Algoritmer og Datastrukturer

Prioritetskøer med Afskæring

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WORST-CASE DATA STRUCTURES FOR THE PRIORITY QUEUE WITH ATTRITION

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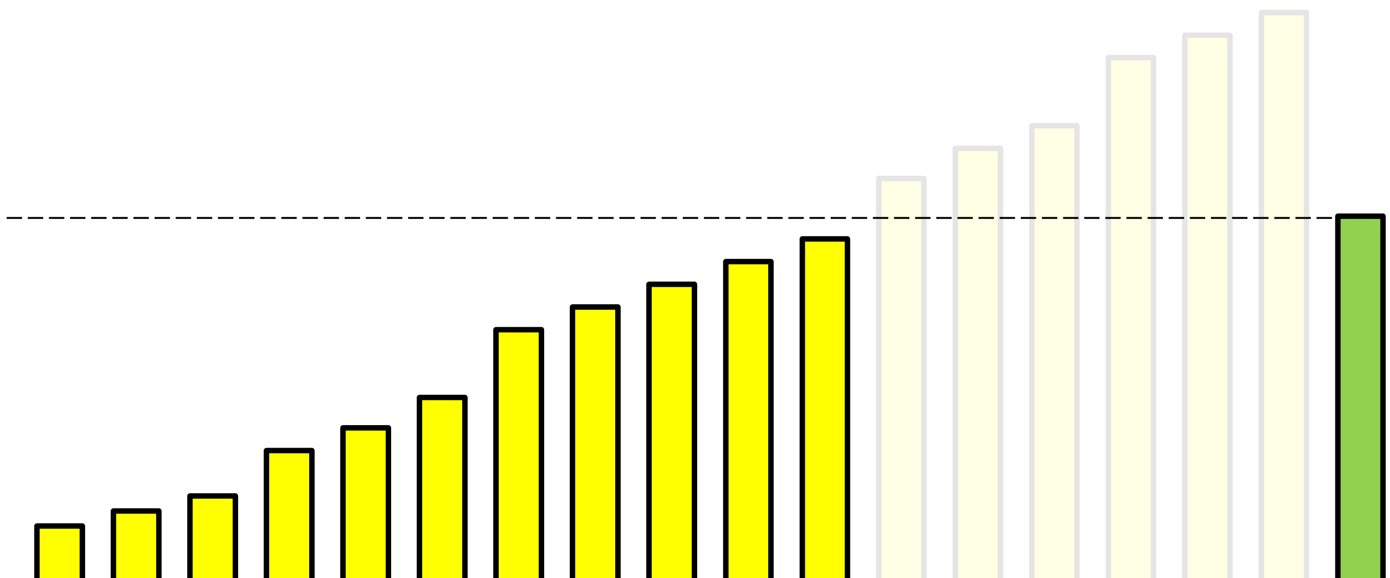
We describe three data structures for the priority queue with attrition (or PQA) that perform each PQA operation in O(1) worst-case time. Previous implementations of the PQA required O(1) amortized time per operation.

Keywords: Priority queue with attrition, worst-case

Operationer

Create	$S := \emptyset$
Insert(*x*)	$S := \{\ x\ \} \cup \{\ y \in S \mid y < x\ \}$
Deletemin	$m := \min(S);\ S := S \setminus \{\ m\ \};$ return m



Insert(4) i {1,3,5,6,9} ?

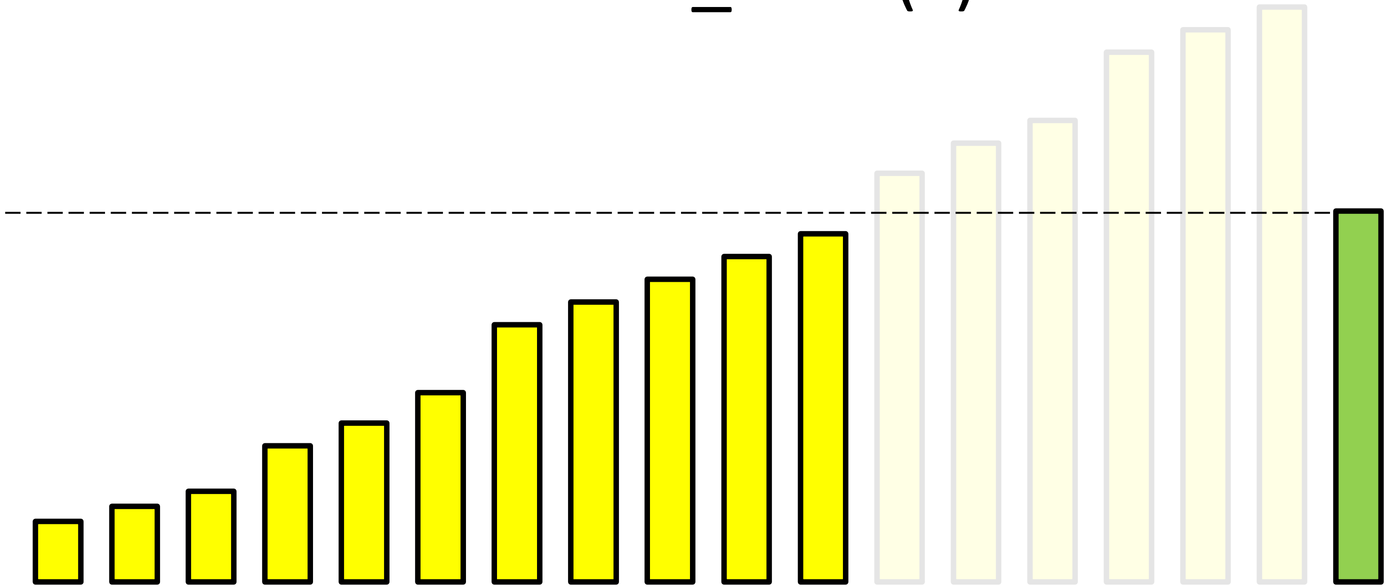
a) {4,5,6,9}
b) {1,3,4,5,6,9}
c) {1,3,4}
d) ved ikke

Løsning: Sorteret Liste

Create S := ()

Insert(*x*) while (|S|>0 and tail(S)≥x) remove_tail(S)
insert_tail(x)

Deletemin return remove_head(S)



Løsning: Sorteret Liste

Create $S := ()$

Insert(*x*) while $(|S|>0$ and $\text{tail}(S) \geq x)$ remove_tail(S)
insert_tail(x)

Deletemin return remove_head(S)

Sætning
Create, Insert og DeleteMin tager **amortiseret** O(1) tid

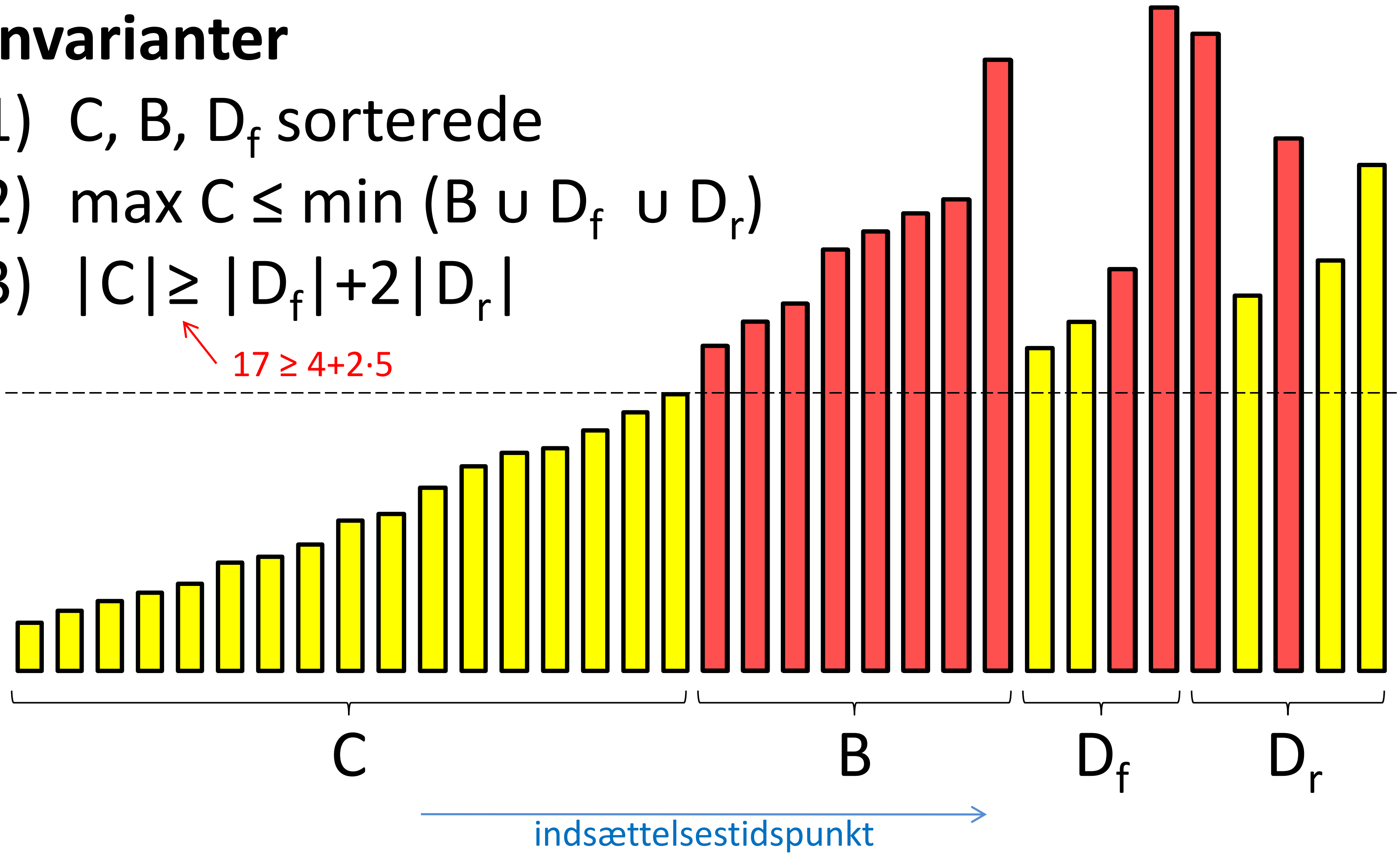
Bevis: $\Phi(S) = |S|$. □

Worst-Case $O(1)$

Løsning: 4 Lister

Invarianter

1) C, B, D_f sorterede
2) max C ≤ min (B ∪ D_f ∪ D_r)
3) $|C| \geq |D_f| + 2|D_r|$



Hvilken mængde repræsenteres ved:

$$(1,2,3,4,5,6)(7,10,14)(8,11)(9,8)$$

C B D_f D_r

a) {1,2,3,4,5,6,7,8,8,9,10,11,14}
b) {1,2,3,4,5,6,7,8,8,9}
c) {1,2,3,4,5,6,7,8,9}
d) {1,2,3,4,5,6,7,8}
e) {1,2,3,4,5,6,7,8,8}
f) ved ikke

Invarianter

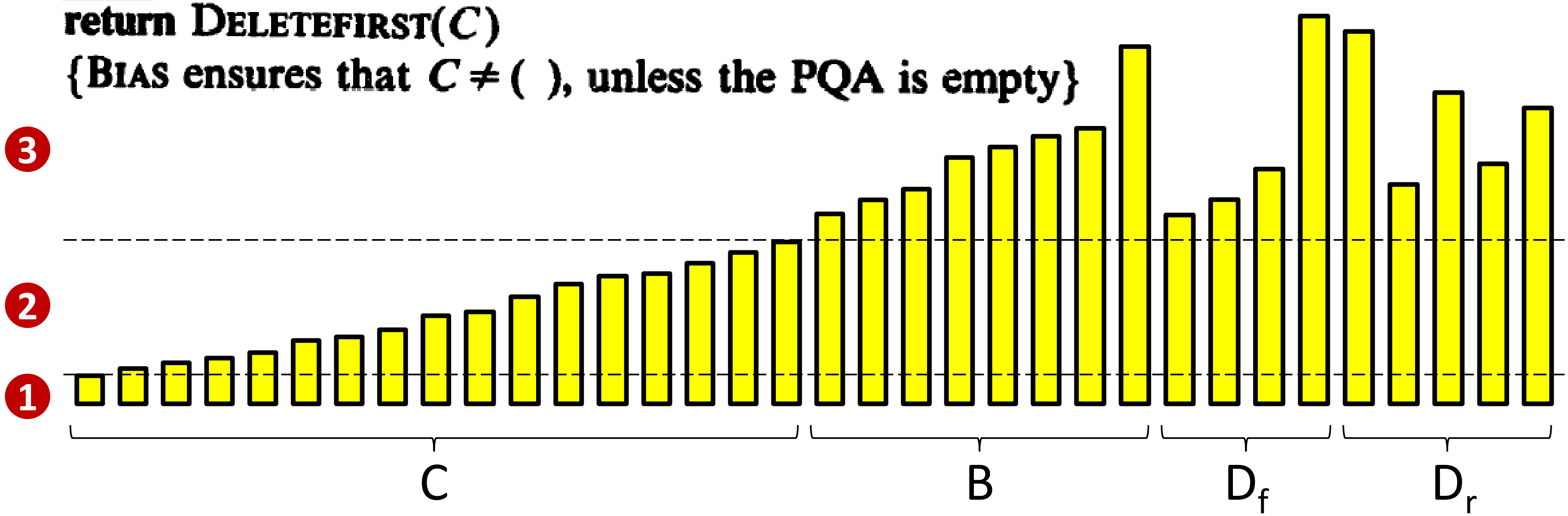
1) C, B, D_f sorterede
2) max C ≤ min (B ∪ D_f ∪ D_r)
3) |C| ≥ $|D_f|$ + 2$|D_r|$

≥ "+1" BIAS

CREATEPQA ≡
    C, B, D_f, D_r := ( ), ( ), ( ), ( )

INSERT(x) ≡
    if $C \neq (\ )$ **cand first**(C) ≥ x **then**
        {Delete all existing items; add x to C}
❶ C, B, D_f, D_r := (x), ( ), ( ), ( )
    else if $C \neq (\ )$ **cand last**(C) ≥ x **then**
        {Empty B, D_f, and D_r; push back rest(C) into B; add x to D_f}
❷ C, B, D_f, D_r := (first(C)), rest(C), (x), ( )
❸   **else** $D_r := D_r \| (x)$; BIAS; BIAS

DELETEMIN ≡
    BIAS;
    return DELETEFIRST(C)
    {BIAS ensures that $C \neq (\ )$, unless the PQA is empty}

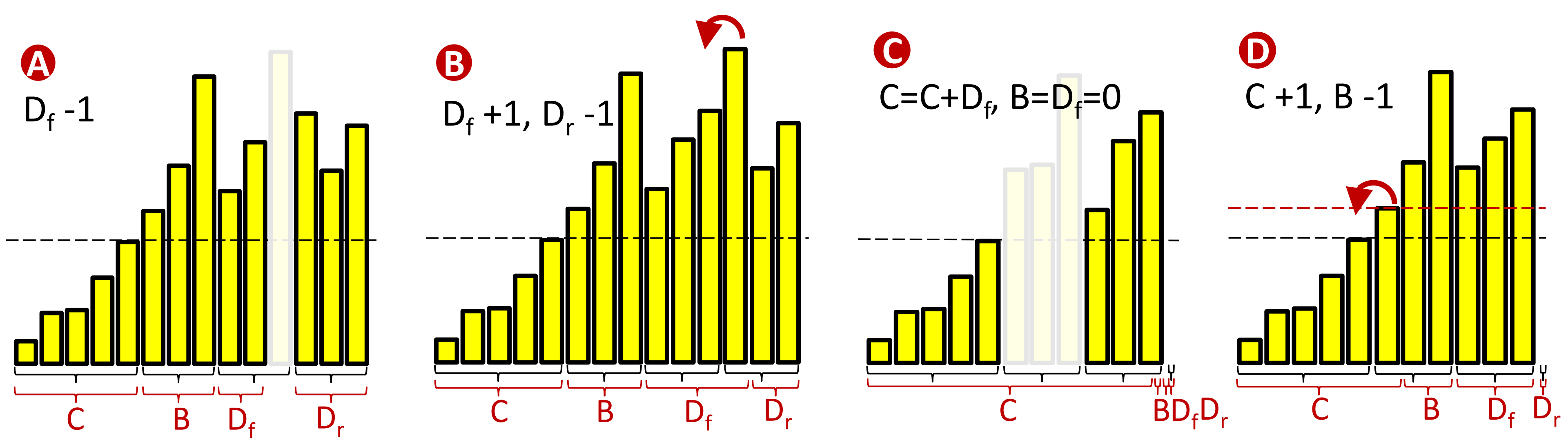


Invarianter

1) C, B, D_f sorterede
2) max C ≤ min (B ∪ D_f ∪ D_r)
3) |C| ≥ $|D_f| + 2|D_r|$

≥ "+1" BIAS

BIAS ≡
- **if** $D_r \neq (\ )$ **then**
  - {Clean-up step}
  - **if** $D_f \neq (\ )$ **cand** last(D_f) ≥ first(D_r) **then**
    - (A) DELETELAST(D_f) {decrease $|D_f|$}
  - (B) **else** PASS(D_r, D_f) {decrease $|D_r|$; increase $|D_f|$}
- **else if** $D_f \neq (\ )$ **cand** ($B = (\ )$ **cor** first(B) ≥ first(D_f)) **then**
  - (C) $D_f, B, C := (\ ), (\ ), C \,\|\, D_f$ {decrease $|D_f|$; increase $|C|$}
- (D) **else if** $B \neq (\ )$ **then** PASS(B, C) {increase $|C|$}
- {else $B = D_f = D_r = (\ )$}



Resultatet af **insert(7)** på:

(1,2,3,4,5,6)(7,10,14)(8,11)(9,8)

C B D_f D_r

a) (1,2,3,4,5,6)(7,10,14)(8,11)(9,8,7)
b) (1,2,3,4,5,6)(7,10,14)(8,9)(8,7)
c) (1,2,3,4,5,6)(7,10,14)(8)(9,8,7)
d) (1,2,3,4,5,6,7)(10,14)(8)(9,8,7)
e) ved ikke

Sætning

Create, Insert og DeleteMin tager **worst-case** O(1) tid

Invarianter

1) C, B, D_f sorterede
2) max C ≤ min (B ∪ D_f ∪ D_r)
3) |C| ≥ |D_f|+2|D_r|