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(or, tips and tricks for index design)

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An overview

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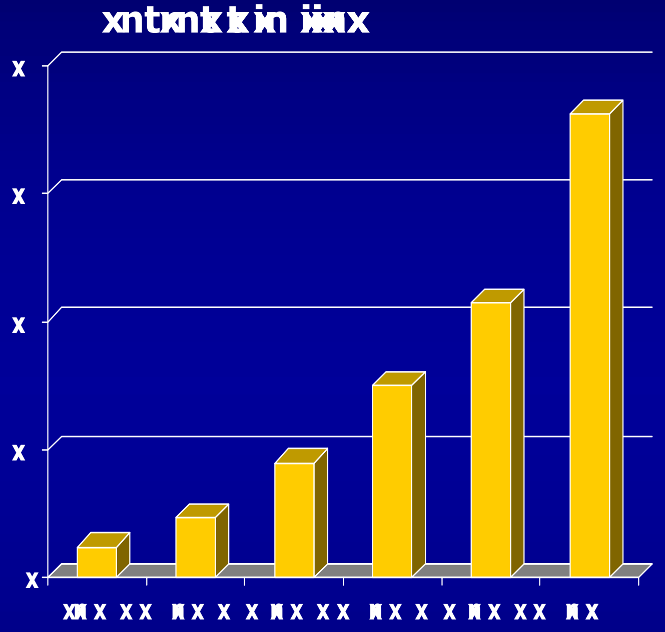

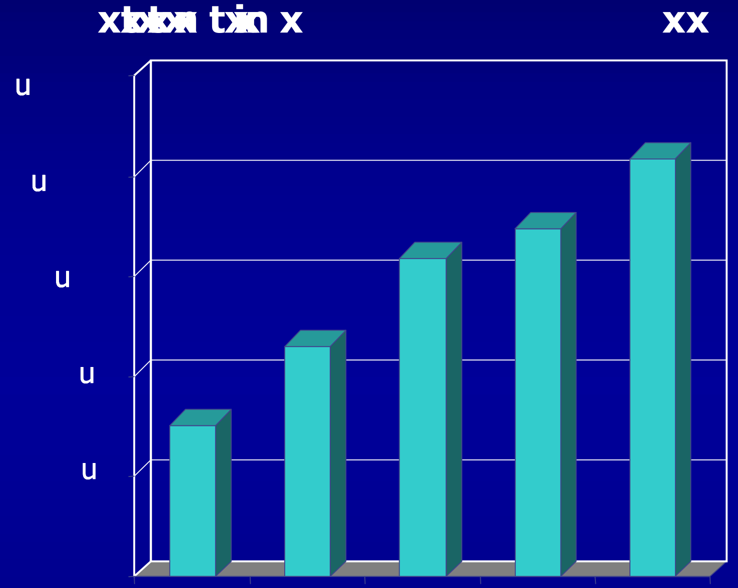

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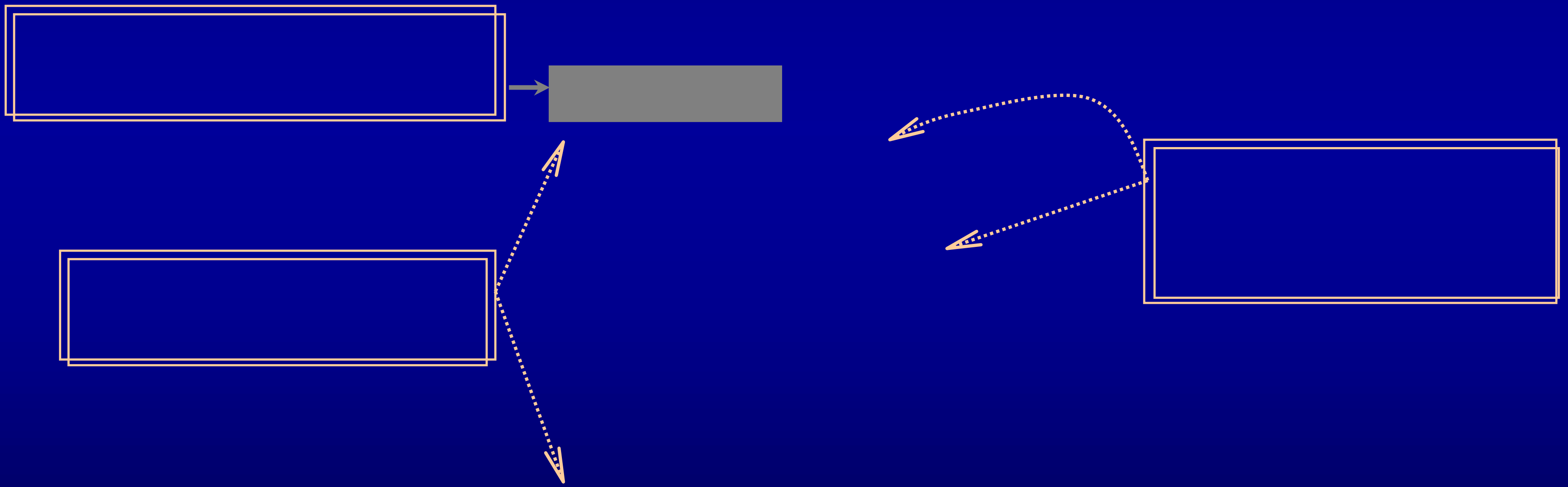

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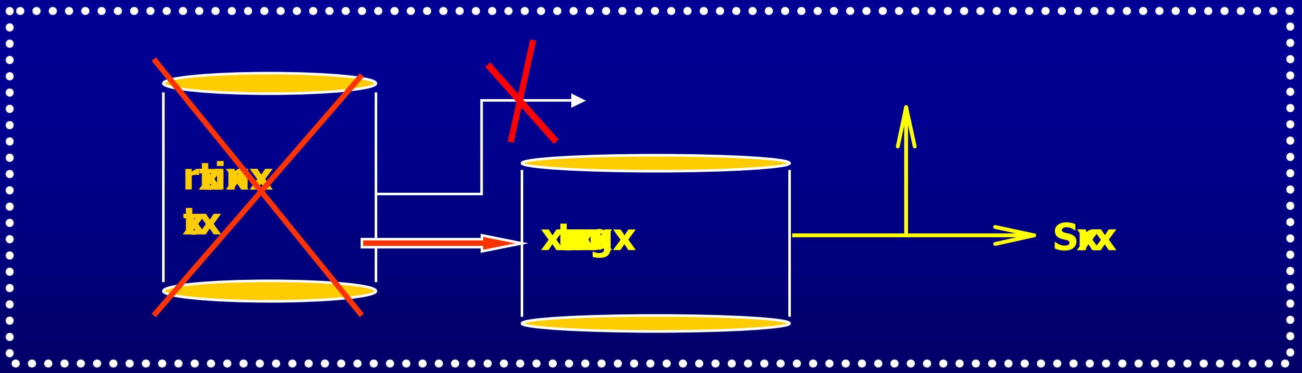

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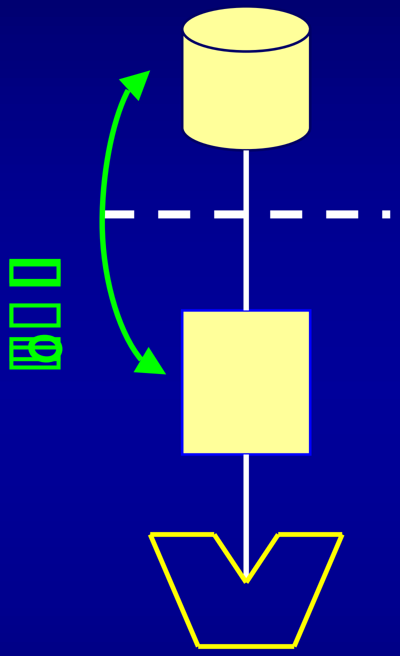

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Types of data

linguistic or tokenizable text
a sequence of characters or bytes

DNA sequences
audio-video files
executables

Types of query

word-based query
character-based query

exact word
word prefix or suffix
phrase

arbitrary substring
complex matches

Two indexing approaches :

Word-based indices, here a concept of word must be devised !
- Inverted files, Signature files or Bitmaps.

Full-text indices, no constraint on text and queries !
- Suffix Array, Suffix tree, Hybrid indices, or String B-tree.

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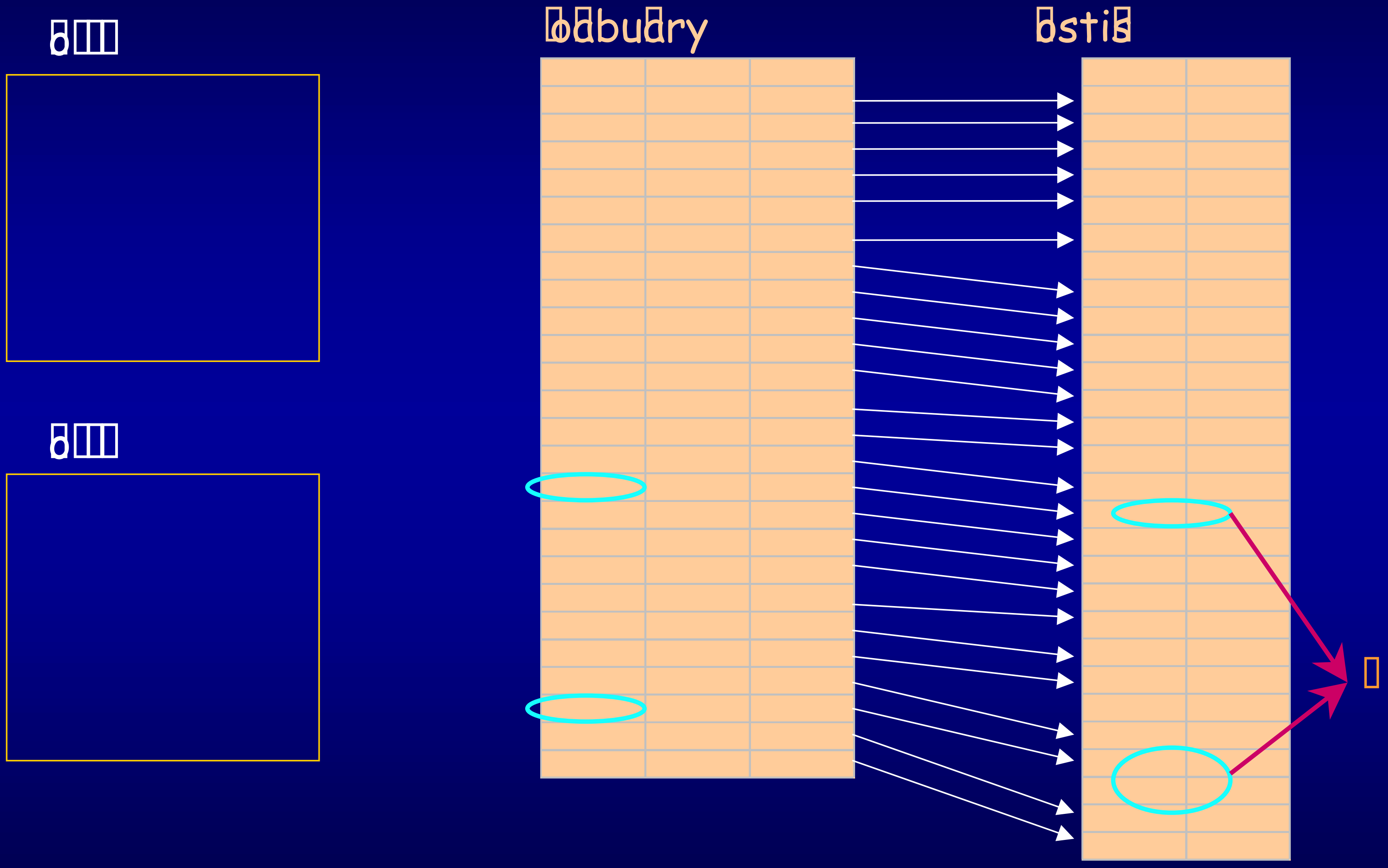

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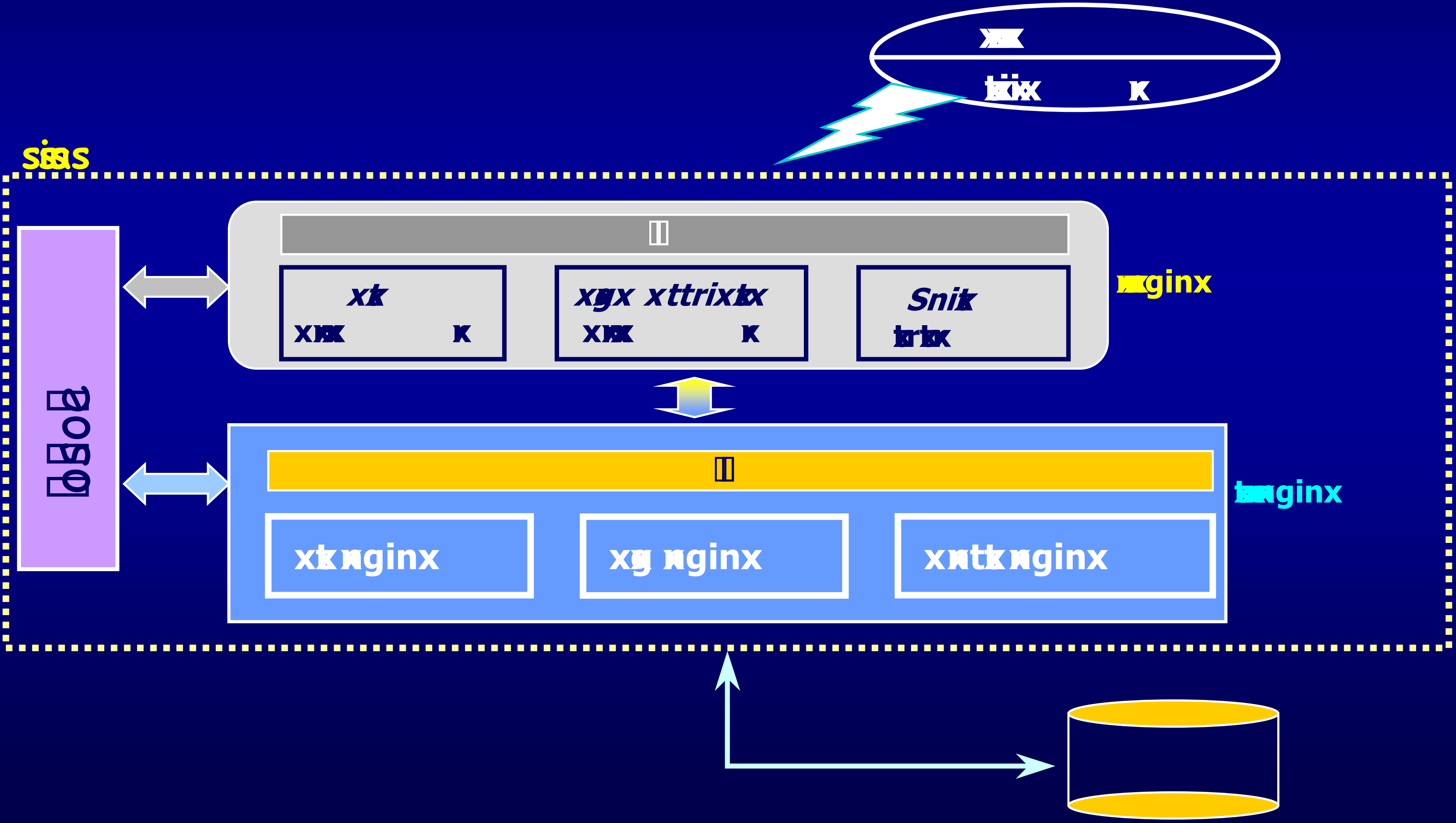

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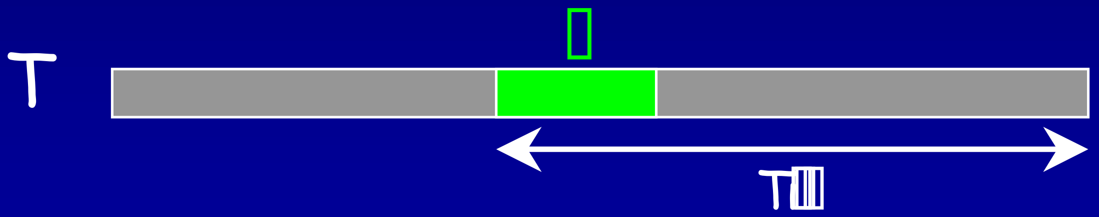
Pattern P occurs at position i of T

iff P is a prefix of the suffix T[i,N]



Occurrences of P in T = All suffixes of T having P as a prefix

T = This is a visual example
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SUF(T) = Sorted set of suffixes of T

SUF(Δ) = Sorted set of suffixes of all texts in Δ

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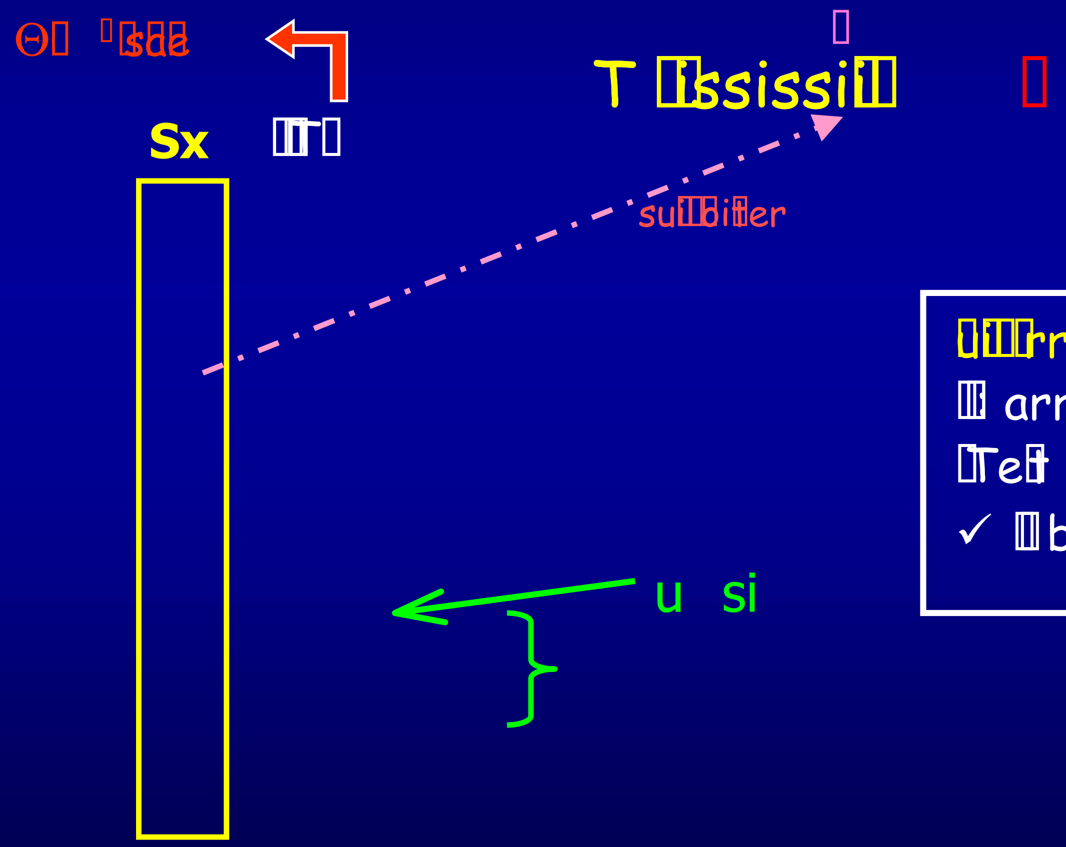

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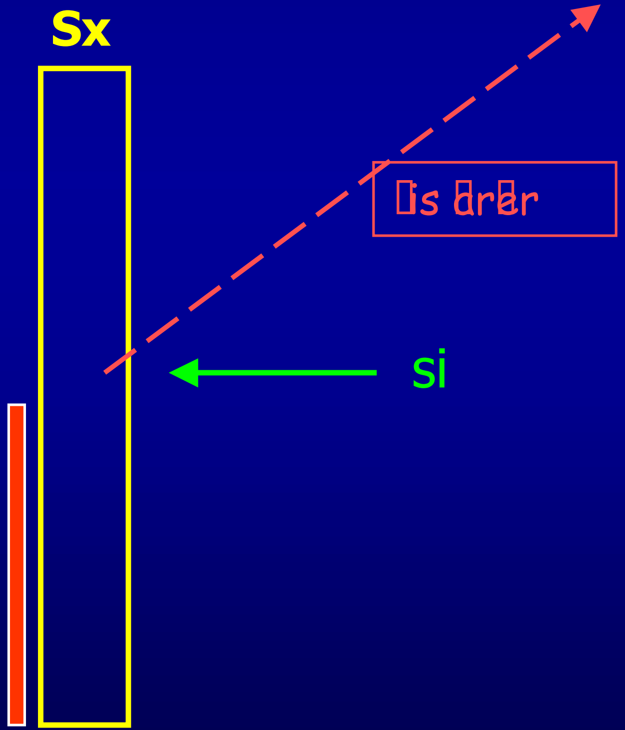

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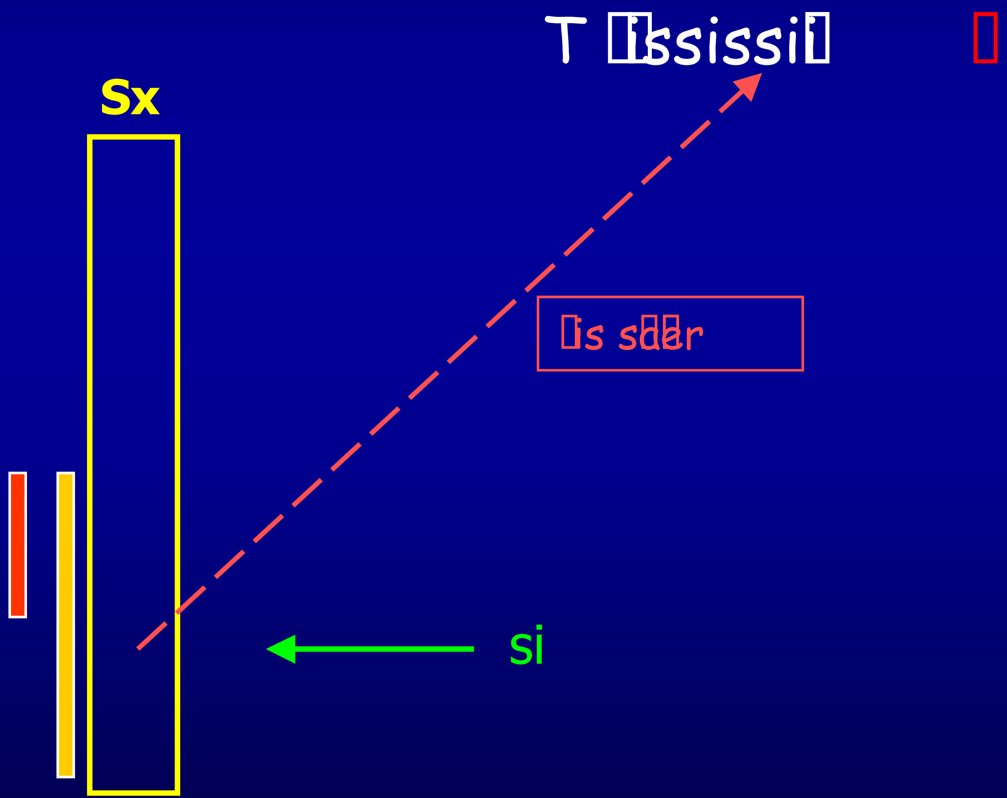

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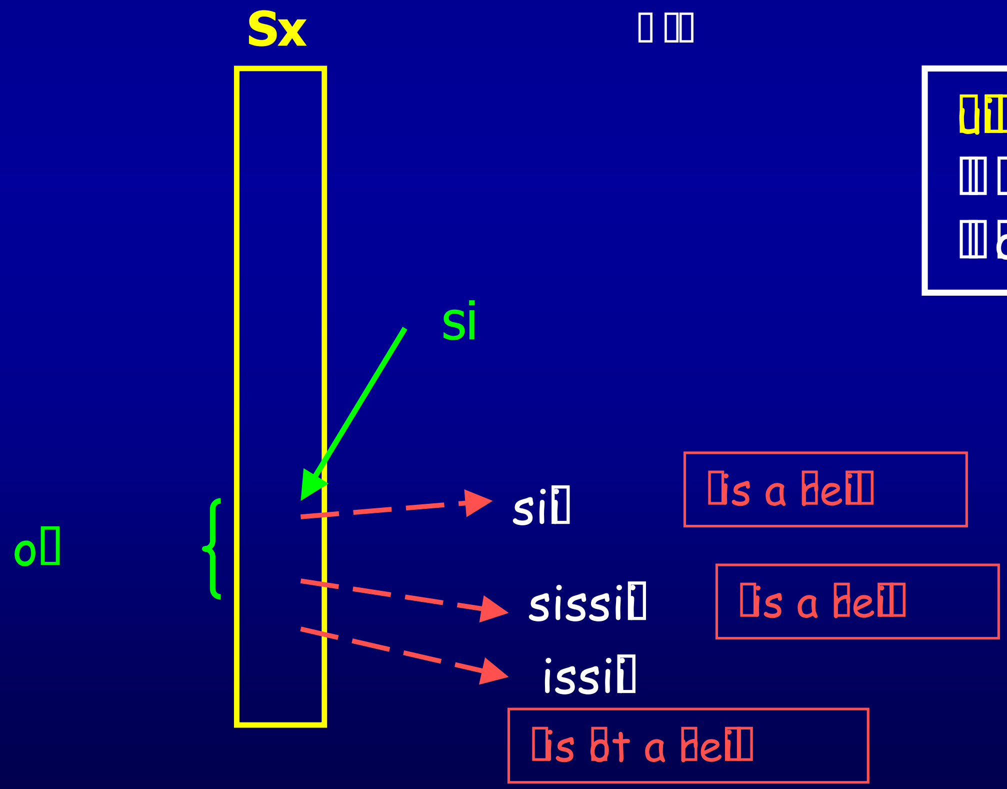

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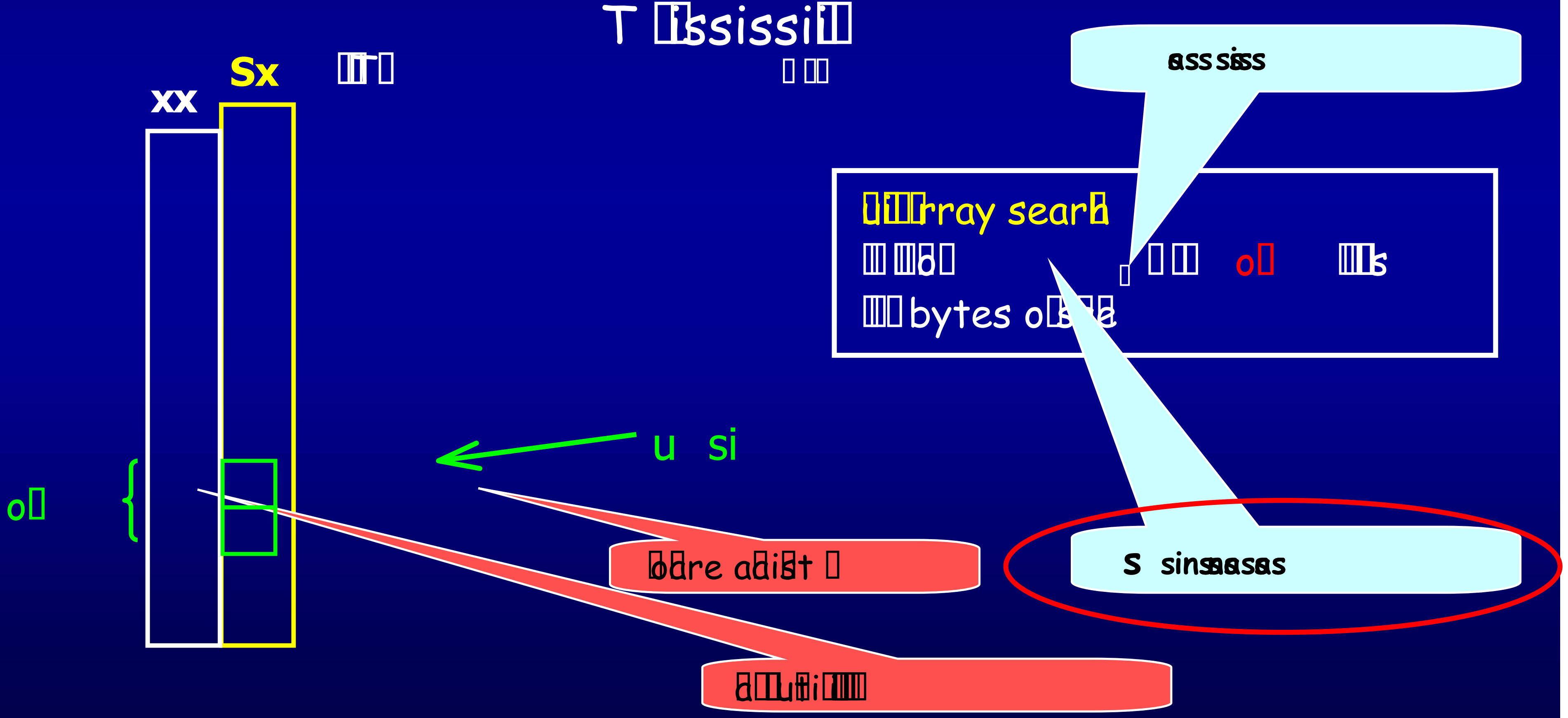

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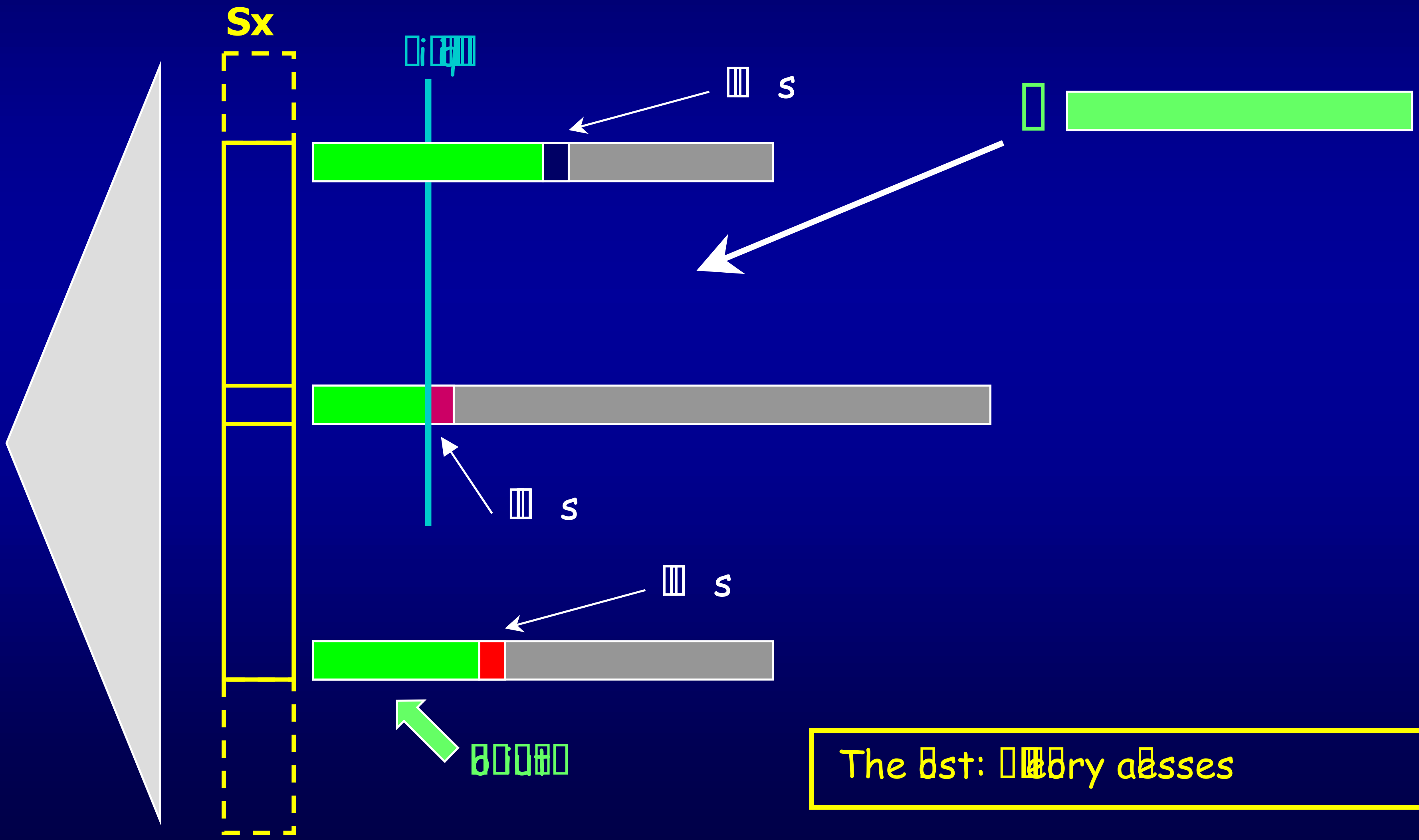

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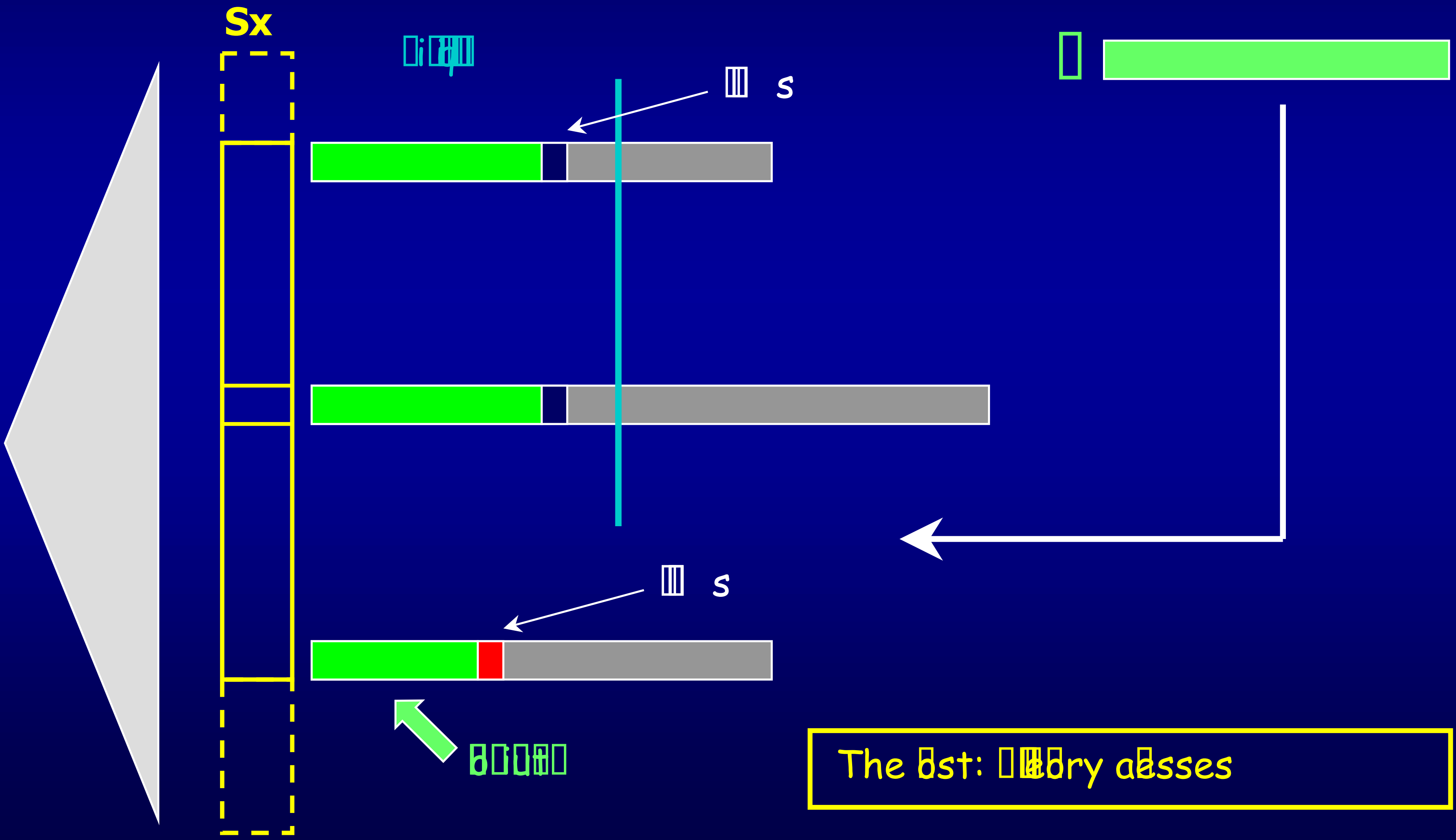

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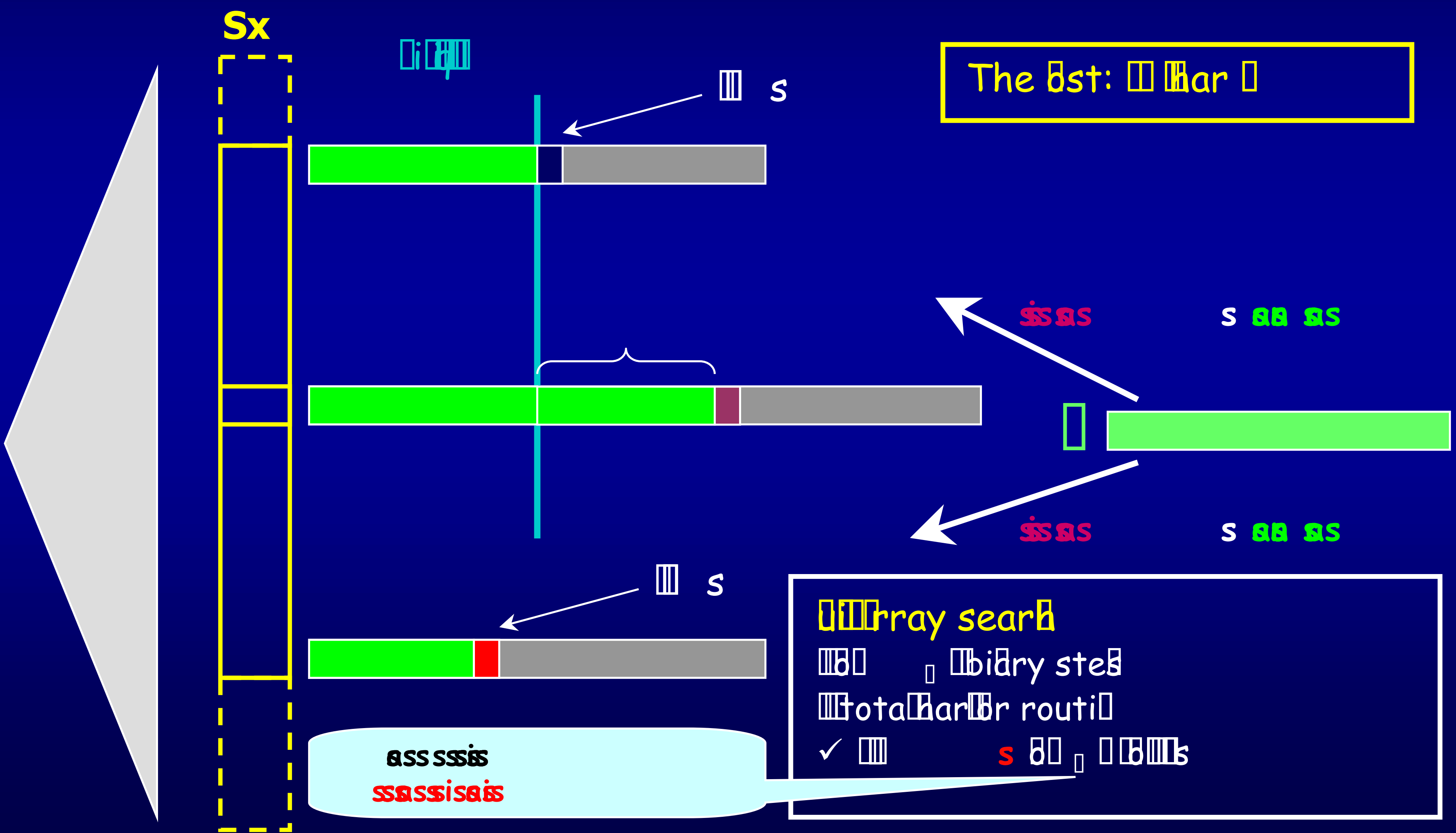

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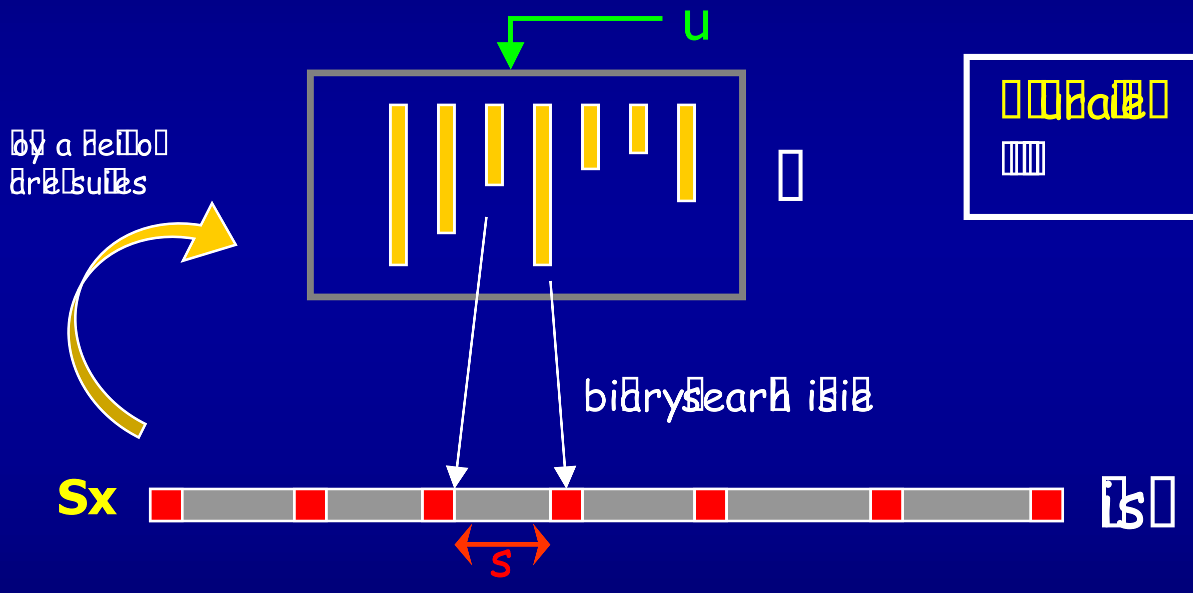
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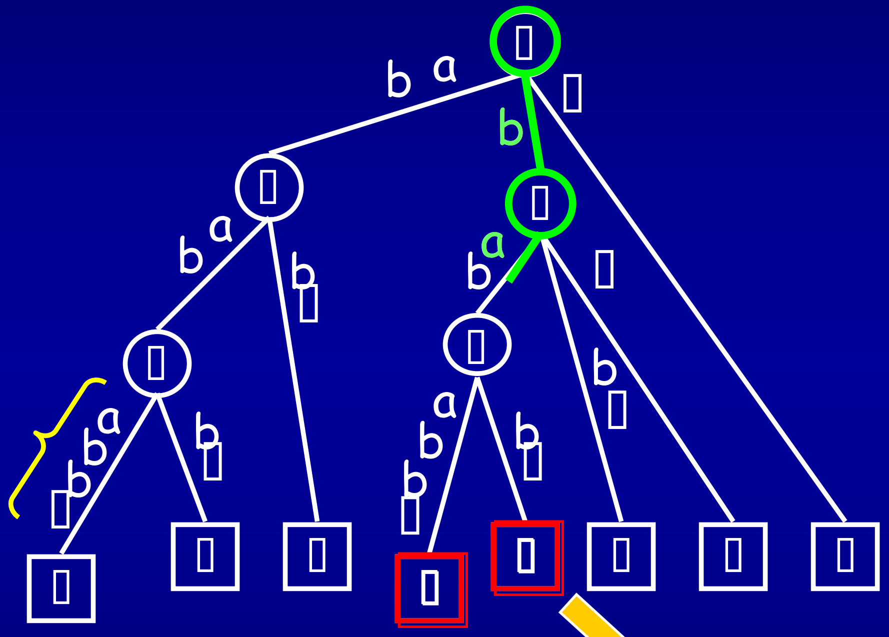

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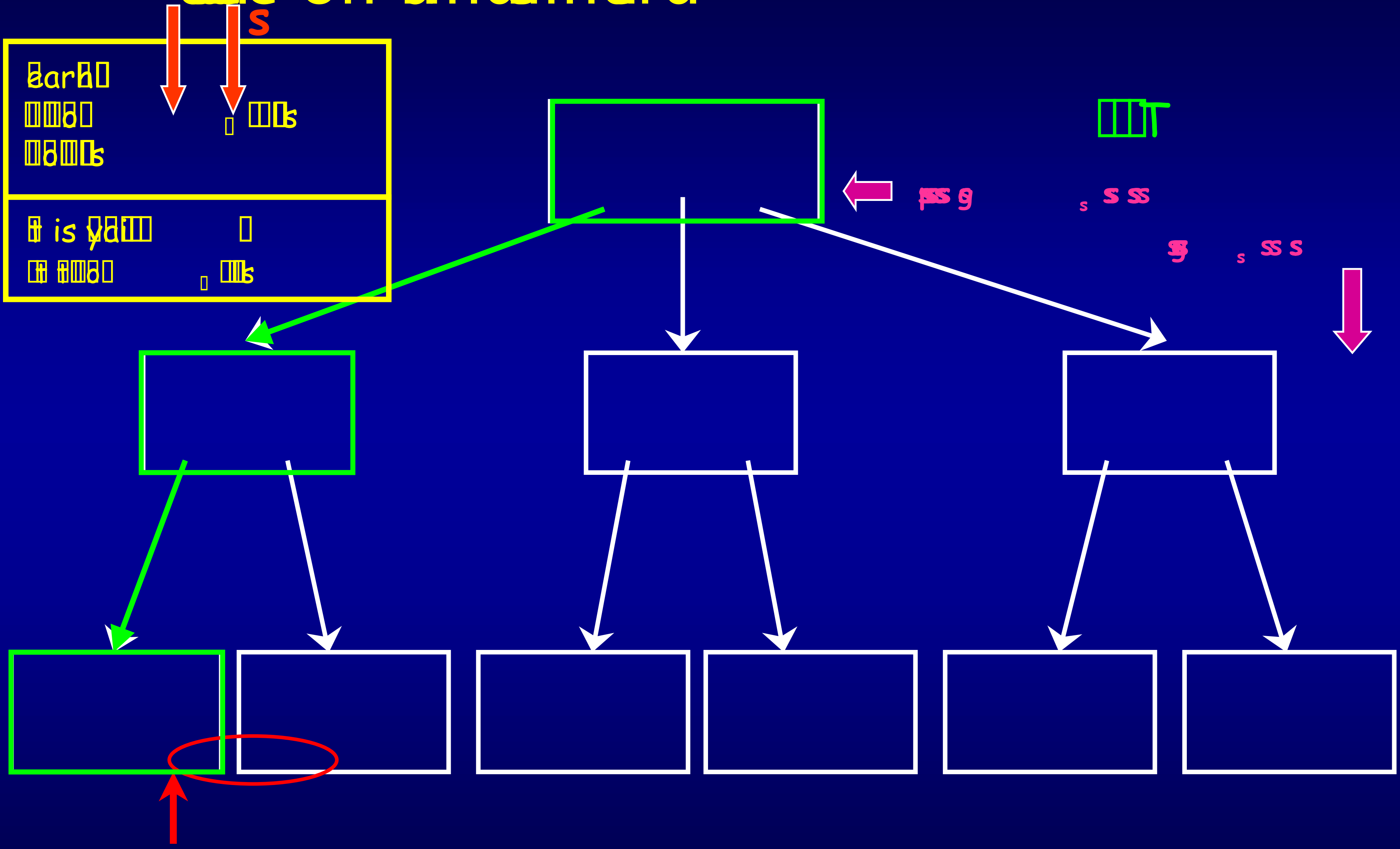

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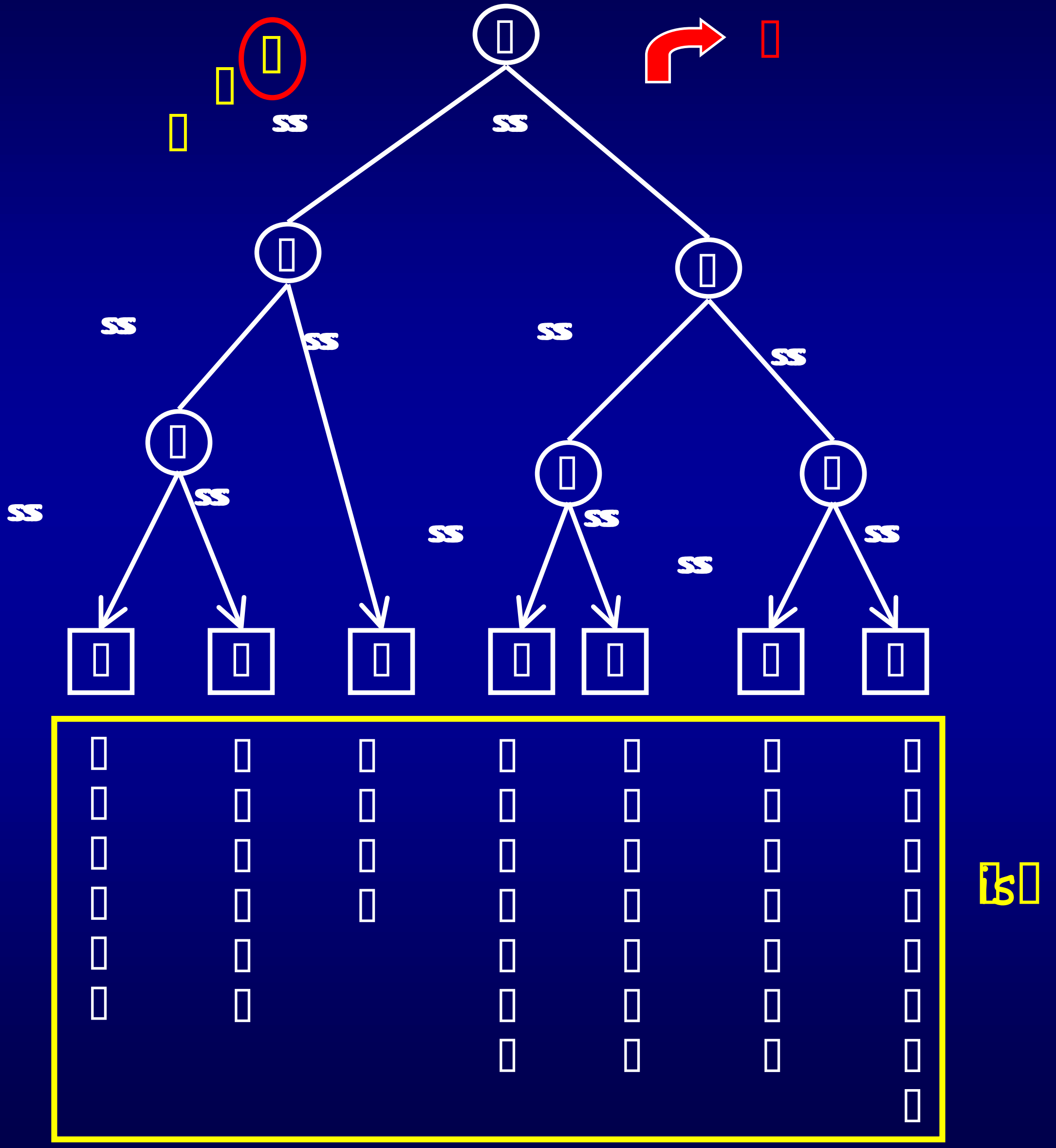

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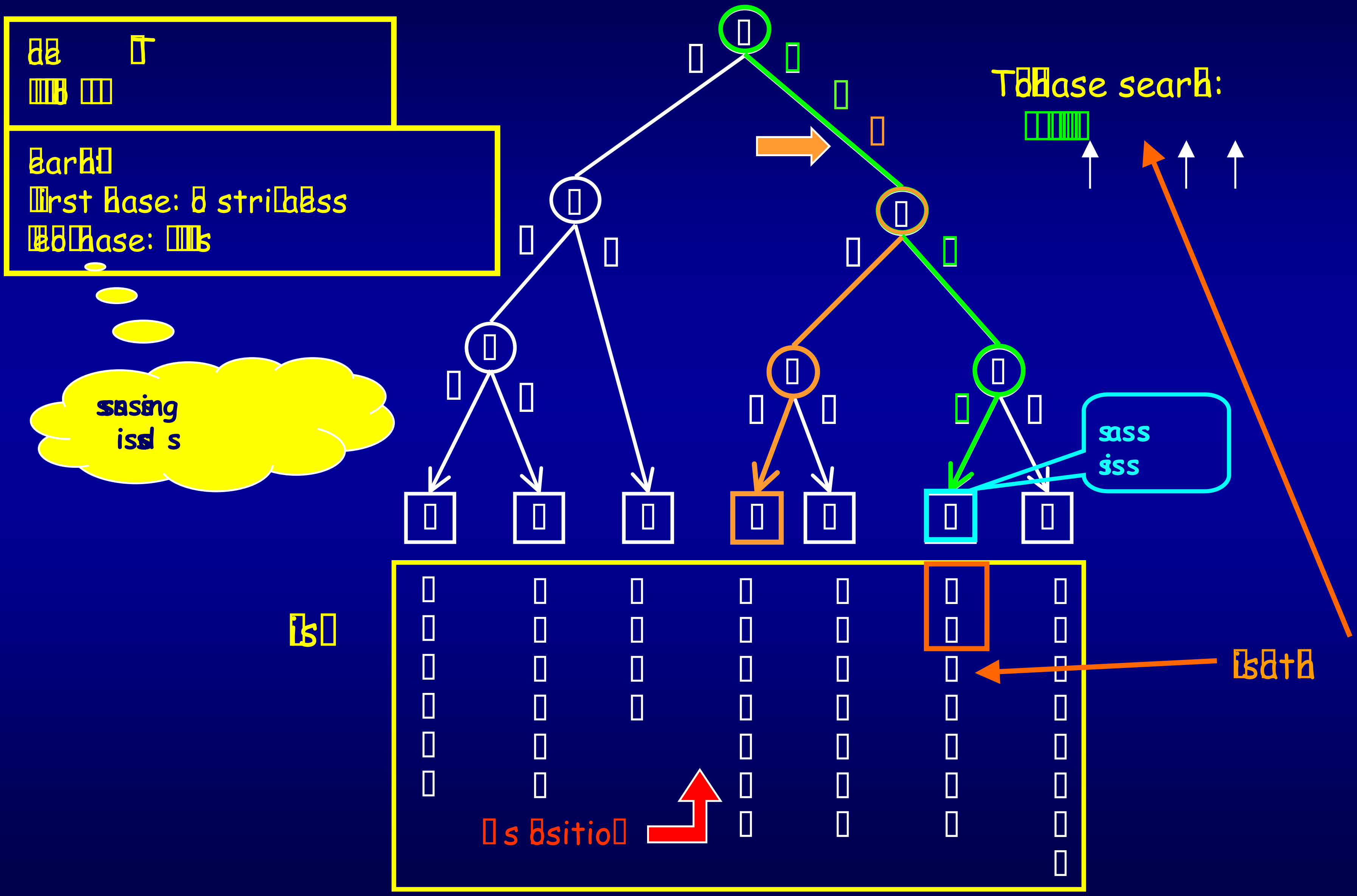

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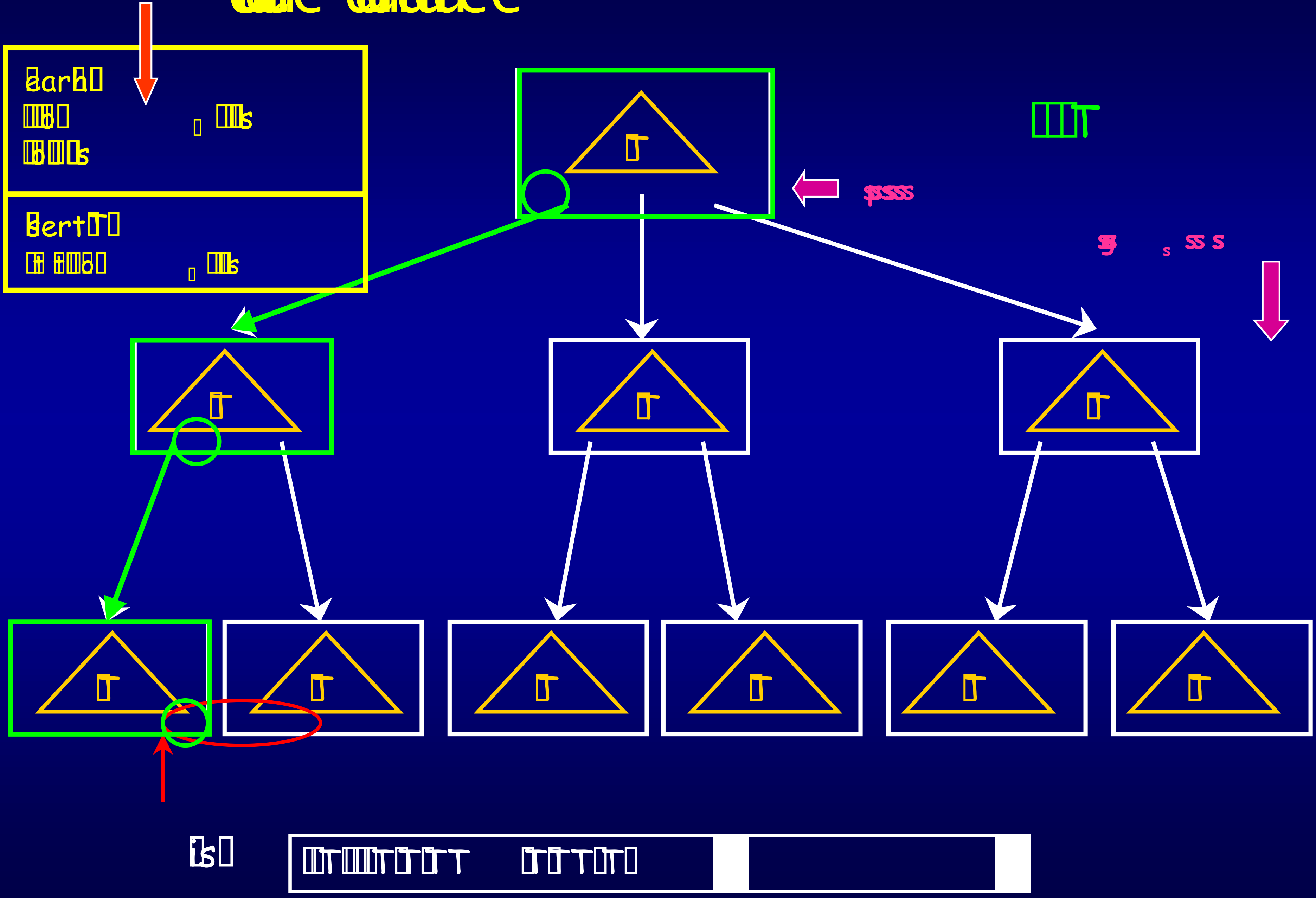

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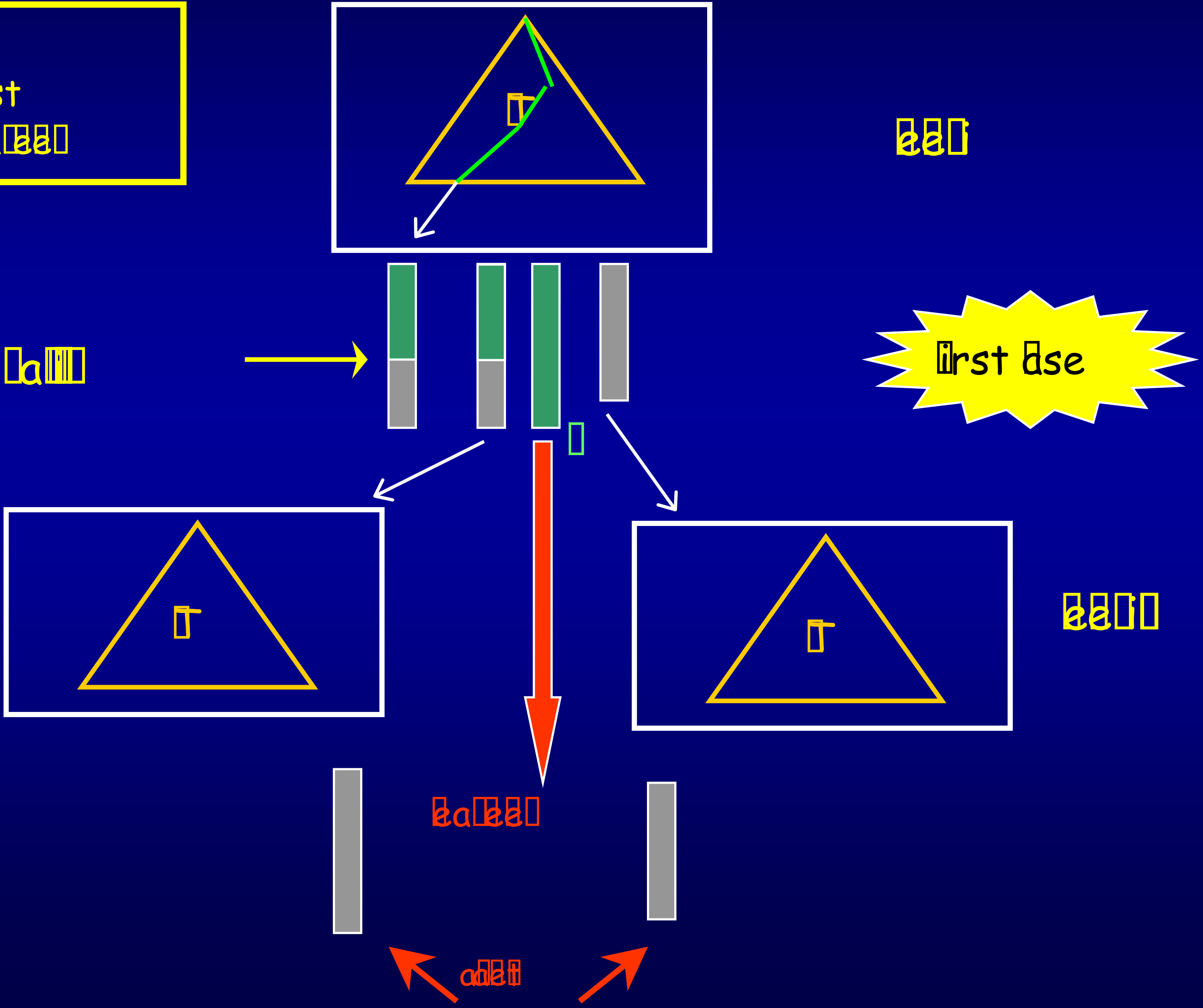

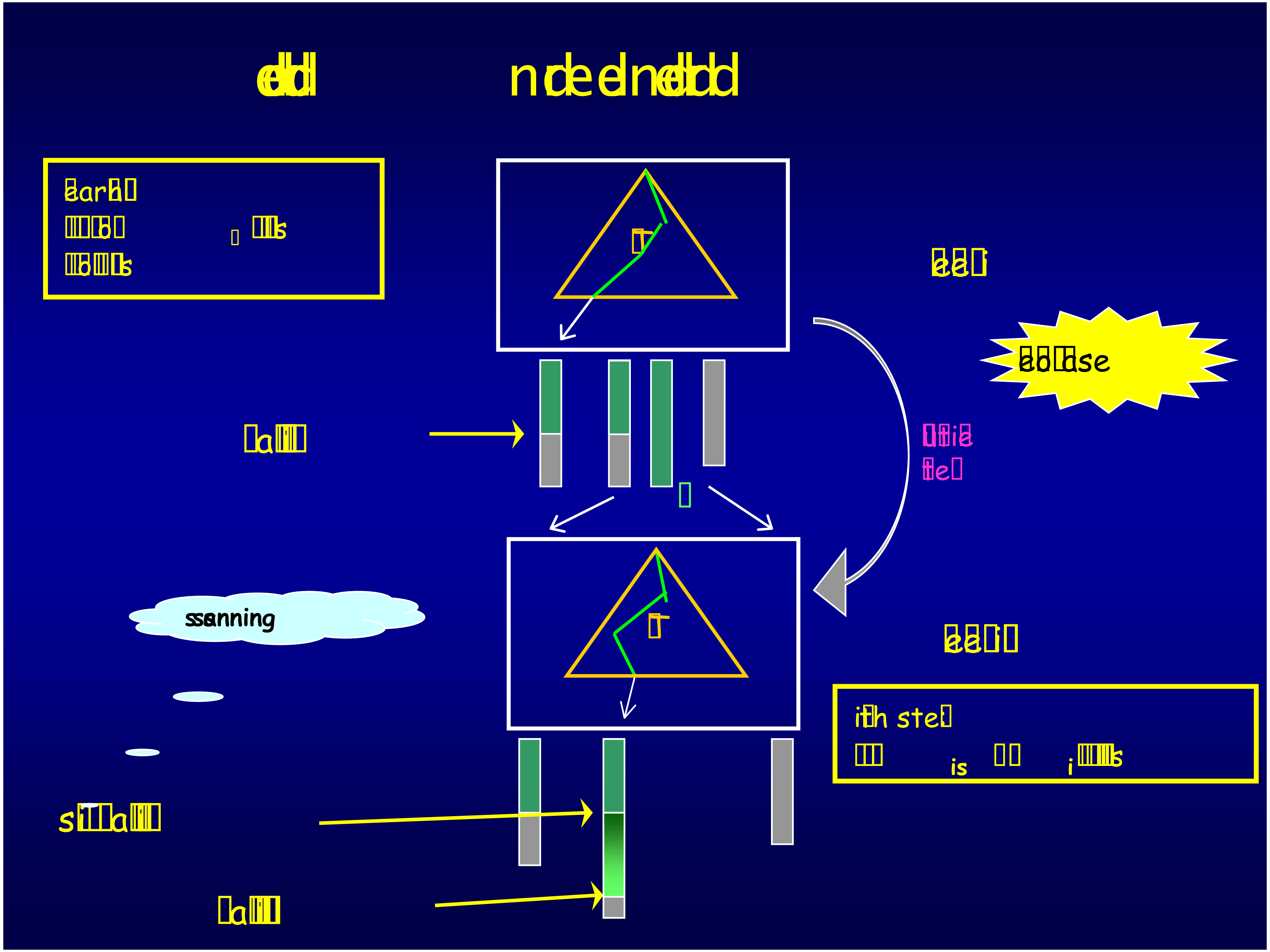

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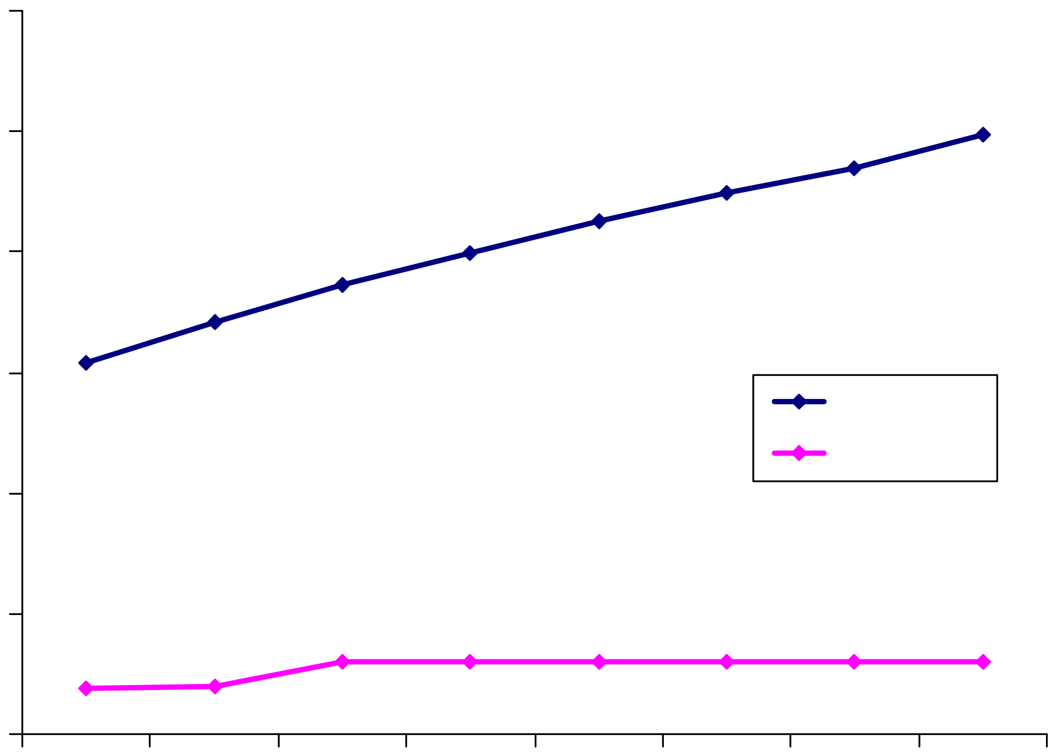

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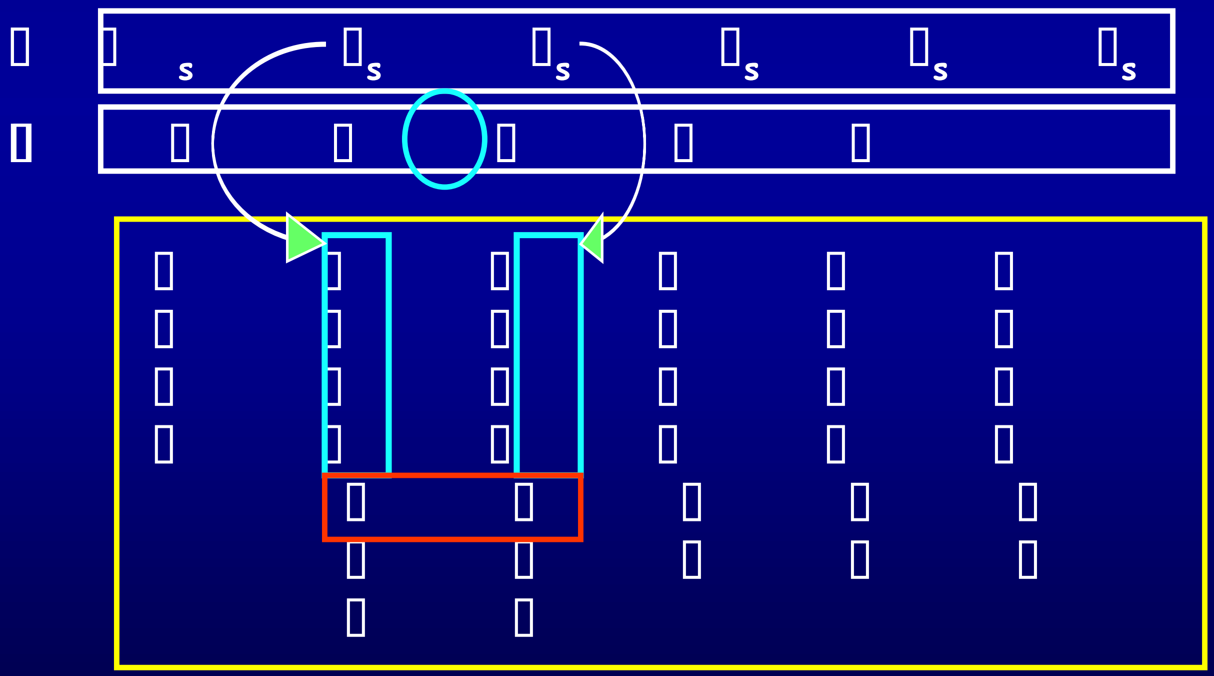

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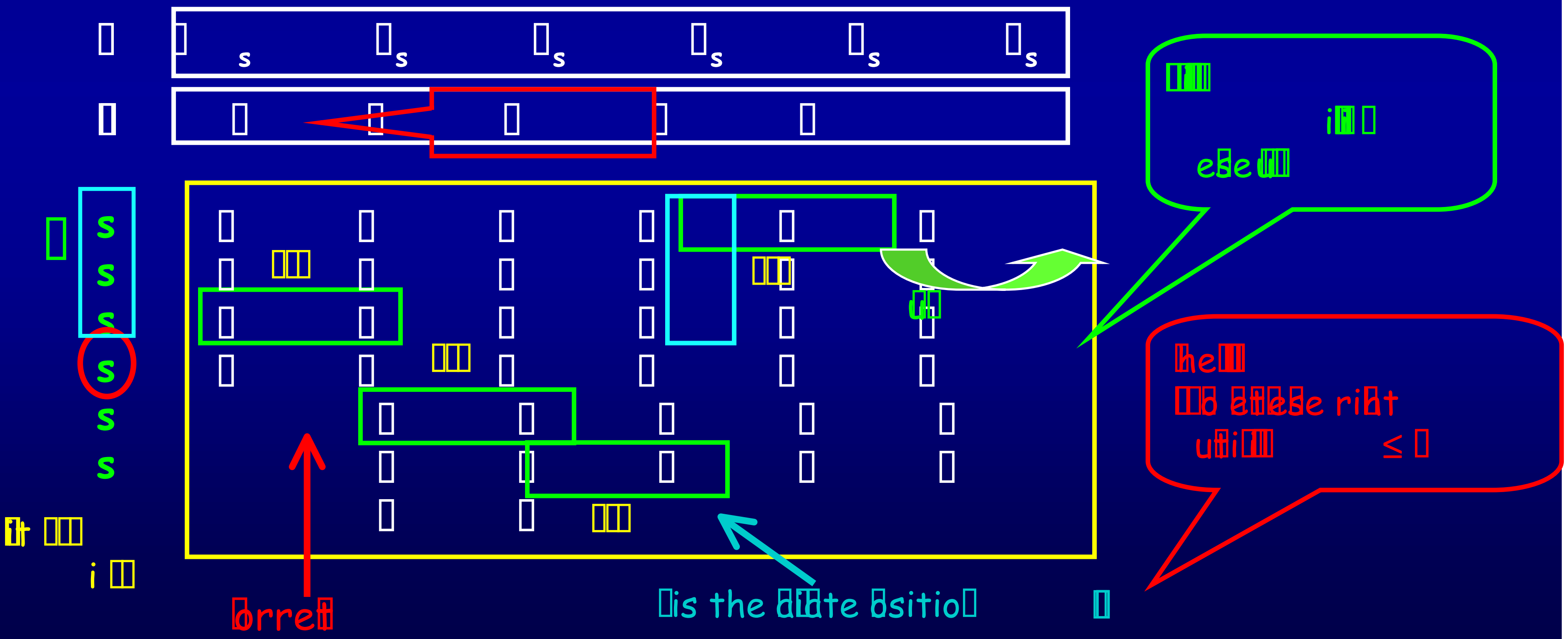

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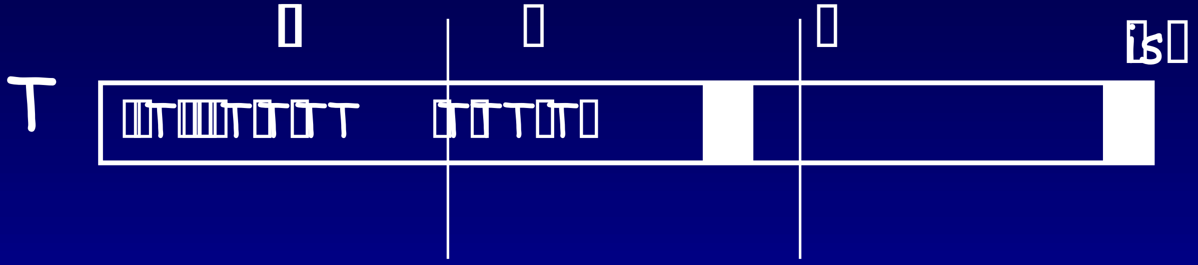

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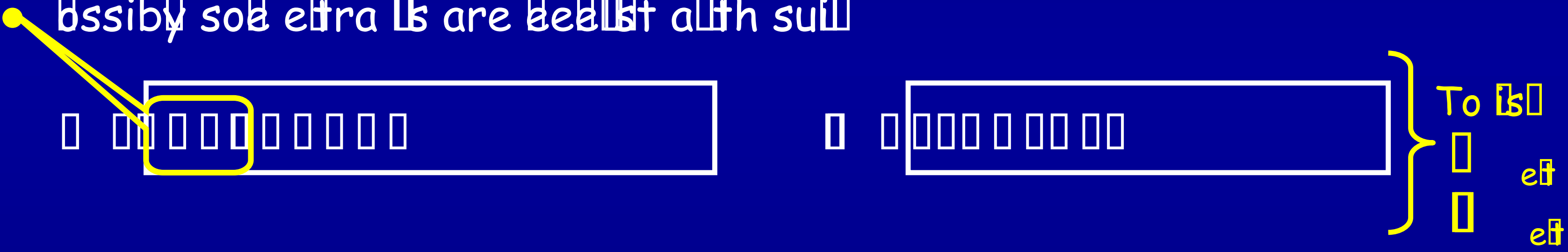

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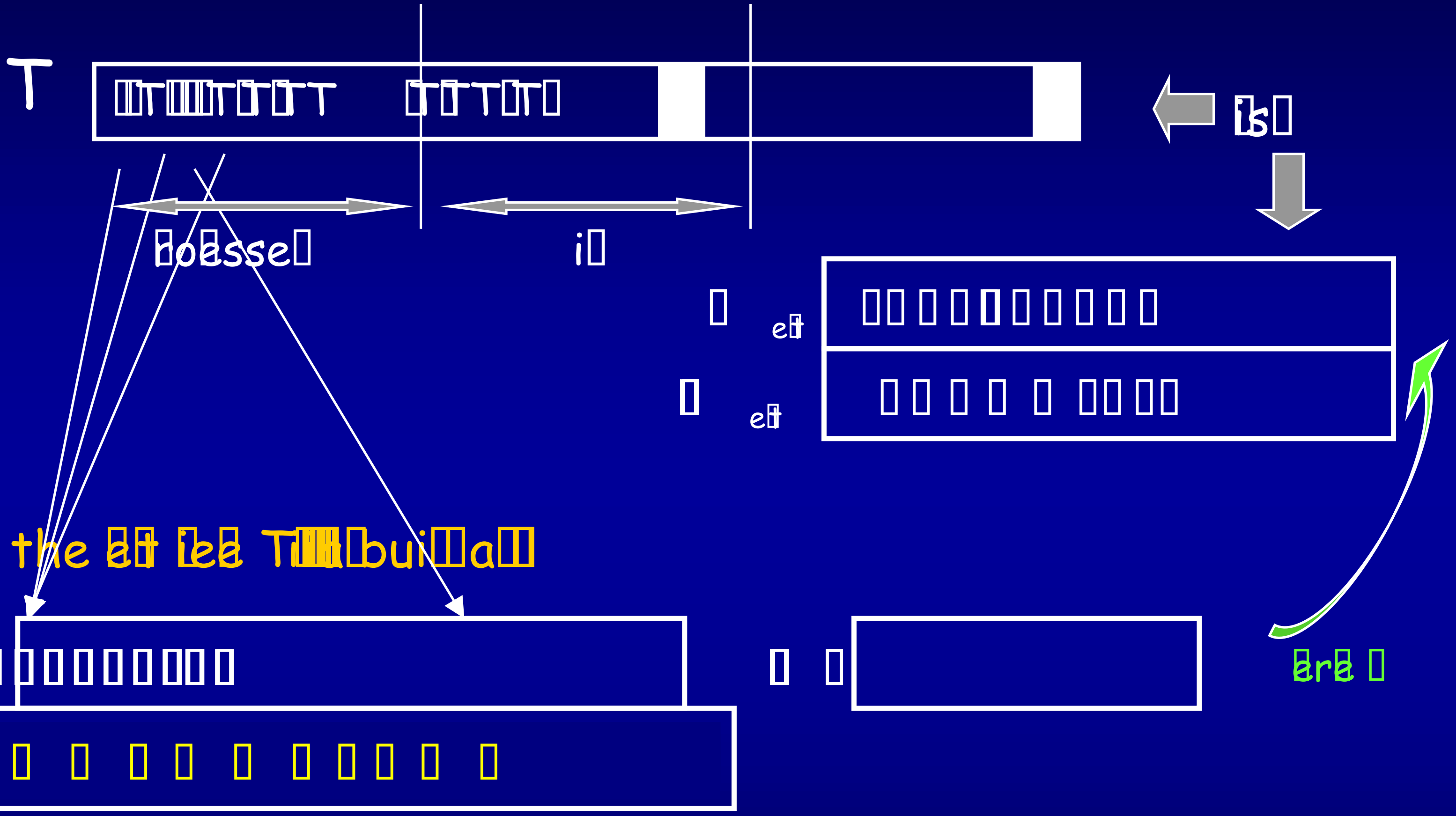

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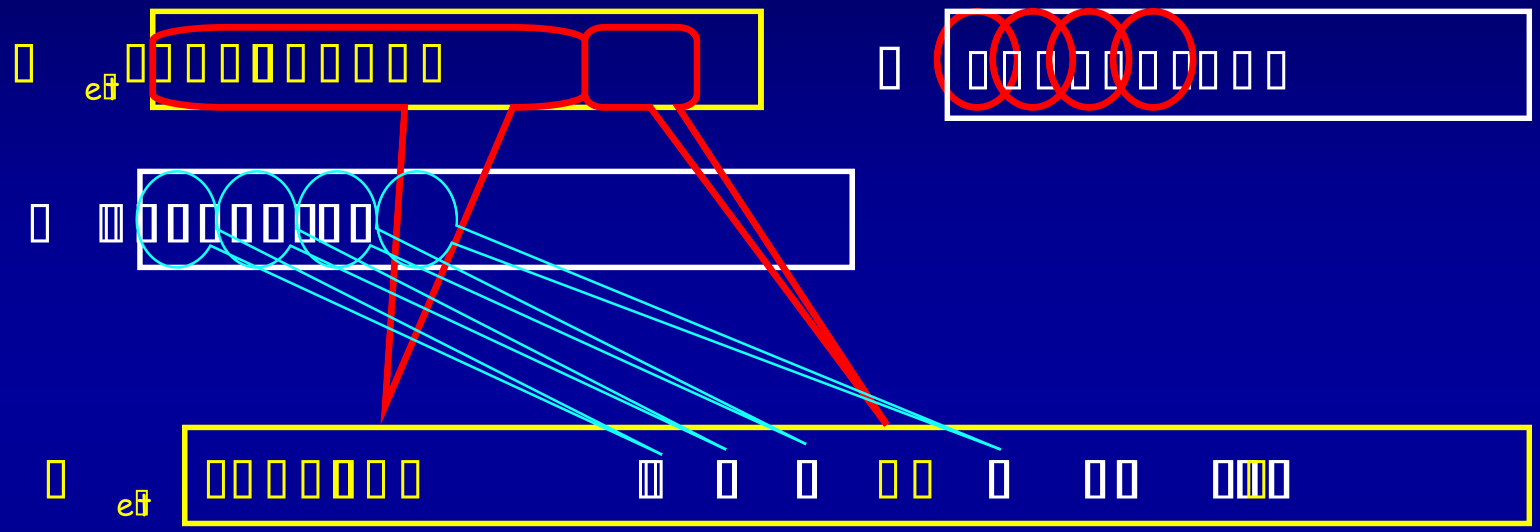

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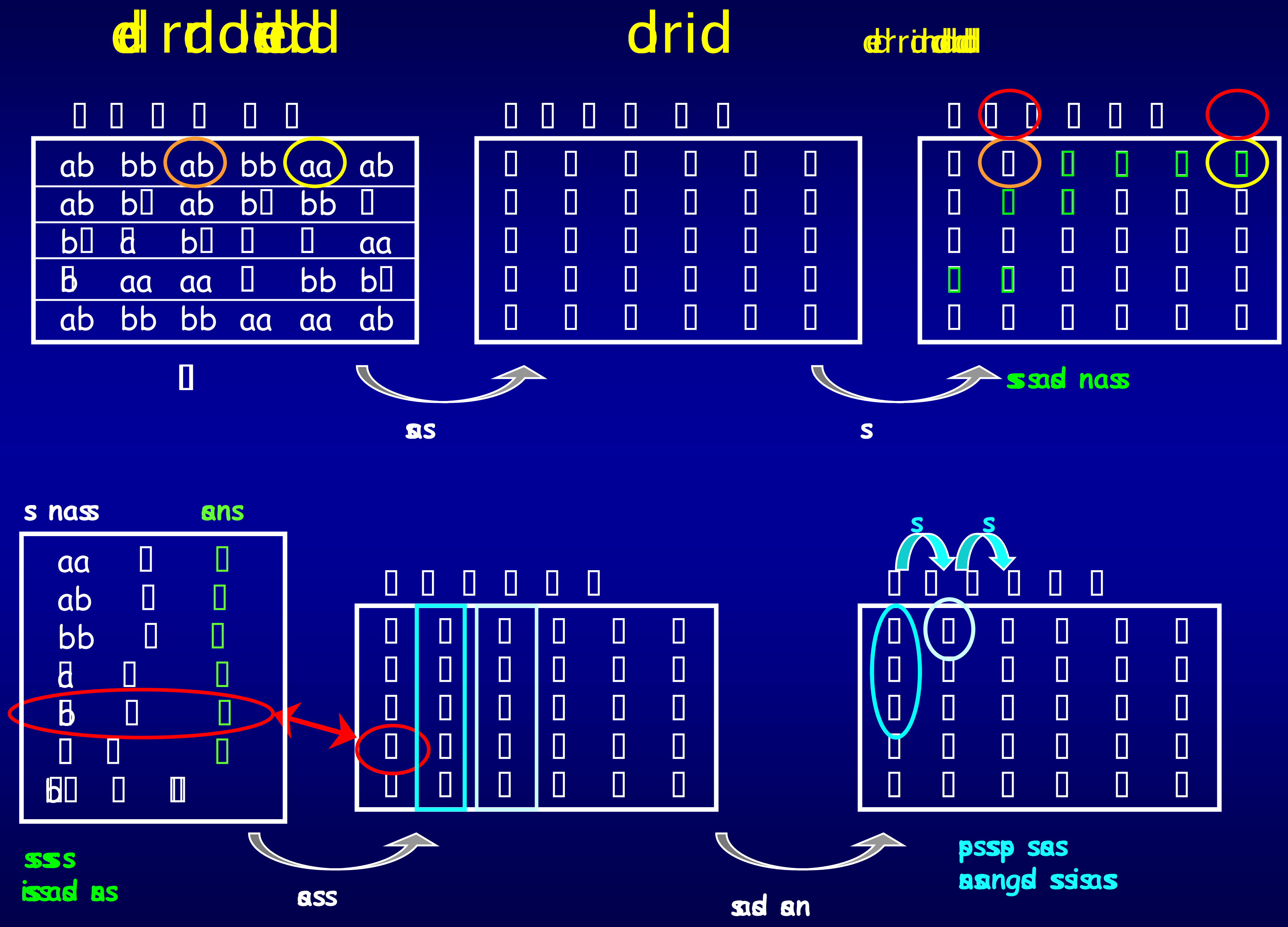

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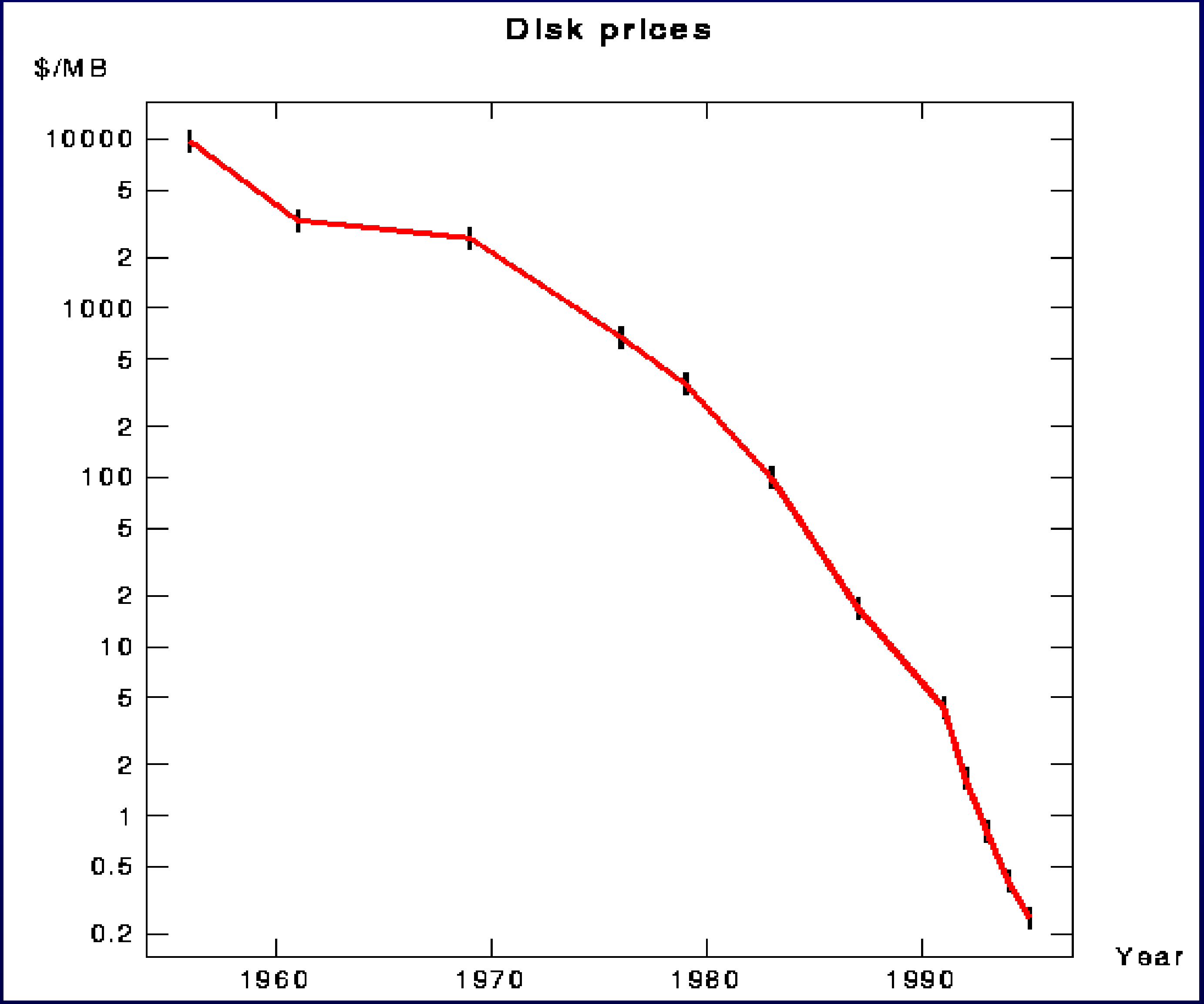

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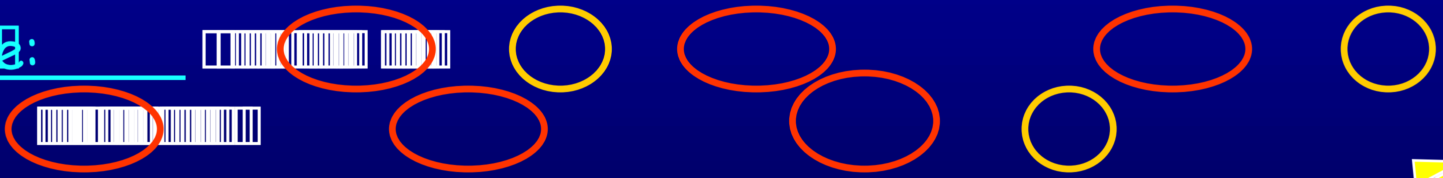

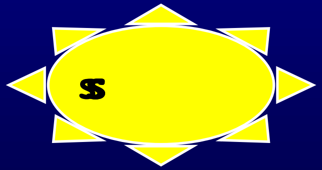

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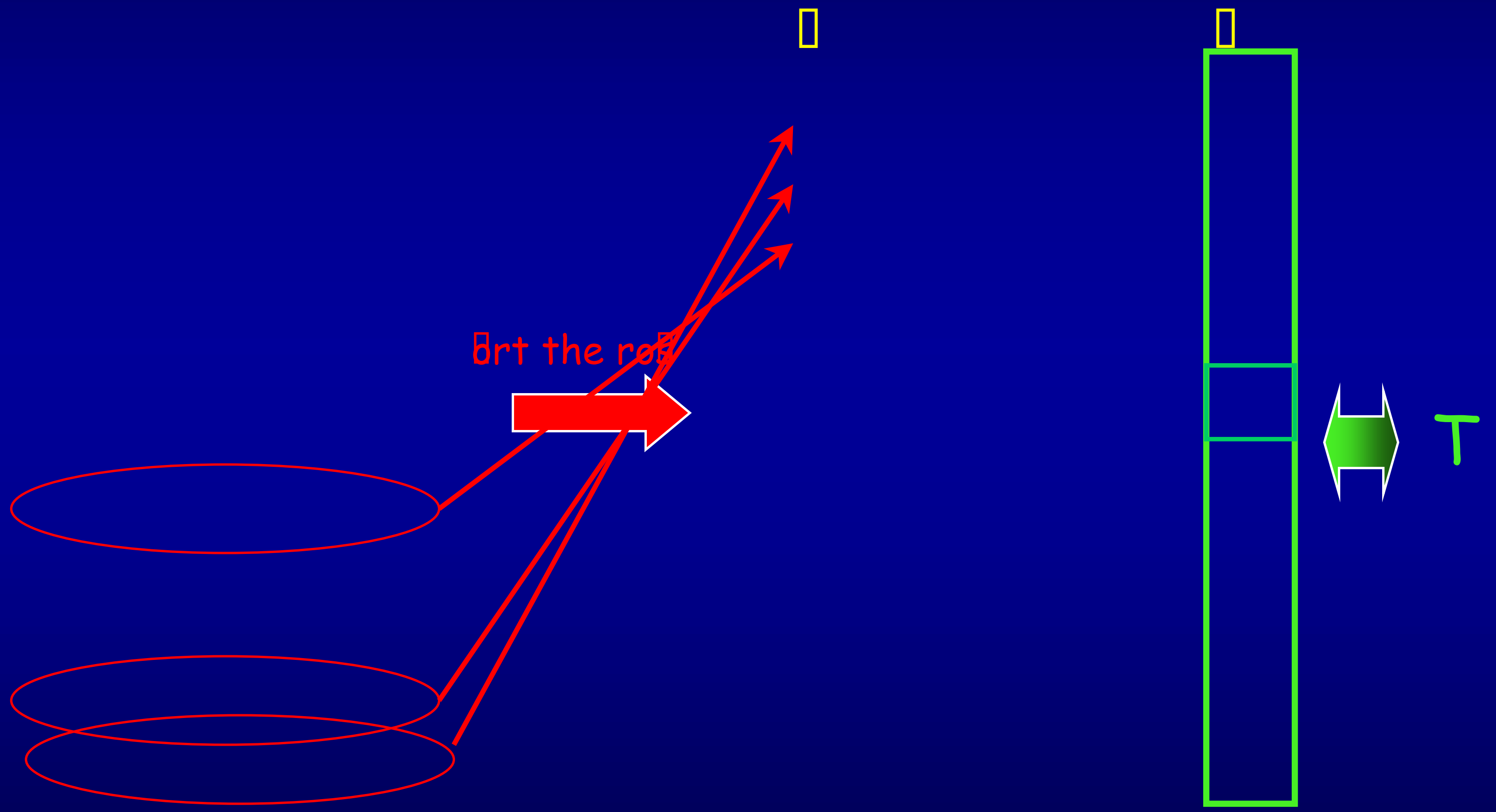

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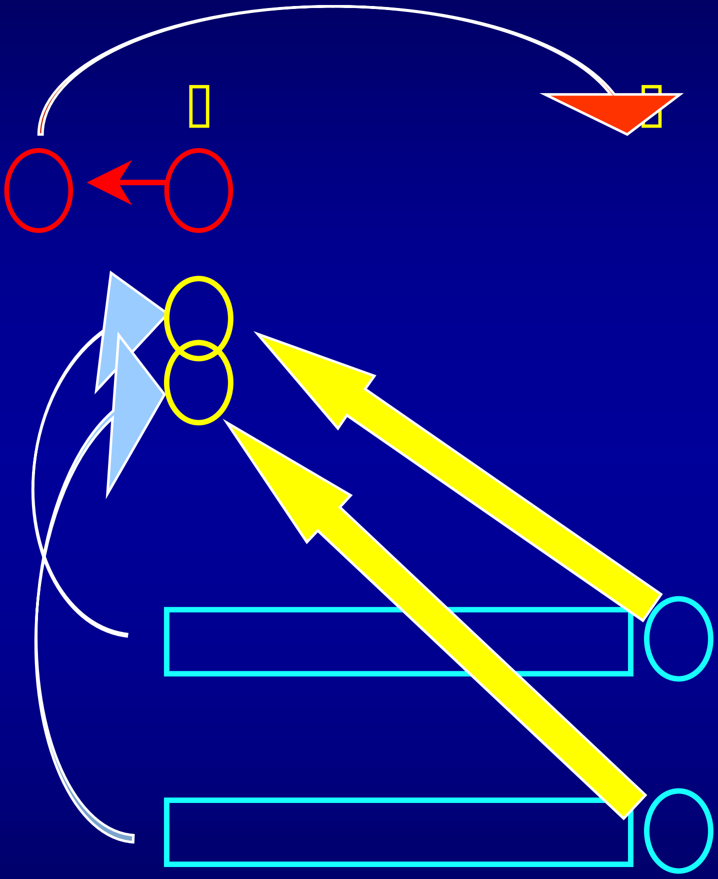

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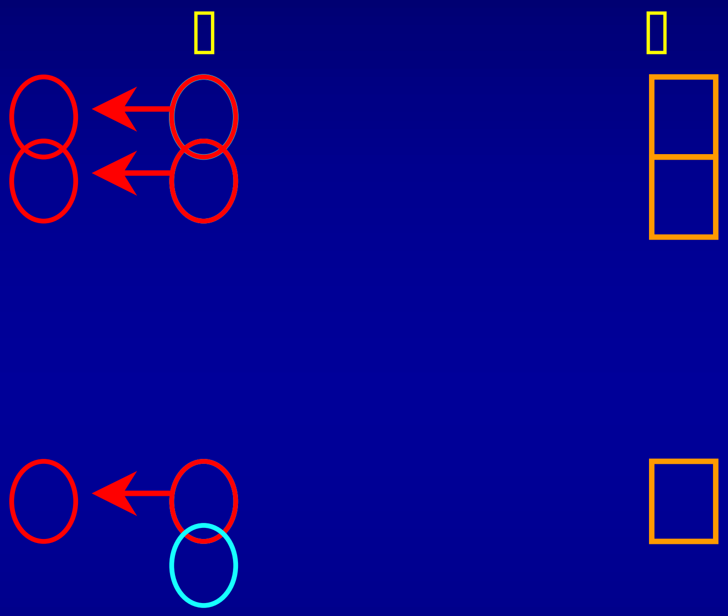

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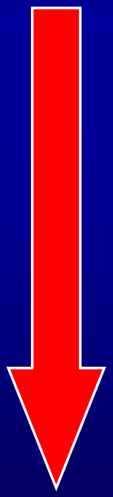

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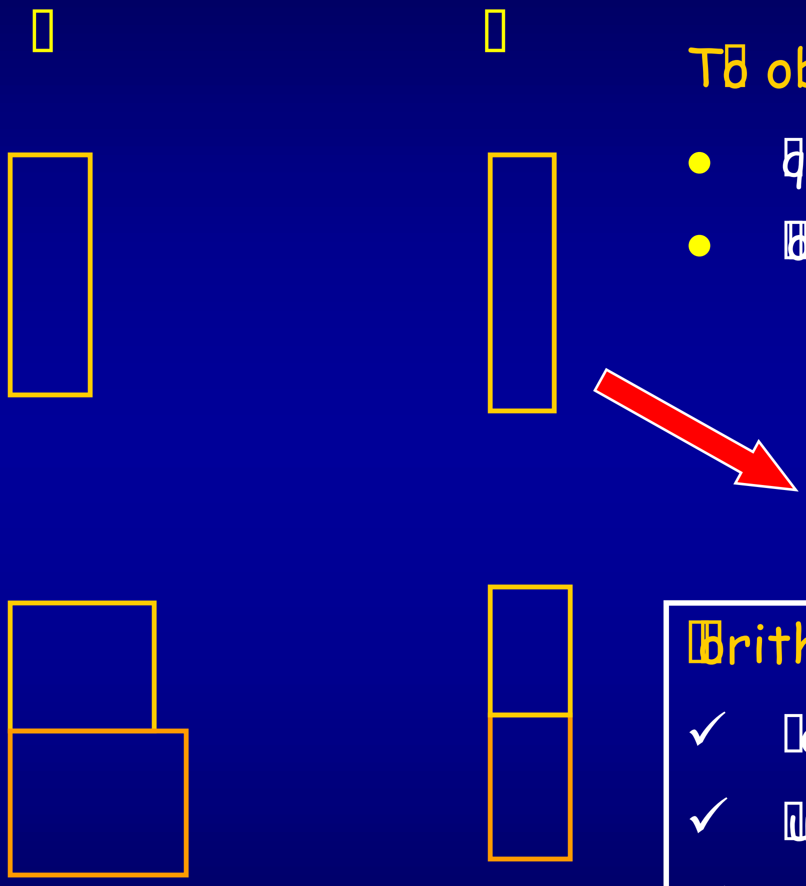

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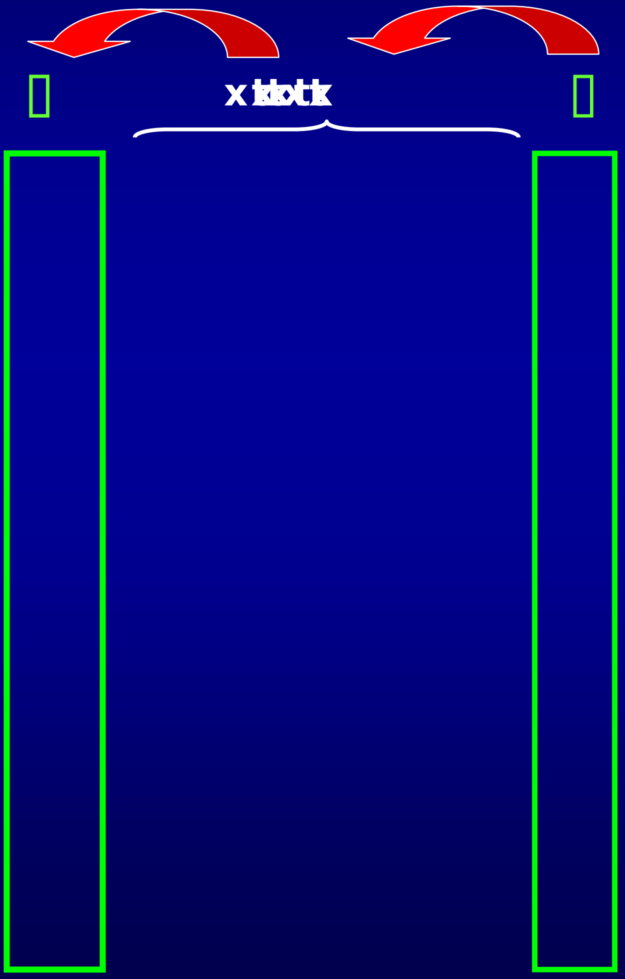

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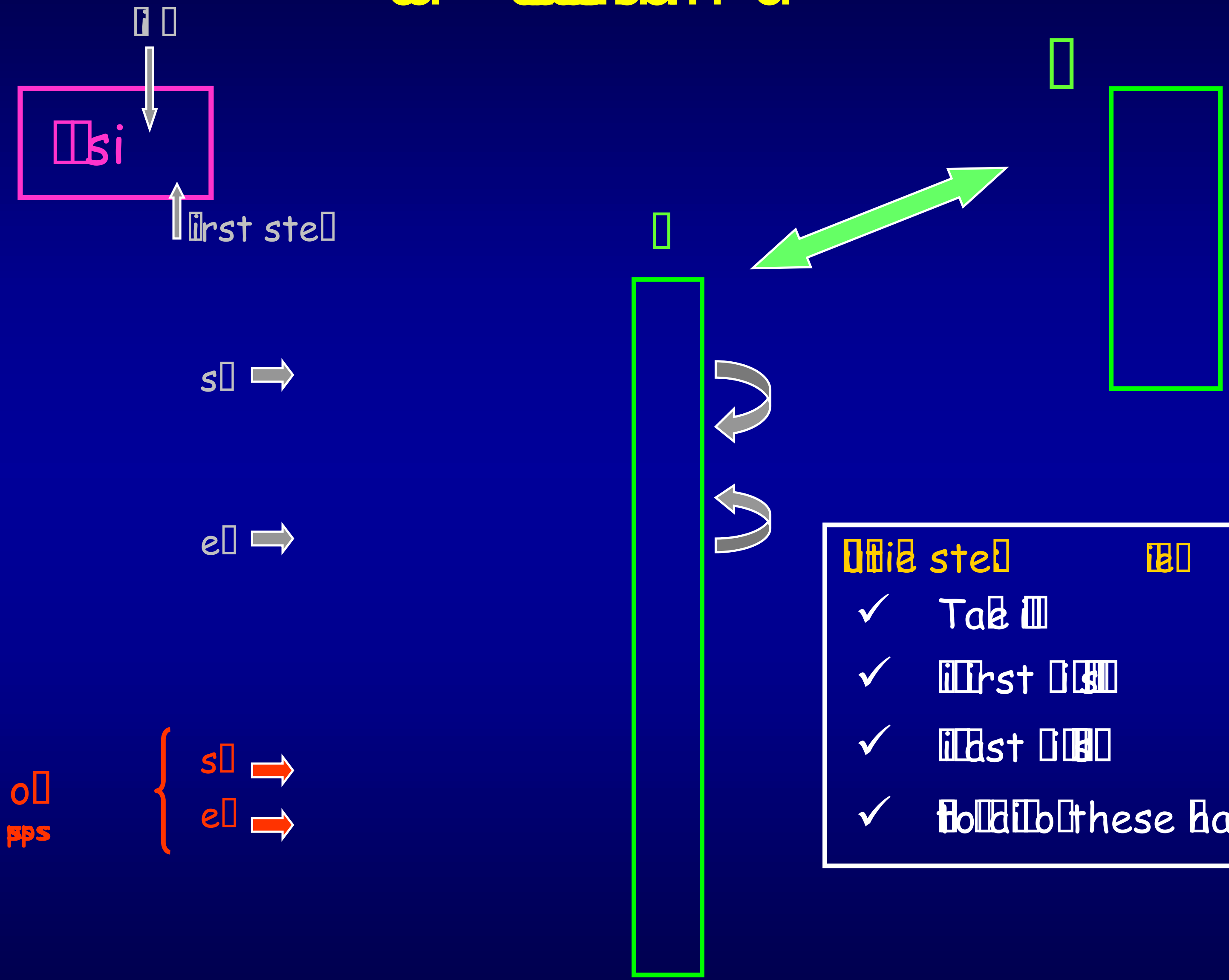

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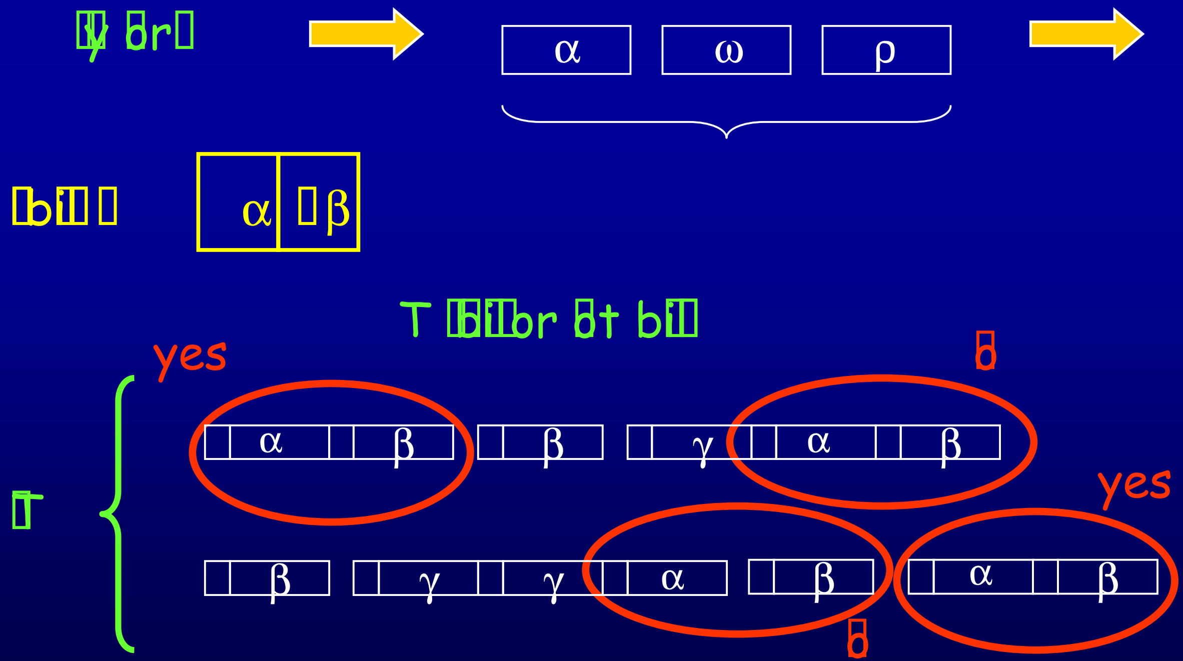

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