Static Program Analysis

Part 6 – path sensitivity and relational analysis

https://cs.au.dk/~amoeller/spa/

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Information in conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  z = z+x;
  if (17>y) { y = y+1; }
  x = x-1;
}
```

The interval analysis (with widening) concludes:

$x = [-\infty, \infty]$, $y = [0, \infty]$, $z = [-\infty, \infty]$

Modeling conditions

Add artifical “assert” statements:

The statement `assert(`*E*`)` models that
E is *true* in the current program state

- it causes a runtime error otherwise
- but we only insert it where the condition will always be true

Encoding conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  assert(x>0);
  z = z+x;
  if (17>y) { assert(17>y); y = y+1; }
  else { assert(!(17>y)); }
  x = x-1;
}
assert(!(x>0));
```

preserves semantics since `asserts` are guarded by conditions

(Alternatively, we could add dataflow constraints on the CFG *edges*)

Constraints for `assert`

- A trivial but sound constraint:

  $[\![v]\!] = JOIN(v)$

- A non-trivial constraint for `assert(`x`>`E`)`:

  $[\![v]\!] = JOIN(v)[x \rightarrow gt(JOIN(v)(x), eval(JOIN(v), E))]$

  where

$$gt([l_1, h_1], [l_2, h_2]) = \begin{cases} [l_1, h_1] & \text{if } l_2 < l_1 \\ [l_2+1, h_1] & \text{if } l_1 \le l_2 < h_1 \\ \bot & \text{otherwise} \end{cases}$$

- Similar constraints are defined for the dual cases
- More tricky to define for other conditions...

Exploiting conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  assert(x>0);
  z = z+x;
  if (17>y) { assert(17>y); y = y+1; }
  else { assert(!(17>y)); }
  x = x-1;
}
assert(!(x>0));
```

The interval analysis now concludes:

$x = [-\infty, 0],\ y = [0, 17],\ z = [0, \infty]$

Branch correlations

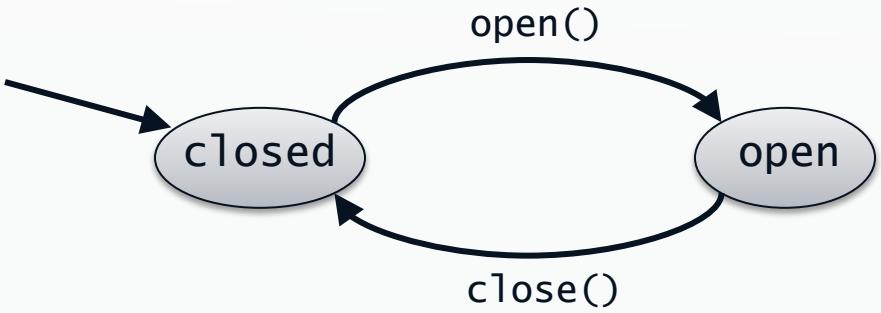
- With assert we have a simple form of *path sensitivity* (sometimes called *control sensitivity*)

- But it is insufficient to handle *correlation* of branches:

```
if (17 > x) { ... }
... // statements that do not change x
if (17 > x) { ... }
...
```

Open and closed files

- Built-in functions `open()` and `close()` on a file
- Requirements:
  - never `close` a closed file
  - never `open` an open file



- We want a static analysis to check this...
  (for simplicity, let us assume there is only one file)

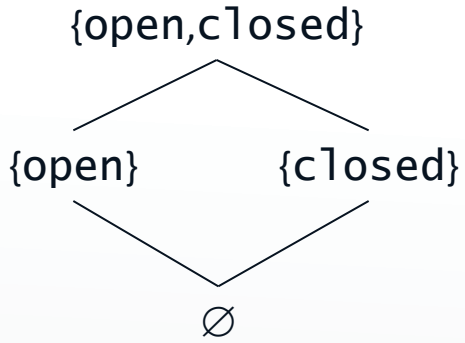
A tricky example

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

The naive analysis (1/2)

- The lattice models the status of the file:

$$L = (\mathcal{P}(\{\texttt{open}, \texttt{closed}\}), \subseteq)$$



- For every CFG node, v, we have a constraint variable $[\![v]\!]$ denoting the status *after* v

- $JOIN(v) = \bigcup_{w \in pred(v)} [\![w]\!]$

The naive analysis (2/2)

- Constraints for interesting statements:

  ⟦*entry*⟧ = {`closed`}

  ⟦`open()`⟧ = {open}

  ⟦`close()`⟧ = {`closed`}

- For all other CFG nodes:

  ⟦v⟧ = *JOIN*(v)

- Before the `close()` statement the analysis concludes that the file is {open,`closed`} ☹

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

The slightly less naive analysis

- We obviously need to keep track of the `flag` variable
- Our second attempt is the lattice:

$$L = (\mathcal{P}(\{\texttt{open}, \texttt{closed}\}) \times \mathcal{P}(\{\texttt{flag=0}, \texttt{flag}\neq\texttt{0}\}), \subseteq \times \subseteq)$$

- Additionally, we add `assert(...)` to model conditionals

- Even so, we still only know that the file is {`open`,`closed`} and that `flag` is {`flag=0`,`flag`≠`0`} ☹

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

Enhanced program

```
if (condition) {
  assert(condition);
  open();
  flag = 1;
} else {
  assert(!condition);
  flag = 0;
}
...
if (flag) {
  assert(flag);
  close();
} else {
  assert(!flag);
}
```

Relational analysis

- We need an analysis that keeps track of *relations* between variables
- One approach is to maintain *multiple* abstract states per program point, one for each *partition*
- For the file example we need the lattice:

L = *Partition* $\rightarrow \mathcal{P}(\{\texttt{open},\texttt{closed}\})$

(isomorphic to L = $\mathcal{P}$(*Partition*$\times\{\texttt{open},\texttt{closed}\}$))

where *Partition* = {flag=0,flag≠0}

Relational constraints (1/2)

- For the file statements:

  $[\![\mathit{entry}]\!] = \lambda p.\{\texttt{closed}\}$

  $[\![\texttt{open()}]\!] = \lambda p.\{\texttt{open}\}$

  $[\![\texttt{closed()}]\!] = \lambda p.\{\texttt{closed}\}$

- For `flag` assignments:

  $[\![\texttt{flag} = 0]\!] = [\texttt{flag}=0 \rightarrow \bigcup_{p \in P} \mathit{JOIN}(v)(p),\ \texttt{flag} \neq 0 \rightarrow \emptyset]$

  $[\![\texttt{flag} = n]\!] = [\texttt{flag} \neq 0 \rightarrow \bigcup_{p \in P} \mathit{JOIN}(v)(p),\ \texttt{flag} = 0 \rightarrow \emptyset]$

  where n is a non-0 constant number

  $[\![\texttt{flag} = E]\!] = \lambda q. \bigcup_{p \in P} \mathit{JOIN}(v)(p)$ for any other E

"infeasible"

Relational constraints (2/2)

- For `assert` statements:

$$[\![\texttt{assert(flag)}]\!] = [\texttt{flag}\neq 0 \rightarrow \mathit{JOIN}(v)(\texttt{flag}\neq 0), \texttt{flag}=0 \rightarrow \varnothing]$$

$$[\![\texttt{assert(!flag)}]\!] = [\texttt{flag}=0 \rightarrow \mathit{JOIN}(v)(\texttt{flag}=0), \texttt{flag}\neq 0 \rightarrow \varnothing]$$

- For all other CFG nodes:

$$[\![v]\!] = \mathrm{JOIN}(v) = \lambda p.\ \bigcup_{w \in \mathit{pred}(v)} [\![w]\!](p)$$

Generated constraints

$[\![entry]\!] = \lambda p.\{\texttt{closed}\}$
$[\![\texttt{condition}]\!] = [\![entry]\!]$
$[\![\texttt{assert(condition)}]\!] = [\![\texttt{condition}]\!]$
$[\![\texttt{open()}]\!] = \lambda p.\{\texttt{open}\}$
$[\![\texttt{flag = 1}]\!] = [\texttt{flag}\neq 0 \rightarrow \cup\, [\![\texttt{open()}]\!](p),\ \texttt{flag}=0 \rightarrow \varnothing]$
$[\![\texttt{assert(!condition)}]\!] = [\![\texttt{condition}]\!]$
$[\![\texttt{flag = 0}]\!] = [\texttt{flag}=0 \rightarrow \cup\, [\![\texttt{assert(!condition)}]\!](p),\ \texttt{flag}\neq 0 \rightarrow \varnothing]$
$[\![\texttt{...}]\!] = \lambda p.([\![\texttt{flag = 1}]\!](p) \cup [\![\texttt{flag = 0}]\!](p))$
$[\![\texttt{flag}]\!] = [\![\texttt{...}]\!]$
$[\![\texttt{assert(flag)}]\!] = [\![\texttt{flag}\neq 0 \rightarrow [\![\texttt{flag}]\!](\texttt{flag}\neq 0),\ \texttt{flag}=0 \rightarrow \varnothing]$
$[\![\texttt{close()}]\!] = \lambda p.\{\texttt{closed}\}$
$[\![\texttt{assert(!flag)}]\!] = [\texttt{flag}=0 \rightarrow [\![\texttt{flag}]\!](\texttt{flag}=0),\ \texttt{flag}\neq 0 \rightarrow \varnothing]$
$[\![exit]\!] = \lambda p.([\![\texttt{close()}]\!](p) \cup [\![\texttt{assert(!flag)}]\!](p))$

Minimal solution

	flag = 0	flag ≠ 0
⟦entry⟧	{closed}	{closed}
⟦condition⟧	{closed}	{closed}
⟦assert(condition)⟧	{closed}	{closed}
⟦open()⟧	{open}	{open}
⟦flag = 1⟧	∅	{open}
⟦assert(!condition)⟧	{closed}	{closed}
⟦flag = 0⟧	{closed}	∅
⟦...⟧	{closed}	{open}
⟦flag⟧	{closed}	{open}
⟦assert(flag)⟧	∅	{open}
⟦close()⟧	{closed}	{closed}
⟦assert(!flag)⟧	{closed}	∅
⟦exit⟧	{closed}	{closed}

We now know the file is `open` before `close()` ☺

Challenges

- The static analysis designer must choose the partitioning
  - often as boolean combinations of predicates from conditionals
  - iterative refinement (e.g. *counter-example guided abstraction refinement*) can be used for gradually finding relevant predicates
- Exponential blow-up:
  - for k predicates, we have 2^k different contexts
  - redundancy often cuts this down
- Reasoning about `assert`:
  - how to update the lattice elements with sufficient precision?
  - possibly involves heavy-weight theorem proving

Improvements

- Run auxiliary analyses first, for example:
  - constant propagation
  - sign analysis

  will help in handling `flag` assignments

- Dead code propagation, change

  $[\![\texttt{open}()]\!] = \lambda p.\{\texttt{open}\}$

  into the still sound but more precise

  $[\![\texttt{open}()]\!] = \lambda p.\text{if } \mathit{JOIN}(v)(p)=\varnothing \text{ then } \varnothing \text{ else } \{\texttt{open}\}$

Trace partitioning

Can we use path sensitivity to avoid joining abstract states at CFG merge points? (See Exercise 7.14)

```
if (condition) {
  x = 0;
  y = 0;
} else {
  x = 1;
  y = 1;
}
z = x-y;
// z is 0
```