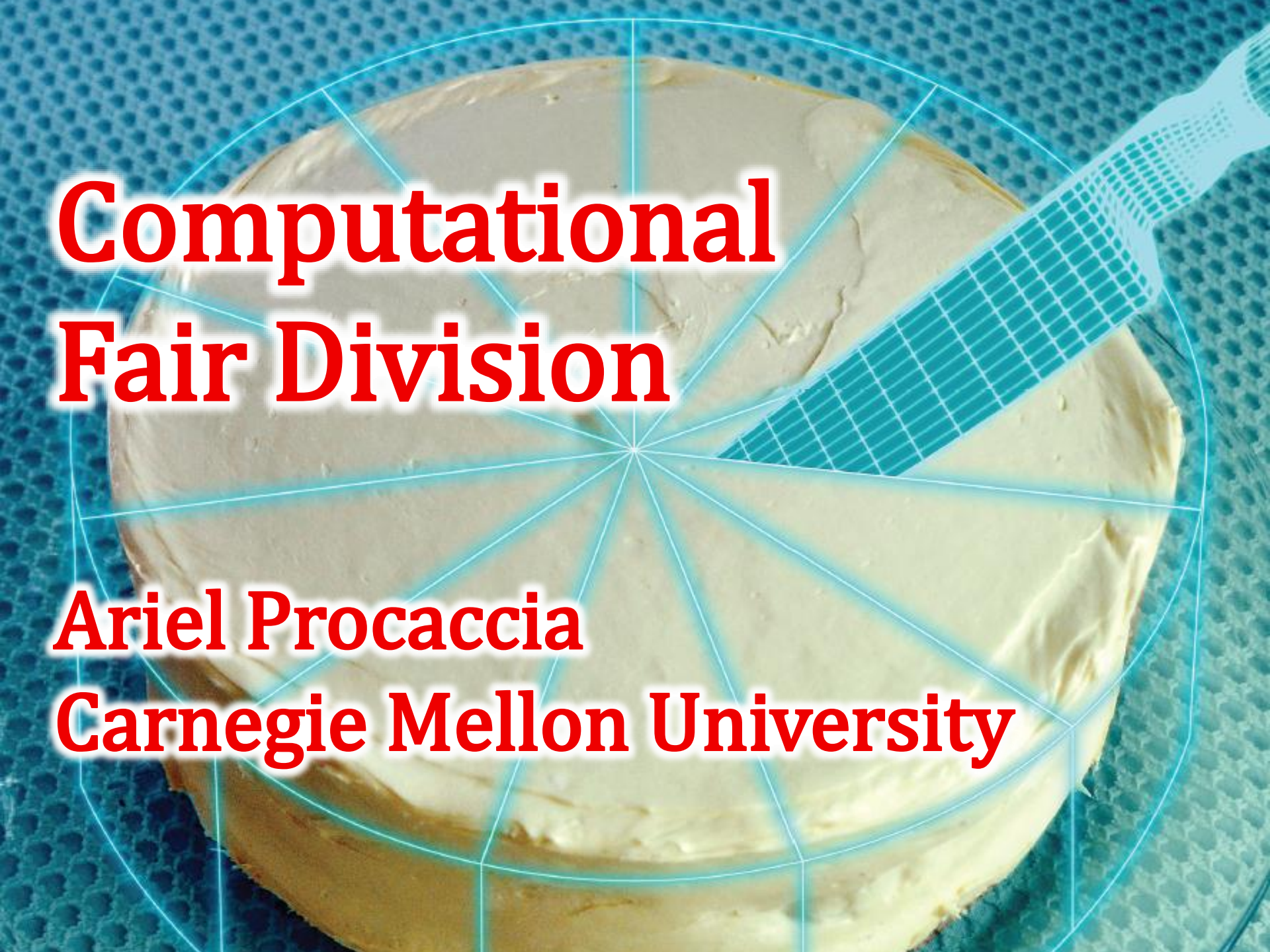

Computational Fair Division

Ariel Procaccia
Carnegie Mellon University

Overview

Rent Division

Computationally efficient algorithms for assigning rooms and dividing rent

Indivisible Goods

A new, approximate notion of fairness and its application in Spliddit

Classroom Allocation

Leximin in the real world: properties, optimization, and implementation

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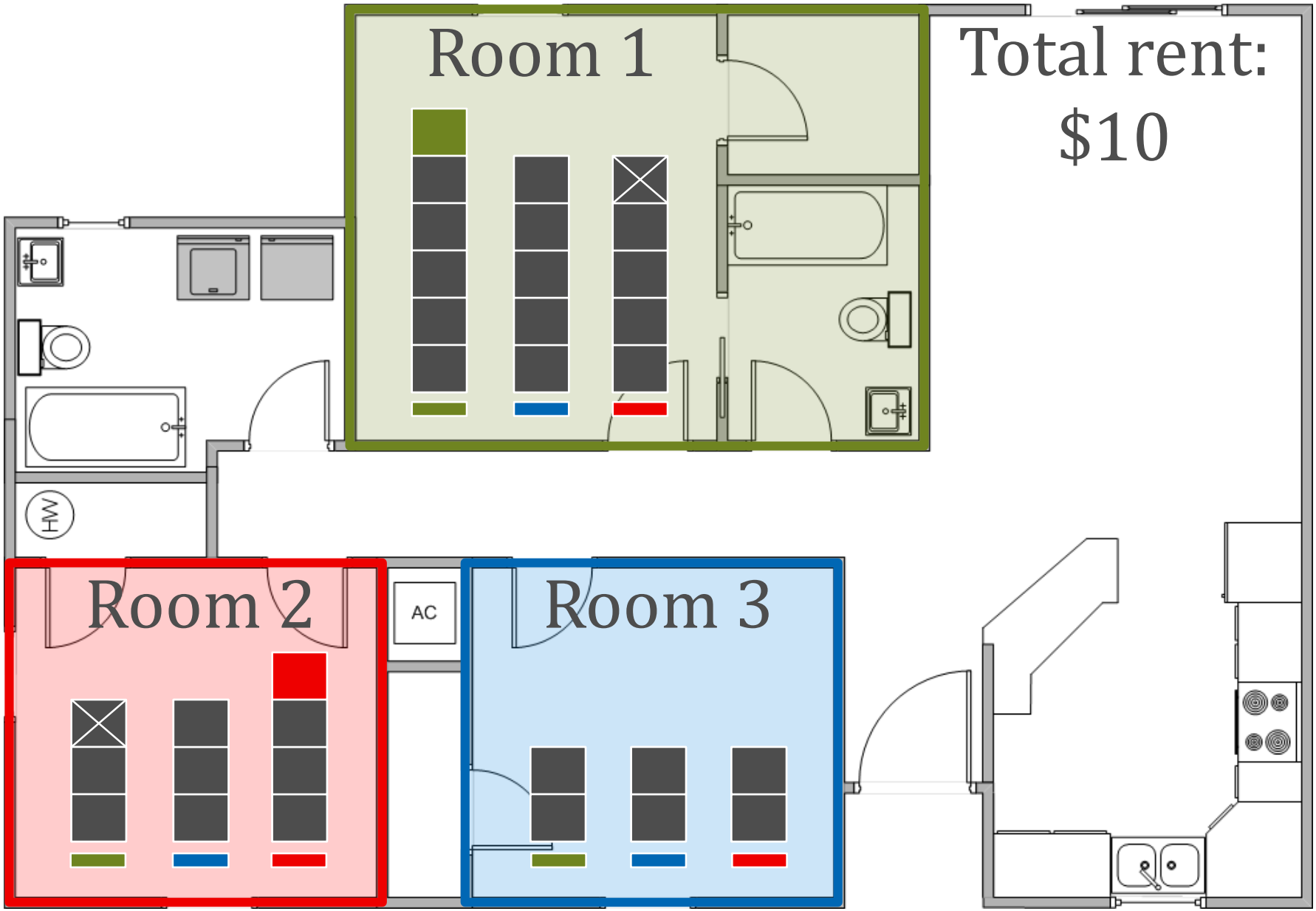
The Whining Philosophers Problem

Nir Ben Moshe

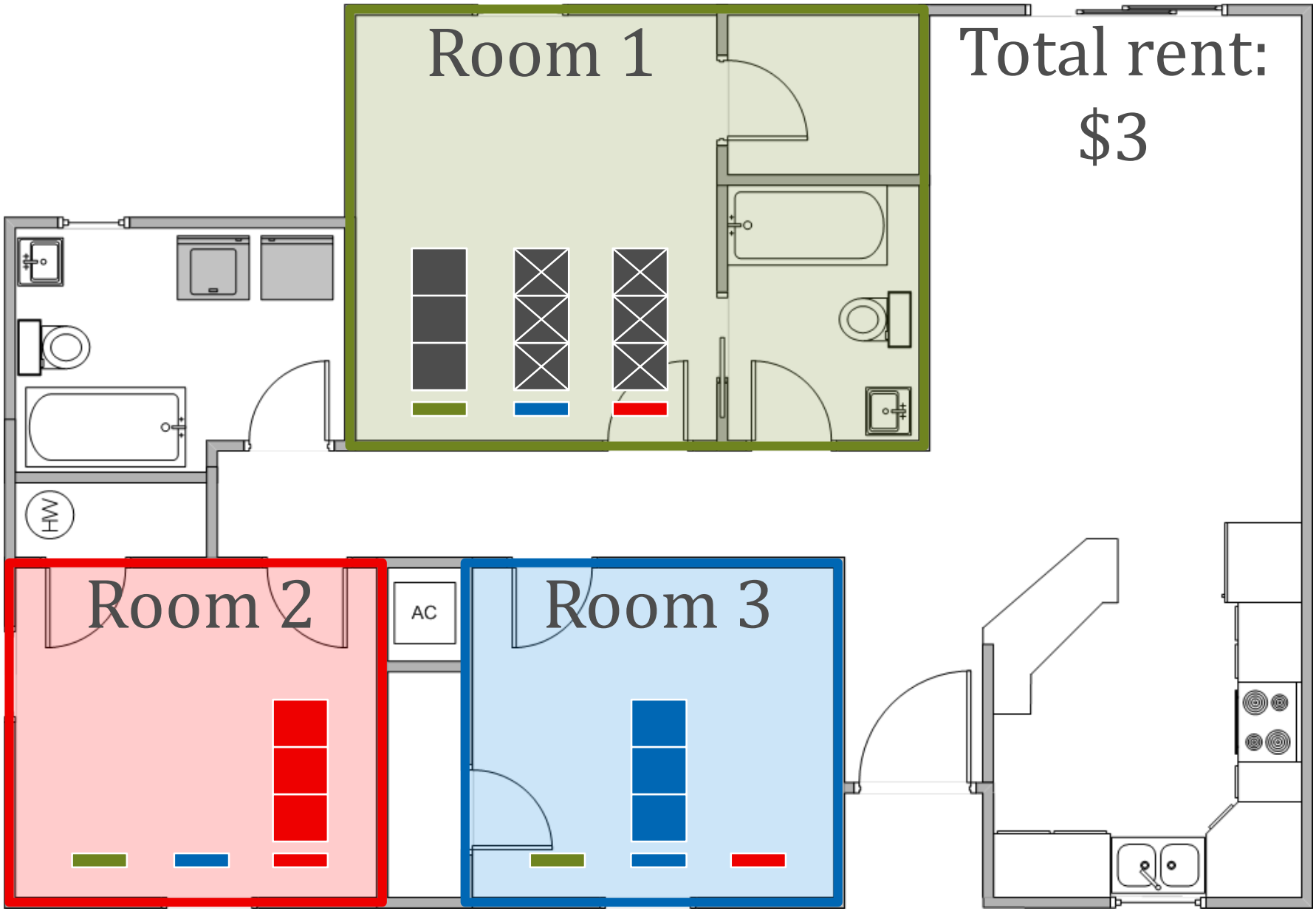
Naomi Sender

Ariel Procaccia


Room 1
Total rent:
$10
Room 2
AC
Room 3
HW

Envy-Free Rent Division

- Theorem [Sevensson 1983]: An envy-free solution always exists
- Theorem [Aragones 1995]: An envy-free solution can be computed in polynomial time


Room 1
Total rent:
$3
Room 2
AC
Room 3
HW

ENVY-FREE RENT DIVISION

- Theorem [Sevensson 1983]: An envy-free solution always exists
- Theorem [Aragones 1995]: An envy-free solution can be computed in polynomial time
- Theorem [Gal, Mash, P, Zick 2015]:
  A solution that maximizes the minimum utility subject to envy-freeness can be found in polynomial time

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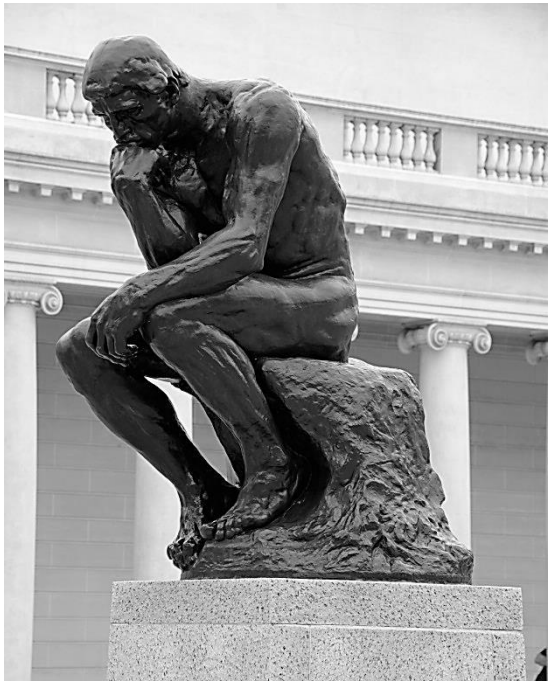
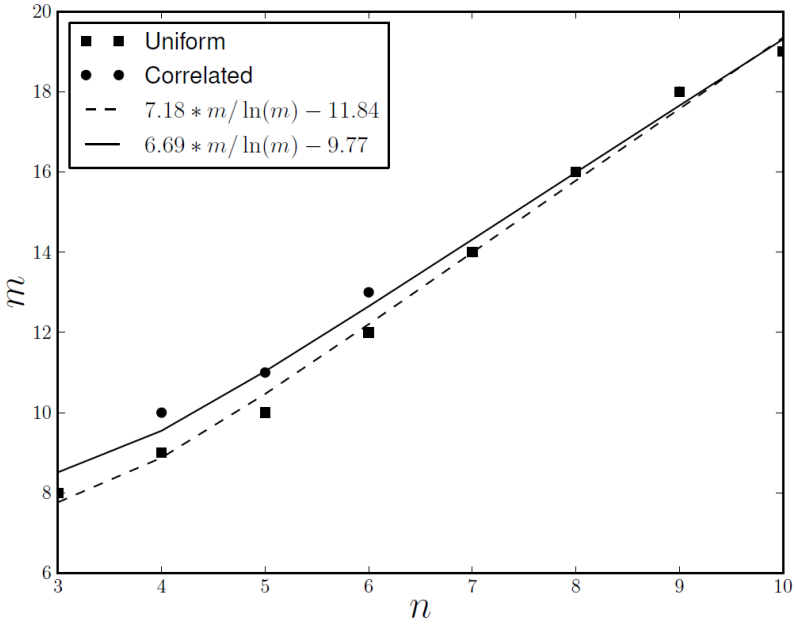


Assume: additive valuations

- EF is infeasible ⇒ random values?
- For each good g, draw values $V_1(g), \ldots, V_n(g)$ from a distribution over $[0,1]^n$
- Theorem [Dickerson et al., 2014]: Under mild technical assumptions, if $m = \Omega(n \cdot \log n)$ then an EF allocation exists w.h.p. as $m \to \infty$



Min value of m such that 99% of instances admit an EF allocation

Maximin Share Guarantee

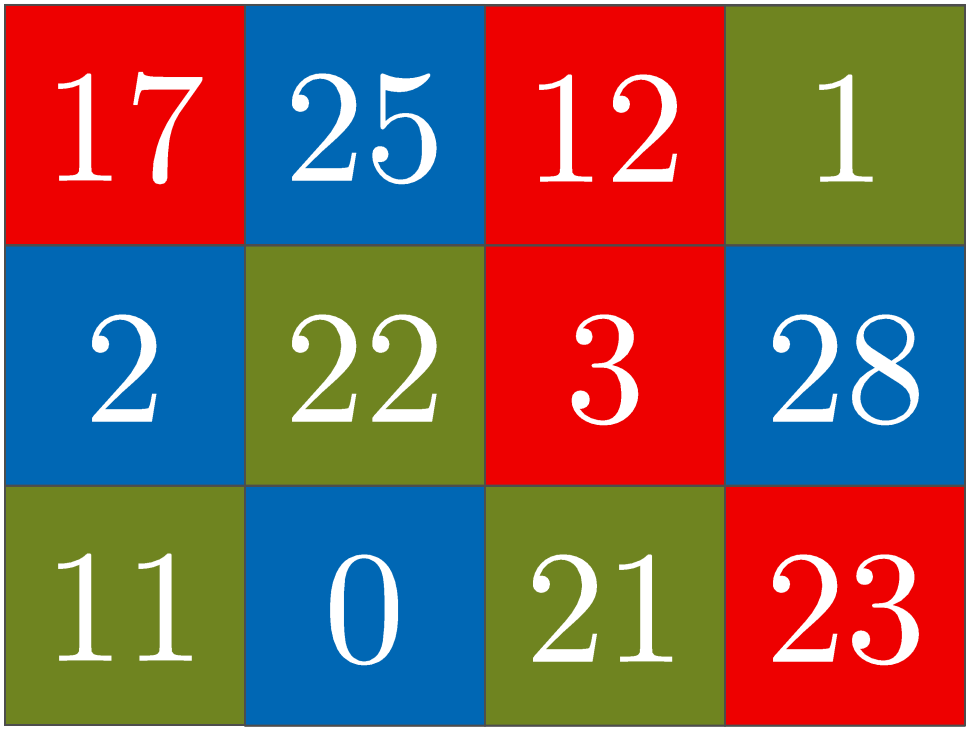
Total: $30	Total: $50	Total: $20				
$30	$50	$2	$5	$5	$3	$5
						
$2	$10	$5	$20	$20	$3	$40
Total: $30				Total: $30		Total: $40

- Maximin share (MMS) guarantee [Budish 2011] of player i:

$$\max_{X_1,\ldots,X_n} \min_j V_i(X_j)$$

- Theorem [P & Wang 2014]: $\forall n \geq 3$ there exist additive valuation functions that do not admit an MMS allocation

Counterexample for $n = 3$



Counterexample for $n = 3$

$$\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline \end{array} \times 10^6 + \begin{array}{|c|c|c|c|} \hline 17 & 25 & 12 & 1 \\ \hline 2 & 22 & 3 & 28 \\ \hline 11 & 0 & 21 & 23 \\ \hline \end{array} \times 10^3 +$$

3	-1	-1	-1
0	0	0	0
0	0	0	0

Player 1

3	-1	0	0
-1	0	0	0
-1	0	0	0

Player 2

3	0	-1	0
0	0	-1	0
0	0	0	-1

Player 3

- Maximin share (MMS) guarantee [Budish 2011] of player i:

$$\max_{X_1,\ldots,X_n} \min_j V_i(X_j)$$

- Theorem [P & Wang 2014]: $\forall n \geq 3$ there exist additive valuation functions that do not admit an MMS allocation
- Theorem [P & Wang 2014]: It is always possible to guarantee each player $2/3$ of his MMS guarantee (in poly time for constant n)

splíddít

Share Rent

Split Fare

Assign Credit

Divide Goods

Distribute Tasks

Suggest an App

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Hervé Moulin

"… the reward of helping people who have a real fair division problem by explaining our solutions, is that they in return pose interesting and difficult new questions, food for our thoughts. … It could be a goldmine of ideas, as well as a costly proposition if there are too many questions!"

Fair division problem in the public school system

[redacted] 5/12/14

to admin

Hello,

I am the analytics specialist for [redacted] Office of Charter Schools.

I read about your company in the New Yo
school districts are tasked with a similar
year in the form of Prop 39
provide any unused space to Charter
more applicants than space available, and
becomes a political process that ends in a
resources.I hope to create a program that
results in the best possible allocation of th
both the school district and the charter sc

"... public school facilities should be shared fairly among all public school pupils, including those in charter schools."

I'm writing in the hopes that there might be an interest in collaborating to create a product that would streamline this process for school districts, allowing more funds to go to students. If not, I was hoping that you might be able to point to literature/contacts that would aid in this endeavor.

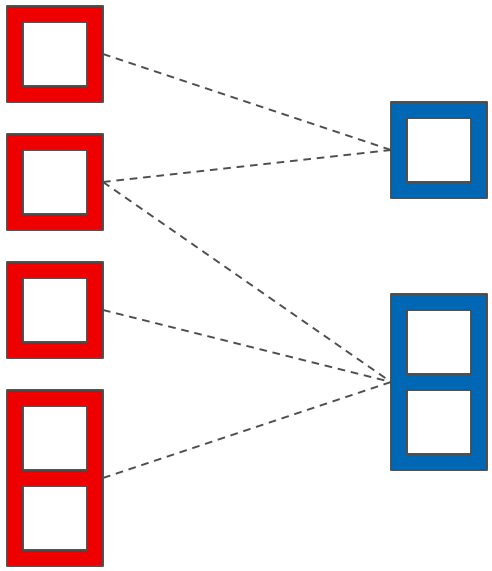
Thank you for your time and best of luck with the launch!

I object, your honor — the method is provably fair!

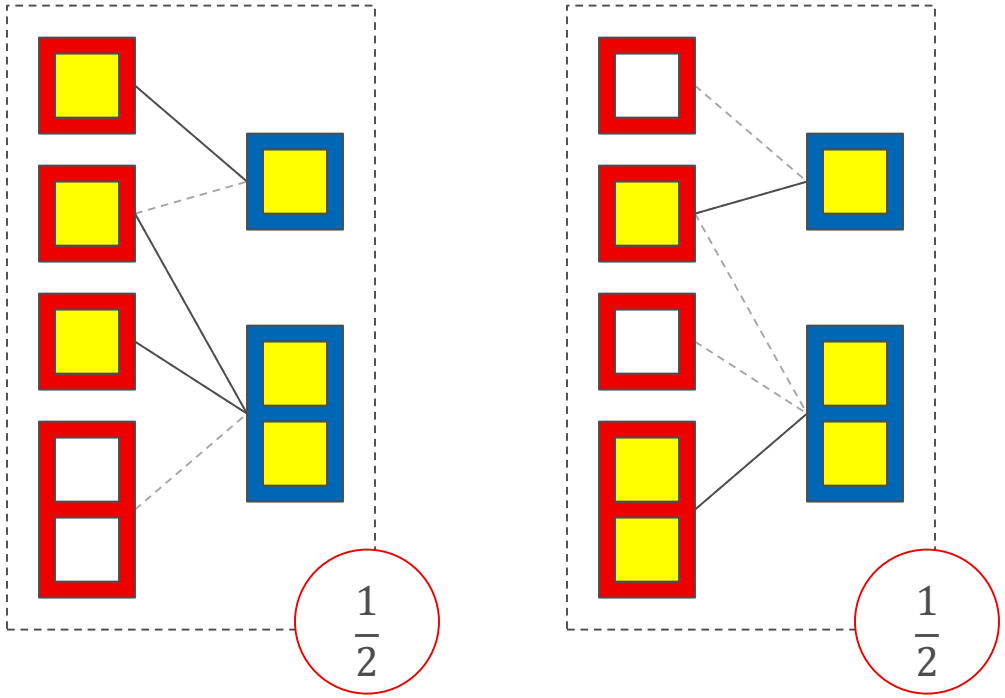
Our approach

- Facilities have capacities
- Players have demands
- Preferences are dichotomous
- Starting point: the Leximin Mechanism [Bogomolnaia and Moulin 2004]

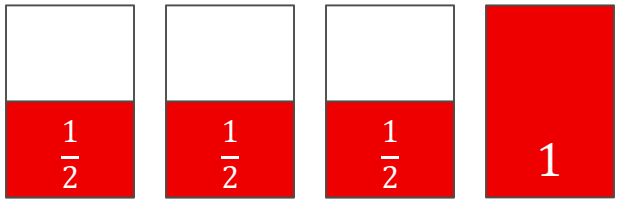
2015/2016 request form: "provide a description of the district school site and/or general geographic area in which the charter school wishes to locate"



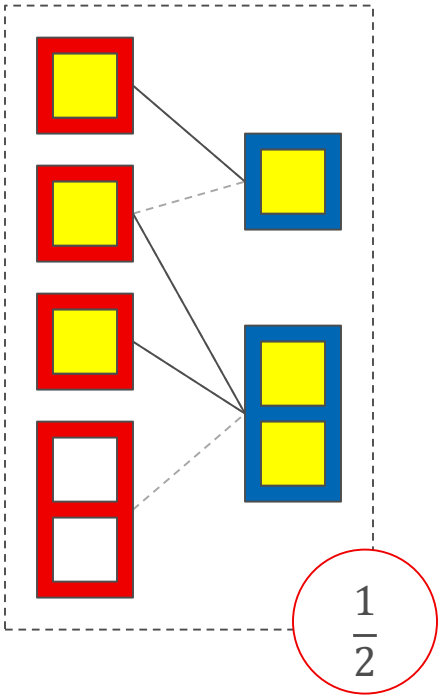
The Leximin Mechanism

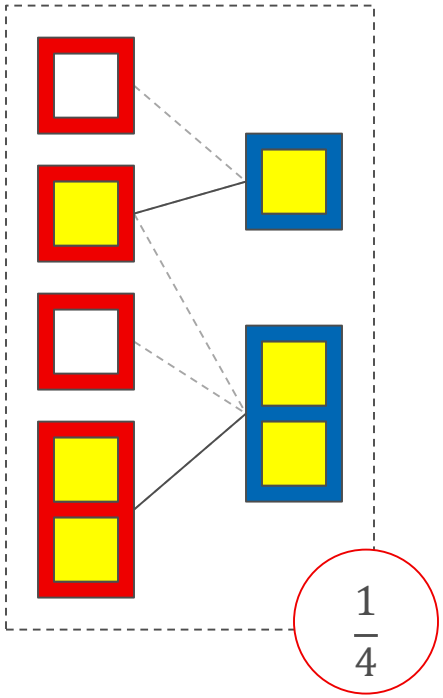


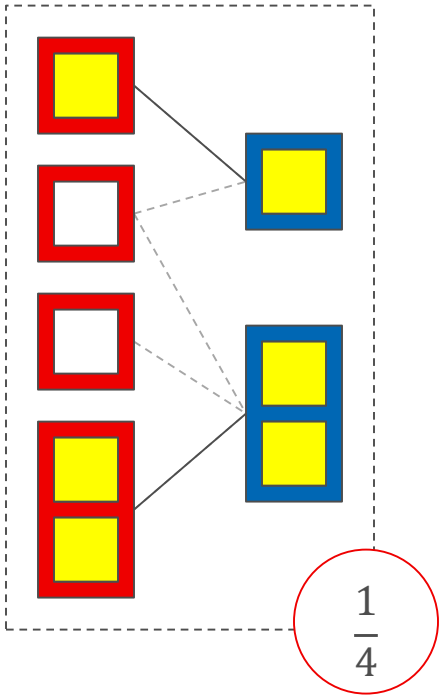
Probabilities:



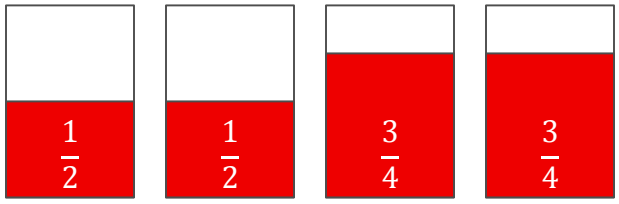
The Leximin Mechanism



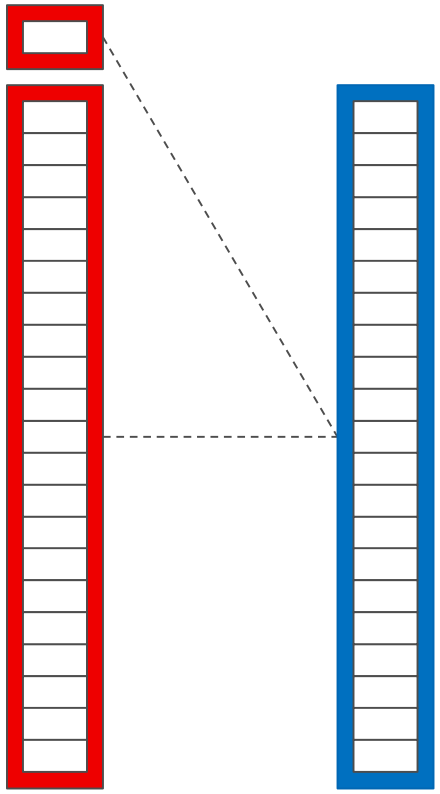


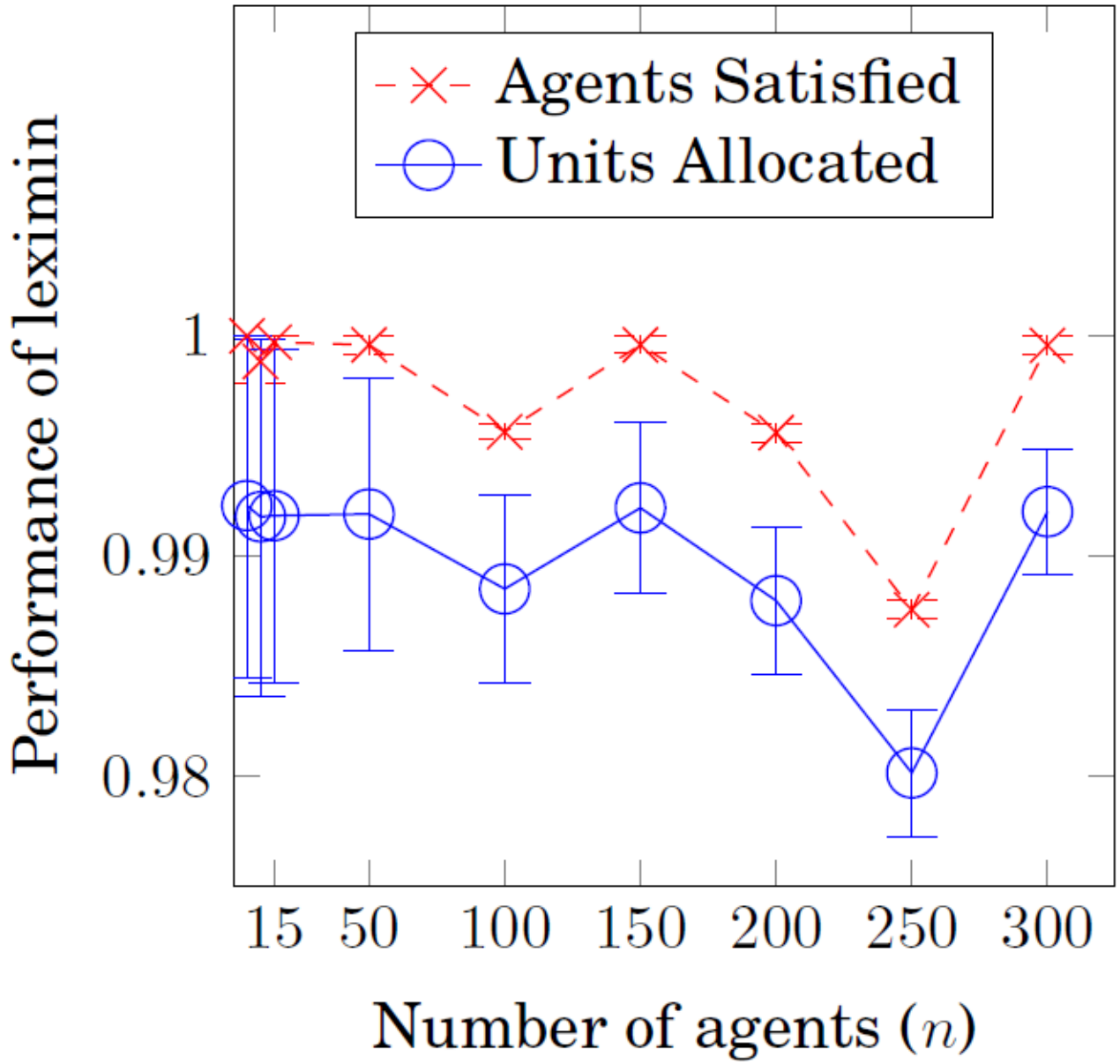


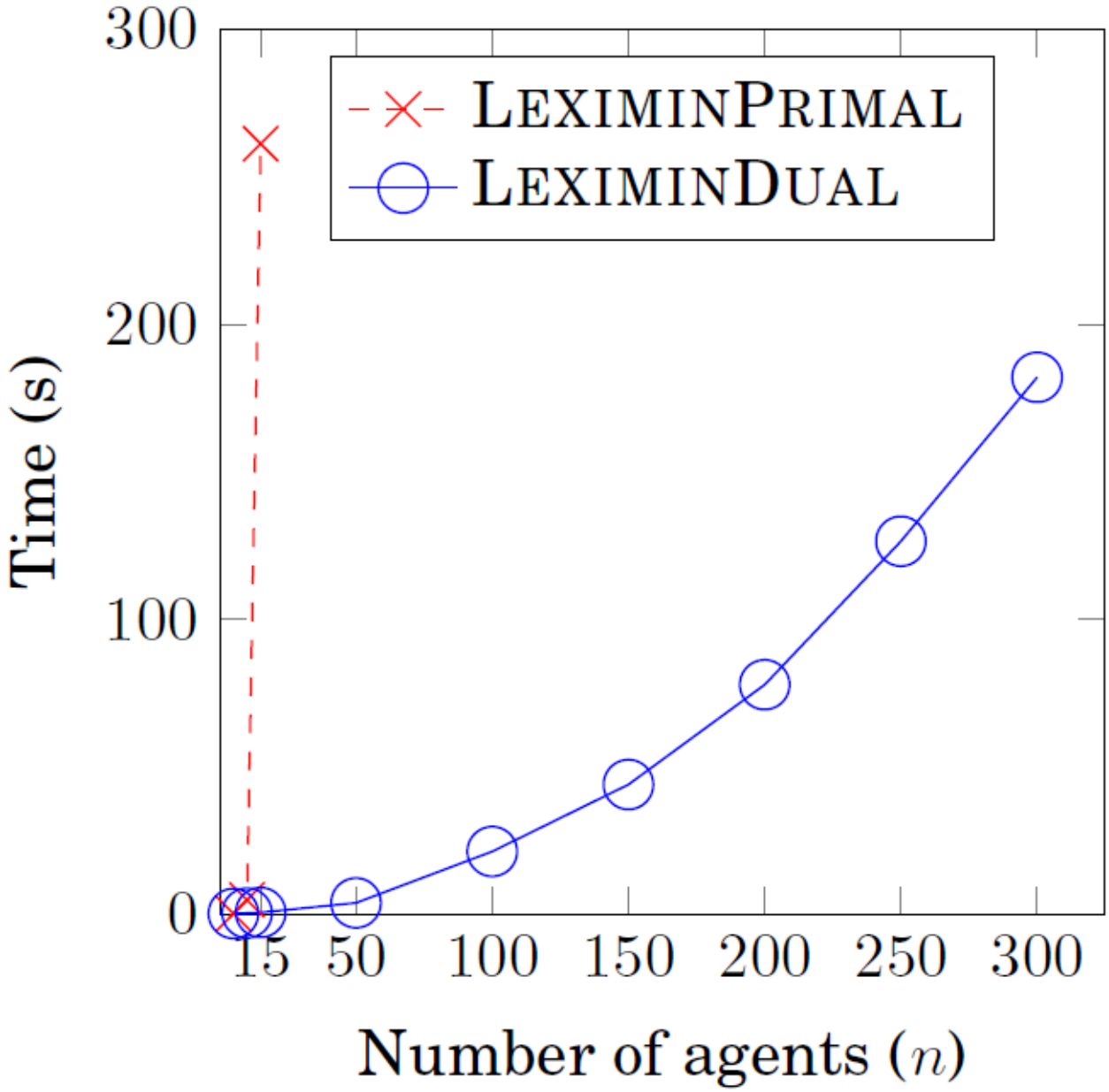
Probabilities:



- Theorem [Kurokawa et al. 2015]: The leximin mechanism satisfies proportionality, envy-freeness, Pareto efficiency, and group strategyproofness
- We actually prove this in a much more general framework
- Theorem [Kurokawa et al. 2015]: The expected number of units allocated by the leximin mechanism 1/4-approximates the maximum number of units that can be allocated simultaneously




Agents Satisfied
Units Allocated
Performance of leximin
1
0.99
0.98
15
50
100
150
200
250
300
Number of agents (n)


300
200
100
0
Time (s)
LeximinPrimal
LeximinDual
15
50
100
150
200
250
300
Number of agents (n)

BIBLIOGRAPHY

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- Kurokawa, Procaccia, and Shah. Leximin Allocations in the Real World. EC 2015.