Ad Exchange: General Envy-free Auctions with Mediators

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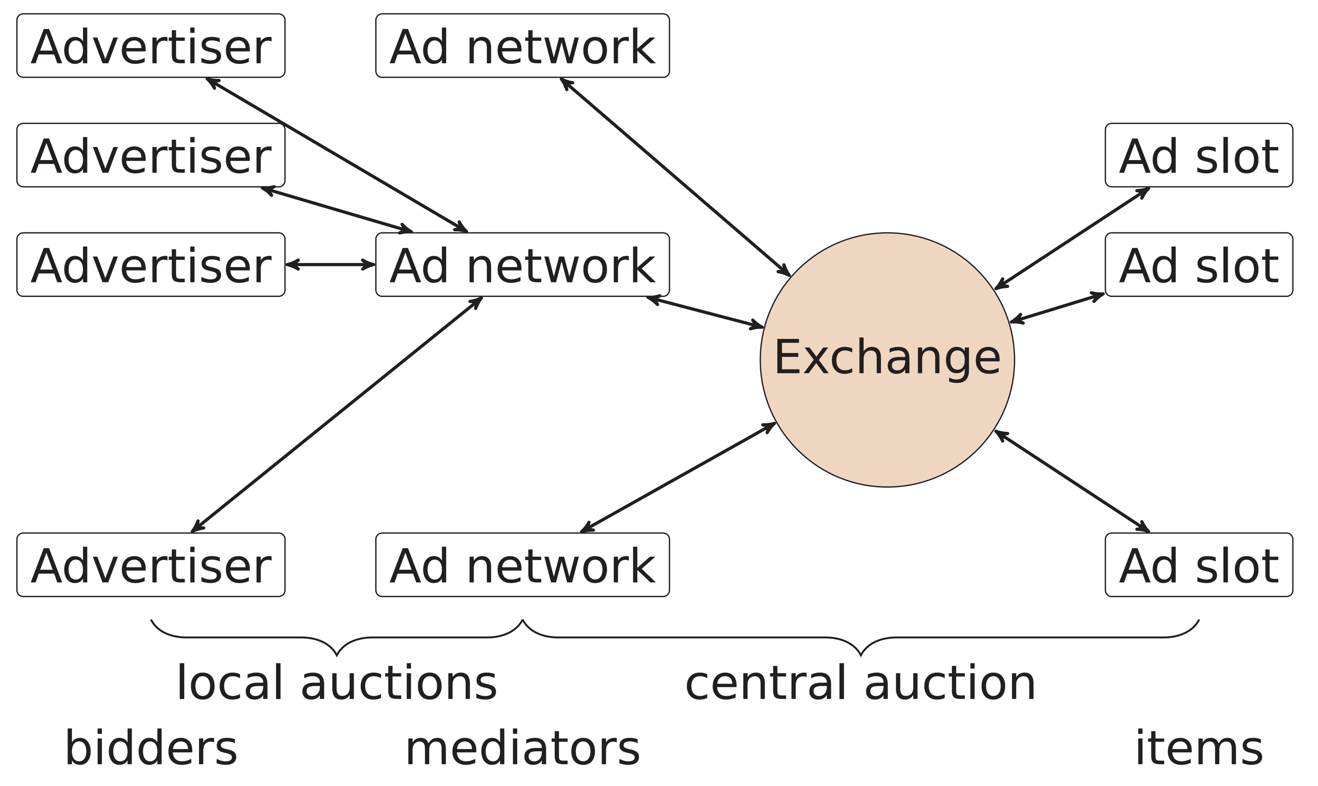
Problem



- started with DoubleClick Ad Exchange (Google) in 2007
- Facebook and Amazon started 2012, Ebay 2013
- market volume recently estimated to $2 billion

- The **utility** of a bidder for an item set S is defined as valuation(S)−price(S).
- The **revenue** of a mediator for item set S is revenue(S) = local auction prices(S) − central auction prices(S) (i.e. money received from bidders minus money paid to ad exchange) if the local auction outcome for item set S is globally envy-free for its bidders and revenue(S) = −1 otherwise.
- The **demand** is the set of item sets with highest utility / revenue.

A **general envy-free** (or **Walrasian**) **equilibrium** is a price vector and an allocation s.t. all bidders and mediators receive a set in their demand and all items with positive price are sold.

- **Does a general envy-free equilibrium always exists?**
- **Can it be computed?**

Main Result

If all bidders have **unit demand** valuations, then there is a way for the mediators to compute their bids for the central auction and the prices for their bidders such that a **general envy-free equilibrium always exists**.

unit demand valuation: $v(S) = \max_{j \in S} v(j)$

Central Auction

- **input:** valuations of bidders (only known to their mediator)
- **result:** assignment μ to mediators, central auction prices p, assignments μ'_{M_i} to bidders, and local auction prices p'_{M_i} s.t. bidders and mediators are envy-free and all items with positive price are sold

each mediator offers $p(j) \leftarrow 0$ to each item j
each item accepts one offer and rejects all others
while some offer rejected **do**
  for all mediators M_i **do**
    for all items j **do**
      if j has accepted M_i's offer **then**
        $p_{M_i}(j) \leftarrow p(j)$
      else
        $p_{M_i}(j) \leftarrow p(j) + 1$
    $D_{M_i} \leftarrow$ demandInclAccepted(p_{M_i}, $D^{=}_{M_i}$)
    offer p_{M_i} to all $j \in D_{M_i}$
  each item accepts one highest offer $p(j)$ and rejects all others

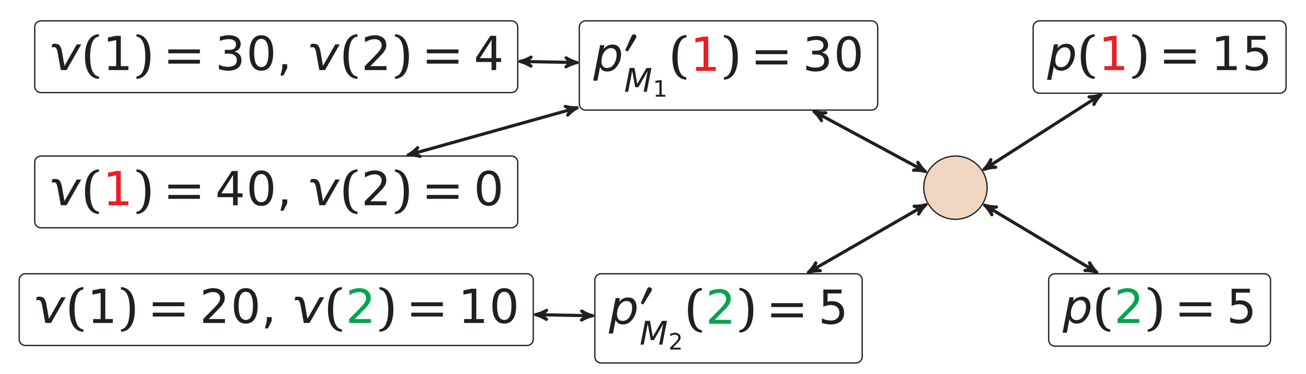
based on *salary-adjustment process* by Kelso and Crawford (1982)

Mediators' Demand

- mediators have to repeat accepted offers
- **input:** central auction prices p_M, set $D^{=}_M$ of accepted items for M
- **result:** returns set D_M in demand of M with $D^{=}_M \subseteq D_M$ and stores result (μ', p') of local auction
- The local auction is run within the subroutine localMinWalrasianEquilibrium. It returns the local Walrasian equilibrium for the bidders of mediator M with the smallest prices $p' \geq p_M$ that matches all items j in $D^{=}_M$ with $p_M(j) > 0$. For this we can use the algorithm and results from Dütting et al. (2011).
- (μ', p') can be initialized with $(\emptyset, 0)$

procedure demandInclAccepted(p, $D^{=}$)
  $\hat{p}(j) \leftarrow \max(p'(j), p(j)) \quad \forall j$
  $\hat{\mu} \leftarrow \{(i, j) \in \mu' \mid j \in D^{=}\}$
  $(\mu', p') \leftarrow$ localMinWalrasianEquilibrium($\hat{\mu}$, $\hat{p}$)
  save (μ', p')
  return $\{j \mid \exists (i, j) \in \mu'\} \vee \{j \in D^{=} \mid p(j) = 0\}$

Example



- $\text{revenue}_{M_1} = 15$, $\text{revenue}_{M_2} = 0$
- competition between ad networks ⇒ revenue for ad exchange
- competition within ad network ⇒ revenue for ad network

Further Results

The minimal demand sets of a mediator form the **bases of a matroid** (for any given price vector).

- similar result for gross-substitute valuations in Gul and Stacchetti (2000)

If all bidders have **additive valuations** $v(S) = \sum_{j \in S} v(j)$, then

- all mediators have additive valuations,
- a Walrasian equilibrium always exists,
- and it can be computed with multiple second price single item auctions.

Open Questions

- Does a strongly polynomial time mechanism exist?
- Can the result be generalized to other valuation classes?
- What if budgets are introduced in the unit demand case?

References and Acknowledgements

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This work was funded by the Vienna Science and Technology Fund (WWTF) through project ICT10-002.