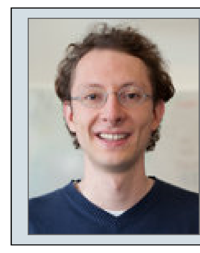
MADALGO – CENTER FOR MASSIVE DATA ALGORITHMICS


Chinmay Hegde
MIT


Piotr Indyk
MIT


Ludwig Schmidt
MIT

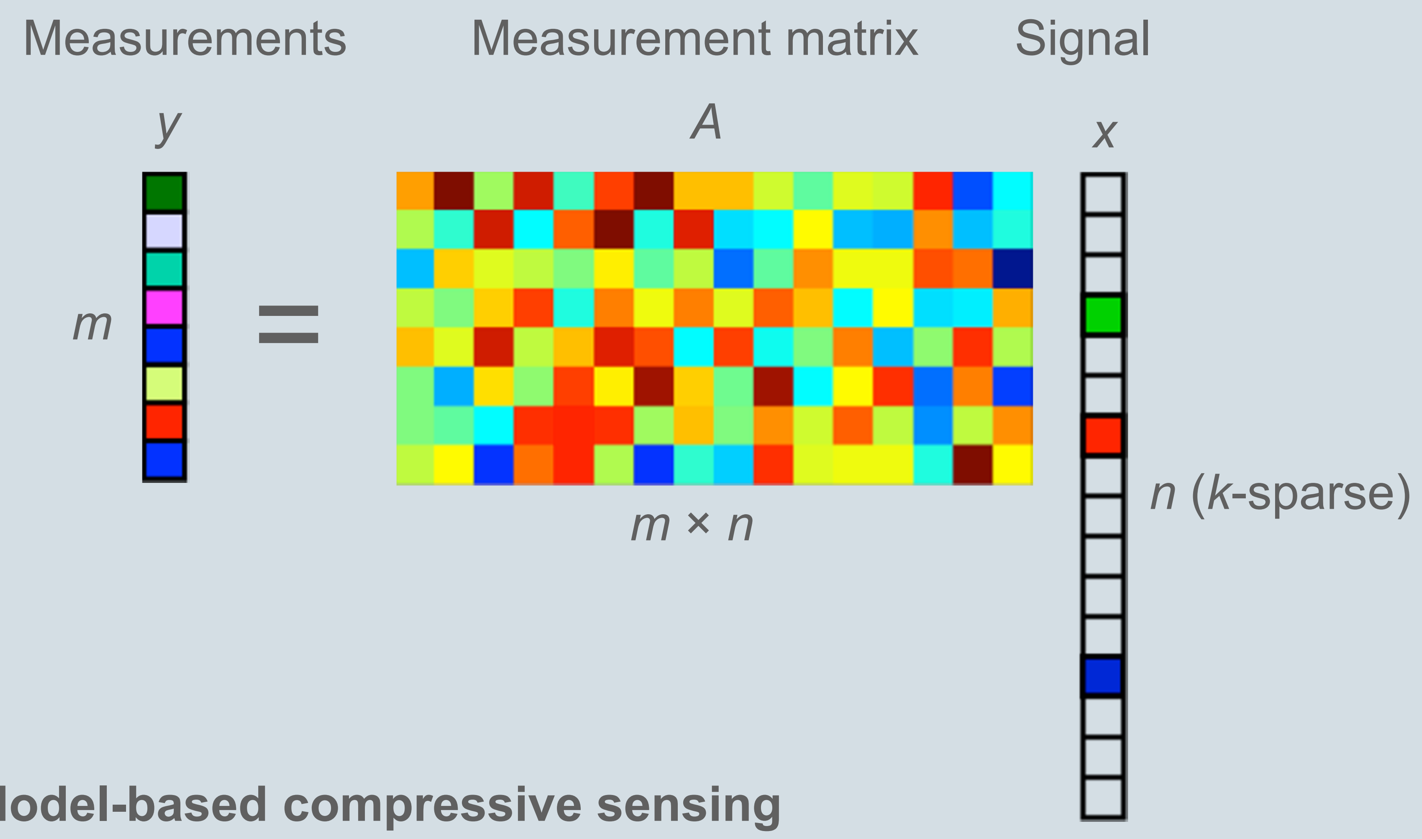
Danmarks Grundforskningsfond
Danish National Research Foundation

A Fast Approximation Algorithm for Tree-Sparse Recovery (ISIT'14)

Model-Based Compressive Sensing

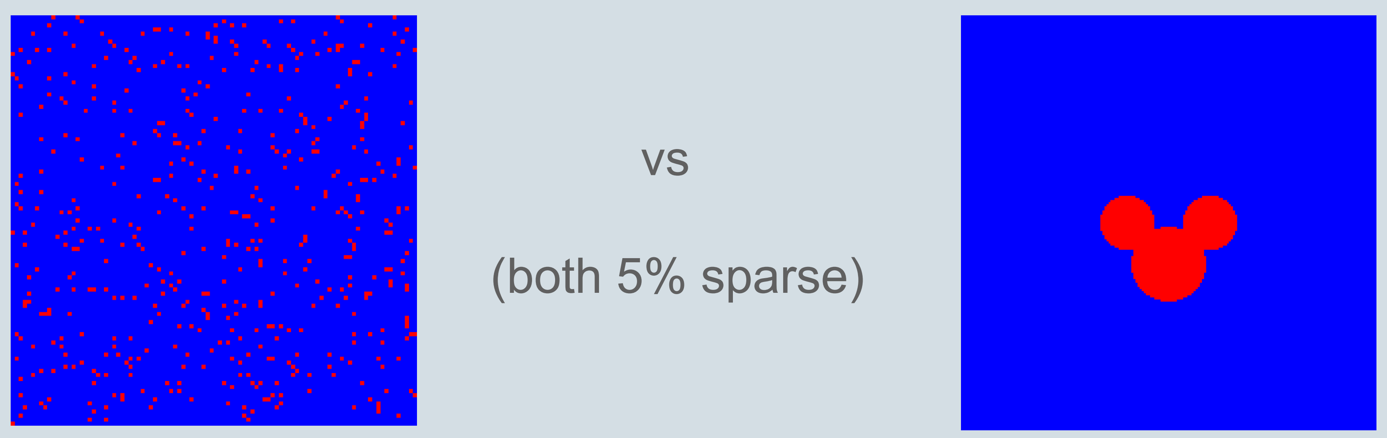
Compressive sensing

Compressive sensing is a method for recording a sparse signal x with few linear measurements of the form $y = Ax$.



Model-based compressive sensing

Model-based compressive sensing is a framework for reducing the number of measurements further by using more structure in the signal.



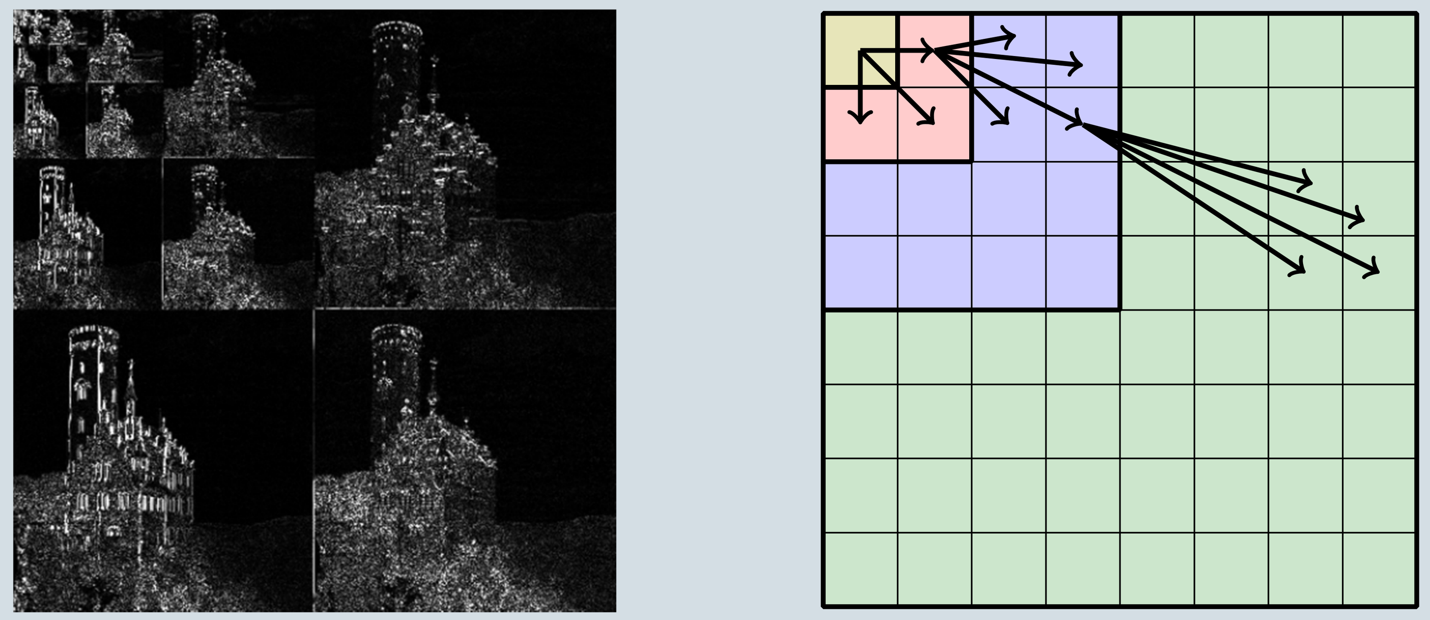
Standard compressive sensing: $\boldsymbol{m = O(k \cdot \log(n/k))}$
Model-based compressive sensing: $\boldsymbol{m = O(k)}$

Motivation: Natural Images in Wavelet Basis

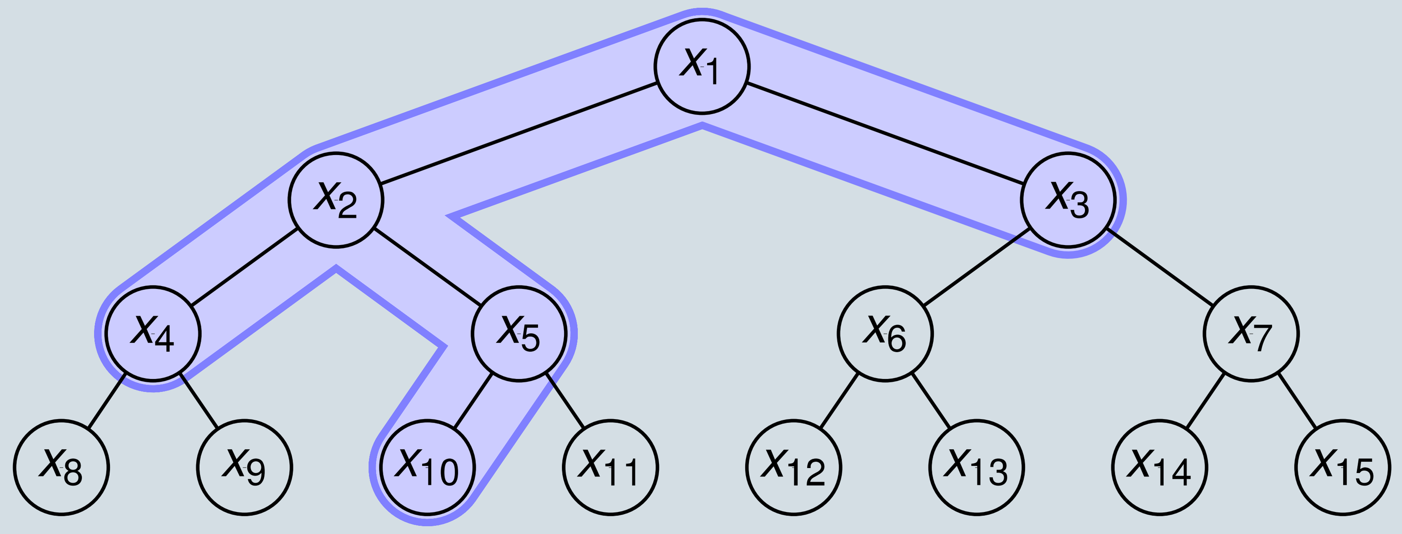
Many natural images are sparse in the wavelet basis.

The Tree Sparsity Model

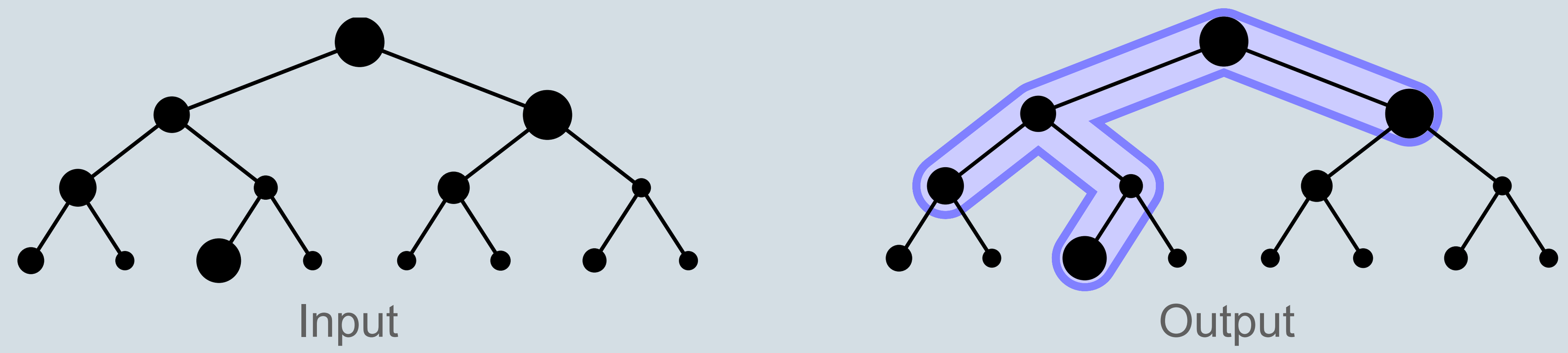
Idea: wavelet coefficients can naturally be arranged as a tree.



Sparsity model: the nonzeros of x form a rooted, connected subtree of size k.



Model projection problem: given an arbitrary signal, find the best approximation in the tree-sparsity model.



Tree-Projection Algorithm

We solve the problem via a **Lagrangian relaxation** of the sparsity constraint:

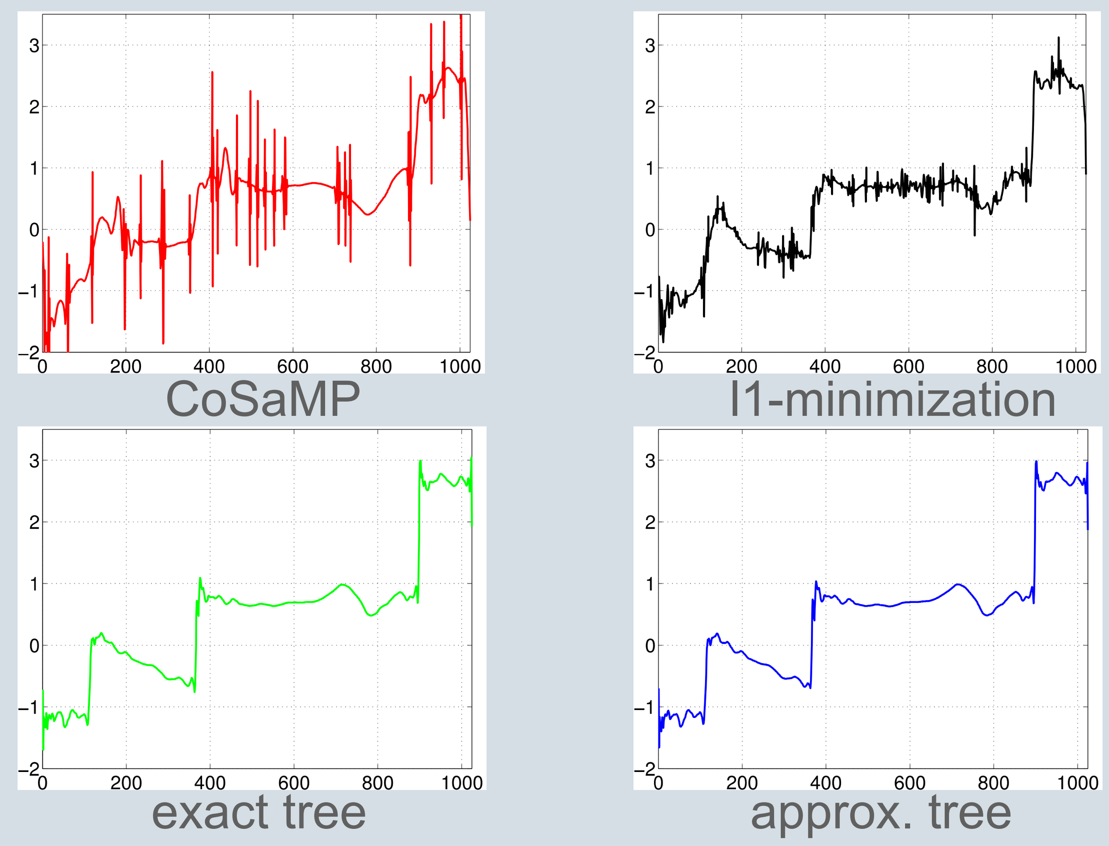
$$\arg\min_{\Omega} \|x - x_{\Omega}\|_2^2 + \lambda|\Omega|$$

where Ω ranges over the supports forming a tree rooted at node 1.

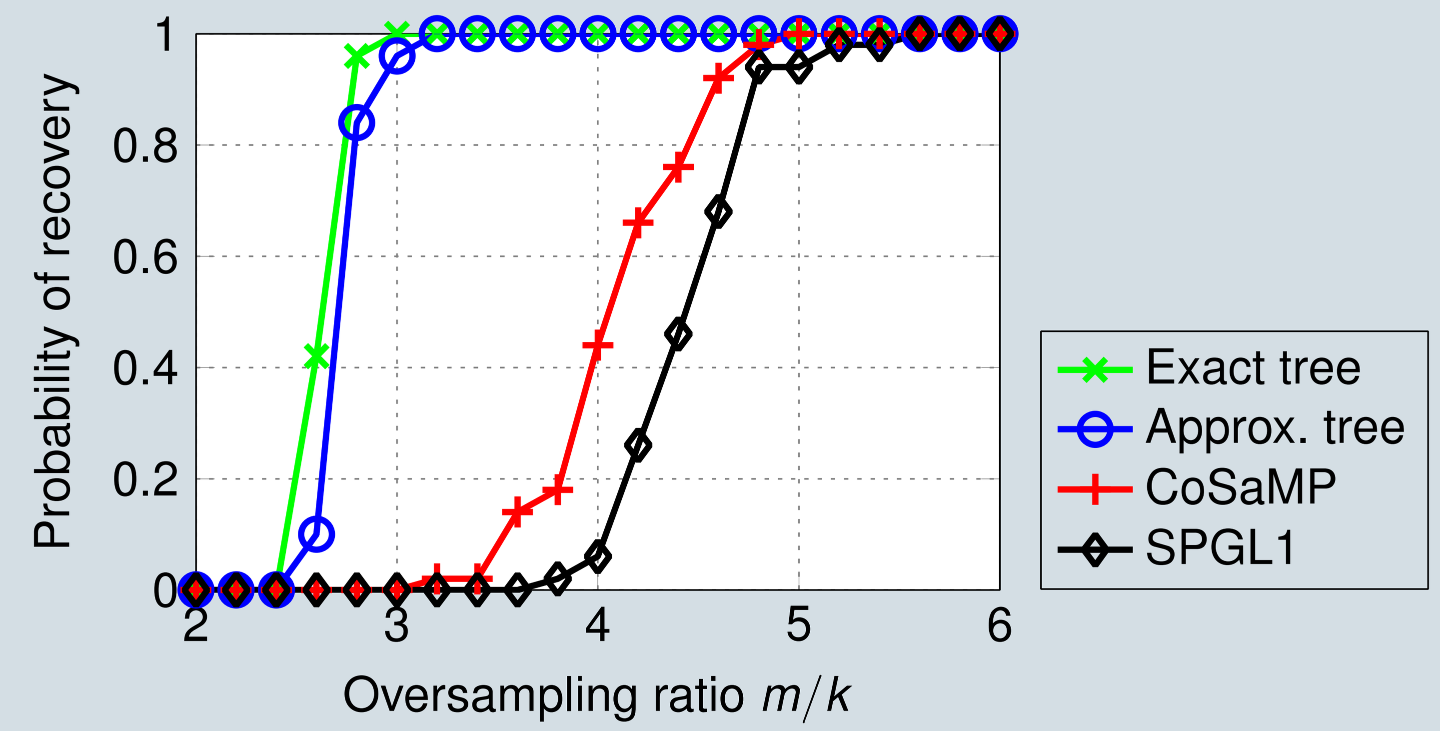
- For a fixed value of the Lagrange parameter λ, we can solve the problem exactly with a dynamic program in $O(n)$ time.
- We perform a binary search over λ in order to find a suitable trade-off between the approximation error and the support size.
- The final approximation error is at most twice as large as the best approximation error with a support in the tree-sparsity model.

Results

Recovery examples (parameters: $n = 1024$, $k = 41$, $m = 144$)



Comparison of recovery algorithms (signals as above, 50 trials)



Running times (single run of the projection algorithm for $n = 2^{18}$)

Algorithm	Exact tree-proj.	Approx. tree-proj.	2 FFTs
Runtime (sec)	13.5470	0.0161	0.0136

Conclusions

- We provide a **fast algorithm** for recovering tree-sparse signals.
- **Empirically**, our algorithm closely matches the sampling performance of the exact tree-projection algorithm, which is significantly slower.
- In follow-up work (ICALP'14), we show how to use our algorithm in a fast recovery scheme with **provable** recovery guarantees.
- The mathematically rigorous recovery scheme relies on the **approximation-tolerant model-based compressive sensing** framework introduced in Hegde-Indyk-Schmidt, SODA'14.

MADALGO – Center for Massive Data Algorithmics, a Center of the Danish National Research Foundation