Succinct Persistent Adaptive Garbled RAM
or
How To Delegate Your Database

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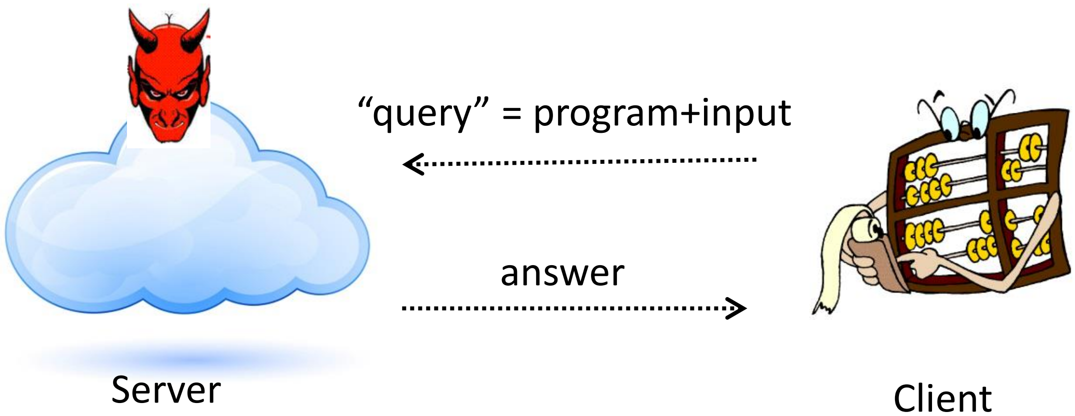
TAU and BU

Based on joint works with

Justin Holmgren, Yilei Chen, Mariana Raykova

ePrint reports 2015/388 and 2015/1074

Delegating Computation



Verifiability + **Privacy** +

Efficiency

Bandwidth

Server

Client

Delegating Computation

"Old-fashioned" Setting: Small input + Big Computations

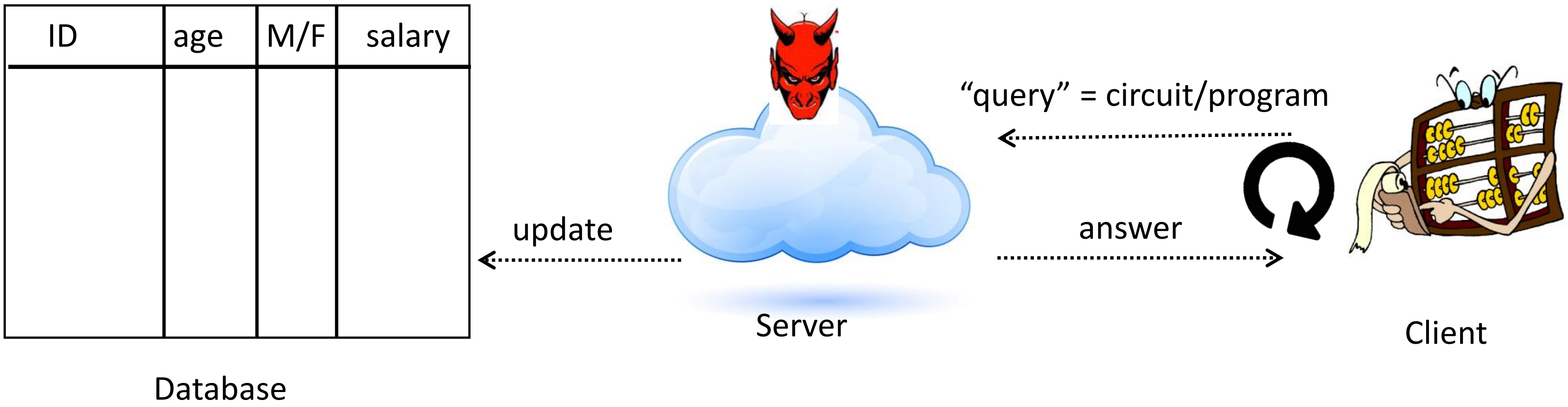
- Verifiable Computation Protocols [Blum-Kannan89, Blum-Luby-Rubinfeld90, Kilian92,Micali00, Ergun-Kumar-Rubinfeld99, Goldwasser-Kalai-Rothblum08, Gennaro-Gentry-Parno10…]

- Fully Homomorphic Encryption [Gentry09 … … …]

➔ Client work + Bandwidth proportional to input size

Today: Big Data + Small Computations

Delegating ~~Computation~~ Databases



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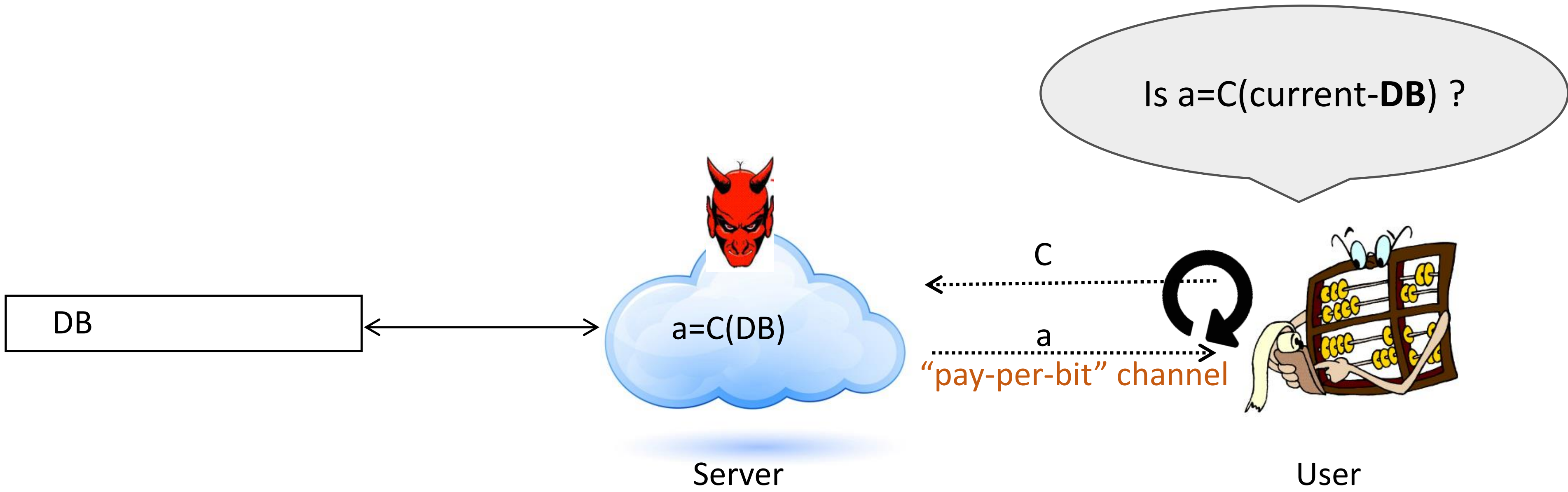
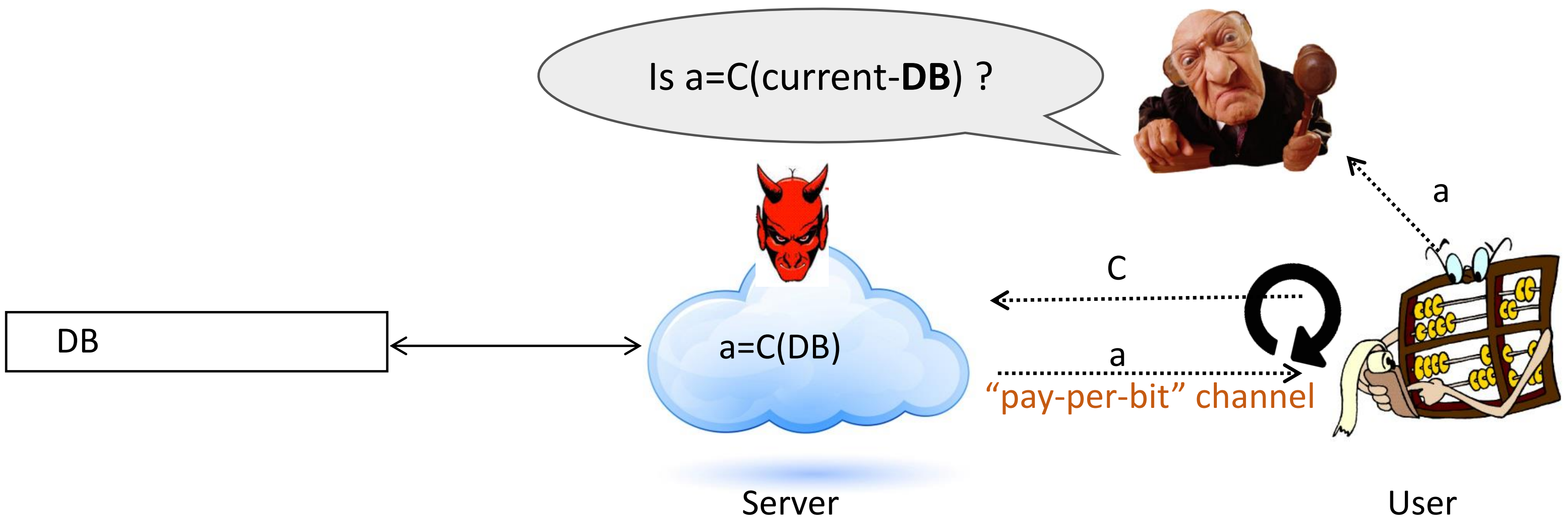
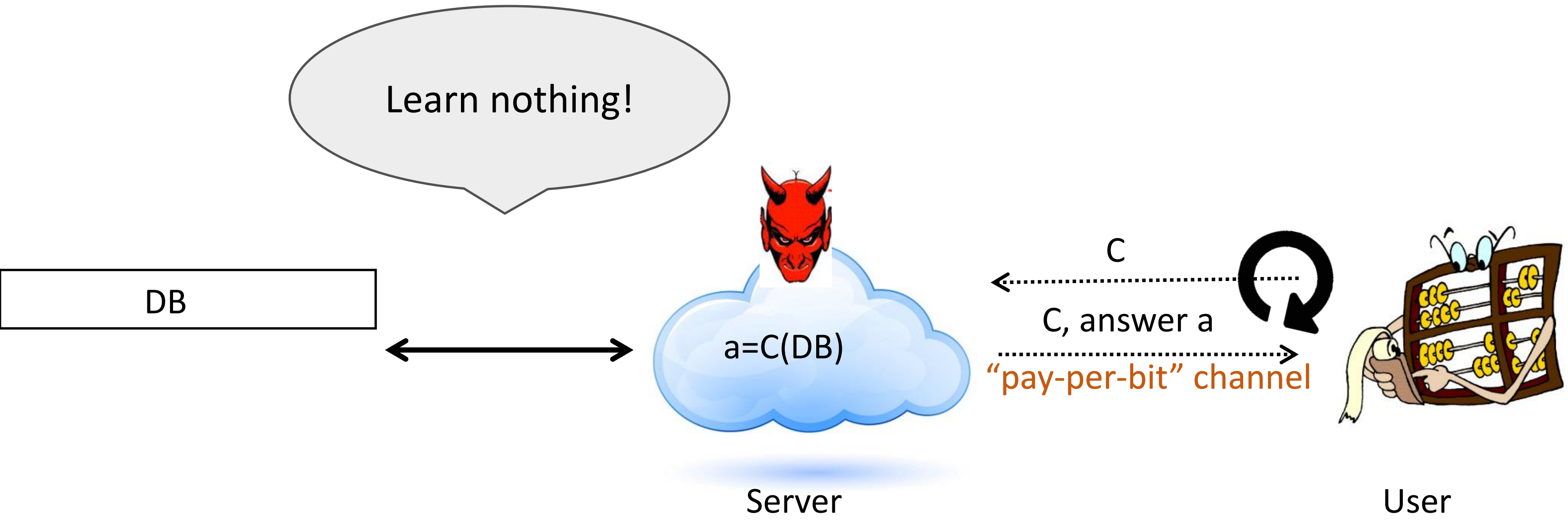
Requirement 1: Verifiability



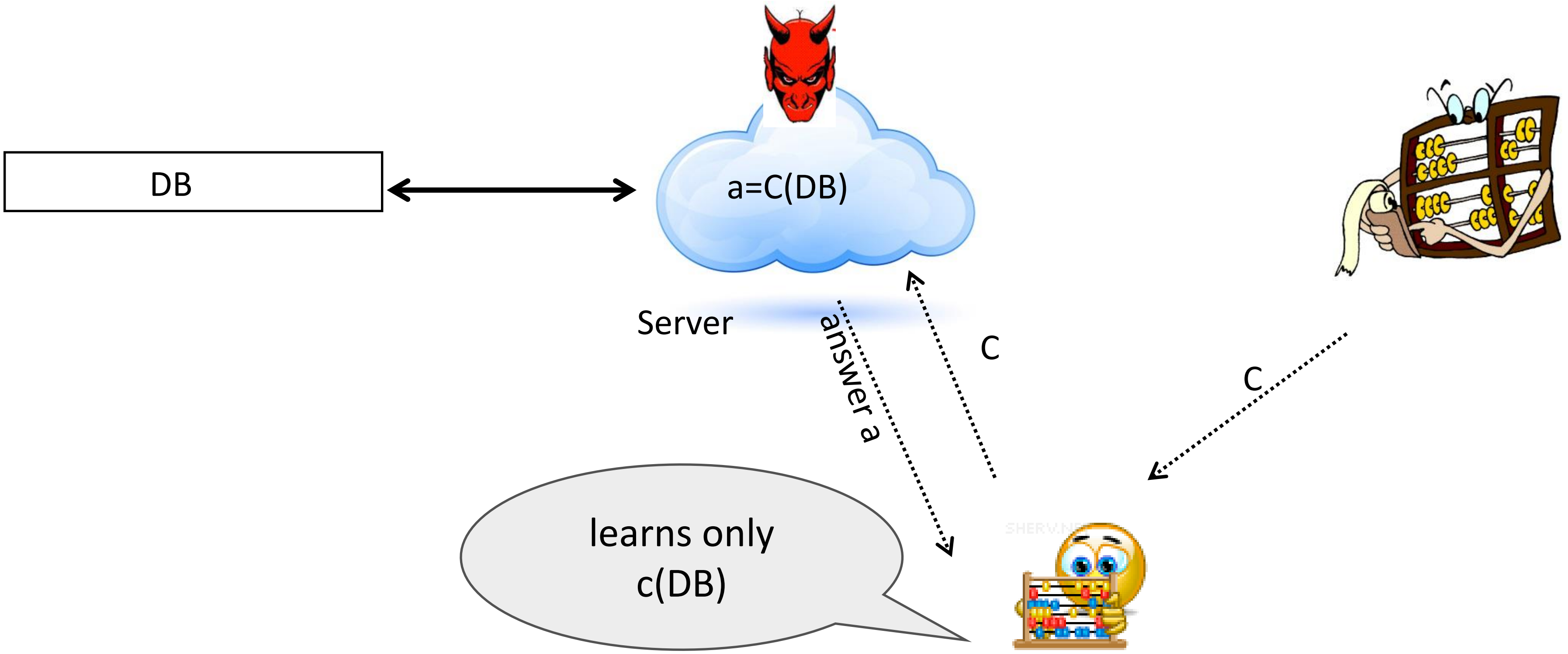
Requirement 1: Public Verifiability



Requirement 2: Privacy



Requirement 3: Query delegation



Putting it all together:
Remote Database ideal functionality

- Obtain DB from owner, reveal size to adv
- Receive (Query, Recipient) from owner:
  - Run Query(DB)
    (potentially updating DB, disclose size & runtime to adv)
- Output answer to Recipient, disclose size to adv
- If Recipient corrupted, Adversary learns (only!) the answer

Requirement 4: efficiency & size

Want:

- Size of query & answer proportional to

  that of "plaintext query and answer"

- All clients are efficient in size of answer
- Database size is comparable to plaintext
- Server runtime proportional to original

A scheme that UC-realizes the above functionality and has the above efficiency requirements is s called a secure database delegation scheme.

Existing solutions

Verifiability:

- Memory delegation [Chung-Kalai-Vadhan]
- SNARKS & Proof Carrying Data [Chiesa-Tromer, Bitansky-C-Chiesa-Tromer,…]
- Accumulators & set computations [Tamassia, Triandopoulos, Papadopoulos,…]
- General RAM computations with persistent memory

  [Kalai-Paneth,Brakerski-Holmgren-Kalai]

  But: no privacy…

Existing solutions

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But: no privacy…

Privacy:

- Homomorphic encryption… but requires Ω(DB) work!
- Searchable encryption (order preserving, token based, CryptDB,…)

But: no verifiability…

Main result:

Assuming circuit IO and const-to-1 CRHFs,
there exist a secure database delegation scheme.

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Use Succinct Persistent Adaptive Garbled RAM…

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[concurrently by Ananth-Chen-Chung-Lin-Lin]

Garbling / Randomized Encoding
[Yao, Ishai-Kushilevitz, Bellare-Hoang-Rogaway]

- Algorithm $Garble$
- $\tilde{f}, \tilde{x} \leftarrow Garble(f, x)$:
  - Correctness: $f(x) = \tilde{f}(\tilde{x})$
  - Security: If $f(x) = f'(x')$, then
    $$Garble(f, x) \approx Garble(f', x')$$
  - Efficiency: Computing $\tilde{f}(\tilde{x})$ is as easy as computing $f(x)$
  - Succinctness: sizes of $\tilde{f}$, $\tilde{x}$ are proportional to the size of f, x

Garbling / Randomized Encoding
[Yao, Ishai-Kushilevitz, Bellare-Hoang-Rogaway]

- Algorithm $Garble$ (Kgen,Fgarble,Igarble)
- $\tilde{f}, \tilde{x} \leftarrow Garble(f, x)$: $k \leftarrow$ Kgen(), $\tilde{f} \leftarrow$ Fgarble(k, f), $\tilde{x} \leftarrow$ Igarble(k, x)
  - Correctness: $f(x) = \tilde{f}(\tilde{x})$
  - Security: If $f(x) = f'(x)$, then
    $$Garble(f, x) \approx Garble(f', x')$$
  - Efficiency: Computing $\tilde{f}(\tilde{x})$ is as easy as $f(x)$
  - Succinctness: sizes of $\tilde{f}$, $\tilde{x}$ are prop. to $x, f(x)$
  - Adaptivity: Adv can choose f $as\ a\ function\ of\ \tilde{x}$, and x as a function of $\tilde{f}$.

Brief History (partial)

- [Yao]: circuit garbling. No succinctness
- …
- [Goldwasser-Kalai-Poppa-Vinod-Zeldovich]: TM garbling. Size Proportional to input size
- [Lu-Ostrovsky, Gentry-Halevi-Raykova-Wichs,…]: RAM machine garbling. Size proportional to runtime.
- [Bellare-Hoang-Rogaway]: adaptive circuit garbling, in ROM
- [Bitansky-Garg-Lin-Pass-Telang, C-Holmgren-Jain-Vinod] : TM/RAM garbling, semi succinct.
- [Koppula-Lewko-Waters]: TM garbling, fully succinct.
- [C-Holmgren,Chen-Chow-Chung-Lai-Lin]: Fully succinct RAM garbling.

Garbling with persistent memory
[Gentry-Halevi-Raykova-Wichs]

- Algorithm $Garble$ = *(Kgen,Fgarble,Igarble)*
- $k \leftarrow$ Kgen(), $\widetilde{x} \leftarrow Igarble(k,x)$ $\widetilde{f}_i \leftarrow Fgarble(k, fi)$, *i=1,2,…*
- Correctness: $f_i(x_i) = \widetilde{f}_i(\widetilde{x}_i)$ for all i
- Security: If $f_i(x_i) = f_i'(x'_i)$, for all i, then

  $\widetilde{x}, \widetilde{f}_1, \ldots \widetilde{f}_i \approx \widetilde{x}', \widetilde{f'}_1, \ldots \widetilde{f'}_i$

- Efficiency: Computing $\tilde{f}(\tilde{x})$ is as easy as $f(x)$
- Succinctness: sizes of $\widetilde{f}_i$, $\widetilde{x}_i$ prop. to size of $x_{i'}, f_i(x_i)$
- Adaptivity: Adv can choose f_i *after seeing* $\widetilde{x}, \widetilde{f}_1, \ldots \widetilde{f_{i-1}}$

From
Succinct Persistent Adaptive Garbled RAM (SPAGRAM)
to database delegation

- To delegate database x: Garble x, send to server.
  Choose keys (sig, ver) for a signature scheme. Post ver.

- To query program C, garble the program:
  "Output C(x), sign using key sig."
  Send to server (or to third party)

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Note: Adaptivity is key!

RAM Garbling with persistent memory: constructions

[GHRW]:   Efficient, non-succinct, non-adaptive, assuming “special purpose public-coins DIO”.

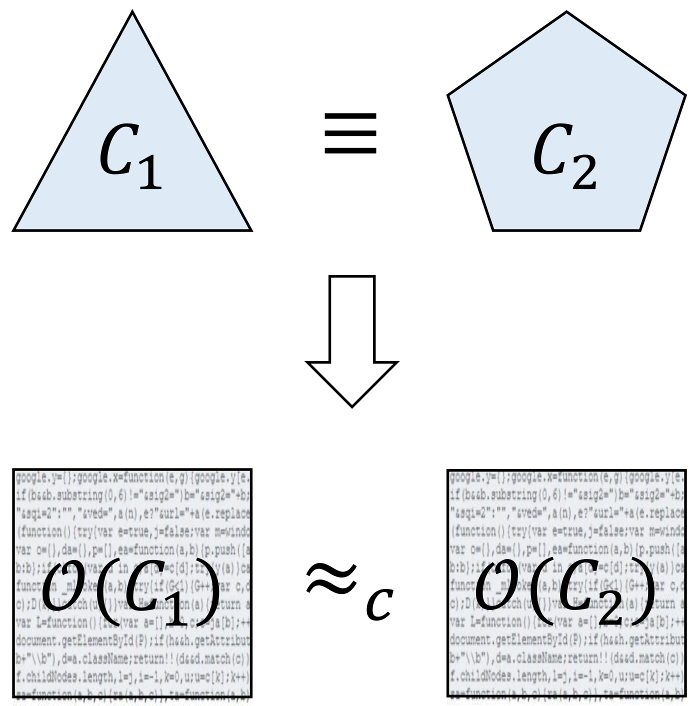
[C-Holmgren, Chung etal]: Succinct, non-adaptive, from IO+OWFs

[CCHR, ACCLL]: Adaptive
(from IO+const-2-1 CRHFs / DDH)


artwork by
Bocian20_10

Indistinguishability Obfuscation (IO)

[Barak-Goldreich-Impagliazzo-Sahai-Rudich-Vadhan-Yang 01, Goldwasser-Rothblum 07]



Several candidate constructions

[Garg-Gentry-Halevi-Raykova-Sahai-Waters 13… … … Lin 16]

The age of IO

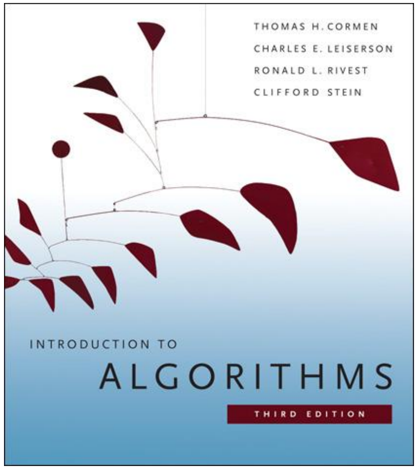
- Amazing concept:
  - Extremely powerful, versatile
  - A whole set of new techniques
  - Elusive… "too good to be true"
- Does it exist?   Under what assumptions?
- Can we show impossibility?
- Can we make it more efficient / realistic?
- How to use it?
- Relaxed/stronger notions?

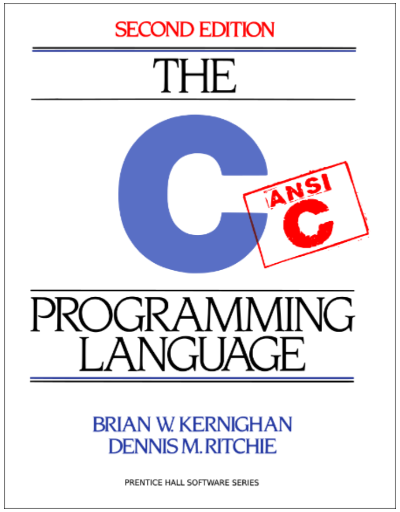
Towards making IO more realistic
(Towards impossibility of IO?)

We Have

Circuit Obfuscation

Real World



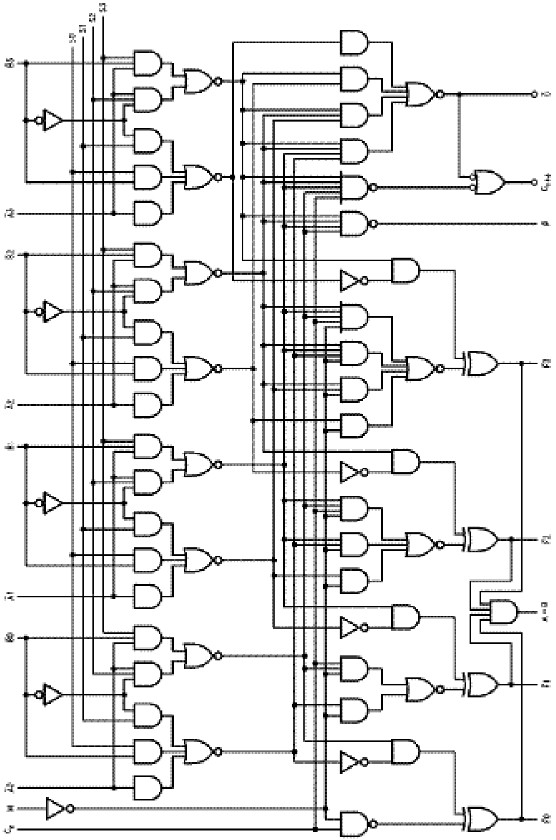


Can we obfuscate more realistic computations?

Trivial "Solution"

```
BINARY-SEARCH(x, T, p, r)
  low = p
  high = max(p, r + 1)
  while low < high
      mid = ⌊(low + high)/2⌋
      if x ≤ T[mid]
          high = mid
      else low = mid + 1
  return high
```

$\log n$



$n \log n$

What We'd Like

- Indistinguishability Obfuscation for a RAM program M directly
- $iO(M)$ should itself a RAM program, with almost the same complexity parameters as M.
- If $M(x) = M'(x)$ for all inputs x, then
$$iO(M) \approx iO(M')$$

Progress So Far

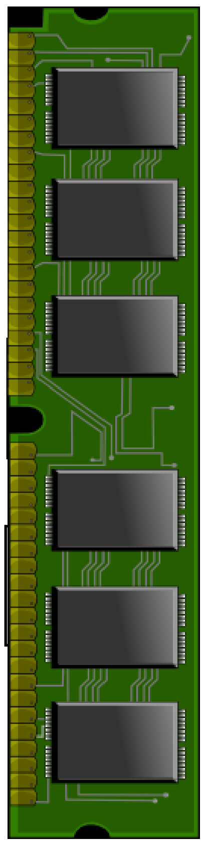
- Turing Machine & RAM obfuscation from non-standard "knowledge assumptions" (DIO and variants) [BCP14,ABGSZ14,GHRW14,IPS14]
- "semi-succinct" TM & RAM obfuscation from subexp-IO and IOWFs: size depends on space of computation. [Bitansky-Garg-Lin-Pass-Telang,C-Holmgren-Jain-Vinod]
- Fully succinct Turing Machine obfuscation from subexp IO and IOWFs [Koppula-Lewko-Waters 14]
- Fully succinct RAM obfuscation from subexp IO and IOWFs [C-H,Chung etal]
- Extension to PRAM [Chung etal]

➔ All recent works obtain succinct garbling as a first step.

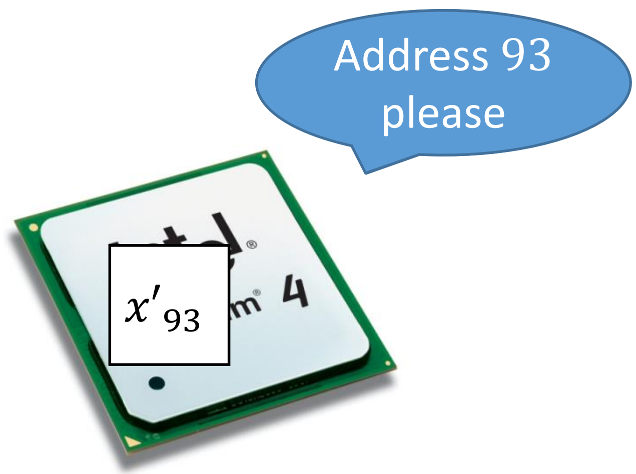
Our Techniques

A Naïve Attempt at RAM garbling

Memory



CPU



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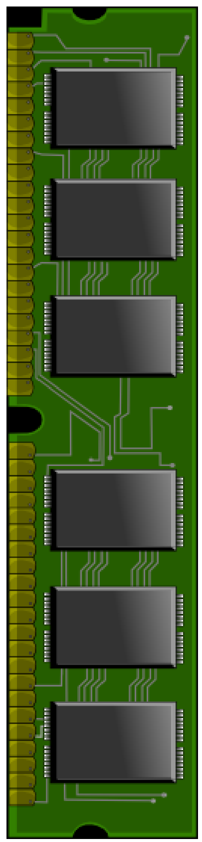

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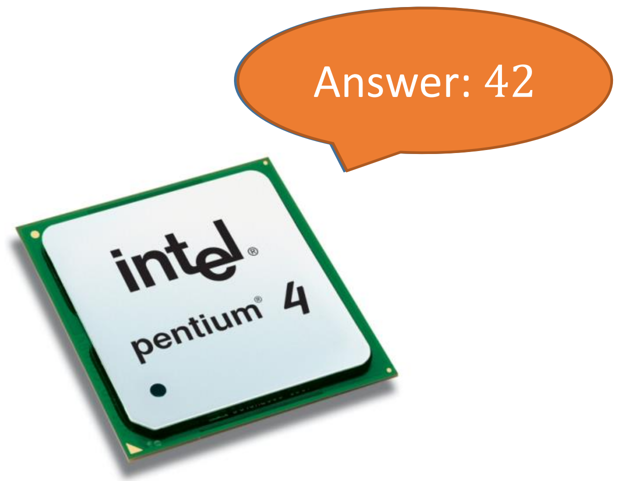
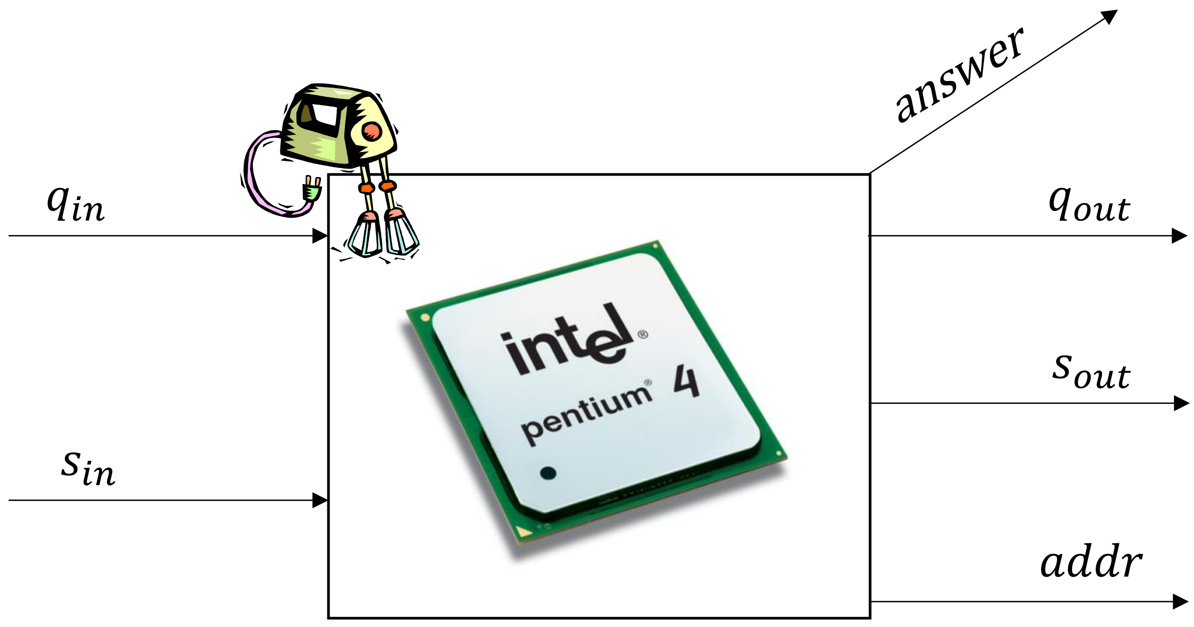


Naïve Attempt at RAM garbling



What’s wrong? Everything

- Doesn’t prevent adversary from giving circuit illegal inputs
- Doesn’t hide any intermediate state
- Doesn’t hide memory addresses accessed

We’ll address these challenges one by one.

Goal: Succinct Garbling
2-step approach

1. Construct a weaker notion of garbling
2. Compile a weak garbler into a full garbler

Roadmap:
How to compile a stronger garbler

Weaken conditions for indistinguishability:

What needs to be the same?

	Final Output	Addresses	Memory Values
Same-Trace	Yes	Yes	Yes
Same-Address	**Yes**	**Yes**	**No**
Full	Yes	No	No

✅ [KLW14]

What's missing?

- Internal RAM state
- Circuit behavior on illegal inputs

Same-Trace Garbling

$Tr(M, x) \stackrel{\text{def}}{\equiv}$

Time	Address	Value Written	Answer
1	a_1	s_1	$\perp$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T - 1$	a_{T-1}	s_{T-1}	$\perp$
T	$\perp$	$\perp$	y

Theorem: There is an algorithm STGarble such that:

If $\mathrm{Tr}(M, x) = \mathrm{Tr}(M', x')$, then

$$\mathrm{STGarble}(M, x) \approx \mathrm{STGarble}(M', x')$$

Same-Trace Garbler Construction

- Obfuscate CPU; to ensure integrity of computation use:
  - signature schemes
  - positional accumulators
  - iterators.

(Essentially follows [KLW14]'s "Message-hiding encoding")

Same-Address Garbling

Goal: If (M, x) and (M', x') access same addresses, then

$$\mathrm{SAGarble}(M, x) \approx \mathrm{SAGarble}(M', x')$$

Simple Case: Addresses are locally computable.

Strategy: Encrypt memory words and apply Same-Trace Garbler

Same-Address Garbling (General Case)

- What if addresses *not* locally computable?

Time	Address	Value Written	Answer
1	a_1	c_1	$\perp$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T-j-1$	a_{T-j-1}	c_{T-j-1}	$\perp$
$T-j$	a_{T-j}	z_{T-j}	$\perp$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T-1$	a_{T-1}	z_{T-1}	$\perp$
T	$\perp$	$\perp$	y

How to access $a_{T-j}, \ldots, a_{T-1}$?

Same-Address Garbling (General Case)

- What if addresses *not* locally computable?
- Solution: double-execution

Time	Address	Value Written	Answer
1	a_1	$c_1 \| \| d_1$	$\perp$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T-1$	a_{T-1}	$c_{T-1} \| \| d_{T-1}$	$\perp$
T	$\perp$	$\perp$	y

$$c_i = (i, F(i||a_i) \oplus s_i)$$
$$d_i = (i, G(i||a_i) \oplus s_i)$$

F and G are puncturable PRFs

(Full) Garbling

RAM machines M, M'; Inputs x, x'

Want: If $M(x) = M'(x')$, then

$$Garble(M, x) \approx Garble(M', x')$$

Difficulty: Hiding memory addresses accessed

Tools:

- Oblivious RAM with "Randomness Locality"
- Same Address Garbler ($SAGarble$)

Oblivious RAM

- Transform RAM machine to have a (distributionally) fixed memory access pattern

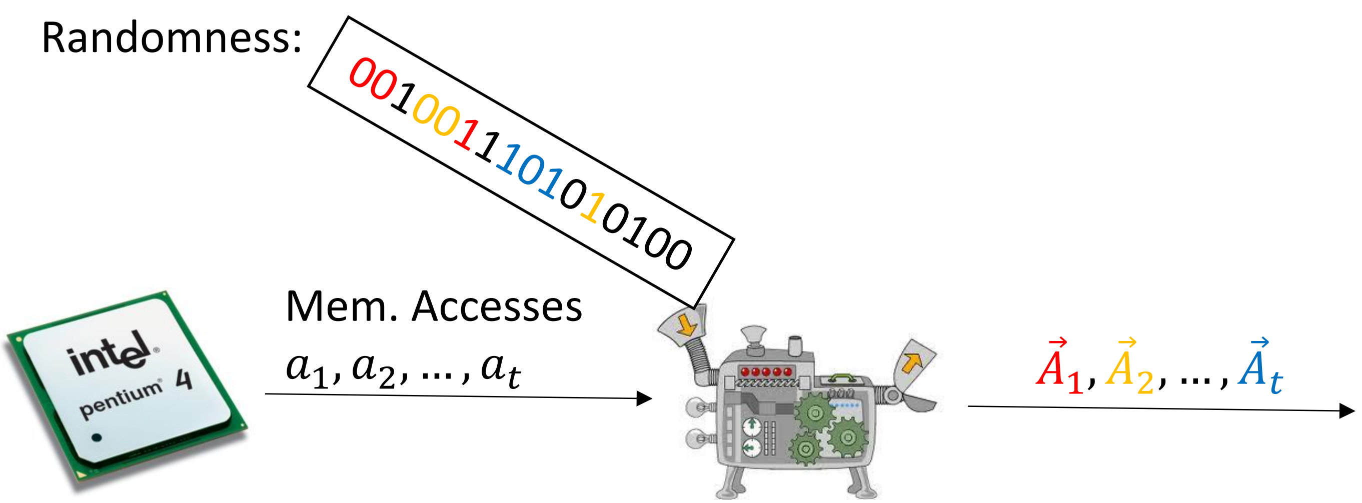
Addresss $a_1, \ldots, a_t$

Addresses $a'_1, \ldots, a'_t$

Addresses $\vec{A}_1, \ldots, \vec{A}_t \approx \overrightarrow{A'}_1, \ldots, \overrightarrow{A'}_t$

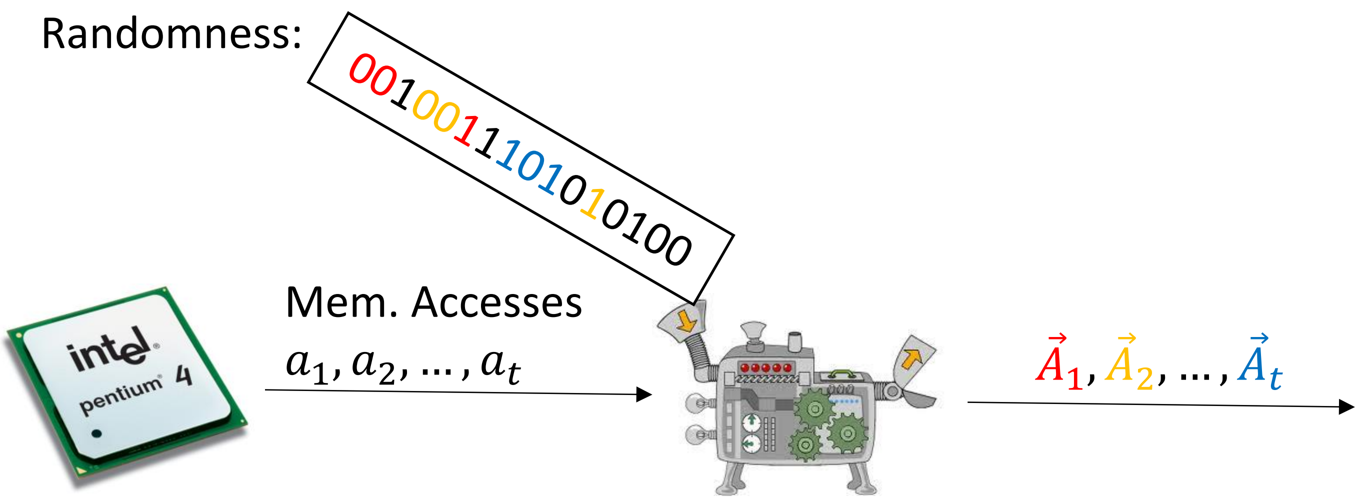
Localized Randomness ORAM

- The vectors of accessed addresses depend (as a function) on small, disjoint subsets of the random bits



Localized Randomness ORAM

- The vectors of accessed addresses depend (as a function) on small, disjoint subsets of the random bits
- Each $\vec{A}_i$ can be efficiently sampled as $OSample(i)$



Localized Randomness ORAM

- The vectors of accessed addresses depend on small, disjoint subsets of the random bits
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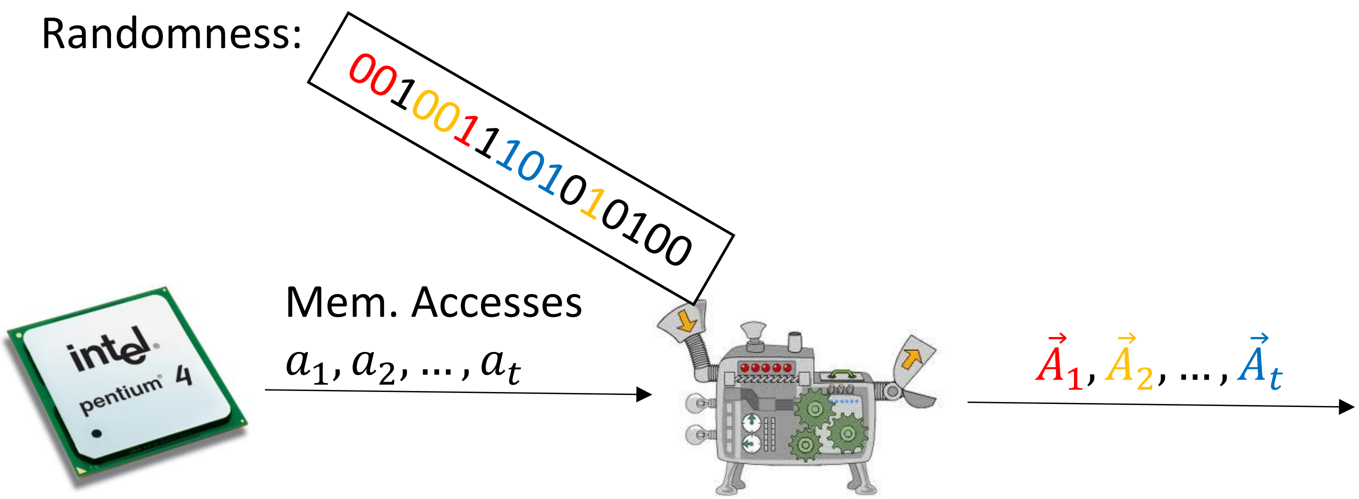
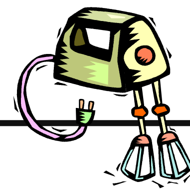
Satisfied by Chung-Pass ORAM



Full Garbling Construction

$Garble(M, x) \stackrel{\text{def}}{=}$



RAM Machine

1. Read initial ORAM state q_{ORAM} from memory
2. Run M obliviously with randomness $F(1), \ldots, F(T)$
3. Output M's answer

Same Address Garbler

Puncturable PRF F

Memory

q_{ORAM}	ORAM encoding x

Persistent Memory

- Same construction, except:
- In initial memory garbling, add “step 0”
- Augment the i-th machine to look for “step i-1” in memory, and overwrite with “step i”.

(all machines use the same parameters for signature, accumulator, iterator, encryption, oram)

- Simulation strategy the same.

Adaptivity

First issue:

Positional accumulator is a static object:

- Guarantees unconditional binding at a single point.
- But point needs to be set ahead of time…

Recall: Positional accumulator

[Hubacek-Wichs, KLW, Okamoto-Pietrzak-Waters-Wichs]

- Geygen -> pk
- Accumulate $(pk, S, i, x) \rightarrow S'$
- Verify $(pk, S, i, x) \rightarrow yes \mid no$
- Fgen (i, x) = $pk_{i,x}$

Properties:

- Computational binding
- Forced binding
- Indistinguishability of forced keys: $pk \sim pk_{i,x}$

➔ Forced locations need to be fixed in advance

Solutions

- First attempt: Reduction guesses location

Doesn't work…   Pos. Acc. not strong enough
[doesn't guarantee consistency with writes]

[ACCLL]: Fix the notion and guess…

Adaptive Positional accumulator

- Geygen -> *ak,vk*
- Accumulate $(ak, S, i, x) \rightarrow S'$
- Verify $(vk, S, i, x) \rightarrow yes \mid no$
- Fgen (ak, i, x) = $vk_{i,x}$

Properties:

- Computational binding
- Forced binding
- Indistinguishability of forced keys: $vk \sim vk_{i,x}$

➔ Forced locations can be chosen adaptively…

Adaptive Positional accumulator

Construction:

- Define "AP-hash": same properties as "APA" but for hash function Use IO
- From AP-hash to APA: Use Merkle paradigm
- Construct AP-hash:

  vk: IO["Check that the input x is consistent with hash value y"]

  $fvk_{i,x,y}$: IO["if input is i',x',y and either i <> i' or x <>x'

  then reject, else run normal check"]

Adaptivity: ORAM

Second issue:

- ORAM + PPRF is a static object:

  Guarantees unconditional secrecy for a single location.

  But location needs to be set ahead of time…

- Solution: Reduction guesses location…

Questions:

IO with persistent memory?

IO with unbounded input?

Succinct garbling without IO?