16              $w = x.p.right$  // case 3
17              $w.color = x.p.color$  // case 4
18              $x.p.color = BLACK$  // case 4
19              $w.right.color = BLACK$  // case 4
20             LEFT-ROTATE( $T, x.p$ ) // case 4
21              $x = T.root$  // case 4
22         else (same as then clause with “right” and “left” exchanged)
23      $x.color = BLACK$ 

```

Dynamisk Ordbog :

Rød-Sorte Træer

Search(S, x)	$O(\log n)$
Insert(S, x)	
Delete(S, x)	

Rød-Sorte Træer i Java og C++

	Java (JDK SE 12.0.1)	C++ (GCC 9.1)
Metode	java.util.TreeMap	std::map
Dokumentation	https://docs.oracle.com/javase/10/docs/api/java/util/TreeMap.html	http://www.cplusplus.com/reference/map/map/
Kildekode	Installer JDK fra www.oracle.com/technetwork/java/javase/downloads/ Fil java.base\java\util\TreeMap.java fra C:\ProgramFiles\Java\jdk-12.0.1\lib\src.zip	https://github.com/gcc-mirror/gcc/blob/master/libstdc++-v3/include/bits/stl_tree.h