Faster Spectral Sparsification of Laplacian and SDDM Matrix Polynomials

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We say that $D - A$ is $\mathcal{T}$-matrix to remark that its type is either Laplacian or SDDM.

Transition matrix: p-Lazy Random Walk

• $W_p = (1-p) \cdot I + p \cdot D^{-1}A, \ \forall\, p \in (0,1)$

Transition matrix: p-L.R.W. of length N

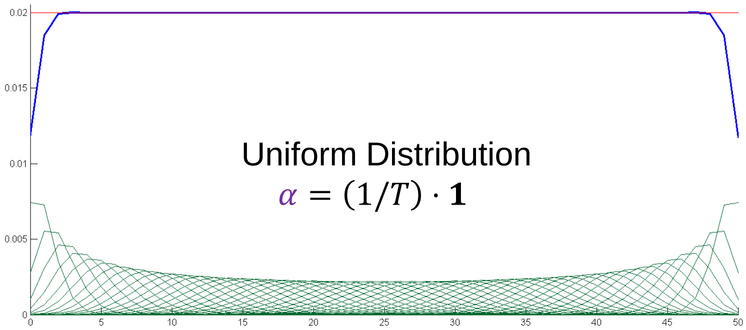
• $W_p^N = \sum_{k=0}^{N} B_{N,k}(p) \cdot (D^{-1}A)^k$, where

• $B_{N,k}(p) = \binom{N}{k} p^k (1-p)^{N-k}$ is Binomial coefficient (Bernstein basis)

Sparsifier of Binomial $\mathcal{T}$-Matrix Polynomial

• $D - \widehat{M} \approx_\epsilon D - DW_p^N$ is $\mathcal{T}$-Matrix

• We give an algorithm with run time $\tilde{O}(\epsilon^{-2} m \log^3 n \cdot \log^2 N + \epsilon^{-4} n \log^4 n \cdot \log^5 N)$



Representational Power of MDBD

• Suppose $w(x)$ is prob. density func. with „nice“ first four derivatives. Then for any $T \geq \Omega(\delta^{-1/2} N^{1+\eta})$ and all $k \in [3:N-3]$

• $\left| \left(1 \pm \frac{1}{100}\right) \frac{1}{N} w\left(\frac{k}{N}\right) - \frac{1}{T} \sum_{i=1}^{T} F_k\left(\frac{i}{T+1}\right) \right| < \frac{\delta}{N}$

• where $F_k(x) = w(x) \cdot B_{N,k}(x)$

Mixture of Discrete Binomial Distributions (MDBD)

• $\{[B(N,p_i), \alpha_i]\}_{i=1}^{T}$ is MDBD such that

  • α is probability distribution over $[1:T]$

  • $B(N,p_i)$ is Binomial distribution

Sparsifier of MDBD $\mathcal{T}$-Matrix Polynomial

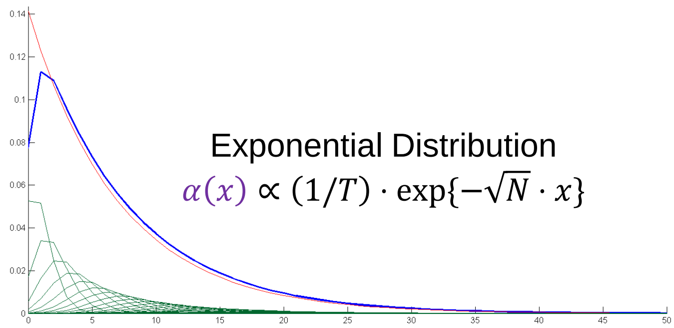
• $D - \widehat{M} \approx_\epsilon \sum_{i=1}^{T} \alpha_i (D - DW_p^N) =$
$= D - D\sum_{k=0}^{N} \gamma_k (D^{-1}A)^k$

• where $\gamma^{\mathrm{T}} = \alpha^{\mathrm{T}} \boldsymbol{B}^{(p)}$ and $\boldsymbol{B}_{ik}^{(p)} = B_{N,k}(p_i)$

• Sparsifier $D - \widehat{M}$ is $\mathcal{T}$-Matrix

• We give faster algorithm with run time $\tilde{O}(\epsilon^{-2} m \log^3 n \cdot \log^2 N + \epsilon^{-4} nT \log^4 n \cdot \log^5 N)$

• [1] Cheng et al. $\tilde{O}(\epsilon^{-2} mN^2 \log^{c_1} n \cdot \log^{c_2} N)$



Exact Recovery of Coefficients α

• Suppose MDBD satisfies

  • $T = N + 1$ and $0 < p_1 < \cdots < p_T < 1$

• Given γ, p we find α in $O(N \log^2 N)$ time.

The first nearly linear time algorithm for analysing *Markov chains* with *transition matrices* $\sum_{k=0}^{N} \gamma_k (D^{-1}A)^k$ where γ is induced by MDBD.

[1] D. Cheng, Y. Cheng, Y. Liu, R. Peng and S. H. Teng, Efficient Sampling for Gaussian Graphical Models via Spectral Sparsification, COLT'15

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