

# The Daycare Assignment Problem: Matching in an Overlapping Generations Model

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## Abstract

In this paper we introduce and study the daycare assignment problem. We take the mechanism design approach to the problem of assigning children of different ages to daycares, motivated by the mechanism currently in place in Denmark. The daycare assignment problem is characterized by an overlapping generations structure, which distinguishes it from the *school choice problem*. For example, children of different ages may be allocated to the same daycare, and the same child may be allocated to different daycares across time. Moreover, the daycares' priorities are history-dependent: a daycare gives priority to children currently enrolled in it, as is the case with the Danish system.

First, we study the concept of stability, and, to account for the dynamic nature of the problem, we propose a novel solution concept, which we call strong stability. With a suitable restriction on the priority orderings of schools, we show that strong stability and the weaker concept of static stability will coincide. We then extend the well known Gale-Shapley deferred acceptance algorithm for dynamic problems and show that it yields a matching that satisfies strong stability. It is not Pareto dominated by any other matching, and, if there is an efficient stable matching, it must be the Gale-Shapley one. However, contrary to static problems, it does not necessarily Pareto dominate all other strongly stable mechanisms. Most importantly, we show that the Gale-Shapley algorithm is not strategy-proof. In fact, one of our main results is a much stronger impossibility result: For the class of dynamic matching problems that we study, there are no algorithms that satisfy strategy-proofness and strong stability.

Second, we show that the also well known Top Trading Cycles algorithm is neither Pareto efficient nor strategy-proof.

We conclude by proposing a variation of the serial dictatorship, which is strategy-proof and efficient.

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