Static Program Analysis

Part 9 – control flow analysis

http://cs.au.dk/~amoeller/spa/

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Agenda

- **Control flow analysis for TIP with first-class functions**
- Control flow analysis for the λ-calculus
- The cubic framework
- Control flow analysis for object-oriented languages

TIP with first-class functions

```
inc(i) { return i+1; }
dec(j) { return j-1; }
ide(k) { return k; }

foo(n,f) {
  var r;
  if (n==0) { f=ide; }
  r = f(n);
  return r;
}

main() {
  var x,y;
  x = input;
  if (x>0) { y = foo(x,inc); } else { y = foo(x,dec); }
  return y;
}
```

Control flow complications

- First-class functions in TIP complicate CFG construction:
  - several functions may be invoked at a call site
  - this depends on the dataflow
  - but dataflow analysis first requires a CFG

- Same situation for other features:
  - function values with free variables (closures)
  - a class hierarchy with objects and methods
  - prototype objects with dynamic properties

Control flow analysis

- A control flow analysis approximates the call graph
  - conservatively computes possible functions at call sites
  - the trivial answer: *all* functions
- Control flow analysis is usually flow-*insensitive*:
  - based on the AST
  - the call graph can be used for an interprocedural CFG
  - a subsequent dataflow analysis may use the CFG
- Alternative: use flow-sensitive analysis
  - potentially on-the-fly, during dataflow analysis

CFA for TIP with first-class functions

- For a computed function call

  $E(E_1 , \ldots , E_n)$

  we cannot immediately see which function is called

- A coarse but sound approximation:
  - assume any function with right number of arguments

- Use CFA to get a much better result!

CFA constraints (1/2)

- Tokens are all functions $\{f_1, f_2, \ldots, f_k\}$
- For every AST node, v, we introduce the variable $[\![v]\!]$ denoting the set of functions to which v may evaluate

- For function definitions $f(\ldots)\{\ldots\}$:

  $f \in [\![f]\!]$

- For assignments $x = E$:

  $[\![E]\!] \subseteq [\![x]\!]$

CFA constraints (2/2)

- For **direct** function calls $f(E_1, \ldots, E_n)$:

  $[\![E_i]\!] \subseteq [\![a_i]\!]$ for i=1,...,n $\wedge$ $[\![E']\!] \subseteq [\![f(E_1, \ldots, E_n)]\!]$

  where f is a function with arguments $a_1, \ldots, a_n$ and return expression E'

- For **computed** function calls $E(E_1, \ldots, E_n)$:

  $f \in [\![E]\!] \Rightarrow ([\![E_i]\!] \subseteq [\![a_i]\!]$ for i=1,...,n $\wedge$ $[\![E']\!] \subseteq [\![(E)(E_1, \ldots, E_n)]\!])$

  for every function f with arguments $a_1, \ldots, a_n$

  and return expression E'

  – If we consider typable programs only:

    only generate constraints for those functions f
    for which the call would be type correct

Generated constraints

$\mathtt{inc} \in [\![\mathtt{inc}]\!]$
$\mathtt{dec} \in [\![\mathtt{dec}]\!]$
$\mathtt{ide} \in [\![\mathtt{ide}]\!]$
$[\![\mathtt{ide}]\!] \subseteq [\![\mathtt{f}]\!]$
$[\![\mathtt{f(n)}]\!] \subseteq [\![\mathtt{r}]\!]$
$\mathtt{inc} \in [\![\mathtt{f}]\!] \Rightarrow [\![\mathtt{n}]\!] \subseteq [\![\mathtt{i}]\!] \wedge [\![\mathtt{i+1}]\!] \subseteq [\![\mathtt{f(n)}]\!]$
$\mathtt{dec} \in [\![\mathtt{f}]\!] \Rightarrow [\![\mathtt{n}]\!] \subseteq [\![\mathtt{j}]\!] \wedge [\![\mathtt{j-1}]\!] \subseteq [\![\mathtt{f(n)}]\!]$
$\mathtt{ide} \in [\![\mathtt{f}]\!] \Rightarrow [\![\mathtt{n}]\!] \subseteq [\![\mathtt{k}]\!] \wedge [\![\mathtt{k}]\!] \subseteq [\![\mathtt{f(n)}]\!]$
$[\![\mathtt{input}]\!] \subseteq [\![\mathtt{x}]\!]$
$[\![\mathtt{foo(x,inc)}]\!] \subseteq [\![\mathtt{y}]\!]$
$[\![\mathtt{foo(x,dec)}]\!] \subseteq [\![\mathtt{y}]\!]$
$\mathtt{foo} \in [\![\mathtt{foo}]\!]$
$\mathtt{foo} \in [\![\mathtt{foo}]\!] \Rightarrow [\![\mathtt{x}]\!] \subseteq [\![\mathtt{n}]\!] \wedge [\![\mathtt{inc}]\!] \subseteq [\![\mathtt{f}]\!] \wedge [\![\mathtt{r}]\!] \subseteq [\![\mathtt{foo(x,inc)}]\!]$
$\mathtt{foo} \in [\![\mathtt{foo}]\!] \Rightarrow [\![\mathtt{x}]\!] \subseteq [\![\mathtt{n}]\!] \wedge [\![\mathtt{dec}]\!] \subseteq [\![\mathtt{f}]\!] \wedge [\![\mathtt{r}]\!] \subseteq [\![\mathtt{foo(x,dec)}]\!]$
$\mathtt{main} \in [\![\mathtt{main}]\!]$

assuming we do not use the special rule for direct calls

(At each call we only consider functions with matching number of parameters)

Least solution

$$[\![\mathtt{inc}]\!] = \{\mathtt{inc}\}$$
$$[\![\mathtt{dec}]\!] = \{\mathtt{dec}\}$$
$$[\![\mathtt{ide}]\!] = \{\mathtt{ide}\}$$
$$[\![\mathtt{f}]\!] = \{\mathtt{inc}, \mathtt{dec}, \mathtt{ide}\}$$
$$[\![\mathtt{foo}]\!] = \{\mathtt{foo}\}$$
$$[\![\mathtt{main}]\!] = \{\mathtt{main}\}$$

(the solution is the empty set for the remaining constraint variables)

With this information, we can construct the call edges and return edges in the interprocedural CFG

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CFA for the lambda calculus

- The pure lambda calculus

$Exp \rightarrow \lambda Id.Exp$		(function definition)
$\mid$	$Exp_1\ Exp_2$	(function application)
$\mid$	Id	(variable reference)

- Assume all λ-bound variables are distinct
- An *abstract closure* λx abstracts the function $\lambda x.E$ in all contexts (values of free variables)
- Goal: for each call site $E_1 E_2$ determine the possible functions for E_1 from the set $\{\lambda x_1, \lambda x_2, \ldots, \lambda x_n\}$

Closure analysis

A flow-insensitive analysis that tracks function values:

- For every AST node, v, we introduce a variable $[\![v]\!]$ ranging over subsets of abstract closures
- For $\lambda x.E$ we have the constraint

$$\lambda x \in [\![\lambda x.E]\!]$$

- For $E_1 E_2$ we have the *conditional* constraint

$$\lambda x \in [\![E_1]\!] \Rightarrow \left([\![E_2]\!] \subseteq [\![x]\!] \wedge [\![E]\!] \subseteq [\![E_1 E_2]\!]\right)$$

  for every function $\lambda x.E$

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The cubic framework

- We have a set of tokens T={t_1, t_2, ..., t_k}
- We have a collection of constraint variables V={x_1, ..., x_n} ranging over subsets of tokens
- A collection of constraints of these forms:
  - $t \in x$
  - $x \subseteq y$
  - $t \in x \Rightarrow y \subseteq z$
- Compute the unique minimal solution
  - this exists since solutions are closed under intersection
- A cubic time algorithm exists!

The solver data structure

- Each variable is mapped to a node in a directed graph
- Each node has a bitvector in $\{0,1\}^k$
  - initially set to all 0's
- Each bit has a list of pairs of variables
  - used to model conditional constraints
- The edges model inclusion constraints
- The bitvectors will at all times directly represent the minimal solution to the constraints seen so far

Implementation: `SimpleCubicSolver`

The solver data structure

- $x.sol \subseteq T$: the set of tokens for x (the bitvectors)
- $x.succ \subseteq V$: the successors of x (the edges)
- $x.cond(t) \subseteq V \times V$: the conditional constraints for x and t
- $W \subseteq T \times V$: a worklist (initially empty)

Adding constraints

- $t \in x$

  addToken(t, x)
  propagate()

- $x \subseteq y$

  addEdge(x, y)
  propagate()

- $t \in x \Rightarrow y \subseteq z$

  if t $\in$ x.sol
    addEdge(y, z)
    propagate()
  else
    add (y, z) to x.cond(t)

```
addToken(t, x):
  if t ∉ x.sol
    add t to x.sol
    add (t, x) to W

addEdge(x, y):
  if x ≠ y ∧ y ∉ x.succ
    add y to x.succ
    for each t in x.sol
      addToken(t, y)

propagate():
  while W ≠ ∅
    pick and remove (t, x) from W
    for each (y, z) in x.cond(t)
      addEdge(y, z)
    for each y in x.succ
      addToken(t, y)
```

Time complexity

- O(n) functions and O(n) applications, with program size n
- O(n) singleton constraints, O(n) subset constraints, O(n^2) conditional constraints
- O(n) nodes, O(n^2) edges, O(n) bits per node
- addToken takes time O(1)
- addEdge takes amortized time O(n)
- Each pair (t, x) is processed at most once by propagate
- O(n^2) calls to addEdge (either immediately or via propagate)
- O(n^3) calls to addToken

Time complexity

- Adding it all up, the upper bound is O(n^3)

- This is known as the *cubic time bottleneck*:
  - occurs in many different scenarios
  - but O(n^3/log n) is possible…

Implementation tricks

- Cycle elimination (collapse nodes if there is a cycle of inclusion constraints)
- Process worklist in topological order
- Interleaving solution propagation and constraint processing
- Shared bit vector representation
- Type filtering
- On-demand processing
- Difference propagation
- Subsumed node compaction
- ...

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Simple CFA for OO (1/3)

- CFA in an object-oriented language:

```
x.m(a,b,c)
```

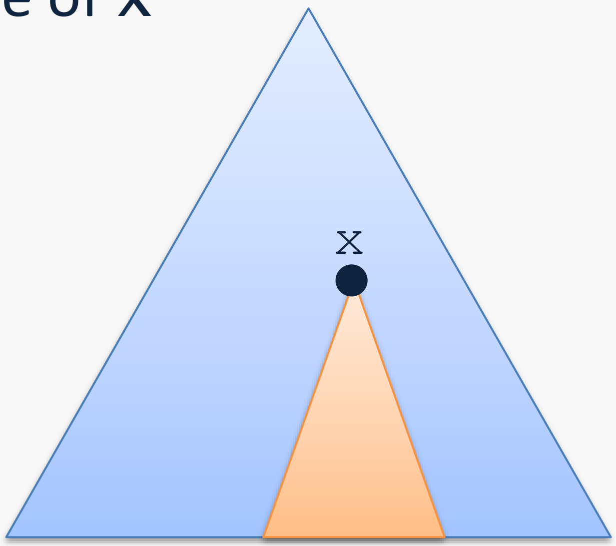
- Which method implementations may be invoked?

- Full CFA is a possibility...
- But the type information enables simpler solutions

Simple CFA for OO (2/3)

- Simplest solution:
  - select all methods named `m` with three arguments
- Class Hierarchy Analysis (CHA):
  - consider only the part of the class hierarchy rooted by the declared type of `x`

```
Collection<T> c = ...
c.add(e)
```



Simple CFA for OO (3/3)

- Rapid Type Analysis (RTA):
  - restrict to those classes that are actually used in the program in `new` expressions
  - start from `main`, iteratively find reachable methods