MADALGO
CENTER FOR MASSIVE DATA ALGORITHMICS


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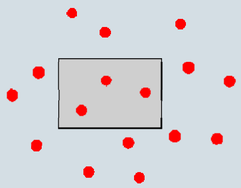
Geometric Range Searching

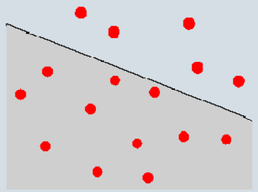
Introduction

Problem

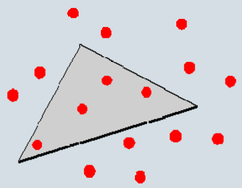
Process a given a set of points such that points inside a given query geometric region can be found efficiently.



Orthogonal Range Searching



Halfspace Range Searching



Simplex Range Searching

Applications

Databases, geographic information systems, graphics and more.
The points can be conceptual representation of cities, employees, objects and so on.

Types of Queries

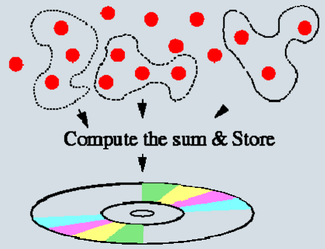
- Counting points (range counting)
- Reporting points (range reporting)
- Deciding if the query is empty (range emptiness)

Models of Computation

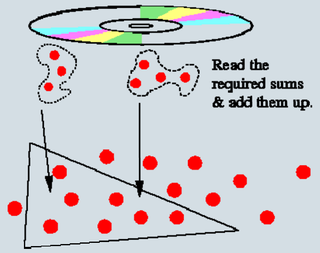
- Pointer Machine (PM)
- RAM (Random Access Machine)
- External Memory (I/O model)
- Cache-Oblivious
- Semi-group Model

Semi-group Model

- Points have weights from a semi-group
- The answer is the sum of the weights in the query
- Data structure is composed of pre-computed sums
- Can answer almost all types of queries



Preprocessing phase



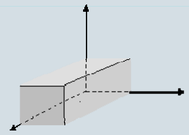
Query phase

Orthogonal Range Reporting in 3D

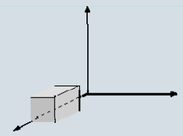
Four important cases of a 3D orthogonal query.

We solve 3-sided queries optimally in PM and I/O models and almost optimally in RAM model [Afshani'08].

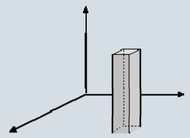
Improve results are achieved for the rest as well.



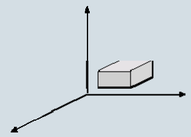
3-sided query



4-sided query



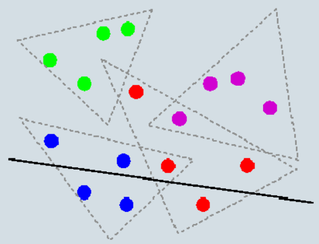
5-sided query

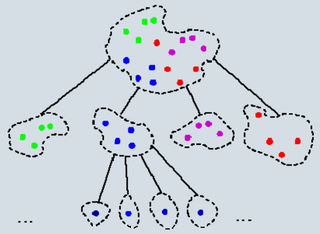


6-sided query

Simplex Range Searching

Partition theorem yields a simplicial partition s.t. any halfspace crosses a few simplices. With recursion, this gives a partition tree, a commonly used tool in simplex and halfspace range searching problems.





3D Approximate Halfspace Range Counting

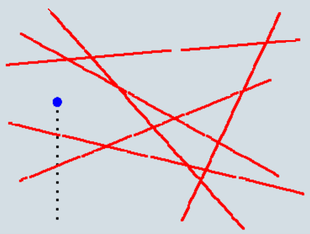
Halfspace range counting is solved using simplex range searching but the performance of the resulting data structure is not satisfactory.

In 3D, performance can be improved significantly through approximations but the previous approximation methods had sub-optimal performance and the correctness of their results were not always guaranteed.

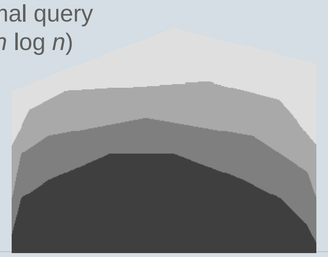
We solve this problem optimally in 3D and our results are always correct [Afshani & Chan'07].

3D Halfspace Range Reporting

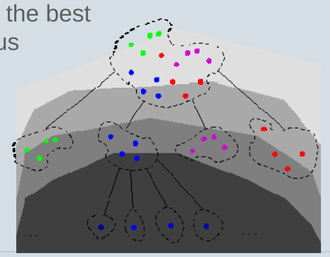
Start in dual space



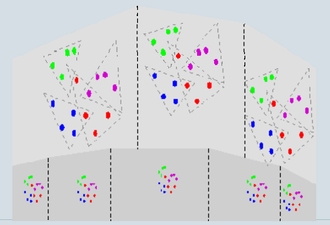
A layered approach based on shallow cutting gives us optimal query with $O(n \log n)$ space.



With simplex range searching it gives us $O(n \text{ loglog } n)$ space, the best previous result.



We solve the problem with optimal space, preprocessing and query [Afshani & Chan'09].



Intersection Searching

Generalization of range searching:

- Objects instead of points

We solve disjoint rectilinear polygon counting [Afshani & Chan'06].

Query time similar to simplex range counting results.