MADALGO
CENTER FOR MASSIVE DATA ALGORITHMICS

Mohammad Abam
Aarhus University

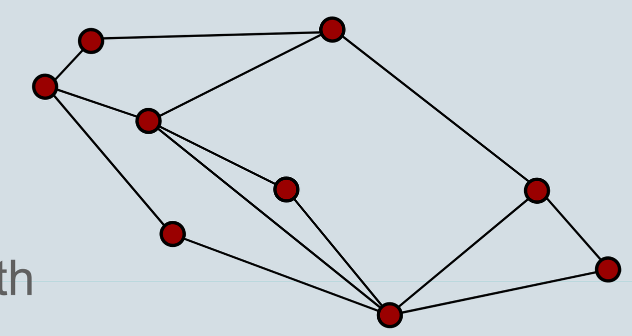
Geometric Spanner Networks

Motivation

Networks play a central role in numerous applications, and the design of good networks is therefore an important topic of study. In general, a good network has certain desirable properties while not being too expensive. In many applications this means one wants a network providing short paths between its nodes, while not containing too many edges. This leads to the concept of spanners.

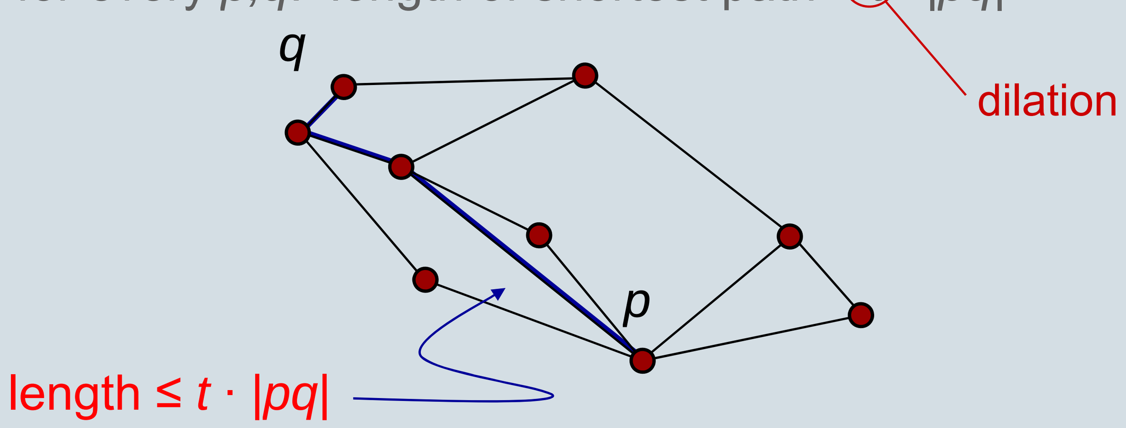
Geometric network

- vertices: set of points in R^d
- edge length: Euclidean length

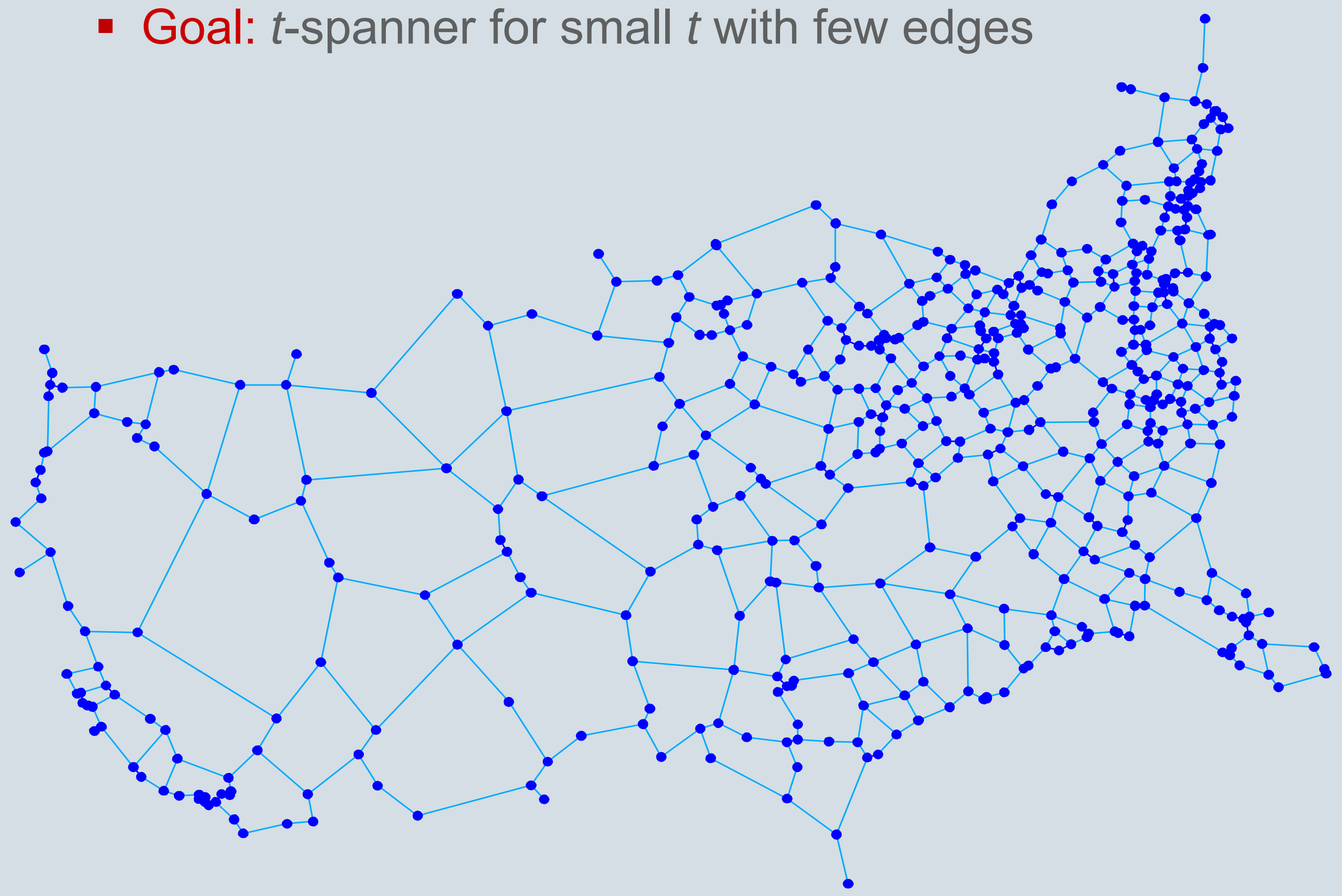


t-spanner

- for every *p*,*q*: length of shortest path $\leq t \cdot |pq|$



- Goal: *t*-spanner for small *t* with few edges



A 1.5-spanner on 532 cities in US

Kinetic Data Structures

Simulate system of moving objects and efficiently maintain discrete geometic attributes of objects such as the convex hull of moving points.

Two parts

- Combinatorial description of the attribute.
- A set of certificates—elementary test on the input objects—with the property that as long as the outcome of the certificates do not change, the attribute does not change.

Structures

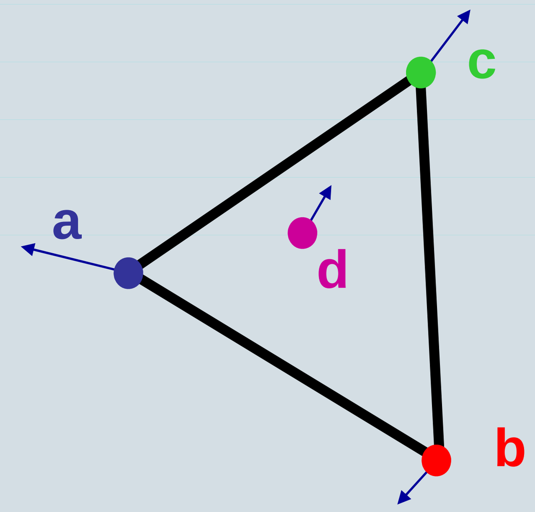
- Proof of correctness of attribute (certificates)
- Priority queue (event queue)

Assumptions

- A simple model for motion: each object follows a known flight plan with rational parameters.
- Certificates are algebraic; failure is next largest root.

Example

Attribute: Convex hull



Certificates

a is to the left of bc
d is to the left of bc
b is to the right of ad
c is to the left of ad

Algebraic equation

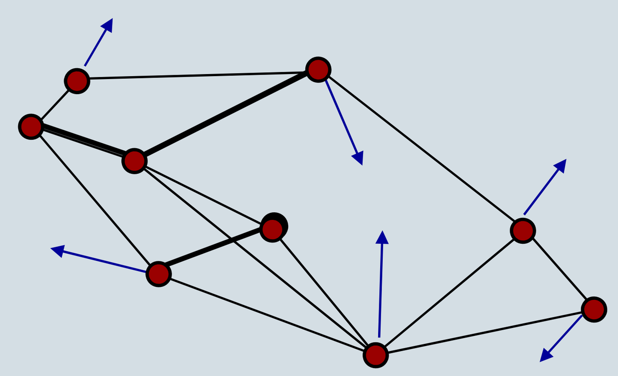
$$\begin{vmatrix} x_c(t) & y_c(t) & 1 \\ x_b(t) & y_b(t) & 1 \\ x_d(t) & y_d(t) & 1 \end{vmatrix} \geq 0$$

KDS Properties

- **Compact**
  If it uses little space in addition to the input.
- **Responsive**
  If data structure invariants can be restored quickly after the failure of a certificate.
- **Local**
  If it can be updated easily if flight plan for an object changes.
- **Efficient**
  If the worst-case number of events handled by the data structure is small compared to some worst case number of external events.

Kinetic Geometric Spanner

One interesting variant in geometric spanners is where points move along continuous trajectories which is inspired by simulations in mobile networks.



Goal

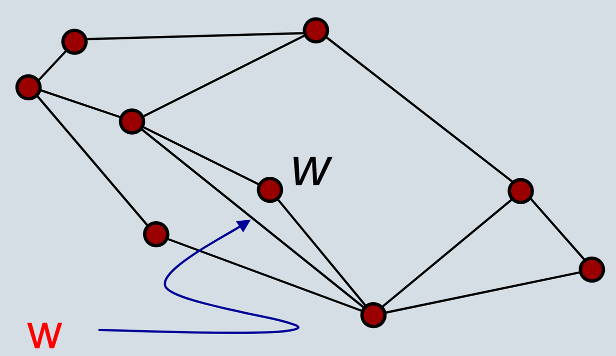
- Need to update spanner to ensure dilation remains $\leq t$
- Stable spanner: not too many events (changes)
- Fast updates at each event

Contribution

We present a new *t*-spanner for sets of *n* points. Our spanner has linear size and low maximum degree. The main advantage of our spanner is that it can be maintained efficiently as the points move. Moreover, our kinetic spanner is local, responsive and compact.

Geometric Spanner for Weighted Point Set

Sometimes the cost of traversing a path in a network is not only determined by the lengths of the edges on the path, but also by delays occurring at the nodes on the path: for example in a computer network a node may need some time to forward a packet to the next node on the path.



Weight w
Added to length of any path passing through this node

Contribution

We present a general method for turning spanner of un-weighted point set into spanner of weighted point set. We can apply our method to obtain 5+*t*-spanners with a linear number of edges. We also describe an alternative method that leads to 2+*t*-spanners using near-linear number of edges. This bound is nearly optimal: it is possible to assign weights to points such that any non-complete graph has dilation larger than 2-*t*.

References

[1] M. Abam and M. de Berg. Kinetic spanners in R^d, *SoCG, 2009.*
[2] M. Abam and M. de Berg, M. Farshi, J. Gudmundsson and M. Smid. *Geometric spanner for weighted point set, submitted.*

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