Efficient Oblivious Transfer Extensions and Applications

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based on joint works with
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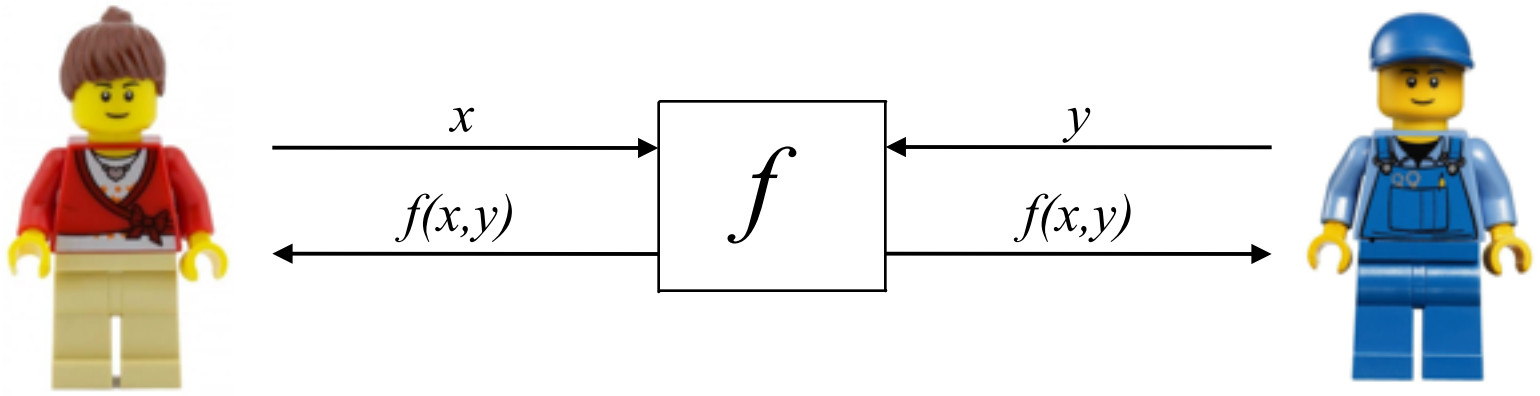
MPC Workshop Aarhus, May 5-9, 2014

Secure Computation



This Talk: **Semi-Honest** Adversaries

Example Privacy-Preserving Applications

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Auctions [NaorPS99], ...

Remote Diagnostics [BrickellPSW07], ...

DNA Searching [Troncoso-PastorizaKC07], ...

Biometric Identification [ErkinFGKLT09], ...

Medical Diagnostics [BarniFKLS**S**09], ...

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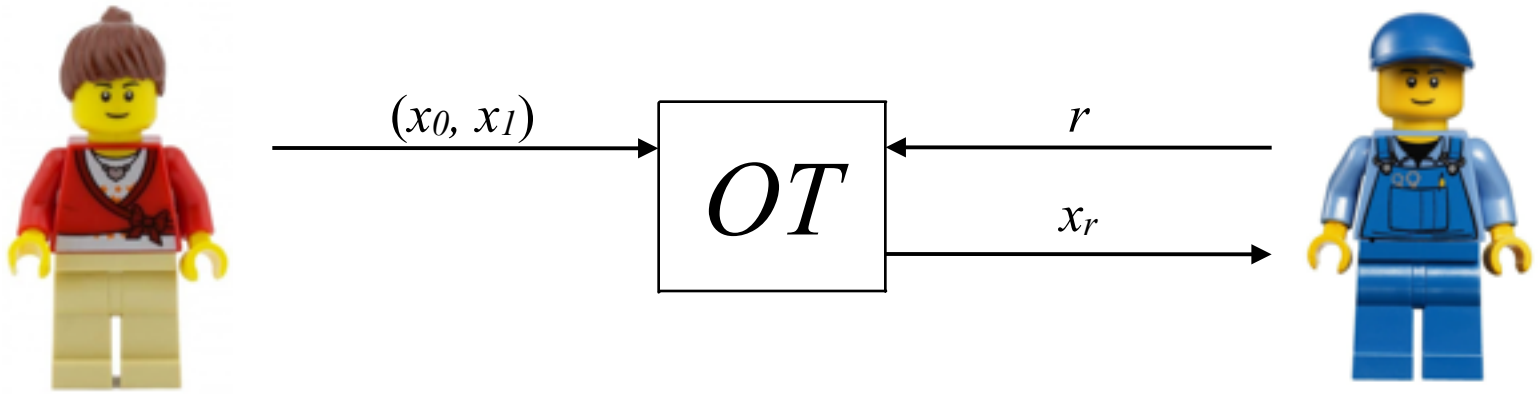
Oblivious Transfer (OT)



OT is used in many secure computation protocols.

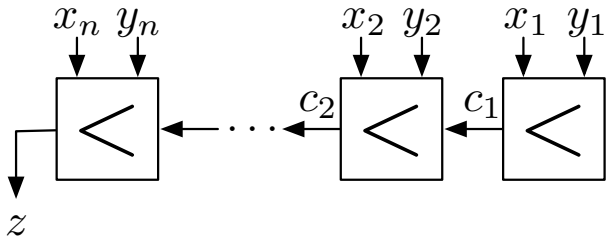
Yao's Garbled Circuits Protocol [Yao'86]

$f(\cdot,\cdot)$ e.g., $\mathbf{x} < \mathbf{y}$

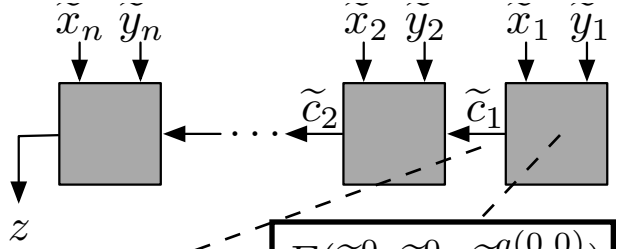
private data $\mathbf{x} = x_1, .., x_n$

private data $\mathbf{y} = y_1, .., y_n$

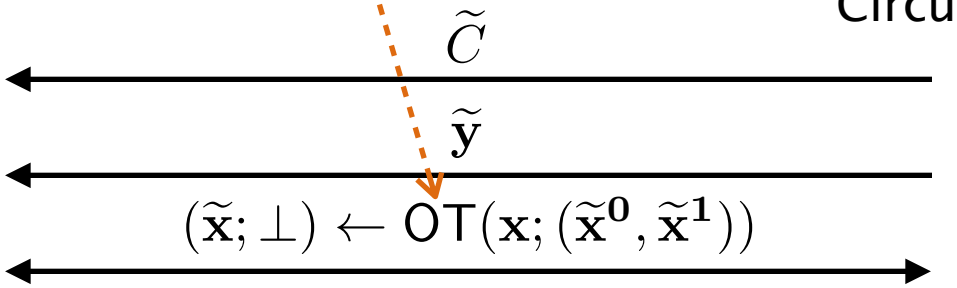
- Circuit



- Garbled Circuit $\widetilde{C}$



OT on keys per Alice's input bit



$f(\mathbf{x},\mathbf{y}) = \widetilde{C}(\widetilde{\mathbf{x}},\widetilde{\mathbf{y}})$

$\widetilde{c}_1^0, \widetilde{c}_1^1$
Garbled Values

$E(\widetilde{x}_1^0, \widetilde{y}_1^0;\ \widetilde{c}_1^{g(0,0)})$
$E(\widetilde{x}_1^0, \widetilde{y}_1^1;\ \widetilde{c}_1^{g(0,1)})$
$E(\widetilde{x}_1^1, \widetilde{y}_1^0;\ \widetilde{c}_1^{g(1,0)})$
$E(\widetilde{x}_1^1, \widetilde{y}_1^1;\ \widetilde{c}_1^{g(1,1)})$

Garbled Table

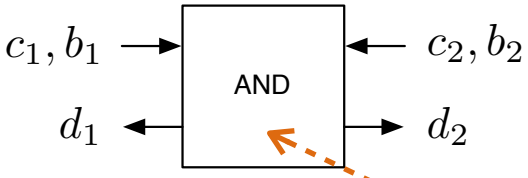
The GMW Protocol
[Goldreich/Micali/Wigderson'87]

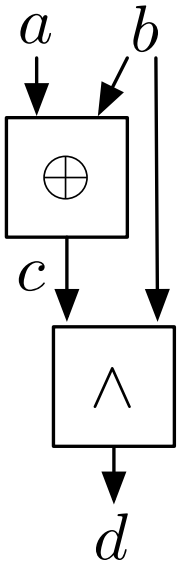
Secret share inputs: $a = a_1 \oplus a_2$

$b = b_1 \oplus b_2$

Non-Interactive XOR gates: $c_1 = a_1 \oplus b_1$; $c_2 = a_2 \oplus b_2$

Interactive AND gates:





Two OTs on bits per AND gate

Recombine outputs: $d = d_1 \oplus d_2$

How to Measure Efficiency of a Protocol?

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- ✓ Runtime (depends on implementation & scenario)
- ✓ Communication
  - # rounds (important for networks with high latency)
  - # bits sent (important for networks with low bandwidth)
- ? Computation
  - Usually: count # crypto operations, e.g.,
    - # modular exponentiations
    - # point multiplications
    - # hash function evaluations (SHA)
    - # block cipher evaluations (AES)
    - # OTP operations

  - **What about non-cryptographic operations?**

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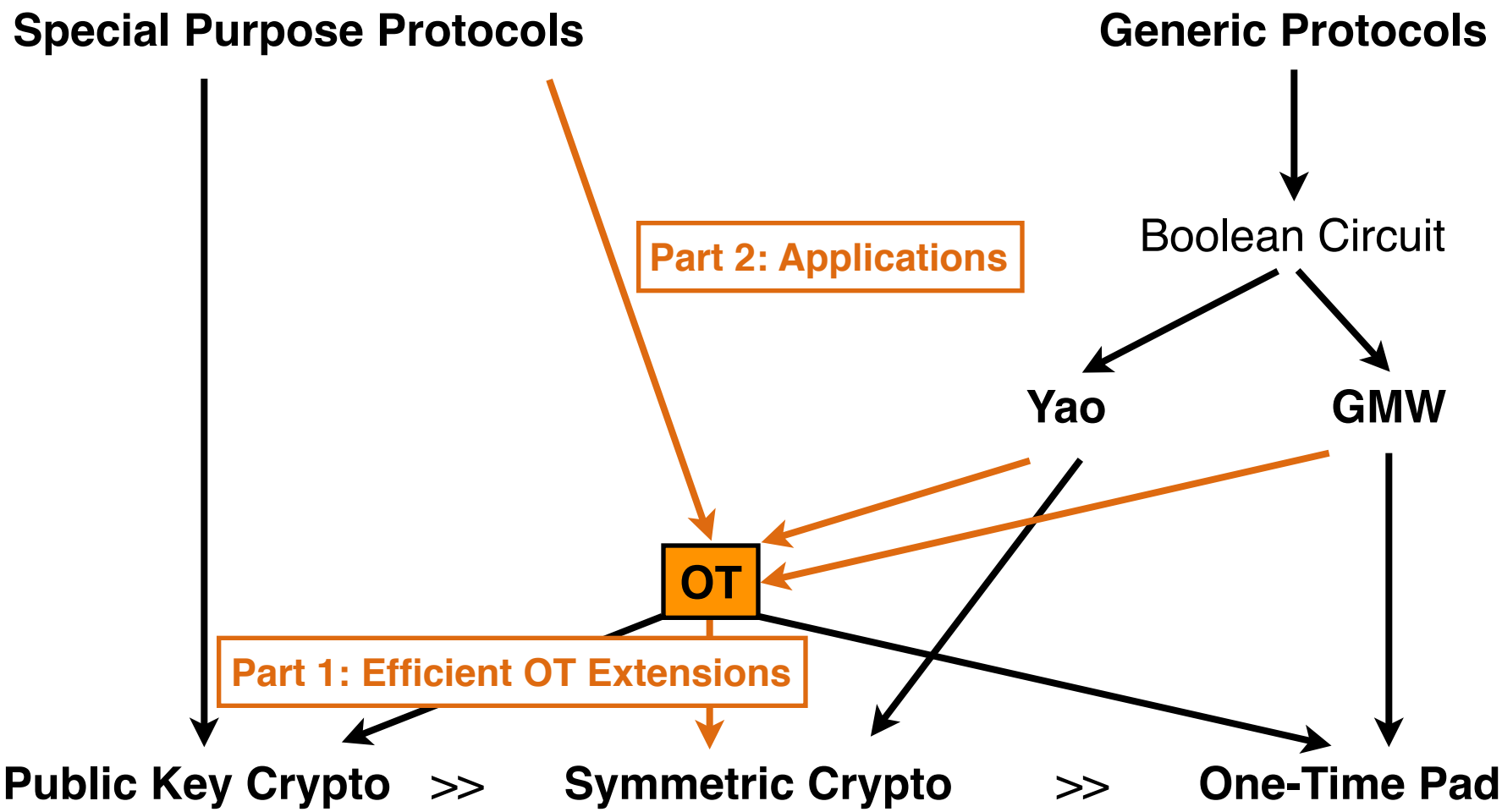
Overview of this talk: Secure Computation



Part 1: Efficient OT Extensions

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G. Asharov, Y. Lindell, T. Schneider, M. Zohner:
More efficient oblivious transfer and extensions for faster secure computation.
In ACM CCS'13.

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OT - Bad News

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- [ImpagliazzoRudich'89]: there's no black-box reduction from OT to OWFs

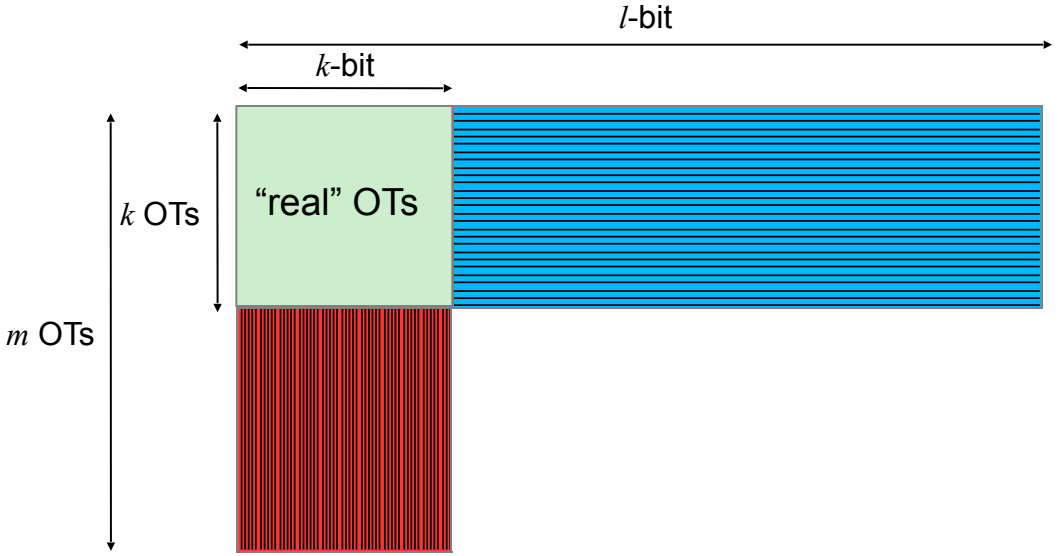
- Several OT protocols based on public-key cryptography
  - e.g., [NaorPinkas'01] yields ~1,000 OTs per second

- Since public-key crypto is expensive, OT was believed to be inefficient

OT - Good News

- [Beaver'95]: OTs can be pre-computed (only OTP in online phase)

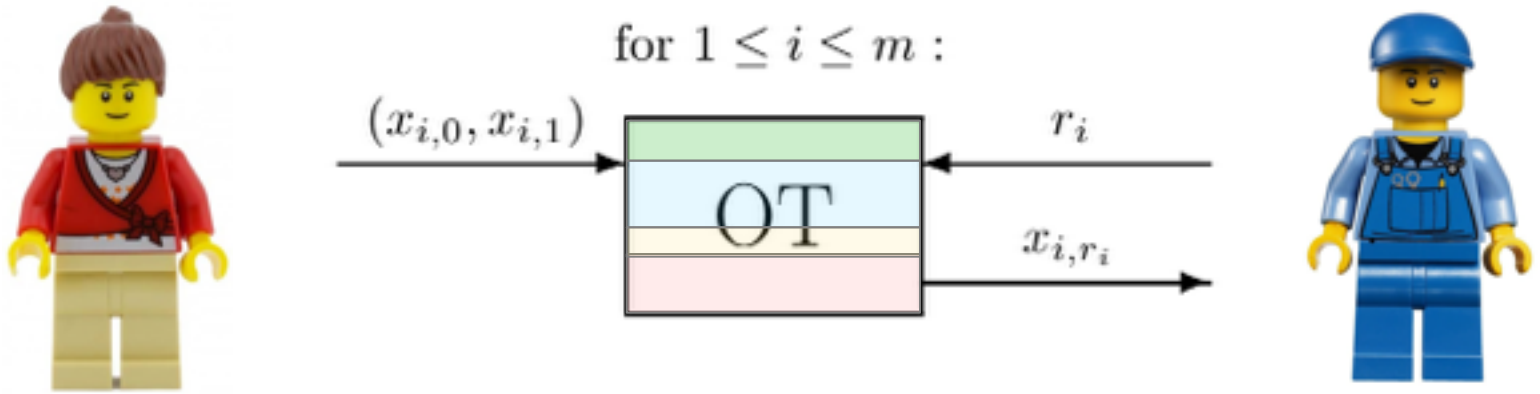
- OT Extensions (similar to hybrid encryption):
  use symmetric crypto to stretch few "real" OTs into longer/many OTs
  - [Beaver'96]: OT on long strings from short seeds
  - [IshaiKNP'03]: many OTs from few OTs



OT Extension of [IKNP'03] (1)

- Alice inputs m pairs of ℓ-bit messages $(x_{i,0}, x_{i,1})$

- Bob inputs m-bit string r and obtains $x_{i,ri}$ in i-th OT



OT Extension of [IKNP'03] (2)

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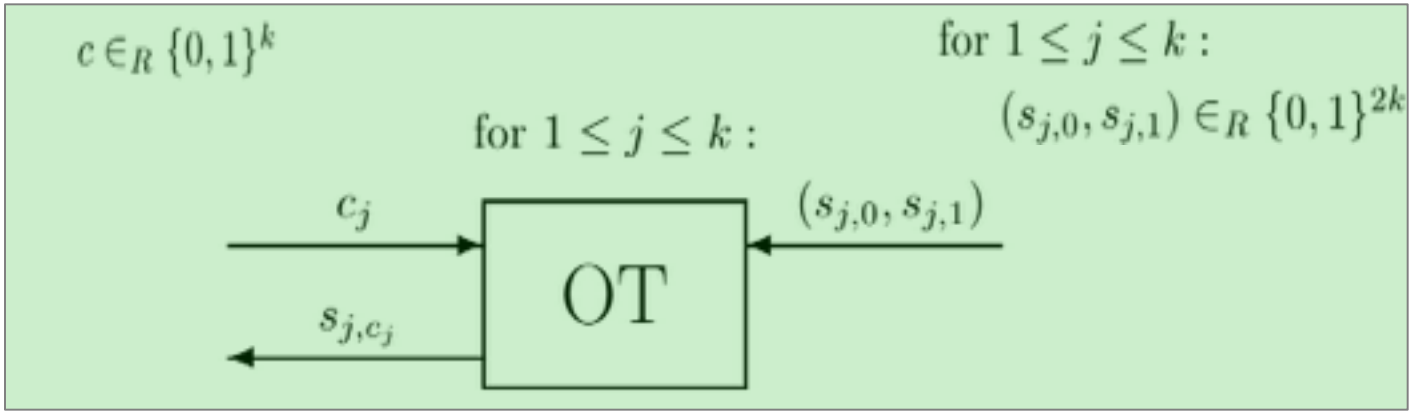
- Alice and Bob perform k "real" OTs on random seeds with reverse roles (k: security parameter)



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OT Extension of [IKNP'03] (3)

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- Bob generates a random m x k bit matrix $\mathbf{T}$ and masks his choices r

- The matrix is masked with the stretched seeds of the "real" OTs

$\mathbf{T} \in_R \{0,1\}^{m \times k}$

for $1 \leq j \leq k$:

$u_{j,0} = PRG(s_{j,0}) \oplus \mathbf{T}[\mathrm{j}]$

$u_{j,1} = PRG(s_{j,1}) \oplus \mathbf{T}[\mathrm{j}] \oplus \mathbf{r}$

$(u_{j,0}, u_{j,1}),\ 1 \leq i \leq k$ ←

for $1 \leq j \leq k$:

$\mathbf{V}[\mathrm{j}] = u_{j,c_j} \oplus PRG(s_{j,c_j})$

PRG: pseudo-random generator (instantiated with AES)

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OT Extension of [IKNP'03] (4)

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- Transpose matrices $\mathbf{V}$ and $\mathbf{T}$

- Alice masks her inputs and obliviously sends them to Bob

$\mathbf{V'} = \mathbf{V}^T$	$\mathbf{T'} = \mathbf{T}^T$
for $1 \leq i \leq m$:	
$y_{i,0} = x_{i,0} \oplus H(i, \mathbf{V'}[\mathrm{i}])$	
$y_{i,1} = x_{i,1} \oplus H(i, \mathbf{V'}[\mathrm{i}] \oplus c)$	
$\xrightarrow{(y_{i,0}, y_{i,1}), 1 \leq i \leq m}$	for $1 \leq i \leq m$:
	$x_{i,r_i} = y_{i,r_i} \oplus H(i, \mathbf{T'}[\mathrm{i}])$

H: correlation robust function (instantiated with hash function)

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Computation Complexity of OT Extension


m pairs $(x_{i,0}, x_{i,1}) \in \{0,1\}^{2l}$ $\quad$ $\mathbf{r} = (r_1, \ldots, r_m) \in \{0,1\}^m$

for $1 \le j \le k$:

$c_j \in_R \{0,1\}$ → OT ← $(s_{j,0}, s_{j,1}) \in_R \{0,1\}^{2k}$

s_{j,c_j}

$\mathbf{T} \in_R \{0,1\}^{m \times k}$

for $1 \le j \le k$:

$u_{j,0} = PRG(s_{j,0}) \oplus \mathbf{T}[j]$

$u_{j,1} = PRG(s_{j,1}) \oplus \mathbf{T}[j] \oplus \mathbf{r}$

$(u_{j,0}, u_{j,1}),\ 1 \le i \le k$ ←

for $1 \le j \le k$:

$\mathbf{V}[j] = u_{j,c_j} \oplus PRG(s_{j,c_j})$

$\mathbf{V}' = \mathbf{V}^T$ $\quad$ $\mathbf{T}' = \mathbf{T}^T$

for $1 \le i \le m$:

$y_{i,0} = x_{i,0} \oplus H(i, \mathbf{V}'[i])$

$y_{i,1} = x_{i,1} \oplus H(i, \mathbf{V}'[i] \oplus c)$

$(y_{i,0}, y_{i,1}), 1 \le i \le m$ →

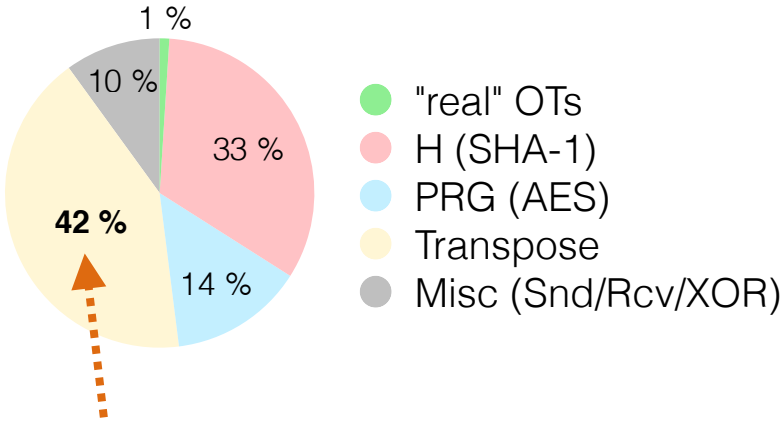
for $1 \le i \le m$:

$x_{i,r_i} = y_{i,r_i} \oplus H(i, \mathbf{T}'[i])$


Per OT:

1	# PRG evaluations	2
2	# H evaluations	1

Time distribution for 10 Mio. OTs (in 21s):



Non-crypto part is bottleneck!!!

Algorithmic Optimization
Efficient Bit-Matrix Transposition

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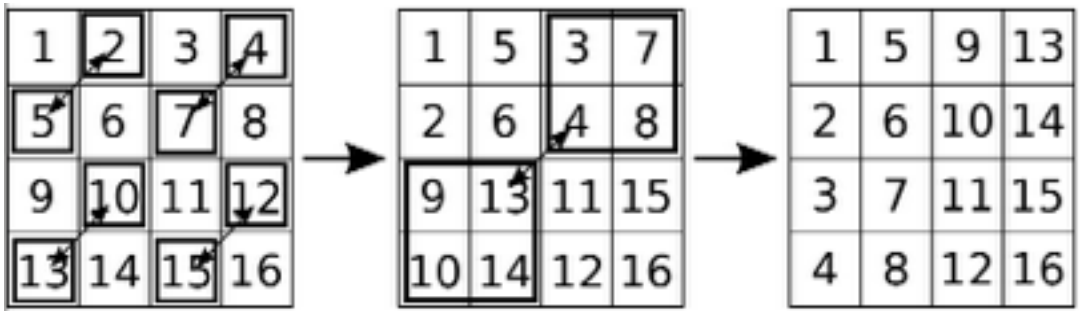
- Naive matrix transposition performs mk load/process/store operations

- Eklundh's algorithm reduces number of operations to O($m \log_2 k$) swaps
  - Swap whole registers instead of bits
  - Transposing 10 times faster



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Algorithmic Optimization
Parallelized OT Extension

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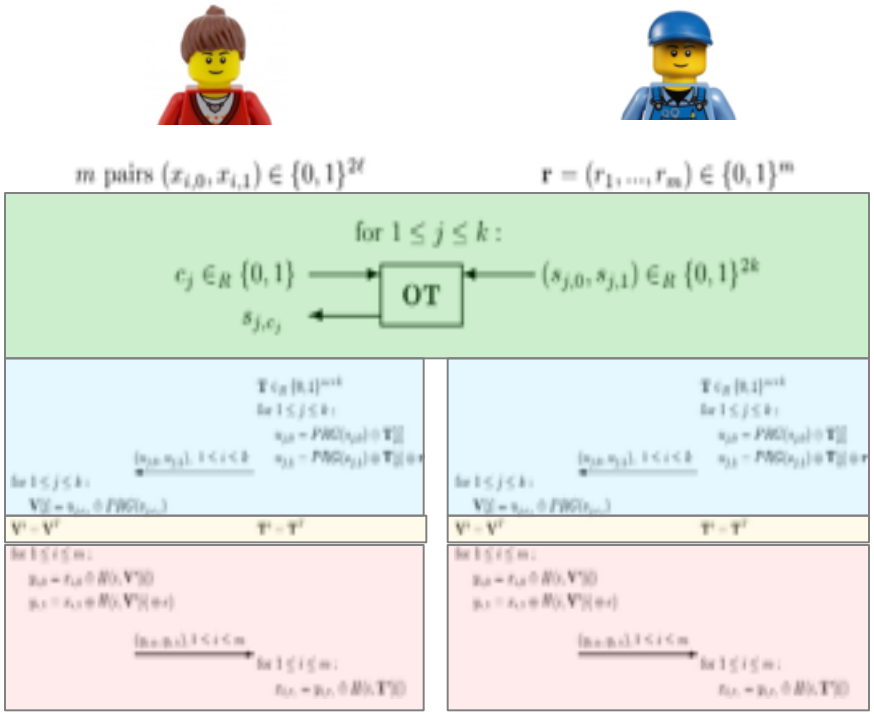
- OT extension can easily be parallelized by splitting the $\mathbf{T}$ matrix into sub-matrices

- Since columns are independent, OT is highly parallelizable



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Communication Complexity of OT Extension


m pairs $(x_{i,0}, x_{i,1}) \in \{0,1\}^{2\ell}$ | $\mathbf{r} = (r_1, \ldots, r_m) \in \{0,1\}^m$

for $1 \le j \le k$:

$c_j \in_R \{0,1\}$ → OT ← $(s_{j,0}, s_{j,1}) \in_R \{0,1\}^{2k}$

s_{j,c_j} ←

$\mathbf{T} \in_R \{0,1\}^{m \times k}$

for $1 \le j \le k$:

$u_{j,0} = PRG(s_{j,0}) \oplus \mathbf{T}[j]$

$u_{j,1} = PRG(s_{j,1}) \oplus \mathbf{T}[j] \oplus \mathbf{r}$

$(u_{j,0}, u_{j,1}),\ 1 \le i \le k$ ←

for $1 \le j \le k$:

$\mathbf{V}[j] = u_{j,c_j} \oplus PRG(s_{j,c_j})$

$\mathbf{V}' = \mathbf{V}^T$ | $\mathbf{T}' = \mathbf{T}^T$

for $1 \le i \le m$:

$y_{i,0} = x_{i,0} \oplus H(i, \mathbf{V}'[i])$

$y_{i,1} = x_{i,1} \oplus H(i, \mathbf{V}'[i] \oplus c)$

$(y_{i,0}, y_{i,1}), 1 \le i \le m$ →

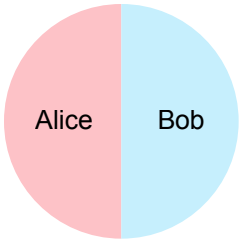
for $1 \le i \le m$:

$x_{i,r_i} = y_{i,r_i} \oplus H(i, \mathbf{T}'[i])$

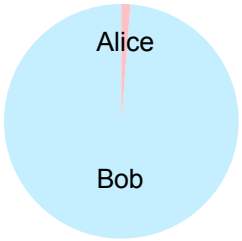

Per OT:

2ℓ Bits sent $2k$

Yao: $\ell = k = 80$



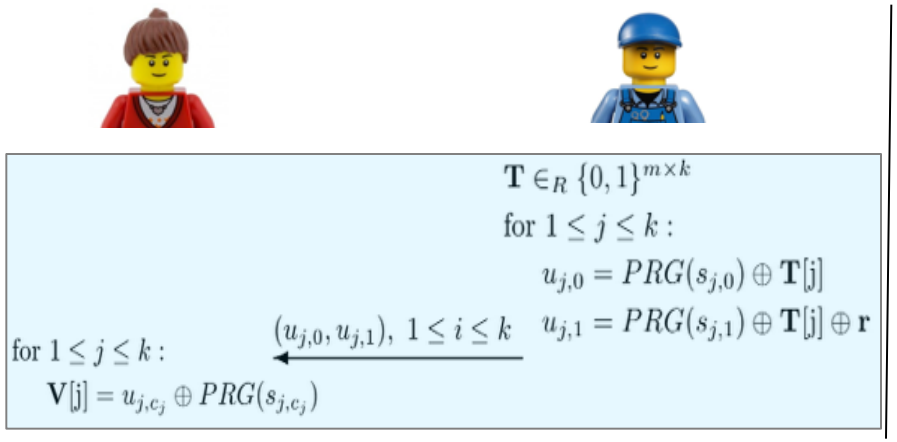
GMW: $\ell = 1,\ k = 80$

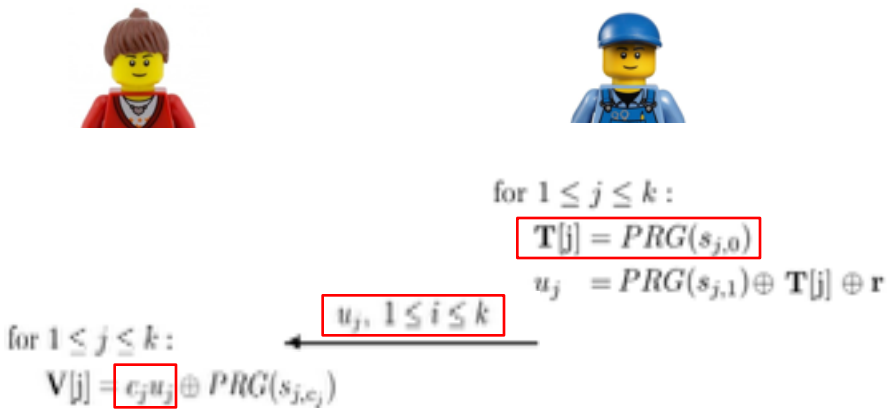


Protocol Optimization
General OT Extension

- Instead of generating a random T matrix, we derive it from $s_{j,0}$

- Reduces data sent by Bob by factor 2





Specific OT Functionalities

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- Secure computation protocols often require a specific OT functionality
  - Yao with free XORs requires strings x_0, x_1 to be XOR-correlated
  - GMW with multiplication triples can use random strings

- Correlated OT: random x_0 and $x_1 = x_0 \oplus x$

Correlated OT

- Random OT: random x_0 and x_1

Random OT

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Specific OT Functionalities
Correlated OT Extension (C-OT)

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- Choose $x_{i,0}$ as random output of H (modeled as RO here)

- Compute $x_{i,1}$ as $x_{i,0} \oplus x_i$ to obliviously transfer XOR-correlated values

- Reduces data sent by Alice by factor 2


$\mathbf{V'} = \mathbf{V}^T$ $\qquad$ $\mathbf{T'} = \mathbf{T}^T$

for $1 \le i \le m$:

$y_{i,0} = x_{i,0} \oplus H(i, \mathbf{V'}[\mathrm{i}])$

$y_{i,1} = x_{i,1} \oplus H(i, \mathbf{V'}[\mathrm{i}] \oplus c)$

$(y_{i,0}, y_{i,1}), 1 \le i \le m \longrightarrow$

for $1 \le i \le m$:

$x_{i,r_i} = y_{i,r_i} \oplus H(i, \mathbf{T'}[\mathrm{i}])$


$\mathbf{V'} = \mathbf{V}^T$ $\qquad$ $\mathbf{T'} = \mathbf{T}^T$

for $1 \le i \le m$:

$x_{i,0} = H(i, \mathbf{V'}[\mathrm{i}])$

$x_{i,1} = x_{i,0} \oplus x_i$

$y_i = x_{i,1} \oplus H(i, \mathbf{V'}[\mathrm{i}] \oplus c)$

$y_i, 1 \le i \le m \longrightarrow$

for $1 \le i \le m$:

$x_{i,r_i} = r_i y_i \oplus H(i, \mathbf{T'}[\mathrm{i}])$

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Specific OT Functionalities
Random OT Extension (R-OT)

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- Choose $x_{i,0}$ and $x_{i,1}$ as random outputs of H (modeled as RO here)

- No data sent by Alice


$\mathbf{V'} = \mathbf{V}^T$ $\qquad$ $\mathbf{T'} = \mathbf{T}^T$

for $1 \le i \le m$:

$y_{i,0} = x_{i,0} \oplus H(i, \mathbf{V'}[\mathrm{i}])$

$y_{i,1} = x_{i,1} \oplus H(i, \mathbf{V'}[\mathrm{i}] \oplus c)$

$(y_{i,0}, y_{i,1}), 1 \le i \le m \longrightarrow$ for $1 \le i \le m$:

$x_{i,r_i} = y_{i,r_i} \oplus H(i, \mathbf{T'}[\mathrm{i}])$


$\mathbf{V'} = \mathbf{V}^T$

for $1 \le i \le m$:

$x_{i,0} = H(i, \mathbf{V'}[\mathrm{i}])$

$x_{i,1} = H(i, \mathbf{V'}[\mathrm{i}] \oplus c)$

$\mathbf{T'} = \mathbf{T}^T$

for $1 \le i \le m$:

$x_{i,r_i} = H(i, \mathbf{T'}[\mathrm{i}])$

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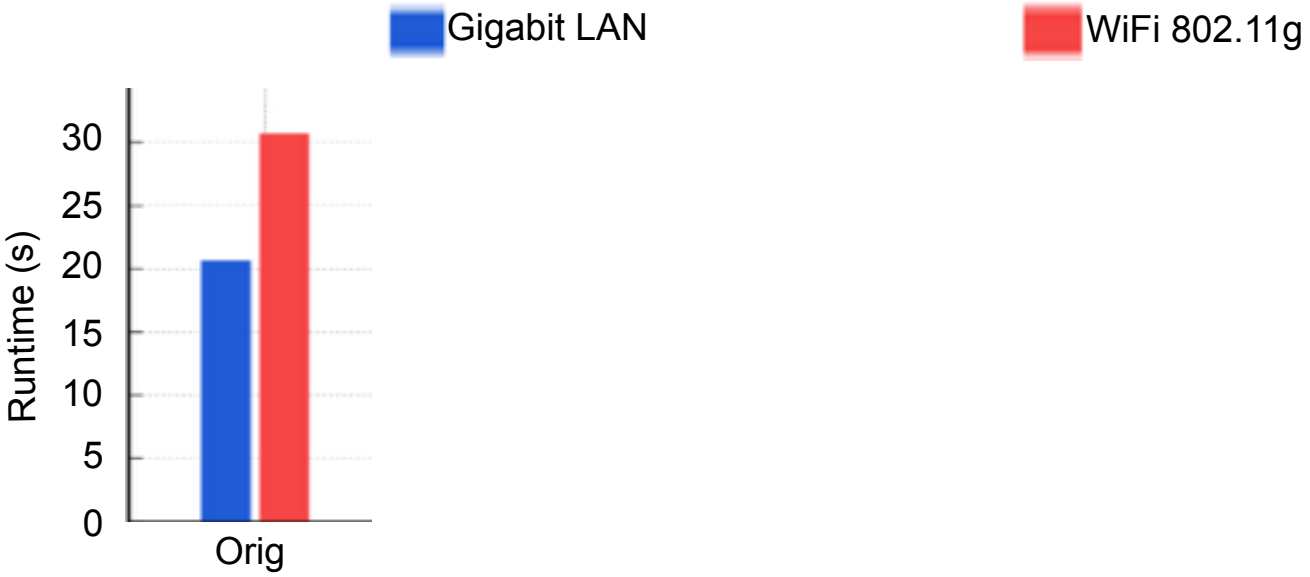
Performance Evaluation
Original Implementation



- C++ implementation of [**S**Z'13] implementing OT extension of [IKNP'03]

- Performance for 10 Mio. OTs on 80-bit strings

Performance Evaluation
Efficient Matrix Transposition

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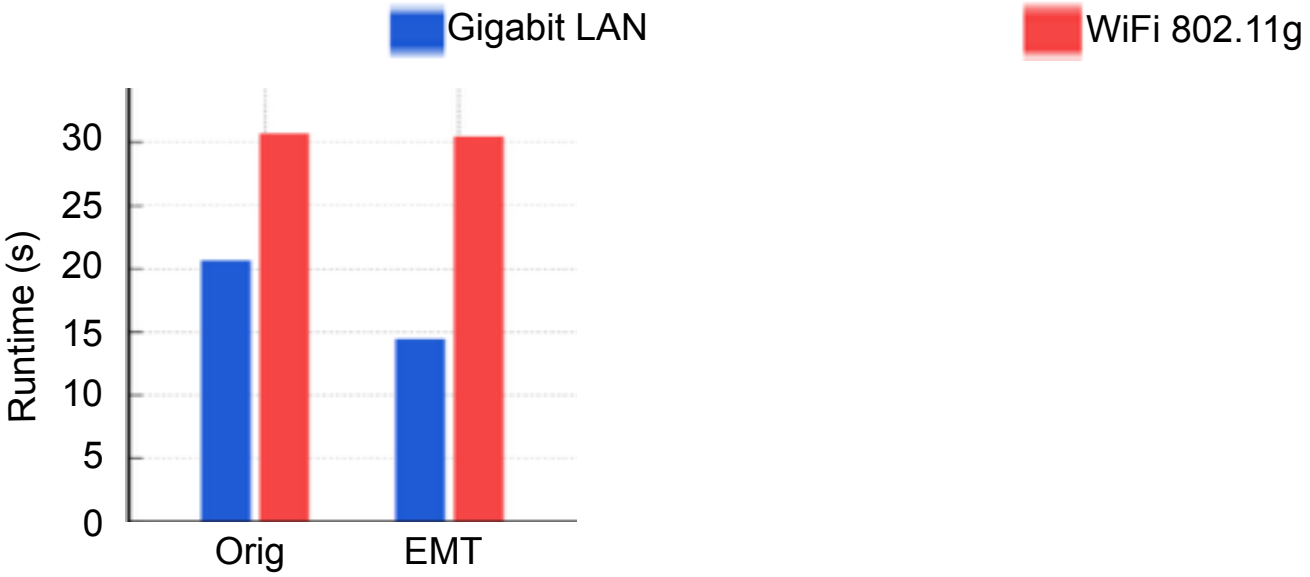

- Efficient matrix transposition – improved computation

- Only decreases runtime in LAN where computation is the bottleneck

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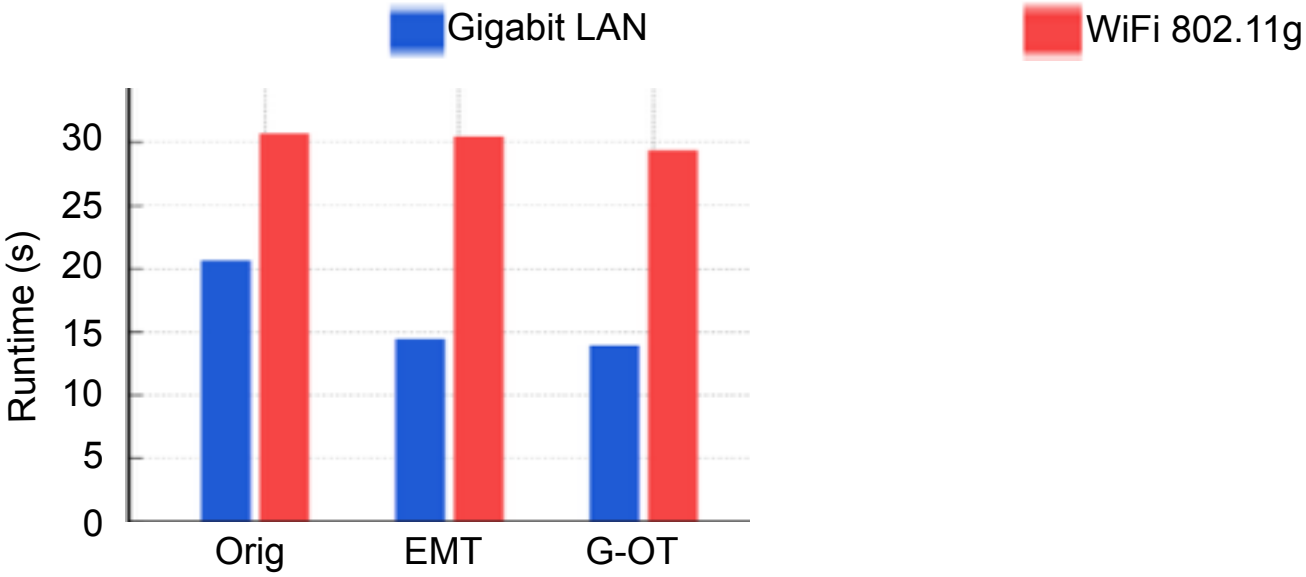
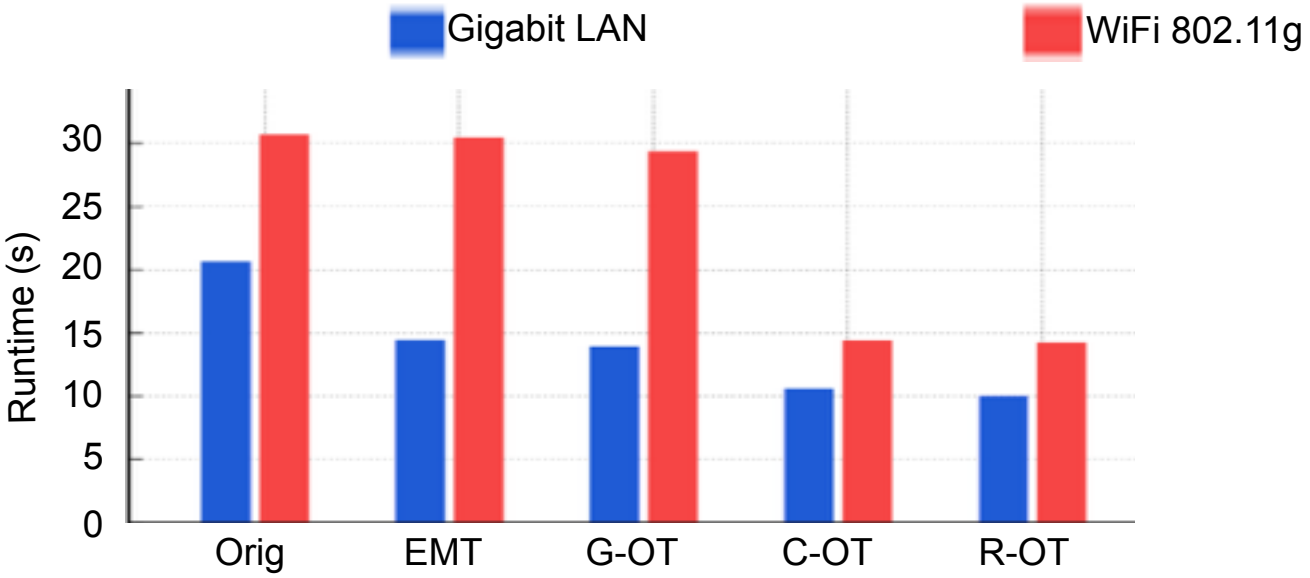
Performance Evaluation
General Oblivious Transfer



- Generate T matrix from seeds – improved communication Bob → Alice

- Runtimes only slightly faster (bottleneck: communication Alice → Bob)

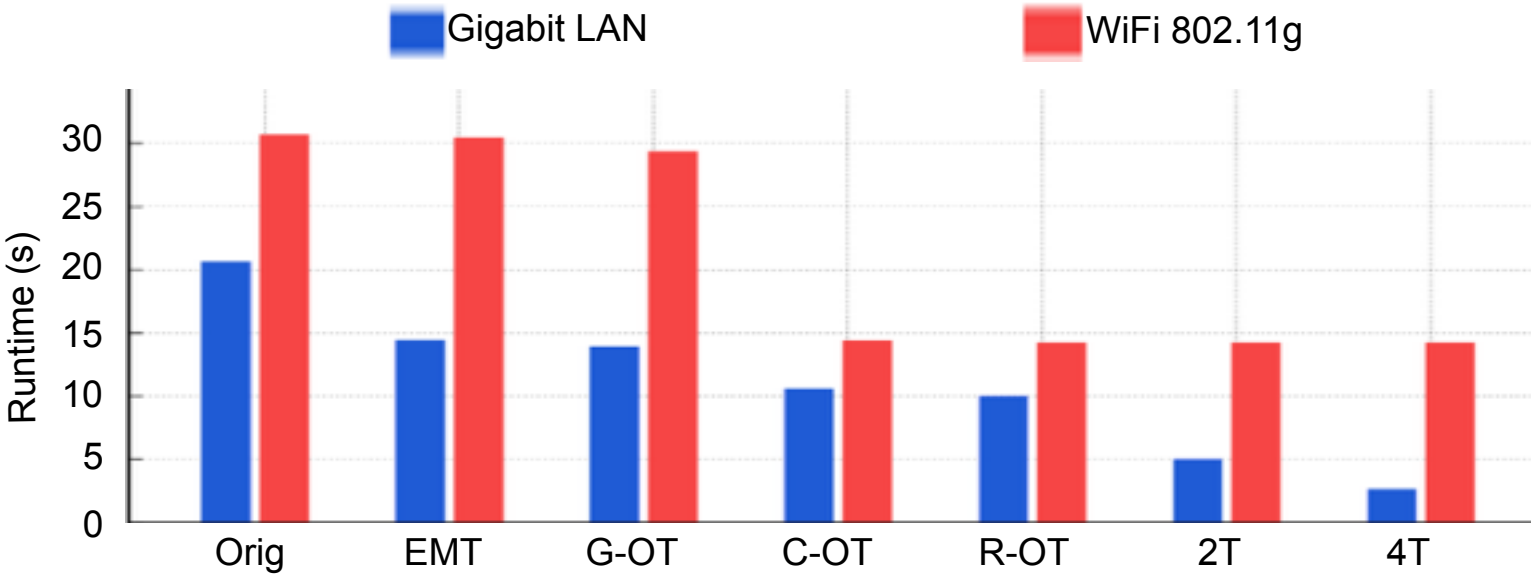
Performance Evaluation
Correlated/Random Oblivious Transfer



- Correlated/Random OT – improved communication Alice → Bob

- WiFi runtime faster by factor 2 (bottleneck: communication Bob → Alice)

Performance Evaluation
Parallelized Oblivious Transfer

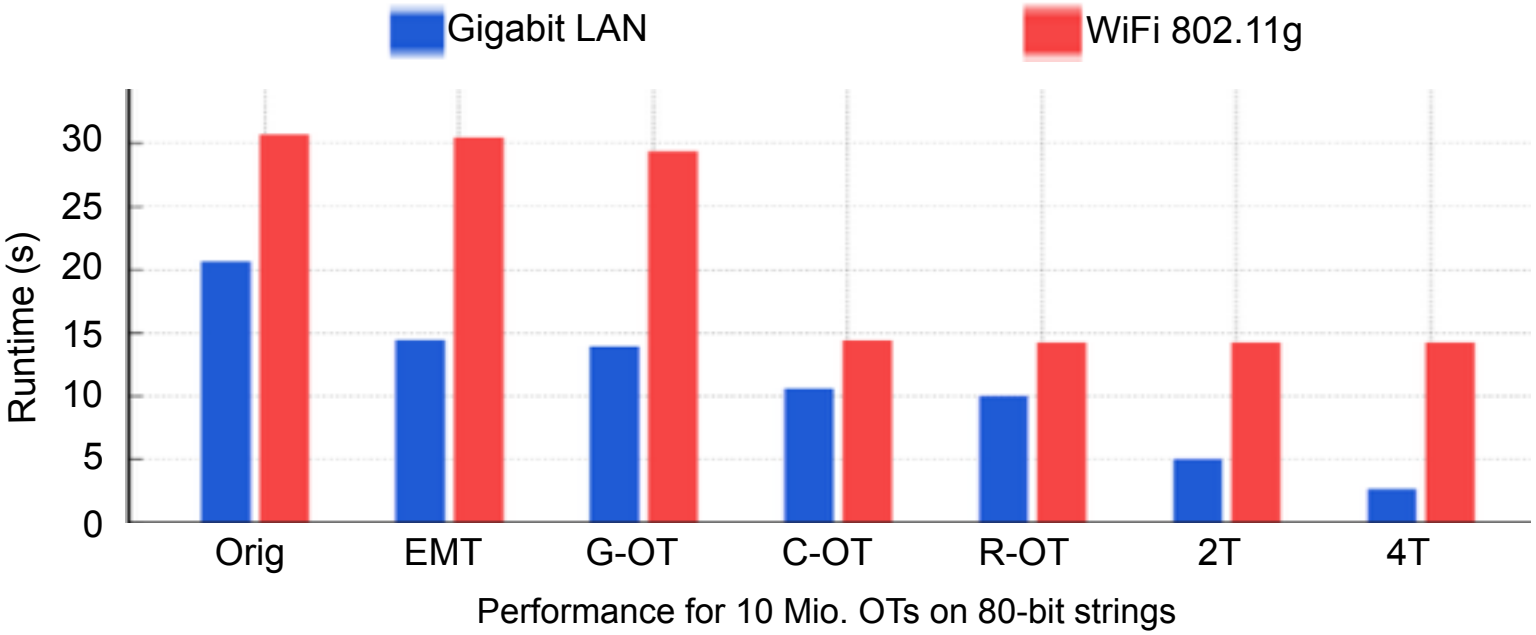


- Parallel OT extension with 2 and 4 threads – improved computation

- LAN runtime decreases linear in # of threads

- WiFi runtime remains the same (bottleneck: communication)

Performance Evaluation
Conclusion



- OT is very efficient

- **Communication** is the **bottleneck** for OT (even without using AES-NI)

Part 2: Applications of OT Extensions

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J. Bringer, H. Chabanne, M. Favre, A. Patey, T. Schneider, M. Zohner*:*
GSHADE: Faster privacy-preserving distance computation and biometric identification.
In ACM IH&MMSEC'14.

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Applications of OT Extensions

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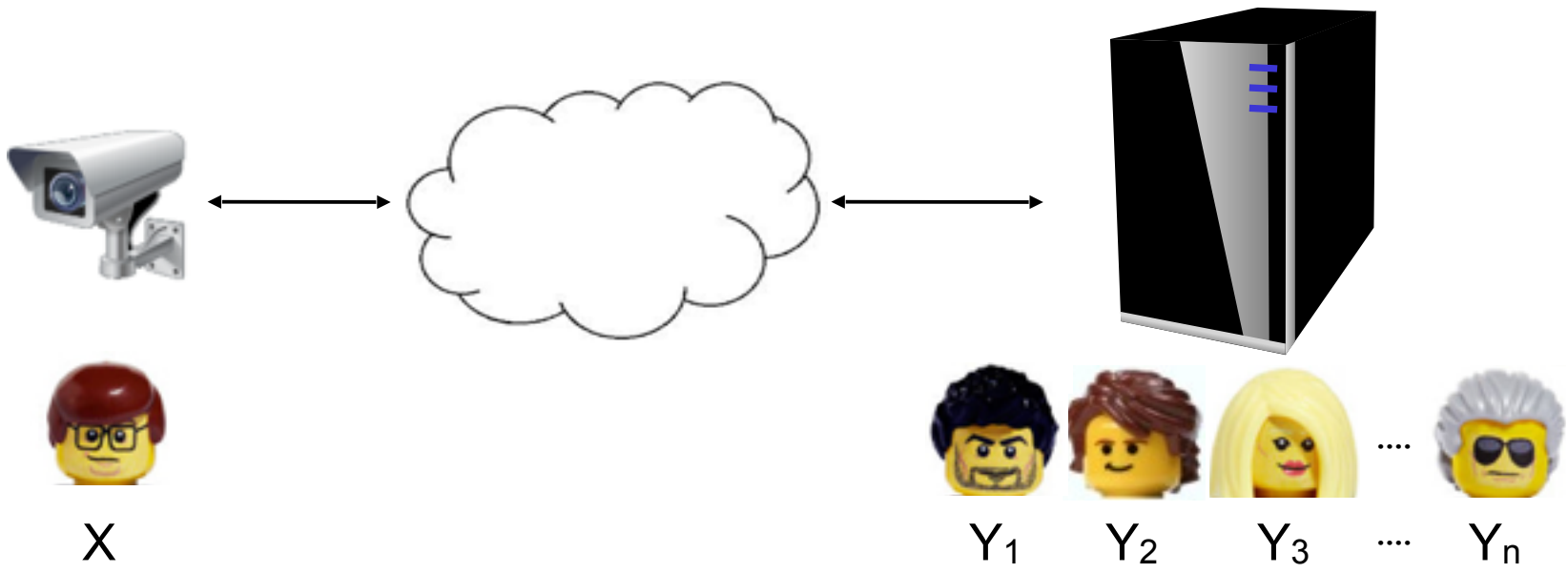
Many secure computation protocols need millions of OTs

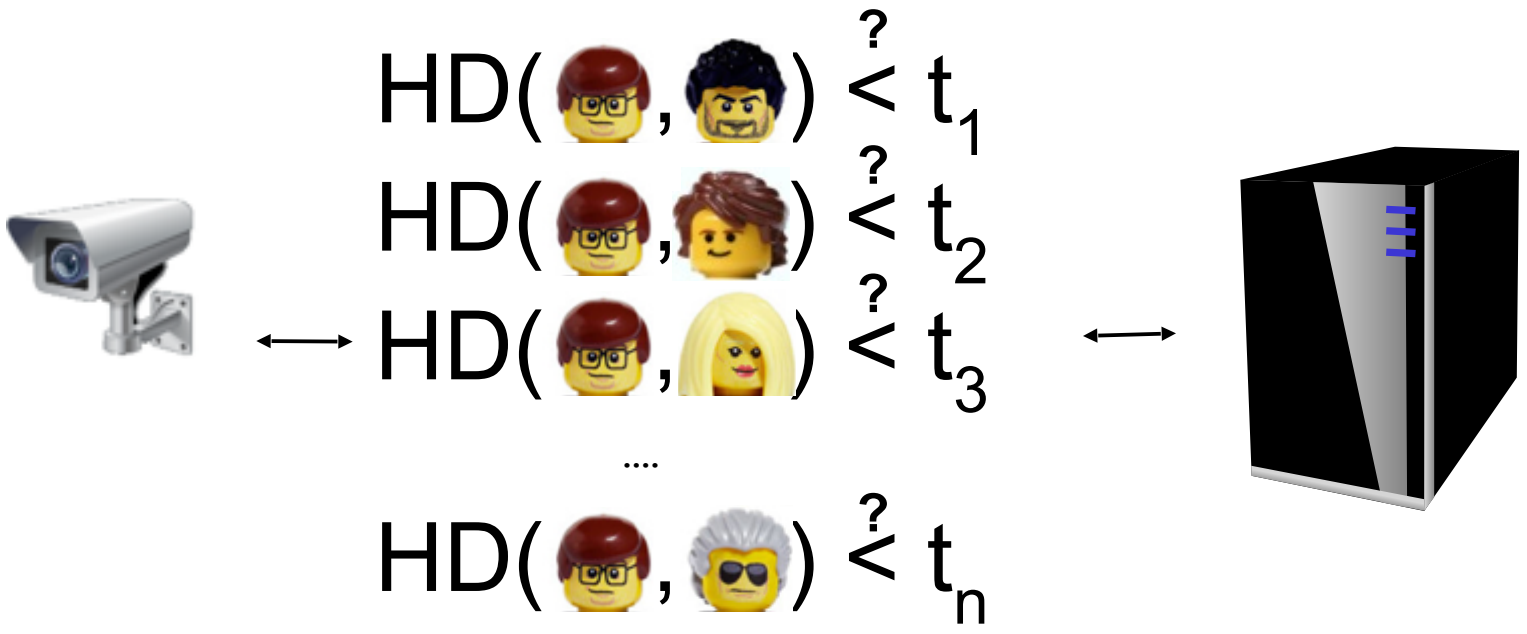
- Generic secure computation protocols based on OT
  - Yao: OT per input
  - GMW: OT per AND gate
- Special purpose protocols based on OT
  - Private Set Intersection (see Benny's talk)
    - Bloom-Filter Set-Intersection [Dong/Chen/Wen CCS'13]
    - Novel PSI Protocols
  - Privacy-Preserving Biometric Identification (see next)
    - Hamming Distance for Face Recognition: SHADE [BringerCP'13]
    - Further Distance Metrics: Generalized SHADE (GSHADE)

Privacy-Preserving Face Recognition



Task: Check if query face is ***similar*** to a face in the DB.
- without revealing the query to the server
- without revealing the DB to the client

The SCiFI Algorithm
[Osadchy/Pinkas/Jarrous/Moskovich S&P'10]



Compute Hamming distance of ℓ=900 bit strings and compare with threshold.

SHADE

Secure Hamming Dist. computation from OT [Bringer/Chabanne/Patey'13]

Goal: compute HD(X,Y) = $\Sigma(x_i \oplus y_i)$, i=1..ℓ

for i=1..ℓ:

choose $r_i \in_R \mathbb{Z}_{\ell+1}$

r_i+y_i ; $r_i+(1-y_i)$ → OT ← x_i

OT → $t_i=r_i+(x_i \oplus y_i)$

$R = \Sigma r_i$

$T = \Sigma t_i = R + HD(X,Y)$

Continue with generic MPC protocol (e.g., Yao or GMW) from T - R = HD(X,Y) …

GSHADE: Optimizations and Generalization of SHADE

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- For multiple HD computations: HD(X,Y_1), HD(X,Y_2), …:
  Same number of OTs, but on longer strings

- Use correlated OT (C-OT) to improve communication

- Generalize to larger class of functions $f(X,Y) = f_X(X) + f_Y(Y) + \Sigma f_i(x_i,Y)$
  - Hamming Distance: $f_X=f_Y=0$, $f_i(x_i,Y)=x_i \oplus y_i$
  - Squared Euclidean Distance (for face & fingerprint recognition):
    $f_X(X)=\Sigma x_i^2$, $f_Y(Y)=\Sigma y_i^2$, $f_i(x_i,Y)=-2x_iy_i$
  - Normalized Hamming Distance (for iris recognition)
  - Squared Mahalanobis Distance
    (for hand shape, keystroke, signature recognition)

GSHADE Protocol

Goal: compute $f(X,Y) = f_X(X) + f_Y(Y) + \Sigma f_i(x_i,Y)$

for $i=1..\ell$:

choose $r_i \in_R \mathbb{Z}_m$

$r_i+f_i(0,Y)$; $r_i+f_i(1,Y)$ → OT ← x_i

$t_i=r_i+f_i(x_i,Y)$ →

$R = - f_Y(Y) + \Sigma r_i$

$T = f_X(X) + \Sigma t_i$

Continue with generic MPC from T - R = f(X,Y) = …

Performance of GSHADE

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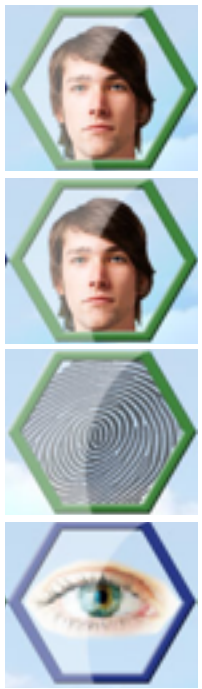
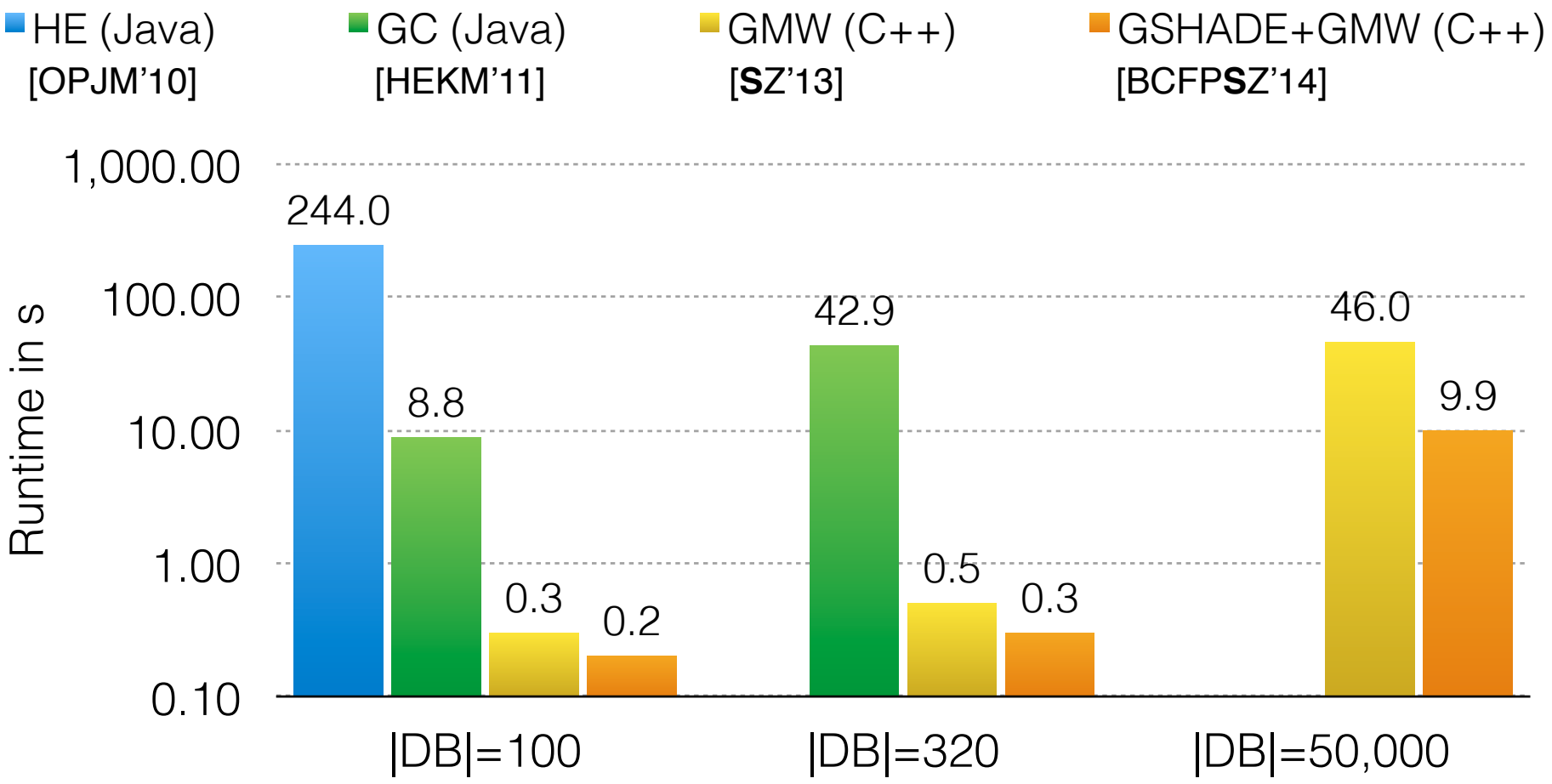
Compare biometric sample with DB of **5,000** entries.



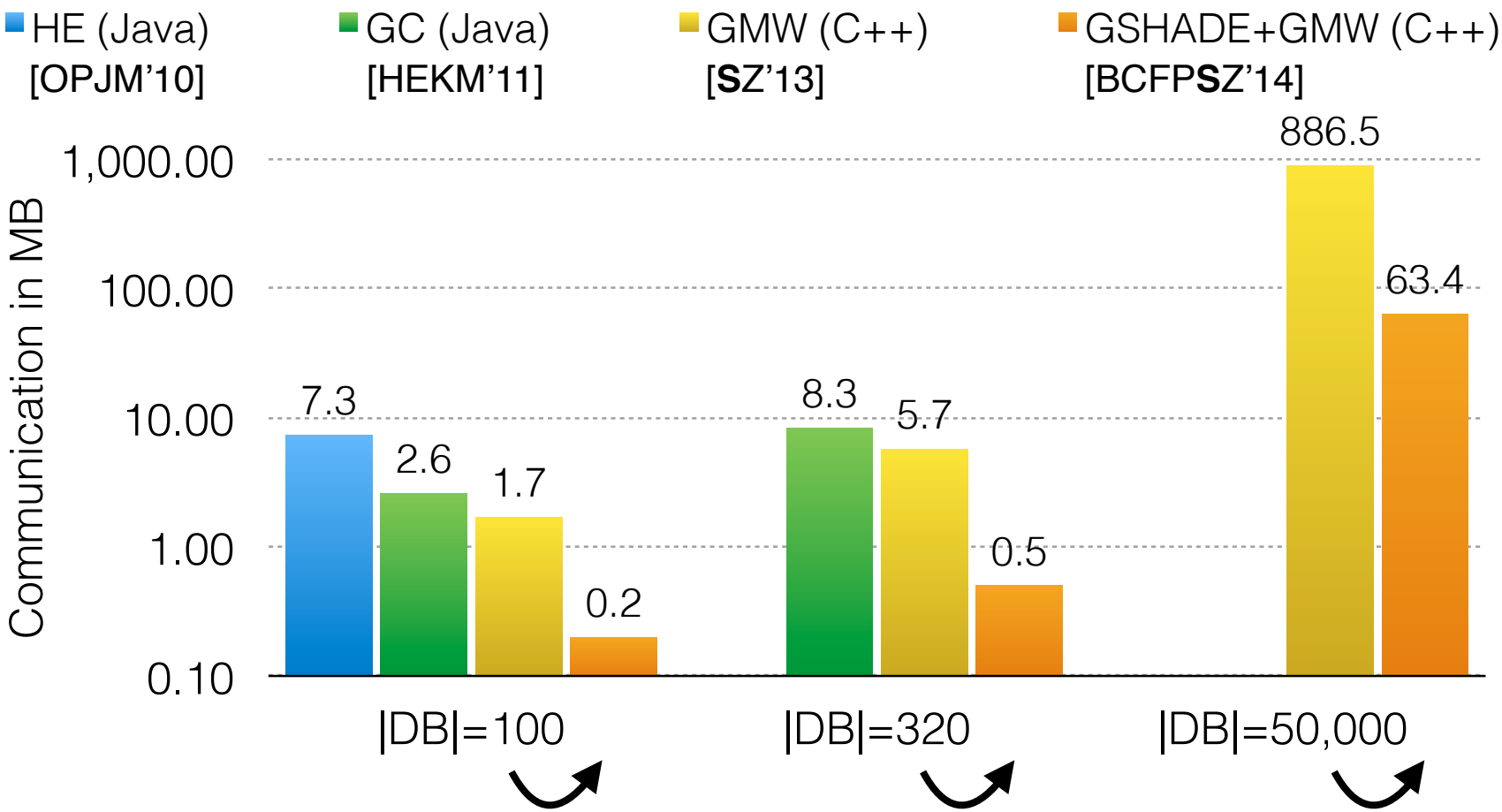
Algorithm	Time in s	Communication in MB
SCiFI	1.0	6.2
Eigenfaces	5.0	83.6
FingerCodes	6.7	67.5
IrisCodes	9.1	56.4

Performance for SCiFI: Runtime

Numbers taken from papers (**not** measured on same machines).



Performance for SCiFI: Communication



Significant improvement by 1 order of magnitude.

Summary

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Conclusion

- OT is very efficient due to OT extensions
- Many interesting applications can be built on OT

Future Work

- Further optimize communication of OT / secure computation
- More applications based directly on OT
- Extend to stronger adversary models

Efficient Oblivious Transfer Extensions and Applications

Thanks for your attention.

Questions?

Contact: http://encrypto.de