Bounding Rationality by Computation Complexity

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A Shopping Trip

Choices

- Cost: $1.39
- Value to you: 2.38

Choices

- Consumer's Goal
  - Maximize total value of goods
  - Stay in budget

Supermarket's Dilemma

- Supermarket wants to set prices to maximize profits
- Supermarket doesn't know the values each customer has for each item.
- Track purchases

- Find consistent values of products over time assuming consumers make their optimal choices.

Choices

- Consumer's Goal
  - Maximize total value of goods
  - Stay in budget
- Find set S that maximizes
    - $\max \Sigma_{i \in S} v_i$
  - subject to
    - $\Sigma_{i \in S} c_i \leq B$

Computational Issues

- Find set S that maximizes
    - max $\sum_{i \in S} v_i$
  - subject to
    - $\sum_{i \in S} c_i \le B$
- Better known as the Knapsack Problem
- NP-complete
- So consumers don't necessarily make their optimal choices.
- Even if consumers made their optimal choices finding consistent valuations is also NP-complete.

Consumer Choices

- Find set S that maximizes
    - max $\Sigma_{i \in S} v_i$
  - subject to
    - $\Sigma_{i \in S} c_i \le B$
- NP-complete but consumers still shop.
- Approximate their optimal choice
- Greedy: Pick next item to max v_i/c_i
- Better optimization by using approximate values

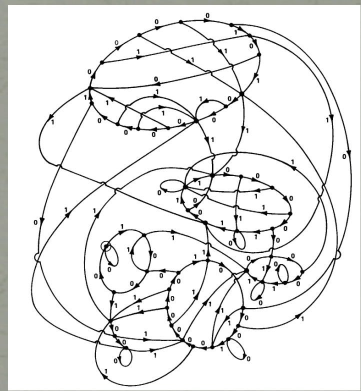
Budinich-Fortnow-Vohra

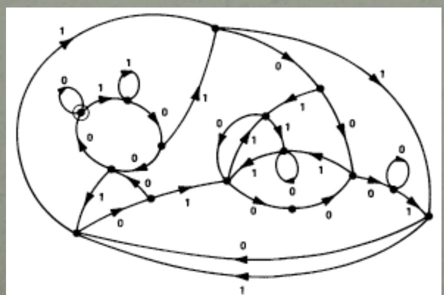
- Given a list of prices and buying behavior spread over several trips for a single consumer.
- We find valuations for each of the products for that consumer.
- Consumer using those valuations might not have that buying behavior even if acting optimally.
- Consumer using those valuations and that buying behavior performs close to optimally.
- Similar results for non-linear valuations.

Philosophy

- Economic models assume agents make optimal choices given the information they have available.
- Often problems agents have to solve are simple so this is a reasonable assumption.
- Other times agents need to solve complex problems.
- Economists in past have looked at simple cost of computation or simple program-size/memory restricted models.
- My goal: Bring in tools of computational complexity into economic theory.

Bounded Rationality

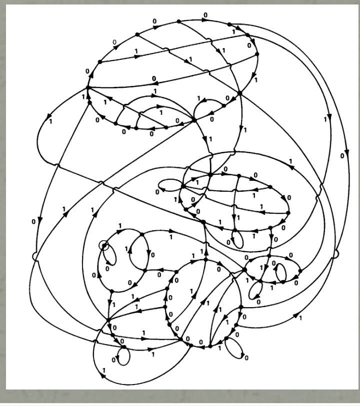




	C	D
C	(3,3)	(0,4)
D	(4,0)	(1,1)

Bounded Rationality

Can get cooperation if we have more rounds than states.



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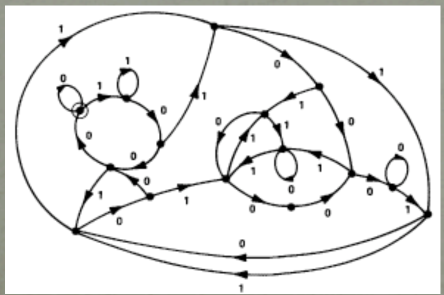
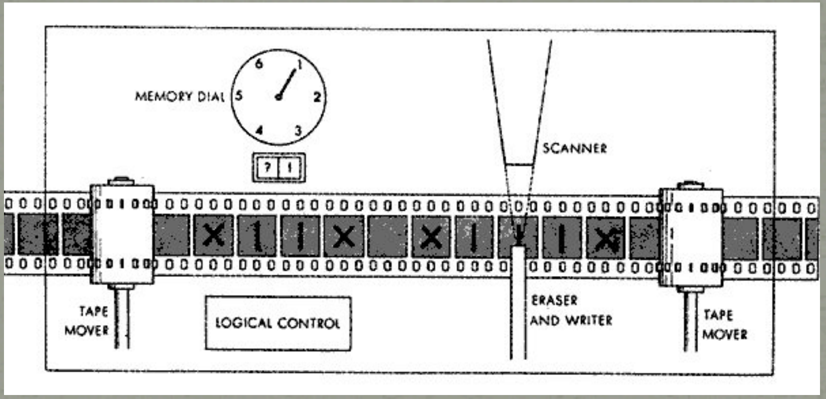


Efficient Computation

- Turing Machine



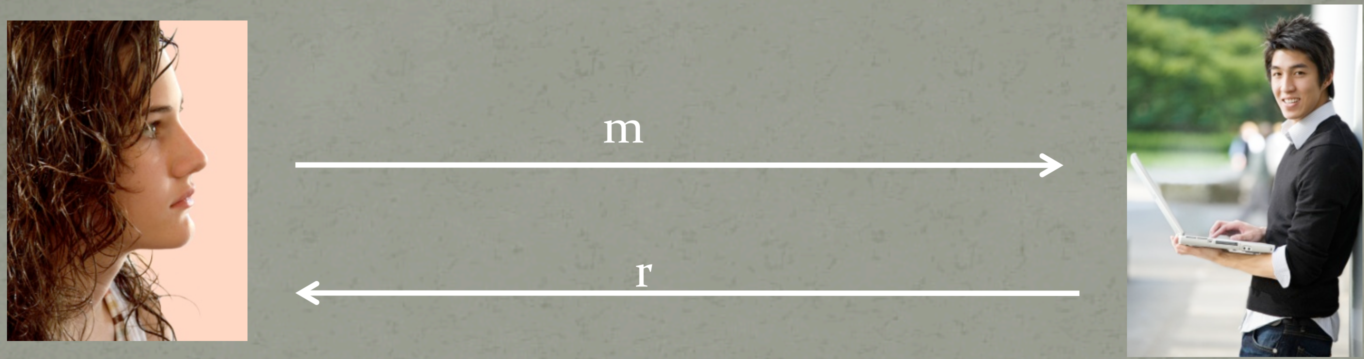
- Advantages
  - Captures Computation (Church-Turing Thesis)
  - Universal Simulation
- Can take very long time, maybe never halt.

Efficient Computation

- Hartmanis-Stearns, Edmonds, Cobham
  - Running time bounded by a polynomial in the length of the input.
  - The "P" in P versus NP.
- Problems in Applying to Economic Models
  - No clear notion of input length.
  - Race condition: Whichever player has the larger polynomial can simulate the other but not vice versa.
  - Hard to predict.

Factoring Game

- Ben-Sasson, Kalai, Kalai

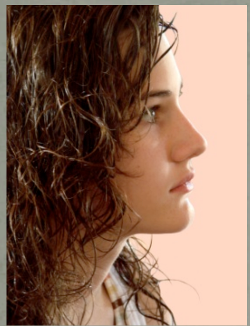


  - m, r are integers at least 2
  - If r is a prime factor of m then
    - Bob’s payoff is 2 and Alice’s payoff is 1
  - Otherwise
    - Alice’s payoff is 2 and Bob’s payoff is 1

Discounted Time

- Computation time in a Turing machine is measure by the number of unit operations.
- Discount payoff of Alice of a game by $\square_A{}^{t_A}$ where t_A is the number of steps used by Alice.
- Similarly for Bob.
- For today's technology: $\square$ $\square$ $1\text{-}10^{-12}$
  - A few million computation steps will have a negligible effect on the payoff.

Factoring Game



m

r

- Fortnow-Santhanam '10
- The following are roughly equivalent
  - Factoring cannot be solved efficiently on average over an easily samplable distribution.
  - For every c, [illegible] >0, there is a [illegible]>0
    - $[illegible]_A$ = (1- [illegible]) $[illegible]_B$ = (1- $[illegible]^c$)
    - In [illegible]-equilibrium, Alice receives at least 2-[illegible]

Certifying a Weatherman

60% 20% 40% 70% 30% 50% 80% 50%

Calibration

60% 20% 40% 70% 30% 50% 80% 50%

Calibration Theorem

- Foster-Vohra
  - There exists a probabilistic forecaster that given *any* sequence will, with high probability, be calibrated on that sequence.

Sandroni's Theorem

- Let T be a test of forecasters that
  - Always passes the "Truth"
  - Says "Pass" or "Fail" after finite rounds.
- For every such test T, there is a probabilistic forecaster that knowing nothing expect the path so far will, with high probability, pass test T.

Fortnow-Vohra '09

- We give a linear time tester
  - Passes "The Truth" on every distribution.
  - For every number m there is some distribution such that if a probabilistic forecaster F fools our tester than we can use the forecaster to factor the number m.
- Base hardness on more difficult problems: PSPACE-hardness (implies NP-hard).

Where should I eat tonight?

236 Restaurants in Aarhus

(According to visitaarhus.com)

How Do We Choose?

- Restaurants you have eaten at before
- Recommendations from Friends and Others
- Reviews
- Restaurants you've walked past
- Advertisements

How Do We Not Choose

- Carefully examine all 236 restaurants and make choice that optimizes the expected happiness from eating there (type and quality of food, atmosphere, …)
- Traditional decision making theory assumes we do make choices in this manner.
- We have a lack of awareness of most restaurants.

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- We have a computational cost of awareness of most restaurants.

Computational Awareness

- Halpern '01
  - It is possible to consider more computationally oriented notions of awareness. The problem is then to come up with interesting notions of awareness that have enough structure to allow for interesting mathematical analysis. I believe it should also be possible to use awareness structures to allow for natural reasoning about awareness and lack of it (so that an agent can reason, for example, about the possibility that she is unaware of certain features that another may be aware of). I am currently working on modeling such reasoning.

Computational Awareness

- Devanur-Fortnow '09
- Informal Definition
  - The amount of unawareness of an object is the time needed to enumerate that object in a certain environment and a context.
- A context is a topic like "restaurant".
- The Environment would consist of ways to find restaurants including our memories, interactions with others, guidebooks, "The Internet", etc.

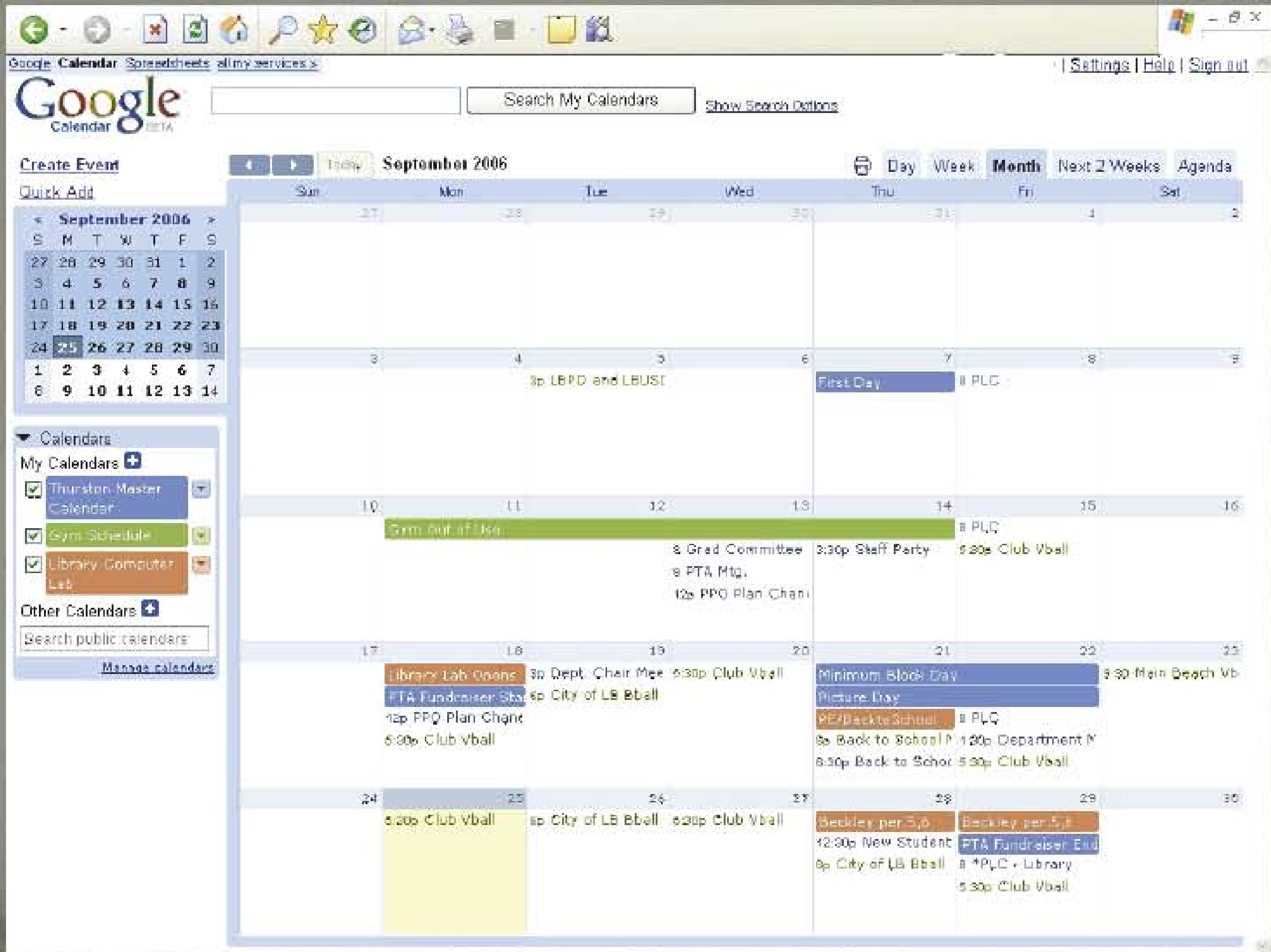
Papers to Referee

High Awareness

Papers to Referee

Low Awareness

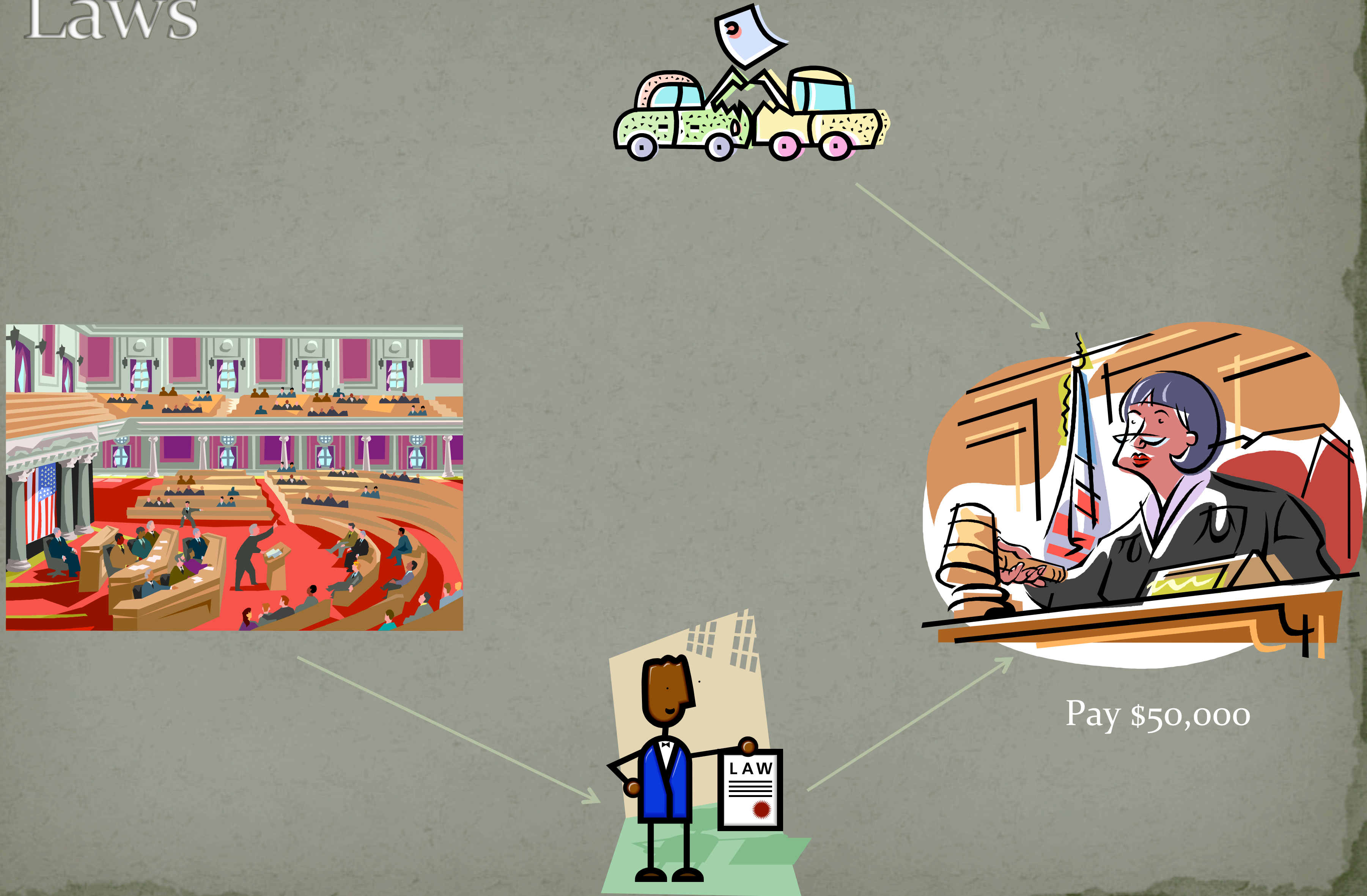
Why do we have calendars?



Organizers

- Why do we keep Calendars?
  - Make us aware of needed information when the context arises.
- To Do Lists
- Address Books
- Email Programs
- Desktop Search
- Photo Albums
- Birthday Reminders

Laws



Loopholes

- Chung-Fortnow ‘07
- Legislature may or may not be unaware of future circumstances.
- Judge is unaware whether legislature was aware of the current circumstance.
- We create simple model of laws and circumstances.
- We show in this model not only do loopholes occur, but legislature may purposely leave in loopholes so that more general unknown circumstances are properly handled.

Predicting the Future

- Will Sarah Palin be the republican nominee in 2012?
- What is the probability that Sarah Palin will be the Republican nominee in 2012?
- There is an 18.8% chance that Sarah Palin will be the Republican nominee.

Prediction Markets

- Security 2012.REP.NOM.PALIN on Intrade.com
  - Pays $100 if Palin nominated
  - Pays $0 if Palin not nominated
- Theory says price indicates probability.

Wisdom of Crowds

- James Surowiecki
- Four Criteria
  - Diversity of Opinion
  - Independence
  - Decentralization
  - Aggregation
- Information Markets capture all four
  - As opposed to committees or "experts"
- Empirically better than polls and experts
  - With the possible exception of Nate Silver

Why do markets perform well?

- Information markets efficiently aggregate individual information.
- Rational Expectations Equilibrium – Nash Equilibrium with players having different partial information will remain an equilibrium if all players have union of all information.
- When can rational equilibrium be achieved?

Simple Model

- Feigenbaum-Fortnow-Pennock-Sami '05
- Let f be a Boolean function on N-variables.
  - $f:\{0,1\}^N \rightarrow \{0,1\}$
- We have one security that pays off \$1 if f(x) = 1 and \$0 if f(x)=0.
- We have n players.
  - Distribution x drawn from is common knowledge.
  - Player i is given x_i.

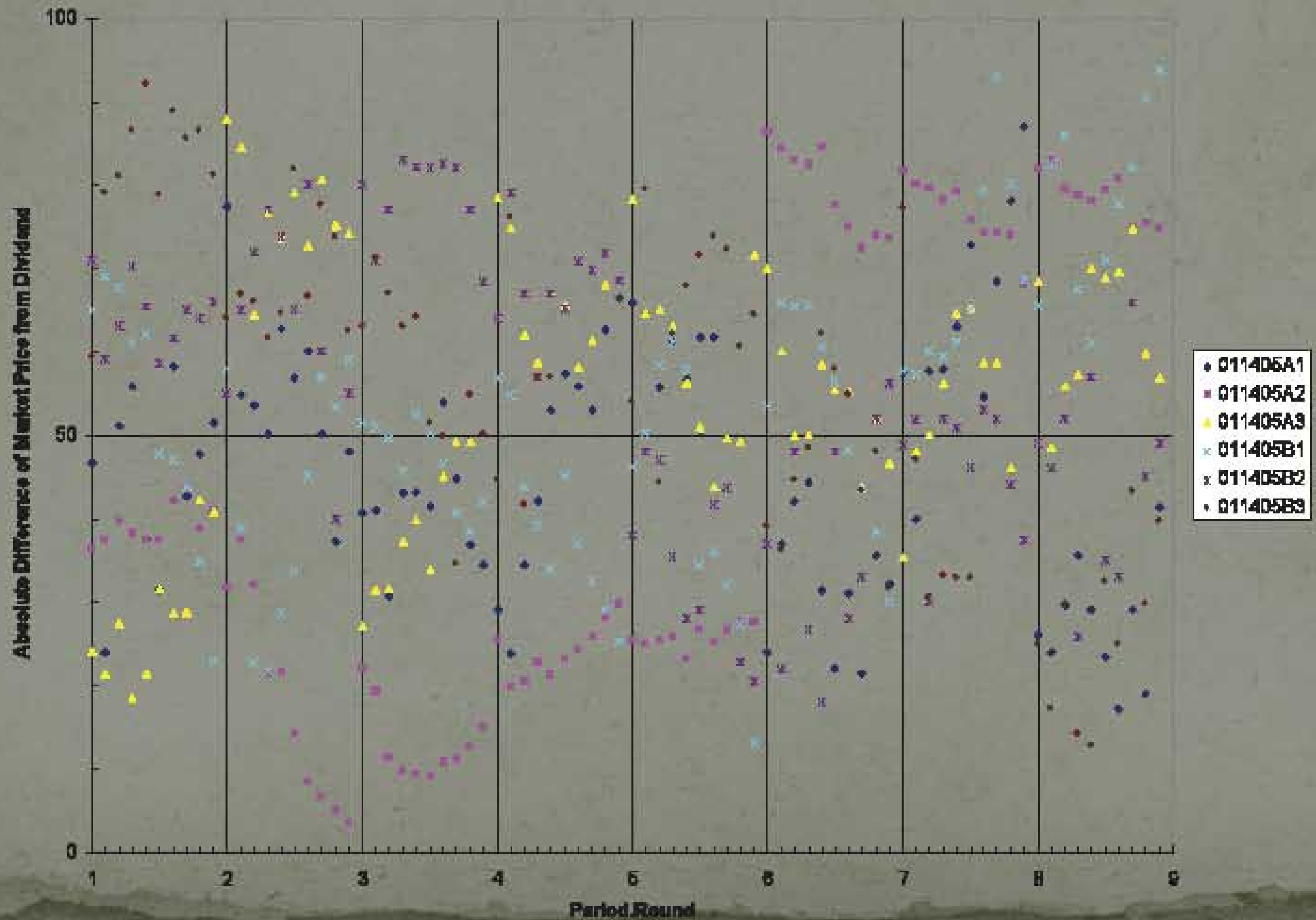
When do prices converge to f?

- For what functions do the prices converge to the correct value of f(x)?
- Theorem: The following are equivalent
  - For any initial distribution, the prices for f(x) eventually converge to f(x).
  - The function f is a weighted threshold function, i.e., f(x) = 1 if $\sum w_i x_i > v$ for fixed real w_i and v.

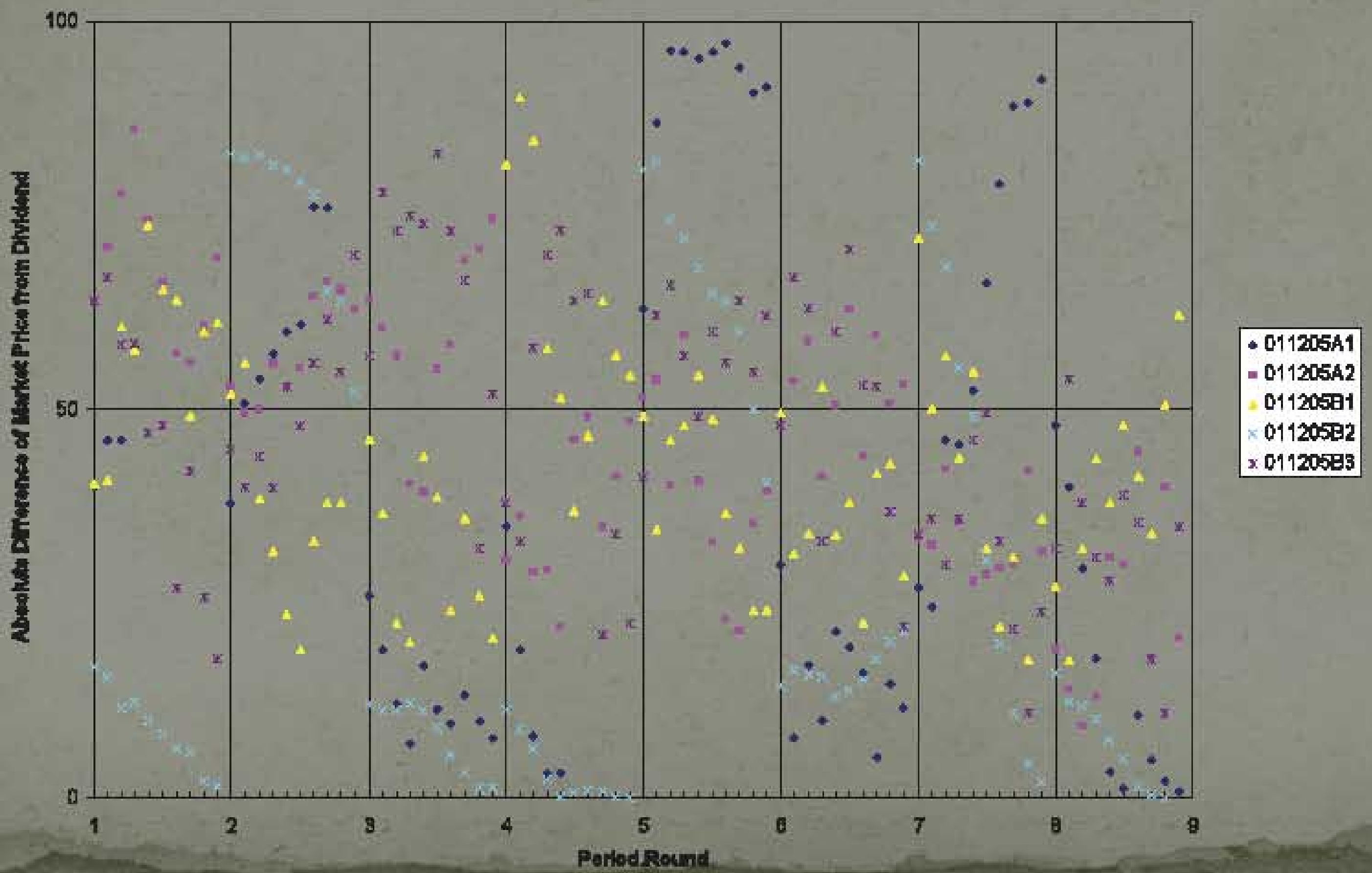
Experimental Results

- Experiments performed at Penn State by Yiling Chen and Tony Kwasnica
- Using five agents (inputs) each gets A/B signal
- Uniform Distribution
- Majority (at least three A)
- Parity (odd)

Parity Treatment



Majority Treatment



How do we handle many securities?

- What if we have lots of securities that reflect different states of the future, yet have some dependence among them?
- Complexity of computing prices and arbitrage
- Fortnow-Kilian-Pennock-Wellman ’05
  - NP-hard and worse for securities of the form
    - Sarah Palin is nominated and Obama is not
- Chen-Fortnow-Lambert-Pennock-Wortman ‘08
  - Securities based on horse racing (permutations)

Other Research

- Complexity of Nash Equilibrium (PPAD)
- Complexity of Voting Schemes
- Halpern and Pass
  - General models for computational bounds on rationality
  - Applications of crypto to game theory
- Arora-Barak-Brunnermeier-Ge '09
  - Use computational hardness to hide information in derivative pricing.

Conclusions

- Just starting to see applications of computational complexity to micro-economics.
- Future Topics
  - No-Trade without priors
  - Patents
  - Randomness
- Ultimate Goal: Efficient Market Hypothesis
  - Strong Version: Market prices reflect all the aggregated public information
  - Weaker Version: No entity can distinguish market prices from properly aggregated prices.