Matching couples with Scarf's algorithm

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Mechanism design in matching markets dates back to the seminal paper of Gale and Shapley [4] on college admissions. They introduced the concept of *stable matching*, that is a fair solution where an application of a student can be rejected by a college only if its quota is filled with better candidates. Gale and Shapley gave an efficient algorithm to find a stable matching in this setting. It turned out that the same method had already been used in the US resident matching program (NRMP) since 1952. This program has been redesigned later [6], partly because the organisers wanted to accommodate the wishes of couples. Since then couples can submit joint preference lists in order to avoid being matched to hospitals far from each other. However, for the *Hospitals Residents problem with couples* (HRC) the existence of a stable matching is not guaranteed any more. Moreover, the related decision problem is NP-hard [5], therefore we need to use heuristics for large markets. Biró et al. [3] compared some old and new heuristics for a setting that is currently present in the Scottish resident allocation program.

Another seminal paper in cooperative game theory is by Scarf [7]. He gave an algorithm to find a core element for any *balanced NTU-game*. Aharoni and Fleiner [1] used this algorithm to find *stable fractional matchings* for problems where the underlying graph is not necessarily bipartite. Biró and Fleiner [2] generalised this result by showing that Scarf's algorithm can be used to find *stable allocations* for NTU-games where both the agents and their cooperations can have capacities. In particular, this method can be used as a heuristic for HRC as well, since if a stable allocation, obtained by Scarf's algorithm, happens to be integral then it corresponds to a stable matching.

In this paper we show how the Scarf algorithm can be used to find stable allocations for HRC efficiently. We then compare the performance of this method and other heuristics described in [6] and [3]. Our main finding is that the Scarf algorithm works very well if the proportion of couples is high. This result may be relevant regarding other applications too, such as the Hungarian higher education matching scheme where students can apply to pairs of courses.

Keywords: Mechanism design, Hospitals Residents problem, couples, Scarf algorithm

References

[1] R. Aharoni and T. Fleiner. On a lemma of Scarf. *Journal of Combinatorial Theory, Series B*, 87(1):72–80, 2003.

[2] P. Biró and T. Fleiner. Fractional solutions for NTU-games. *In Proceedings of COMSOC 2010: 3rd International Workshop on Computational Social Choice*, pp. 283-294, 2010.

[3] P. Biró and R.W. Irving and I. Schlotter. Stable matching with couples – an empirical study. *To appear in ACM Journal of Experimental Algorithmics*, 2011.

[4] D. Gale and L.S. Shapley. College admissions and the stability of marriage. *American Mathematical Monthly*, 69:9-15, 1962.

[5] E. Ronn. NP-complete stable matching problems. *Journal of Algorithms*, 11:285–304, 1990.

[6] A. E. Roth and E. Peranson. The redesign of the matching market for American physicians: Some engineering aspects of economic design. *American Economic Review*, 89(4):748–780, 1999.

[7] H. E. Scarf. The core of an N person game. *Econometrica*, 35:50–69, 1967.

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