Approximation and Bayesian Mechanism Design

Jason Hartline

September 6, 2011

This tutorial surveys four recent directions for approximation in Bayesian mechanism design. Result 1: reserve prices are approximately optimal in single-item auctions. Result 2: posted-pricings are approximately optimal multi-item mechanisms. Result 3: optimal auctions can be approximated with a single-sample from the prior distribution. Result 4: BIC mechanism design reduces to algorithm design.

Goals for Mechanism Design Theory

Mechanism Design: how can a social planner / optimizer achieve objective when participant preferences are private.

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- *Descriptive:* predict/affirm mechanisms arising in practice.
- *Prescriptive:* suggest how good mechanisms can be designed.
- *Conclusive:* pinpoint salient characteristics of good mechanisms.
- *Tractable:* mechanism outcomes can be computed quickly.

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Informal Thesis: *approximately optimality* is often descriptive, prescriptive, conclusive, and tractable.

Example 1: Gambler's Stopping Game

A Gambler's *Stopping Game*:

- *sequence* of n games,
- *prize* of game i is distributed from F_i,
- *prior-knowledge* of distributions.

On day i, gambler plays game i:

- *realizes* prize $v_i \sim F_i$,
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Question: How should our gambler play?

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Discussion:

- *Complicated:* n different, unrelated thresholds.
- *Inconclusive:* what are properties of good strategies?
- *Non-robust:* what if order changes? what if distribution changes?
- *Non-general:* what do we learn about variants of Stopping Game?

Threshold Strategies and Prophet Inequality

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Discussion:

- *Simple:* one number t.
- *Conclusive:* trade-off "stopping early" with "never stopping".
- *Robust:* change order? change distribution above or below t?
- *General:* same solution works for similar games: invariant of "tie-breaking rule"

Prophet Inequality Proof

0. Notation:

- $q_i = \textbf{Pr}[v_i < t]$.
- $x = \textbf{Pr}[\text{never stops}] = \prod_i q_i$.

1. Upper Bound on $\textbf{E}[\max]$:

2. Lower Bound on $\textbf{E}[\text{prize}]$:

3. Choose $x = 1/2$ to prove theorem.

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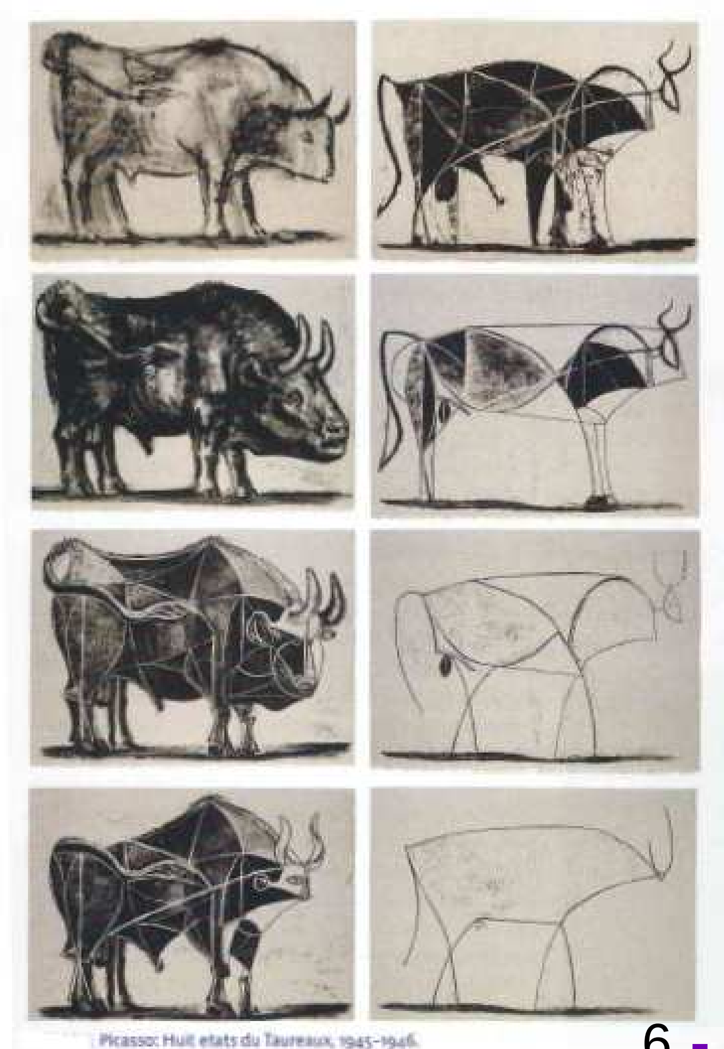

[Picasso's Bull 1945–1946 (one month)]

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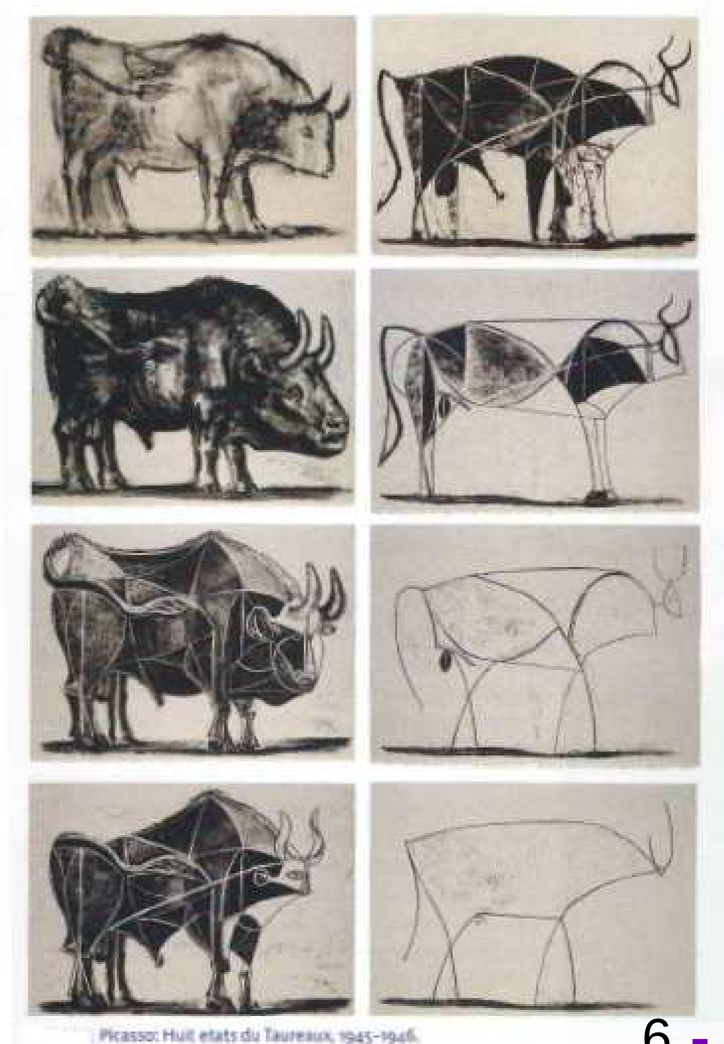

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- Practitioner can apply intuition from theory.
- Exact optimization is often impossible.
  (information theoretically, computationally)

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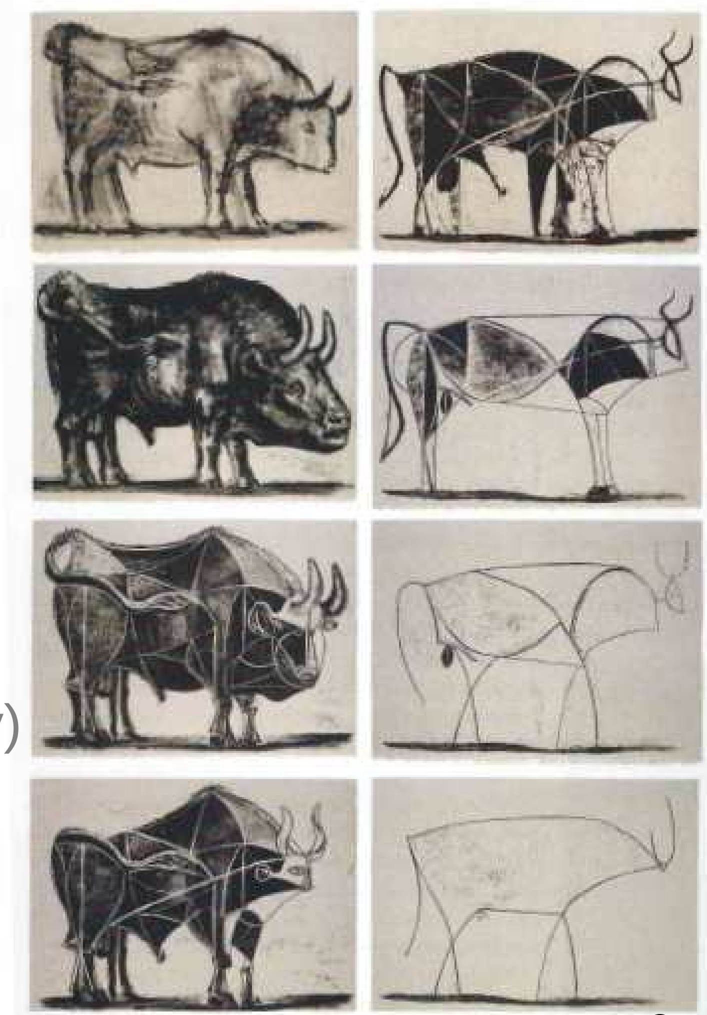

Picasso: Huit etats du Taureaux, 1945-1946.

Questions?

Overview

0. Review of auction theory

1. Single-dimensional preferences

   (e.g., single-item auctions)

2. Multi-dimensional preferences.

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3. Prior-independent mechanisms.

4. Computationally tractable mechanisms.

Part 0: Review of Auction Theory

[Vickrey '61, Myerson '81, etc.]

Single-item Auction Problems:

Given:

- one item for sale.
- n bidders (with unknown private values for item, $v_1, \ldots, v_n$)
- Bidders' objective: maximize *utility* $=$ value $-$ price paid.

Design:

- Auction to solicit bids and choose winner and payments.

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Possible Auction Objectives:

- Maximize *social surplus*, i.e., the value of the winner.
- Maximize *seller profit*, i.e., total payments.

Example Auctions

First-price Auction

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Second-price Auction

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Questions:

- what are equilibrium strategies?
- what is equilibrium outcome?
- which has higher surplus in equilibrium?
- which has higher profit in equilibrium?

Second-price Auction Equilibrium Analysis

Second-price Auction

1. Solicit sealed bids.
2. Winner is highest bidder.
3. Charge winner the second-highest bid.

Theorem: [Vickrey ’61] “bidding your value” is a *dominant strategy* in the second-price auction.

First-price Auction Equilibrium Analysis?

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Note: first-price auction has no equilibrium in dominant strategies.

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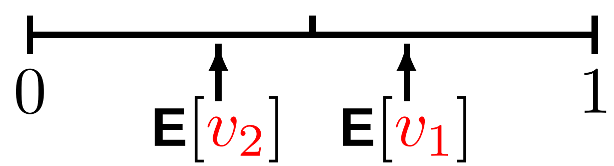

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Conclusion 2: bidder with highest value wins.

Conclusion 3: first-price auction maximizes. social surplus!

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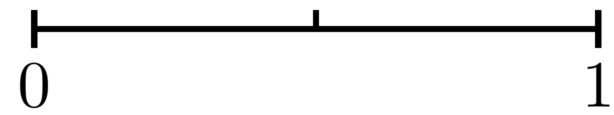

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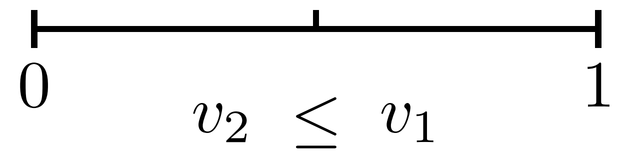

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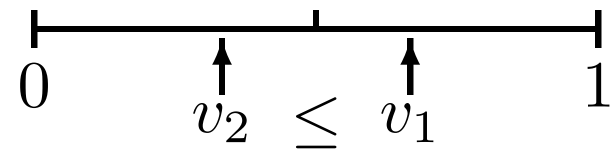

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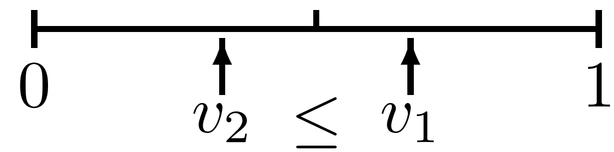

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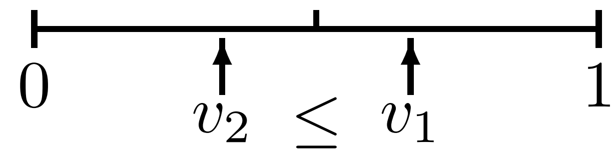

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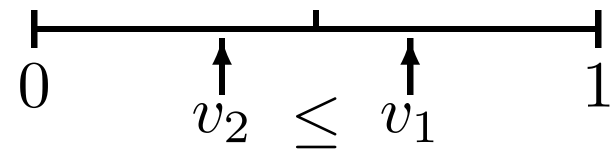

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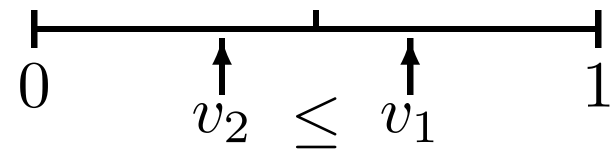

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- $\mathbf{E}[\text{Profit}] = \mathbf{E}[v_1]/2 = 1/3.$

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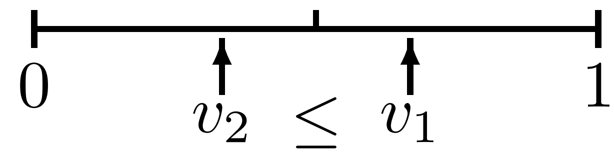

- Sort values.
- In expectation, values evenly divide unit interval.
- $\textbf{E}[\text{Profit}] = \textbf{E}[v_2] = 1/3.$

What is profit of first-price auction?

- $\textbf{E}[\text{Profit}] = \textbf{E}[v_1]/2 = 1/3.$

Notice: second-price and first-price auctions have same expected profit.

Revenue Equivalence

Revenue Equivalence Theorem: [Myerson '81] auctions with the same equilibrium allocation have the same equilibrium revenue.

Part I: Approximation for single-dimensional Bayesian mechanism design

(where agent preferences are given by a private value for service, zero for no service; preferences are drawn from a distribution)

Example 2: Single-item auction

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Question: What is optimal auction?

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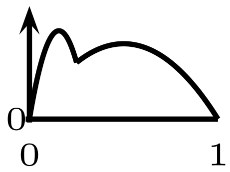

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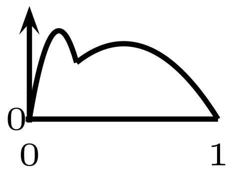

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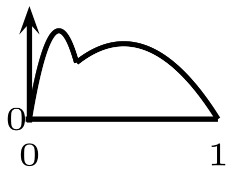

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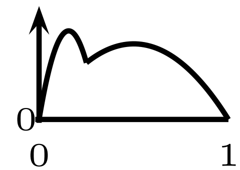

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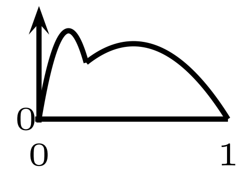

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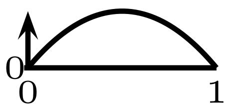

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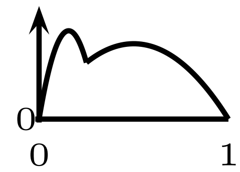

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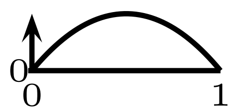

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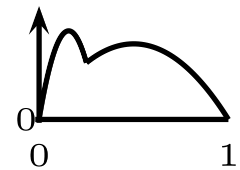

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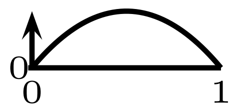

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8. **Cor:** for iid, regular dists, optimal auction is *Vickrey with reserve price* $\varphi^{-1}(0)$.

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Discussion:

- iid, regular case: seems very special.
- general case: optimal auction rarely used. (too complicated?)

Approximation with reserve prices

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prizes	virtual values
threshold t	virtual price
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E[max prize]	**E**[optimal revenue]
E[prize for t]	**E**[Vickrey revenue]

Discussion:

- constant virtual price $\Rightarrow$ bidder-specific reserves.
- *simple:* reserve prices natural, practical, and easy to find.
- *robust:* posted pricing with arbitrary tie-breaking works fine, collusion fine, etc.

Anonymous Reserves

Question: for non-identical distributions, is *anonymous reserve* approximately optimal?

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Discussion:

- theorem is not tight, actual bound is in $[2, 4]$.
- justifies wide prevalence.

Extensions

Beyond single-item auctions: *general feasibility constraints*.

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Thm: non-identical (possibly irregular) distributions, *posted pricing mechanisms* are often constant approximations.

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Proof technique:

- optimal mechanism is a virtual surplus maximizer.
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Basic Open Question: to what extent do simple mechanisms approximate (well understood but complex) optimal ones?

Challenges: non-downward-closed settings, negative virtual values.

Questions?

Part II: Approximation for multi-dimensional Bayesian mechanism design

(where agent preferences are given by values for each available service, zero for no service; preferences drawn from distribution)

Example 3: unit-demand pricing

Problem: Bayesian Unit-Demand Pricing

- a single, unit-demand consumer.
- n items for sale.
- a dist. $\mathbf{F} = F_1 \times \cdots \times F_n$ from which the consumer's values for each item are drawn.

Goal: seller optimal *item-pricing* for $\mathbf{F}$.

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Optimal Pricing

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Discussion:

- little conceptual insight and
- not generally tractable.

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4. *Instantiation:* SD-PRICING $\geq \frac{1}{\beta}$SD-AUCTION

   (virtual surplus approximation)

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- *robust* to agent ordering, collusion, etc.
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Open Question: identify upper bounds beyond unit-demand settings:

- analytically tractable and
- approximable.

Questions?

Part III: Approximation for prior-independent mechanism design.

(mechanisms should be good for any set of agent preferences, not just given distributional assumptions)

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Question: can we design good auctions without knowledge of prior-distribution?

Optimal Prior-independent Mechs

Optimal Prior-indep. Mech: (a.k.a., non-parametric implementation)

1. agents report value and prior,
2. shoot agents if disagree, otherwise
3. run optimal mechanism for reported prior.

Discussion:

- *complex*, agents must report high-dimensional object.
- *non-robust*, e.g., if agents make mistakes.
- *inconclusive*, begs the question.

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- “recruit one more bidder” is prior-independent strategy.
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- *conclusive:* competition more important than optimization.
- *non-generic*: e.g., for k-unit auctions, need k additional bidders.

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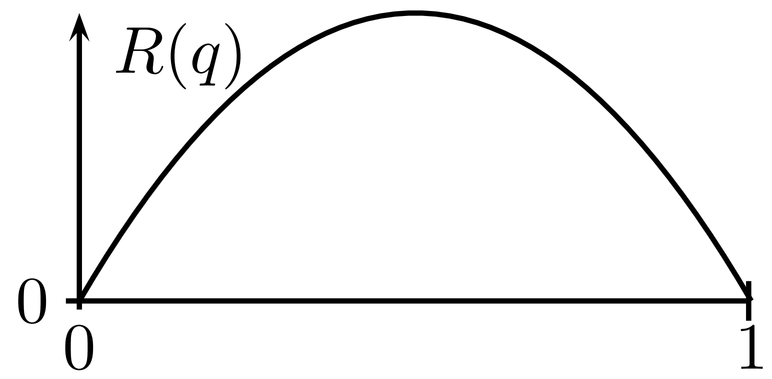

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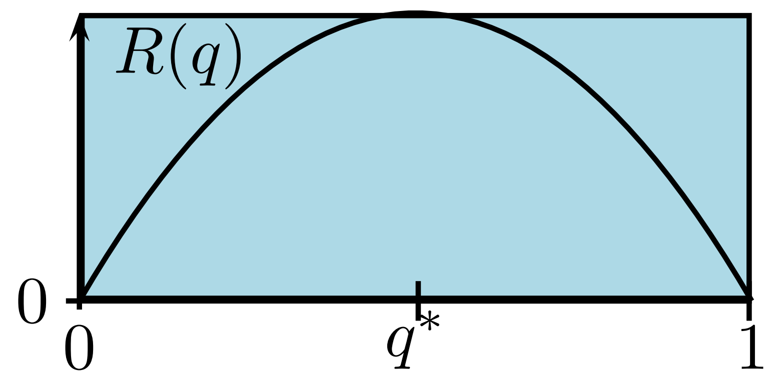

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Recall: revenue curve

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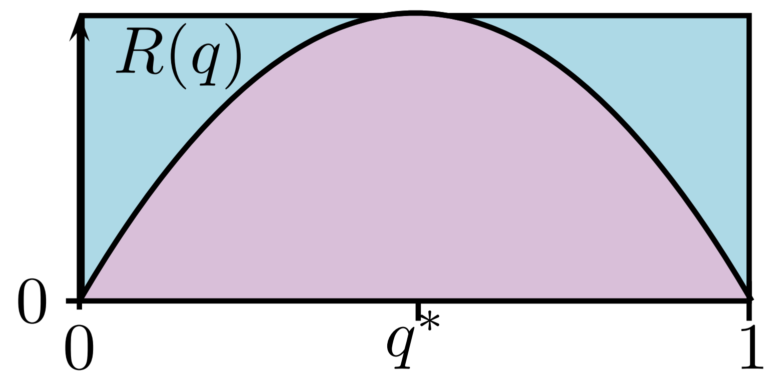

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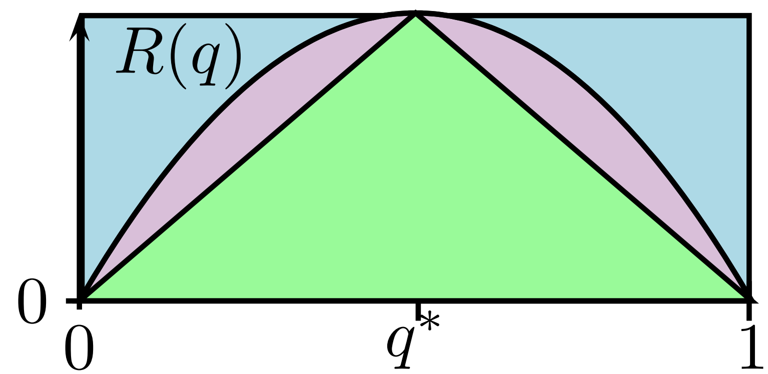

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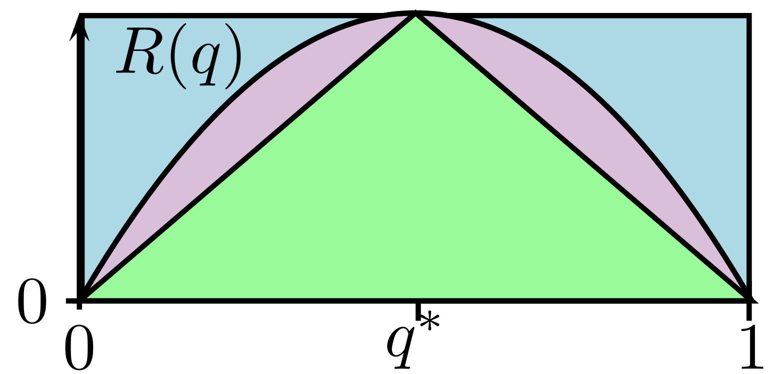

- So Vickrey with two bidders $\geq$ optimal revenue from one bidder.

Example 4: digital goods

Question: how should a profit-maximizing seller sell a *digital good* (n bidder, n copies of item)?

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Discussion:

- optimal,
- simple, but
- not prior-independent

Approximation via Single Sample

Single-Sample Auction: (for digital goods)

[Dhangwatnotai, Roughgarden, Yan ’10]

1. pick random agent i as sample.
2. offer all other agents price v_i.
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Proof: from geometric argument.

Discussion:

- *prior-independent*.
- *conclusive*,
  - learn distribution from reports, not cross-reporting.
  - don't need precise distribution, only need single sample for approximation. (more samples can improve approximation/robustness.)
- *generic*, applies to general settings.

Extensions

Recent Extensions:

- non-identical distributions. [Dhangwatnotai, Roughgarden, Yan ’10]
- position auctions, matroids, downward-closed environments.
  [Hartline, Yan ’11; Ha, Hartline ’11]

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Open Questions:

- non-downward-closed environments?
- multi-dimensional preferences?

Questions?

Part IV: Computational Tractability in Bayesian mechanism design

(where the optimal mechanism may be computationally intractable)

Example 5: single-minded combinatorial auction

Problem: Single-minded combinatorial auction

- n agents,
- m items for sale.
- Agent i wants only bundle $S_i \subset \{1, \ldots, m\}$.
- Agent i's value v_i drawn from F_i.

Goal: auction to maximize *social surplus* (a.k.a., welfare).

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Goal: auction to maximize *social surplus* (a.k.a., welfare).

Question: What is optimal mechanism?

Optimal Combinatorial Auction

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1. allocate to maximize reported surplus,
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2. charge each agent their "externality".

Discussion:

- distribution is irrelevant (for welfare maximization).
- Step 1 is NP-hard *weighted set packing* problem.
- Cannot replace Step 1 with approximation algorithm.

BNE reduction

Question: Can we convert any algorithm into a mechanism without reducing its social welfare?

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Approach:

- Run $\mathcal{A}(\sigma_1(v_1), \ldots, \sigma_n(v_n))$.
- σ_i calculated from *max weight matching* on i's type space.
  - stationary with respect to F_i.
  - $x_i(\sigma_i(v_i))$ monotone.
  - welfare preserved.

Example: σ_i

Example:

$f(v_i)$	v_i	$x_i(v_i)$
.25	1	0.1
.25	4	0.5
.25	5	0.4
.25	10	1.0

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.25	4	0.5	5
.25	5	0.4	4
.25	10	1.0	10

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Note:

- σ_i is from max weight matching between v_i and $x_i(v_i)$.
- σ_i is stationary.
- σ_i (weakly) improves welfare.

BNE reduction discussion

Thm: Any algorithm can be converted into a mechanism with no loss in expected welfare. Runtime is polynomial in size of agent's type space.
[Hartline, Lucier '10; Hartline, Kleinberg, Malekian '11; Bei, Huang '11]

Discussion:

- applies to all algorithms not just worst-case approximations.
- BNE incentive constraints are solved independently.
- works with multi-dimensional preferences too.

Extensions

Extension:

- impossibility for dominant strategy reduction.

[Chawla, Immorlica, Lucier '11]

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- impossibility for dominant strategy reduction.

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Open Questions:

- non-brute-force in type-space? e.g., for product distributions?
- other objectives, e.g., makespan? [Chawla, Immorlica, Lucier '11]

Questions?