Efficient Algorithms for Graph Problems in the Verification and Synthesis of Systems

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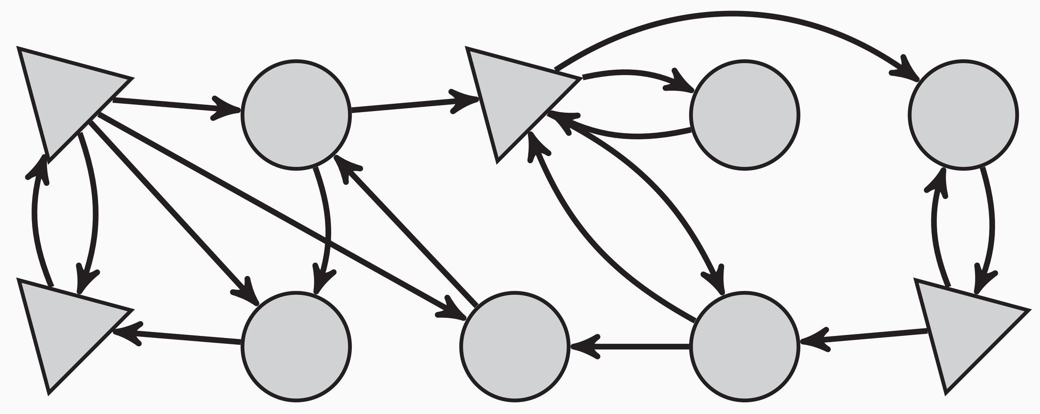
Game Graphs and MDPs



- 2 types of vertices: player 1 owns circles, player 2 owns triangles
- Game: token moved along edges, owner chooses outgoing edge
→ forms infinite path
- Standard graph: no player 2 vertices
- Markov Decision Process (MDP): player 2 chooses edge randomly

Verification and Synthesis of Systems

Graphs model closed systems or open systems with uncontrollable inputs

MDPs additionally model probabilistic behavior

Game Graphs model open systems with both controllable and uncontrollable inputs

Verification Does the system satisfy its specification?

Synthesis Generate a system according to the specification.

The specification can be expressed (e.g.) as Streett objective.

Objectives

Objective a set of infinite paths; is satisfied when one of the paths contained in the objective is played (MDPs: with probability 1)

Winning Set set of vertices from which a player can ensure that her objective is satisfied, no matter how the second player plays

Zero-sum Game the objective of player 2 is the complement of the player 1 objective

Objectives considered here:

Reachability Is some vertex of the set U reached at least once?

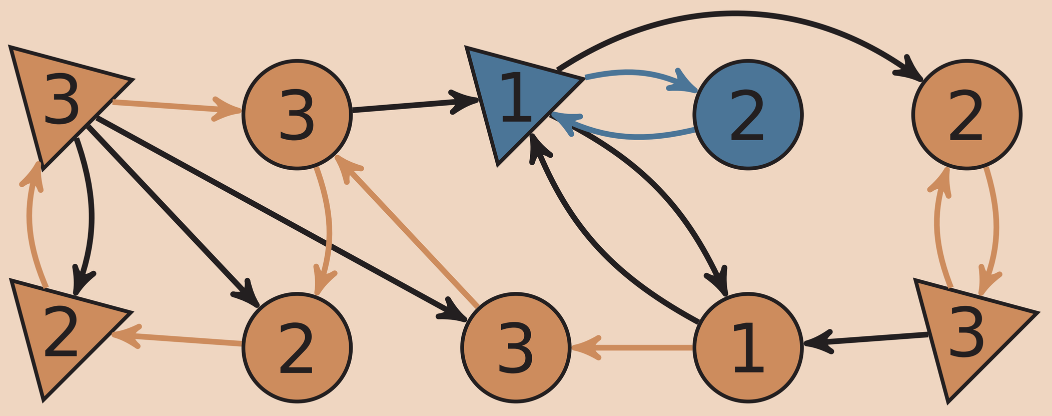
Disjunctive Reachability Does Reachability hold for at least one of the sets $U_1, \ldots, U_k$?

Büchi Is some vertex of the set U reached infinitely often?

Streett Does it hold for all pairs $(L_1, U_1), \ldots, (L_k, U_k)$ that whenever some vertex of L_i is reached infinitely often, then some vertex of U_i is reached infinitely often?

Parity For parity each vertex is assigned some natural number $\leq d$, its priority. Is the lowest priority that is visited infinitely often even?

Example A parity game with 3 different priorities:



Player 1, the "even player", wins when the game starts at an orange vertex; player 2, the "odd player", wins from blue vertices. The colored edges represent optimal strategies of the two players.

Results

We developed algorithms with improved asymptotic upper bounds on the running time for the following problems:

Problem	old	new
Disj. Reach. on MDPs	$O(k \cdot \text{MEC})$	$O(km + \text{MEC})$ [7]
Streett on graphs[1]	$O(\min\{m^{1.5}, mn\})$ [4]	$O(\min\{m^{1.5}, n^2\})$ [2]
Streett on MDPs[1]	$O(n \cdot \text{MEC})$	$O(\min\{m^{1.5}, n^2\})$ [7]
Parity games with $d = 3$	$O(mn)$ [5]	$O(n^{2.5})$ [2]
General Parity Games[1]	$O(mn^{d/3})$ [6]	$O(n^{(4+d)/3})$ [2]

[1]simplified running time

- runtimes for input graph with m edges and n vertices
- MEC denotes the best running time of $O(\min\{m^{1.5}, n^2\})$ for computing the maximal end-component decomposition of an MDP
- results from [2] are improvements for dense graphs only
- parity games with 3 priorities are equivalent to 1-pair Streett games

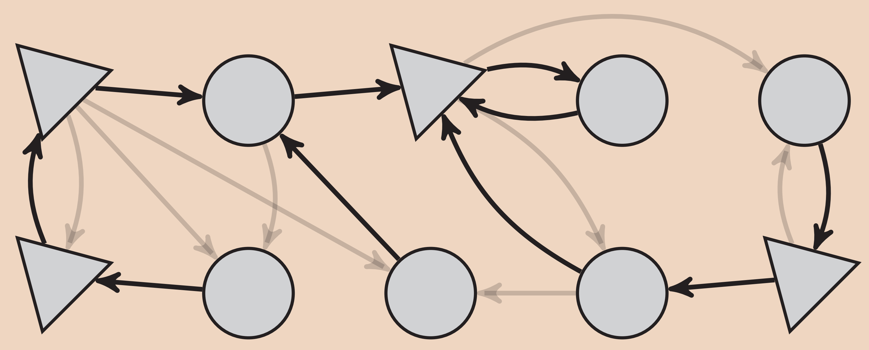
Important Techniques

Sparse graphs local graph exploration "in parallel"

Dense graphs degree-based hierarchical graph decomposition

- one of few sparsification techniques for directed graphs

Hierarchical Graph Decomposition [3, 1]



- graph G_i contains $O(n \cdot 2^i)$ edges, $i \in \{0, 1, \ldots, \log n\}$
→ e.g. first 2^i outgoing edges of each vertex
- find winning parts of size 2^i in G_i and increase i if nothing found
→ if e.g. time in G_i proportional to edges, then $O(n)$ time per vertex

Open Questions

- Parity games are in UP ∩ coUP. Is there a polynomial time algorithm?
- Can reachability (and MEC) in MDPs be solved in linear time (like reachability and strongly connected components in graphs)?
- Are Streett objectives easier on graphs than on MDPs?

References

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