The reverse Product-Mix Auction and further extensions (*Extended Abstract*)

Luis Boscán *

ABSTRACT

In this paper I propose a reverse Product-Mix Auction, initially described by Klemperer (2010) as a forward auction. In my setting, bidders approximate supply functions and the auctioneer chooses demand functions of imperfectly substitutable and indivisible commodities. While this format is applicable to a wide range of settings, I am motivated by its potential application to the problem of power system flexibility procurement. I extend this setup in two directions, namely, to the case in which there are finitely many goods traded (i.e., the N varieties case) and to a double auction format, in which many buyers and many sellers bid. While these extensions have been described in general terms by Klemperer (2010), and Baldwin and Klemperer (2012), the existing literature has not studied them in detail, solution techniques have not been sufficiently analyzed, nor applications (besides the Bank of England's repo operations) have been documented so far. With an emphasis on the computational issues behind its implementation, this paper aims at filling some of these gaps.

Keywords: Multi-unit auctions, Reverse auctions, Procurement.

JEL: D44, D47, L94

*PhD Fellow. Department of Economics, Copenhagen Business School (CBS). Porcelænshaven 16A. Frederiksberg 2000. Denmark. E-mail: lbo.eco@cbs.dk

INTRODUCTION

Motivated by the need to act swiftly as the financial crisis began in August 2007, the Bank of England sought the expert advice of Paul Klemperer to design a solution that would allow them to supply liquidity to troubled financial institutions, while accounting for the different quality of collateral backing their repo operations, such that interest rates for strong collateral were lower than those for weak collateral. While Simultaneous Ascending auctions (Milgrom, 2000) were a well understood framework that could provide a solution, multiple rounds were considered impractical because time was of the essence. His proposal, put forward in Klemperer (2010), is a proxy implementation of multiple round auctions in which bidders compete to supply multiple, imperfectly substitutable items which are to be allocated and priced in a single round. With mutually exclusive bids, which give the possibility to express preferences for only one, some or all of the items sold, the Product Mix auction reduces the incentive of bidders to collude, diminishes the dependence of bids between rounds and facilitates a straightforward solution.

Given its simplicity, the Product Mix auction is applicable to a wide range of settings where, like in financial markets, time is essential too. Flexibility, that is, the ability that a power system has to modify consumption or generation in response to variability, expected or otherwise (IEA, 2014) is one clear example. A growing body of, mostly engineering-oriented, literature[1]describes flexibility as the the main resource that power systems aiming at integrating significant shares of renewables must enable. The availability of Smart Grid solutions, which are helping to decrease transaction costs associated to harnessing flexibility coming from different sources in the power system, is acting as a catalyst in the emergence of new business models that increase flexibility and, ultimately, increase the adaptability of power systems to sudden changes thus reducing the barriers for renewable energy integration.

However, the concept of flexibility remains very technical and its features as a tradable commodity are just beginning to be understood. In related work, Bogetoft et al. (2015) and Boscán and Poudineh (2015) argue that flexibility is a distinct – although related – commodity than capacity or energy, which is inherently *differentiated*, as it involves modifying supply or demand in *time*, *quantity* or both dimensions. Whereas flexibility is aimed at facilitating integration of renewables,

[1]See, for example, Morales et al. (2014), Silva (2010), and Nicolosi (2012).

capacity is related to solving the challenge of resource adequacy. And although a rapidly dispatchable capacity resource may solve the short-term response required by flexibility, both resources are generally not equivalent. There are further but subtle differences among these concepts. For example, capacity is measured in units of power available and it is procured as an option, establishing an upfront fixed fee and a production fee (Schummer and Vohra, 2003). In contrast, flexibility is measured in units of energy but, unlike the electricity traded in conventional wholesale markets, it is to be traded as an agreement that grants the buyer the *right* to determine a specific amount and a specific time of delivery.[2]

The canonical example of flexibility trading, where the reverse Product-Mix auction is to be applied, features a Distribution System Operator as a *buyer* of flexibility who manages a network characterized by decentralized, renewable sources of generation and is required to manage the resulting congestion, or else face the considerably high investment cost of reinforcing the network. *Sellers of flexibility* are aggregators or large-scale consumers who make imperfectly substitutable flexibility offers.

By way of a simple two variety example, this extended abstract explains the reverse product mix auction. This paves the way to formulating the questions that this research project aims at responding, which I explain in the second part of this document.

1 The reverse product mix auction

Consider K sellers who compete to supply N imperfectly substitutable commodities to a single buyer. Each seller has a convex cost function $v : B \mapsto \mathbb{R}^N$, where $B \subsetneq \mathbb{N}^N$ is a finite set of bundles. Sellers, who have quasi-linear preferences, are paid a price vector $\mathbf{p} \in \mathbb{R}^N$ for the bundle $\mathbf{q} \in B$ they sell. His supply set is thus defined as:

$$S_v(\mathbf{p}) := \underset{\mathbf{q}\in B}{\arg\max} \{\mathbf{p} \cdot \mathbf{q} - v(\mathbf{q})\}$$

Following Baldwin and Klemperer (2012), I define T_v as the tropical hypersurface (TH) associ-

[2] From a technical perspective, this description matches the concept of *flexibility object* introduced by Šikšnys et al. (2012). From a financial perspective, this agreement is an american derivative that gives the buyer some optionality regarding the amount and time of delivery but, unlike a conventional american option, the holder has an *obligation* to deliver.

ated with cost v and prices $\mathbf{p}$:

$$T_v := \{\mathbf{p} \in \mathbb{R}^N \mid \#S_v(\mathbf{p}) > 1\}$$

That is, T_v is the locus that depicts the supply set at different prices.

A *bid* is a price and bundle vector pair $(\mathbf{p}, \mathbf{q})$ and bids in a *bid set* are *mutually exclusive*. Note that T_v identifies a bid completely.

To illustrate these concepts, but without loss of generality, I focus on an example with $N = 2$ commodities. I describe in detail the bids placed by three specific bidders:

1. *Bidder 1* has two bid sets. The first one is for the bundle $(2, 0)$ at price vector $(15, 0)$ which simply indicates that the bidder is willing to sell two units of good 1 (and none of good 2) if the price p_1 of commodity 1 is greater than 15, i.e. $p_1 > 15$, regardless of the price p_2 of the second commodity. This bid set has a straightforward geometric interpretation given by the associated TH:

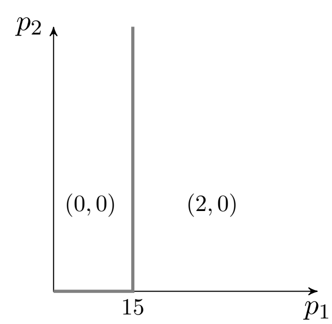

Figure 1: TH associated to bid set $\{(\mathbf{p} = (15, 0), \mathbf{q} = (2, 0))\}$

The second set of bids offers to sell one unit of good 1, i.e. bundle (1,0), *or* one unit of good 2, i.e. bundle (0,1), at price vector $(25, 27)$. That is, if bidder 1 is offered $p_1 > 25$ (and $p_2 - p_1 < 2$), it will sell bundle (1,0) but it will sell (0,1) if $p_2 > 27$ and $p_2 - p_1 > 2$. This bid set has a similar geometric representation (figure 2).

When these two geometric objects are "added", i.e. tropically factorized, bidder 1 approximates an aggregate, piecewise, supply function which clearly states its selling preferences at different prices (see figure 3). That is, if $p_1 < 15$ and $p_2 < 27$, bidder 1 would prefer to stay out of business. If $p_2 - p_1 < 2$ and $15 < p_1 < 25$, it will sell two units of good 1 and none

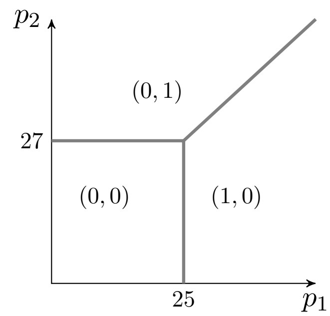

Figure 2: TH associated to bid set $\{(\mathbf{p} = (25, 27), \mathbf{q} = (1, 0))$ OR $(\mathbf{p} = (25, 27), \mathbf{q} = (0, 1))\}$

of good 2, but if $p_1 > 25$ and $p_2 < 27$, it will sell three units of good 1 and none of good 2. Also, if $p_2 > 27$ and $p_1 < 15$, it will sell one unit of good 2 and none of good 1, but if $p_1 > 15$ (and $p_2 > 27$), it will sell two units of good 1 and one unit of good 2. All this information is summarized in a very simple way, in the TH associated to the aggregated bid sets of bidder 1:

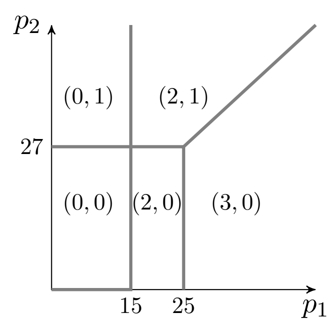

Figure 3: Bidder 1's aggregate bid sets

2. *Bidder 2*, in contrast, is interested in selling *only* units of good 2 and nothing of good 1. His first set of bids consist, therefore, in selling one unit of good 2 if $p_2 > 21$ and nothing otherwise, regardless of the value that p_1 takes. The TH associated with this bid set is:

   This bidder is also willing to sell an *additional* unit of good 2 if it is paid 11 monetary units more. Therefore his second bid is $(0, 1)$ for price $(0, 32)$. This is shown in the associated TH (figure 5).

   Bidder 2's aggregate set of bids indicate that it will sell nothing if $p_2 < 21$, will sell one unit of good 2 if $21 < p_2 < 32$ and will sell two units if $p_2 > 32$. The aggregate bids for bidder 2 are depicted in the corresponding TH (figure 6).

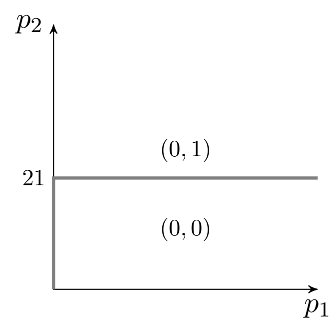

Figure 4: TH associated with bid set $\{(\mathbf{p} = (0, 21), \mathbf{q} = (0, 1))\}$

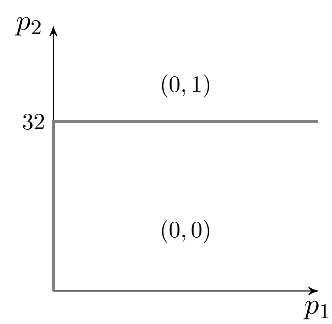

Figure 5: TH associated with bid set $\{(\mathbf{p} = (0, 32), \mathbf{q} = (0, 1))\}$

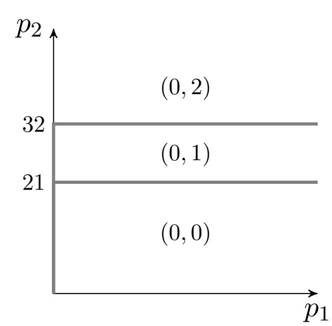

Figure 6: Bidder 2's aggregate bids

3. *Bidder 3* submits only one bid set which states that it will sell one unit of good 1 if $p_1 > 29$ and $p_1 - p_2 > 11$ but will sell one unit of good 2 if $p_2 > 18$ and $p_1 - p_2 < 11$. This is summarized in its corresponding TH (figure 7).

For illustration purposes, let us assume now that the universe of all bids submitted in this reverse product mix auction are depicted in price space, as in figure 8. The least expensive price combinations are in the southwest, while the most expensive price combinations are in the northeast of the graph. The auctioneer sees all the bids and associated quantities (in circles) and must choose which bids to reject, what prices to choose and which bids to assign to each category.

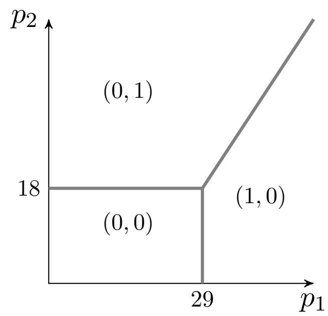

Figure 7: Bidder 3's bids

While there is not a unique way of finding a solution, as this depends on the auctioneer's preferences for each variety of good, it is possible for her to select a *market clearing price vector* $\mathbf{p^*}$ that sets the maximum, uniform price to be paid for each variety. All bids *above* this vector are losing bids, while bids *below* are winning bids. The auctioneer will allocate the commodity that gives the bidder the higher surplus, measured as the one that has the greatest distance from $\mathbf{p^*}$. In this way, bidders participating in the product mix auction are guaranteed that they will always be allocated the profit maximizing bundles, given market clearing prices.

As an example, let $\mathbf{p^*} = (24, 32)$ (marked in black in figure 9) be the price vector that corresponds to the auctioneer's preferences and observe the resulting allocation of bids . The rectangle drawn northeasterly indicates the set of rejected bids (in red). Note, as well, that there are rejected bids outside the rectangle which correspond to single commodity bids. The rectangle extends southwesterly, such that there is a constant price difference between the two varieties. Bids in yellow are allocated to the second variety, while those in green are allocated to the first. Note that, likewise to bidders, the auctioneer is effectively selecting the TH associated with its demand preferences. There is a marginal bid at price (23,31), which could be allocated to any of the two varieties, since the bidder is indifferent in this facet of the TH. We assume here that this bid was allocated to variety 1.

2 Open questions

In this paper, I aim at answering the following two questions, which are mostly concerned with the practical implementation of this auction format. Specifically:

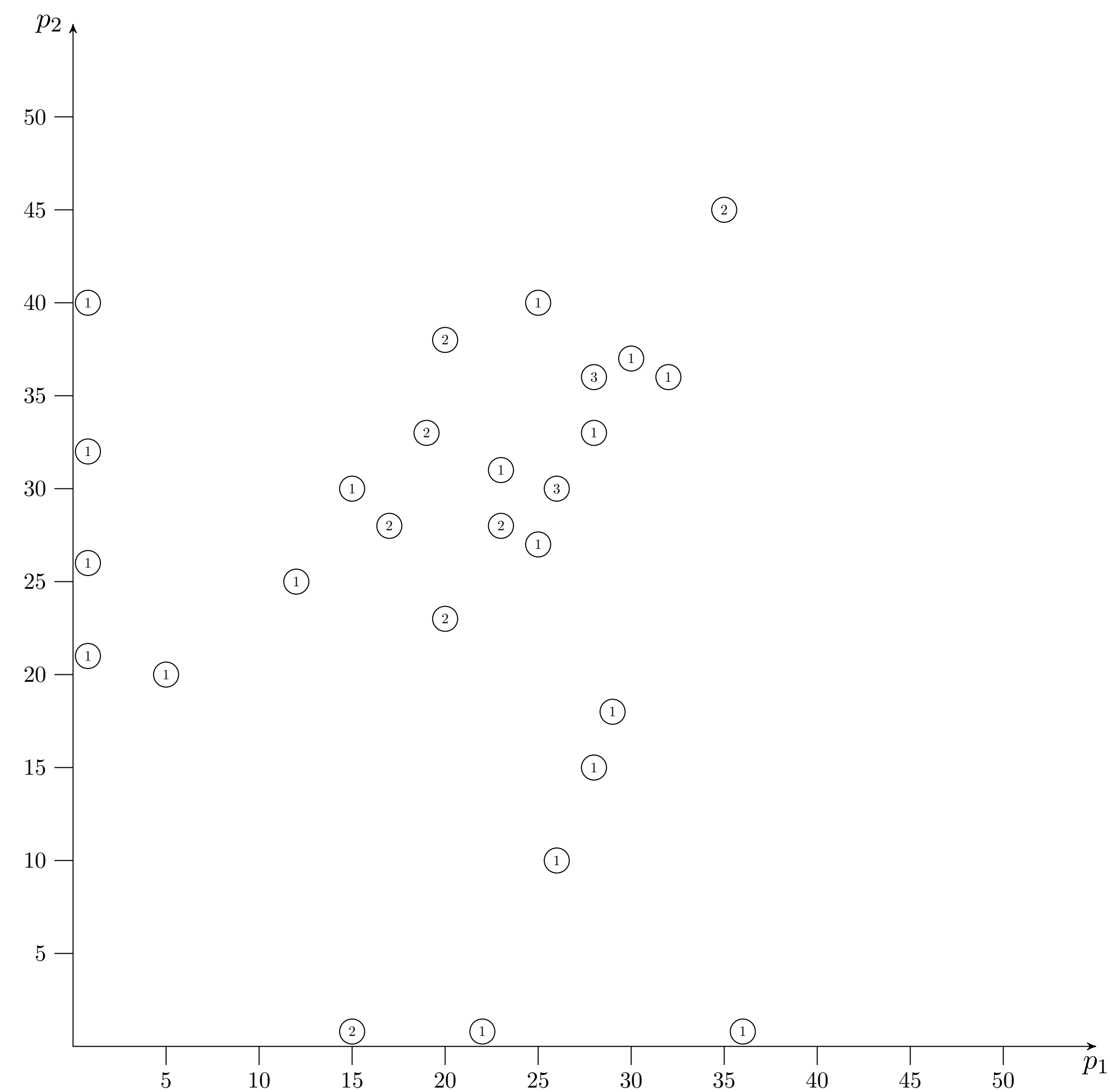

Figure 8: All bids submitted

1. *How can the reverse Product Mix auction be extended to multiple varieties and what are the associated computational issues?* Reportedly, the Bank of England runs a multiple variety (forward) Product Mix Auction. In related work, Milgrom (2009) and Milgrom and Goldband (2014) have provided answers to a similar problem but, to the best of my knowledge, there is no specific literature aimed at formulating or solving the allocation problem associated to the Product Mix Auction. In fact, Baldwin and Klemperer (2012) show their concern for "solution techniques for Product-Mix Auctions, both when we need integer solutions and when rationing is permitted".

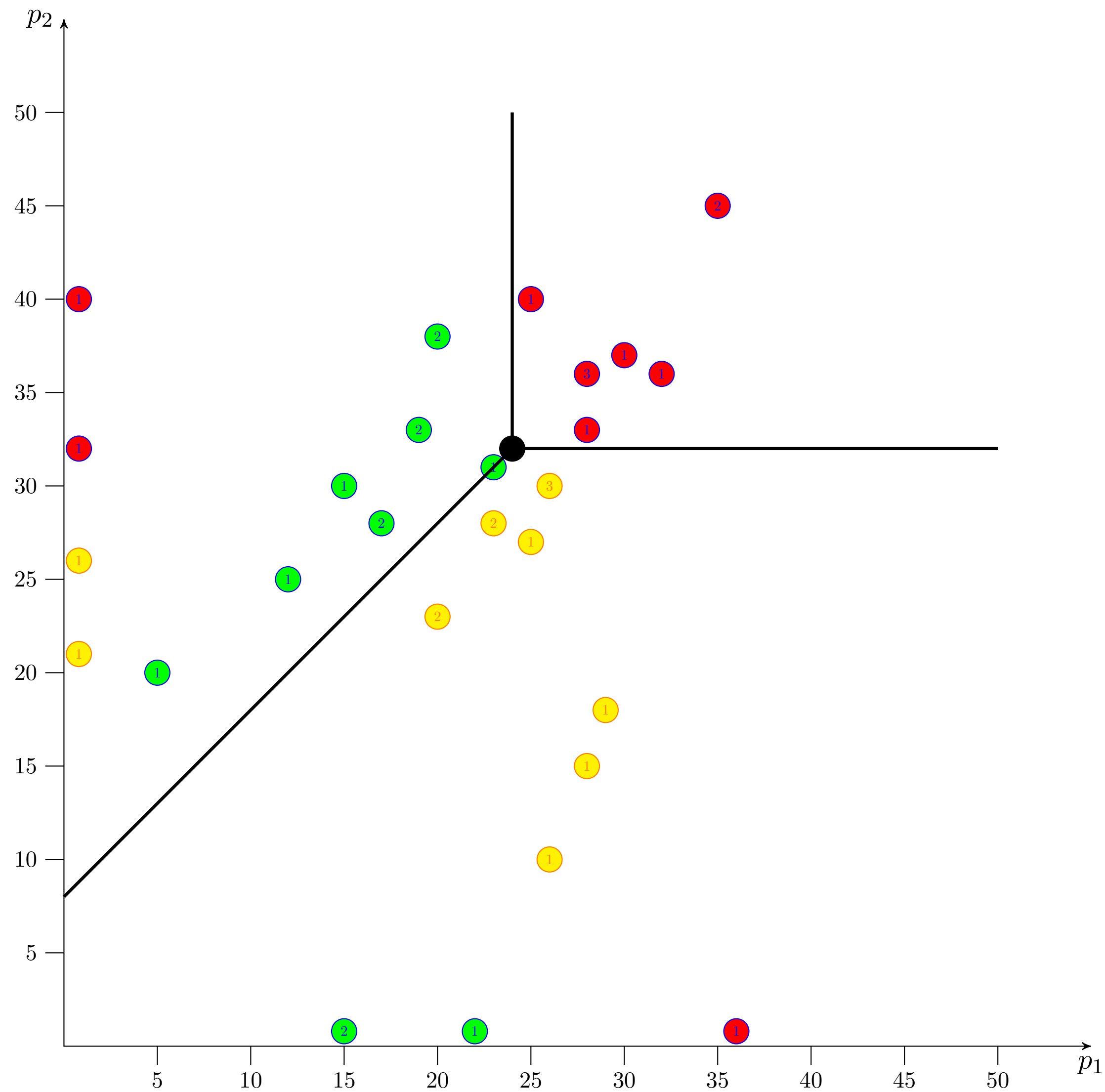

Figure 9: Market clearing in the reverse Product-Mix auction

2. *How can the Product Mix Auction be extended to a double auction format and what are the associated computational techniques to achieve this goal?*

To answer both questions, I formulate the related integer program and formulate it as a network flow problem (see Vohra (2011)). In this way, I establish links with the exisiting literature. Particularly, with results by Shapley and Shubik (1971) and Milgrom (2009)

References

Baldwin, E. and Klemperer, P. (2012). Tropical geometry to analyse demand. Technical report, Working paper, Oxford University.

Bogetoft, P., Boscán, L., and Møllgaard, P. (2015). Trading demand-side flexibility in power markets. Unpublished Mimeo.

Boscán, L. and Poudineh, R. (2015). Flexibility-enabling contracts in electricity markets. Unpublished Mimeo.

IEA (2014). *The Power of Transformation. Wind, Sun and the Economics of Flexible Power Systems.* International Energy Agency. Paris, France.

Klemperer, P. (2010). The product-mix auction: A new auction for differentiated goods. *Journal of the European Economic Association*, 8(2-3):526–536.

Milgrom, P. (2000). Putting auction theory to work: The simultaneous ascending auction. *Journal of Political Economy*, 108(2):245–272.

Milgrom, P. (2009). Assignment messages and exchanges. *American Economic Journal: Microeconomics*, 1(2):95–113.

Milgrom, P. R. and Goldband, S. (2014). System for configuration and implementation of an assignment auction or exchange. US Patent 8,788,364.

Morales, J., Conejo, A., H., M., Pinson, P., and Zugno, M. (2014). *Integrating Renewables in Electricity Markets. Operational Problems.* Springer.

Nicolosi, M. (2012). *The Economics of Renewable Electricity Market Integration. An Empirical and Model-Based Analysis of Regulatory Frameworks and their Impacts on the Power Market.* PhD thesis, University of Cologne.

Schummer, J. and Vohra, R. V. (2003). Auctions for procuring options. *Operations Research*, 51(1):41–51.

Shapley, L. S. and Shubik, M. (1971). The assignment game i: The core. *International Journal of Game Theory*, 1(1):111–130.

Šikšnys, L., Khalefa, M. E., and Pedersen, T. B. (2012). Aggregating and disaggregating flexibility objects. In *Scientific and Statistical Database Management*, pages 379–396. Springer.

Silva, V. (2010). *Value of flexibility in systems with large wind penetration*. PhD thesis, Imperial College London.

Vohra, R. V. (2011). *Mechanism Design: a linear programming approach*, volume 47. Cambridge University Press.