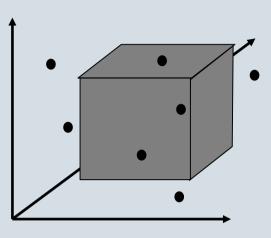
mapalgo ---**CENTER FOR MASSIVE DATA ALGORITHMICS**

Orthogonal Range Reporting: Optimal 3-d Structures and Query Lower Bounds

Problem

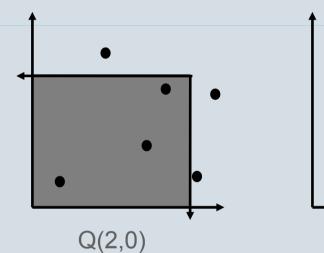
Input: *n* points in *d* dimensional space.

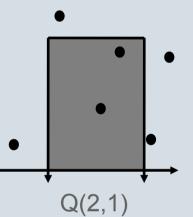


Support reporting the *t* points in a query box.

Variations

Query range may be unbounded in some dimensions:



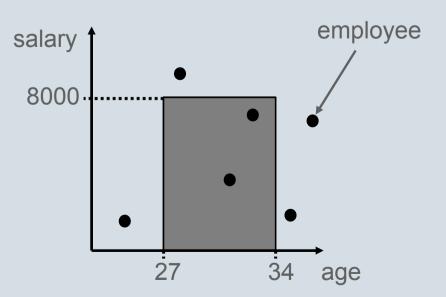


Q(d,k): d dimensional points, query ranges are finite in *k* dimensions.

Why important: more efficient solutions.

Motivation

Database queries:



"Give me all employees of age between" 27 and 34, earning at most 8000 per month."

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The

	Result	S	Concurrent	Range Re
e best previous result:			We solve harder variants of orthogonal range reporting. We call these concurrent $Q(d,k)$ (CQ(d,k)).	Solution for (and Q(<i>d</i> +
Туре	Query	Space	We call these concurrent $Q(u, h)$ ($CQ(u, h)$).	
Q(2,1)	lg <i>n+t</i>	n	$ p = \{ \circ \circ \circ \} $ $C = \{ \circ \circ \circ \circ \} $	
Q(2,2)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>	$\bullet \qquad \bullet \qquad \bullet \qquad P=\{\bullet, \bullet \bullet, \bullet \bullet \bullet \bullet \bullet \}$	
Q(3,0)	lg <i>n+t</i>	n	• •	
Q(3,1)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>		
Q(3,2)) lg <i>n+t</i>	n (lgn/lglgn) ²	Points have colors from set of colors C.	Range tree
Q(3,2)) lg²n/lglgn+t	<i>n</i> lg <i>n</i> /lglg <i>n</i>	Query: $Q(d,k)$ box q + set of colors p from a set $P \subseteq 2^{C}$.	Q(<i>d</i> +1, <i>k</i> +1
Q(3,3)) lg <i>n+t</i>	n (lgn/lglgn) ³	Must report points in q with color in p.	Height of t
Q(3,3)) lg²n/lglgn+t	n (lgn/lglgn)²	Solving Concurrent Q(3,0)	
Q(d,d)) lg <i>n</i> (lg <i>n</i> /lglg <i>n</i>) ^{d-3} +t	n (lgn/lglgn) ^d		
Q(<i>d</i> , <i>d</i>)) lg <i>n</i> (lg <i>n</i> /lglg <i>n</i>) ^{d-2} +t	n (lgn/lglgn) ^{d-1}	Ig <i>n</i> -shallow cuttings: O(<i>n</i> /Ig <i>n</i>) Q(3,0) boxes s.t. queries outputting < Ig <i>n</i> points are inside at least one box. Each	Query unb dimension
	r bound Ω(<i>n</i> (lg <i>n</i> /lgl wer bound beyond :			
Туре	Query	Space		
Q(2,1)	lg <i>n+t</i>	n	Build Ig <i>n</i> -shallow cuttings for every color. Store Q(3,0) structure inside each box, and for every	The grid hand hand hand hand hand hand hand han
Q(2,2)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>	color.	Points sna
Q(3,0)	lg <i>n+t</i>	n		Structures
Q(3,1)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>		 CQ(3,0
Q(3,2)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>		 For even
	lg <i>n+t</i>	n (lgn/lglgn)²		coordin Linked
Q(d,d)) lg <i>n</i> (lg <i>n</i> /lglg <i>n</i>) ^{d-3} +t	n (lgn/lglgn) ^{d-1}	Given query q , find boxes containing it using 2-d point	by z co
e lower k ution for	oound of Ω((lg <i>n</i> /lglg all 3-d variations. Referen	Ces	 enclosure in time O(lg<i>n</i>+ <i>C</i>). For every color: If <i>q</i> inside a box <i>b</i>, query Q(3,0) structure inside <i>b</i>. Otherwise, query Q(3,0) structure on all points. 	Recursi
Afshani, Arge, Larsen. Orthogonal Range Reporting In Three and Higher Dimensions. FOCS 2009. Afshani, Arge, Larsen. Orthogonal Range Reporting: Query Lower Bounds, Optimal Structures in 3-d, and				Queries de In recursiv two CQ(3,

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Results e best previous result:			S	Concurrent Range Re	
				We solve harder variants of orthogonal range reporting.	Solution fo
	Гуре	Query	Space	We call these concurrent $Q(d,k)$ (CQ(d,k)).	(and Q(<i>d</i> +
(ຊ(2,1)	lg <i>n+t</i>	n	$ p = \{ \circ \circ \circ \} $ $C = \{ \circ \circ \circ \circ \} $	
C	ຊ(2,2)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>	$ P=\{\bullet, \bullet \bullet, \bullet \bullet \bullet, \bullet \bullet \bullet \bullet\} $	
C	ຊ(3,0)	lg <i>n+t</i>	n		
C	ຊ(3,1)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>		
(Q(3,2)	lg <i>n+t</i>	n (lgn/lglgn)²	Deinte have calone from out of calons O	
(Q(3,2)	lg²n/lglgn+t	<i>n</i> lg <i>n</i> /lglg <i>n</i>	Points have colors from set of colors C. Query: $Q(d,k)$ box q + set of colors p from a set $P \subseteq 2^{C}$.	Range tree Q(<i>d</i> +1, <i>k</i> +1
C	2(3,3)	lg <i>n+t</i>	n (lgn/lglgn) ³	Must report points in q with color in p .	Height of t
C	2(3,3)	lg²n/lglgn+t	<i>n</i> (lg <i>n</i> /lglg <i>n</i>)²	Solving Concurrent $O(3.0)$	
C	Q(<i>d</i> , <i>d</i>)	lg <i>n</i> (lg <i>n</i> /lglg <i>n</i>) ^{d-3} +t	n (lgn/lglgn) ^d	Solving Concurrent Q(3,0)	
C	Q(<i>d</i> , <i>d</i>)	lg <i>n</i> (lg <i>n</i> /lglg <i>n</i>) ^{d-2} +t	n (lgn/lglgn) ^{d-1}	Ig <i>n</i> -shallow cuttings: O(<i>n</i> /Ig <i>n</i>) Q(3,0) boxes s.t. queries outputting < Ig <i>n</i> points are inside at least one box. Each	Query unb dimension
ace que	lower	old are optimal. bound Ω(<i>n</i> (lg <i>n</i> /lglg ver bound beyond		box contains O(lg <i>n</i>) input points.	
	Гуре	Query	Space		
		lg <i>n+t</i>	n	Build Ig <i>n</i> -shallow cuttings for every color. Store Q(3,0) structure inside each box, and for every	The grid hand in the grid hand in the grid hand in the second sec
C	ຊ(2,2)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>	color.	Points sna
C	ຊ(3,0)	lg <i>n+t</i>	n		Structures
C	ຊ(3,1)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>		 CQ(3,0
C	ຊ(3,2)	lg <i>n+t</i>	<i>n</i> lg <i>n</i> /lglg <i>n</i>		 For even
C	ຊ(3,3)	lg <i>n+t</i>	n (lgn/lglgn)²		coordin Linked
C	Q(d,d)	lg <i>n</i> (lg <i>n</i> /lglg <i>n</i>) ^{d-3} +t	n (lgn/lglgn) ^{d-1}	Given query q , find boxes containing it using 2-d point	by z co
e lo utio	wer bo n for a ani, Ar	ound of Ω((lg <i>n</i> /lglg all 3-d variations. Referen c	Ces gonal Range Reporting	 enclosure in time O(lg<i>n</i>+ <i>C</i>). For every color: If <i>q</i> inside a box <i>b</i>, query Q(3,0) structure inside <i>b</i>. Otherwise, query Q(3,0) structure on all points. 	 Recursi Queries de
Afshani, Arge, Larsen. Orthogonal Range Reporting: Query Lower Bounds, Optimal Structures in 3-d, and			gonal Range Reporting:		In recursiv two CQ(3,

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- [1] Af: in
- [2] Afs QL Higher-dimensional Improvements. SoCG 2010.

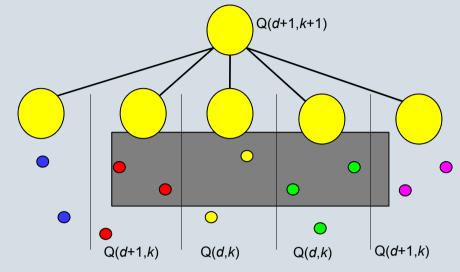


Concurrent Range Reporting

MADALGO – Center for Massive Data Algorithmics, a Center of the Danish National Research Foundation



for CQ(d,k) gives solution for Q(d,k+1)(+1,*k*+1)) by paying lg*n*/lglg*n* in space (and query time).

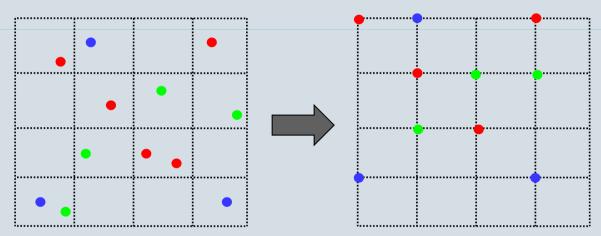


ee with fanout lg^ɛn.

- -1) becomes CQ(d,k) with colors { $\circ \circ$ }.
- tree lgn/lglgn.

Solving Concurrent Q(3,2)

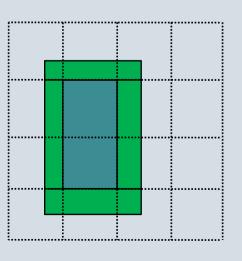
bounded in z-dimension. Build grid on x and y ns.

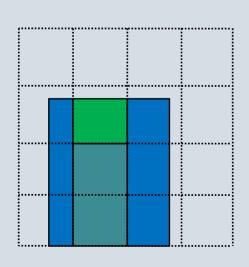


has size w by w where $w = \sqrt{n/|C||P||g^2n}$. s in each horizontal and vertical slab. napped to grid.

es stored:

- ,0) structure on points in every slab.
- very p in P, a Q(3,2) structure on points with least zinate from every color in every grid cell.
- I list for every color in every grid cell with points sorted oordinate.
- sive structure inside each slab.





decompose into four CQ(3,1) queries and a grid query. ive steps, they become one recursive CQ(3,1) query, 3,0) queries and one grid query.