

Title: Optimal commitments in auctions with incomplete information

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We are interested in the problem of optimal commitments (aka. Bayesian Stackelberg equilibrium) in a general class of single-item auctions that include first price and all-pay auctions. Our main contribution is a general methodology for solving optimal commitment in this class of auctions, for any continuous type distributions. Applying our method, we are able to solve optimal commitments for first-price and all-pay auctions in *closed-form* for fairly general settings.

The optimal commitments functions in these auctions reveal two seemingly opposite insights. In the optimal commitment of first price auction, the leader bids passively when he is in a low type. We interpret this as a credible way to alleviate competition and to collude. In contrast, when his type is sufficiently high enough, the leader sometimes would go so far as to overbid his own value. We interpret this as a credible way to threaten. We show via concrete examples that the leader is indeed willing to do so to secure more utility when his type is in between. Similar insights extend to all-pay auctions.

Our main approach consists of a series of innovations. In particular we put forward a concept called equal-bid function that connects both players' strategies, as well as a concept called equal-utility curve that smooths any leader strategy into a weakly better continuous and differentiable strategy. We believe these techniques and insights are of independent interest.