NEW ALGORITHMS FOR NONNEGATIVE MATRIX FACTORIZATION AND BEYOND

ANKUR MOITRA

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Challenge: develop tools for automatic comprehension of data

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- Annotate documents according to these topics
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Parceling Out a Nest Egg, Without Emptying It

By PAUL SULLIVAN

What clients often forget are fixed costs — homes, cars, insurance — that must come down but take time to reduce, she said. Beyond that is her clients' skittish approach to risk; putting all of their money in cash may make them feel safe, she said, but it probably will not support the lifestyle they want for decades.

A generational disconnect is at work here: most people plan to retire at 65, the retirement age established for Social Security in 1935, when the average life expectancy was 61. Today the average is over 80 for men and women with a college degree.

So the $5.12 million gift exemption — created in a compromise between President Obama and Congress in 2010 — presents the well-off with a decision laden with short- and long-term consequences. How much should they give heirs now — and thus avoid giving the government in estate taxes later — while maintaining their lifestyle over a probably longer but still unpredictable remaining life span?

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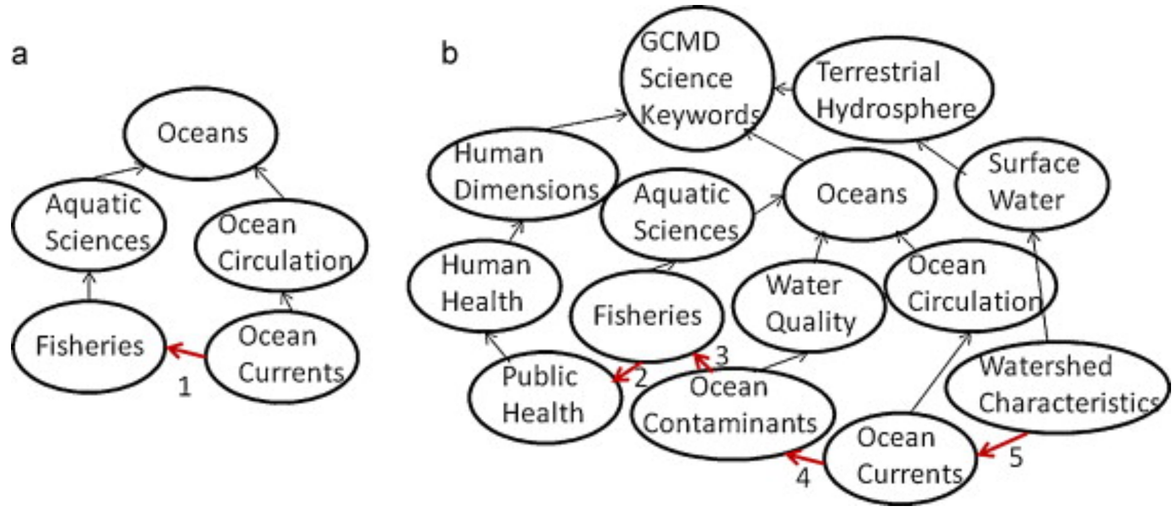

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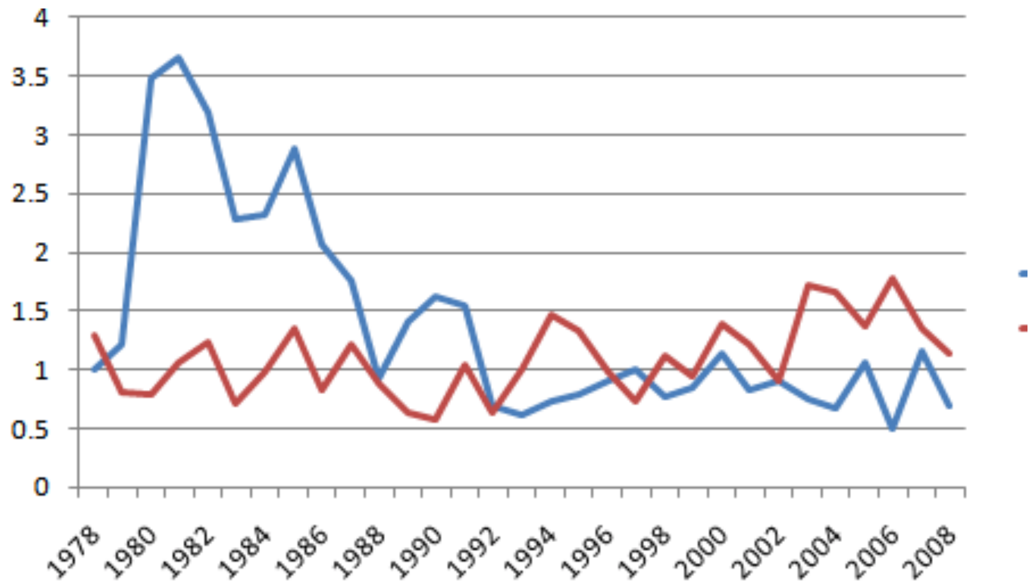

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- Each **document** is a distribution on **topics**
- Each **topic** is a distribution on words

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- Nonnegative Matrix Factorization
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Part II: A Bayesian Perspective

- Topic Models (e.g. LDA, CTM, PAM, …)
- Algorithms for Inferring the Topics
- Experimental Results

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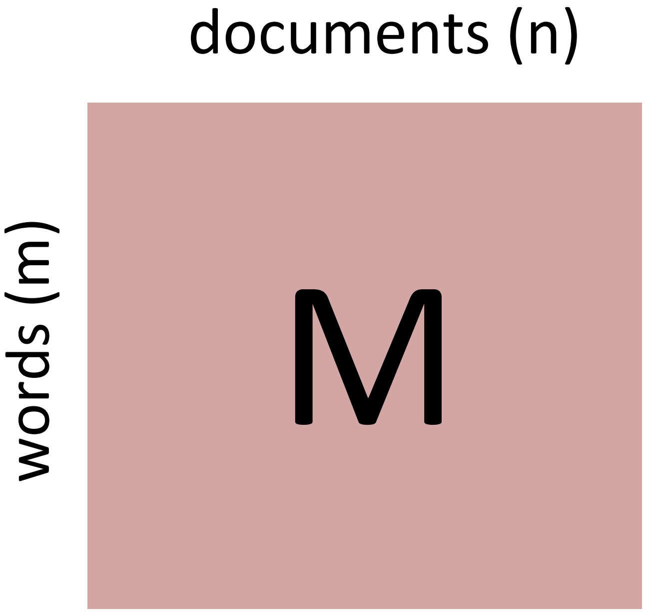

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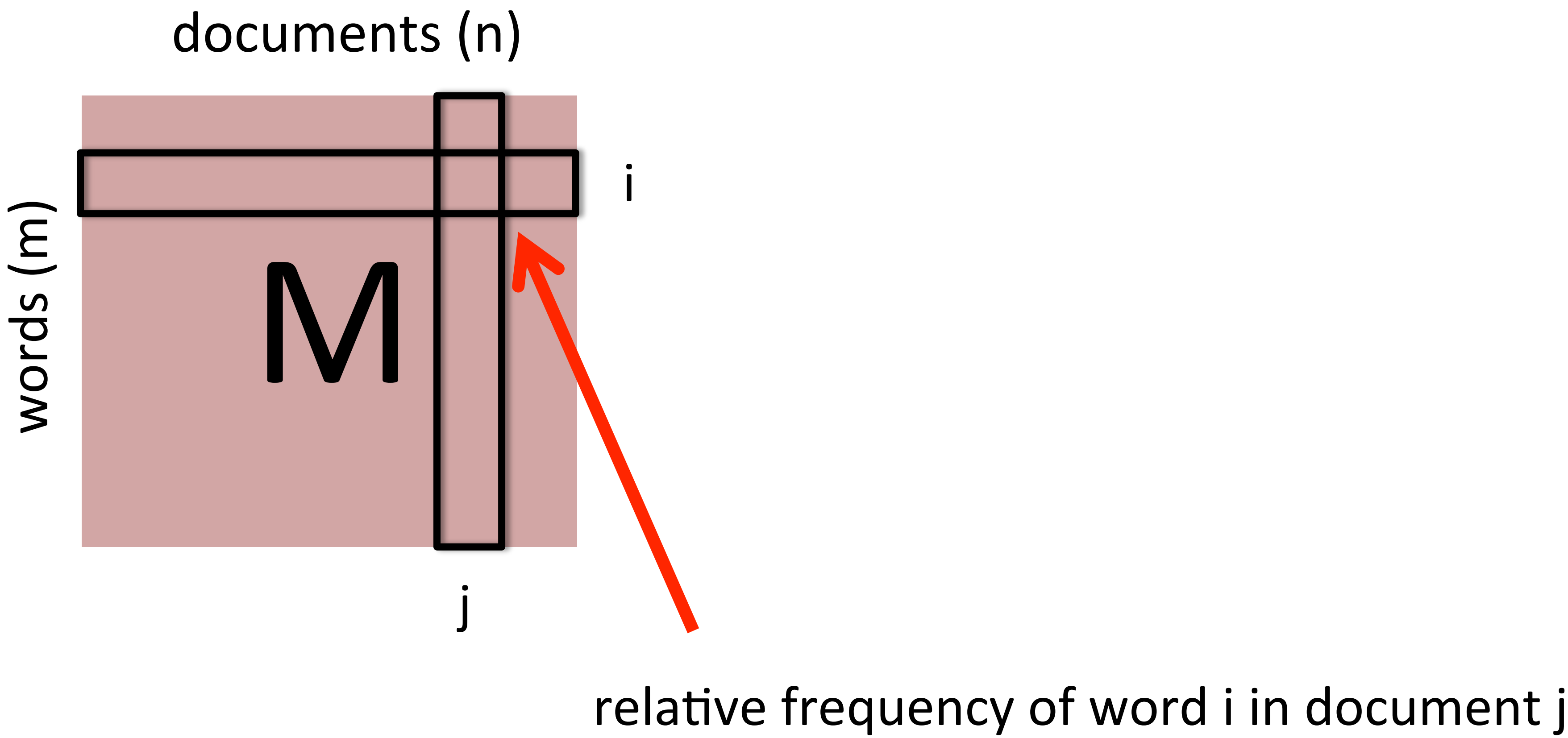

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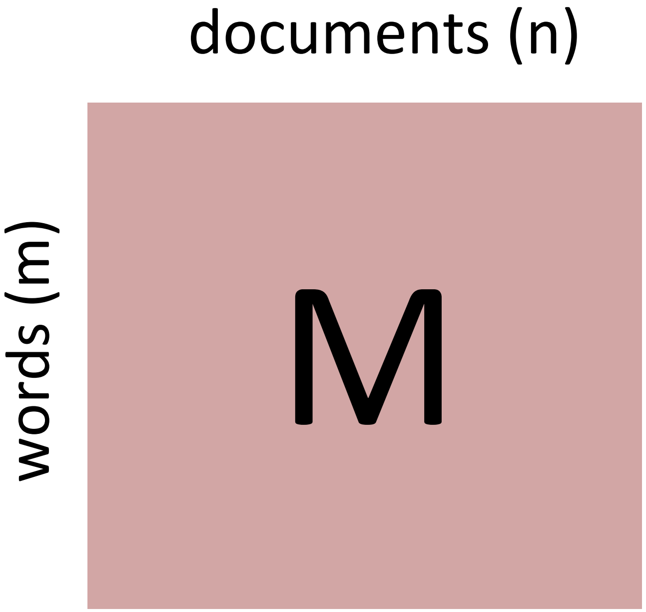

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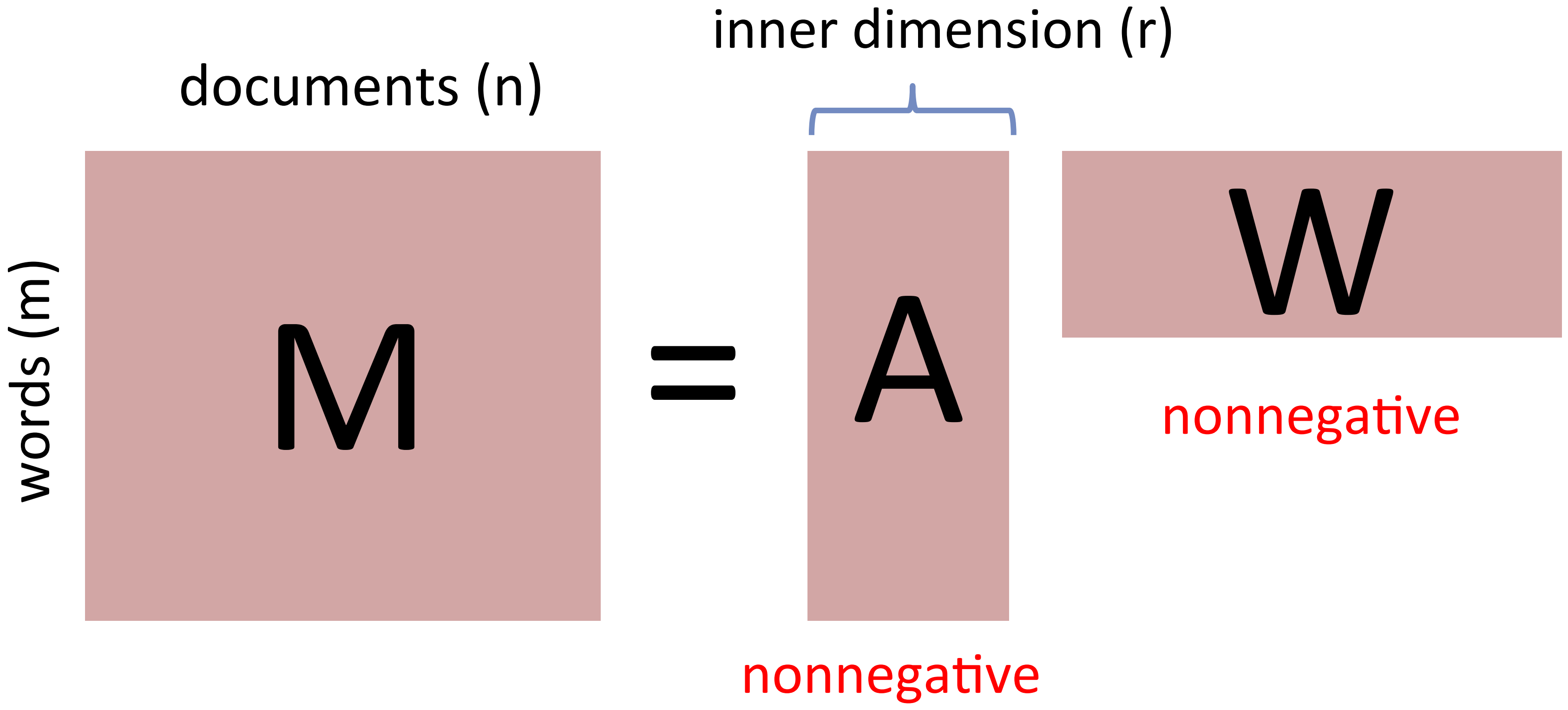

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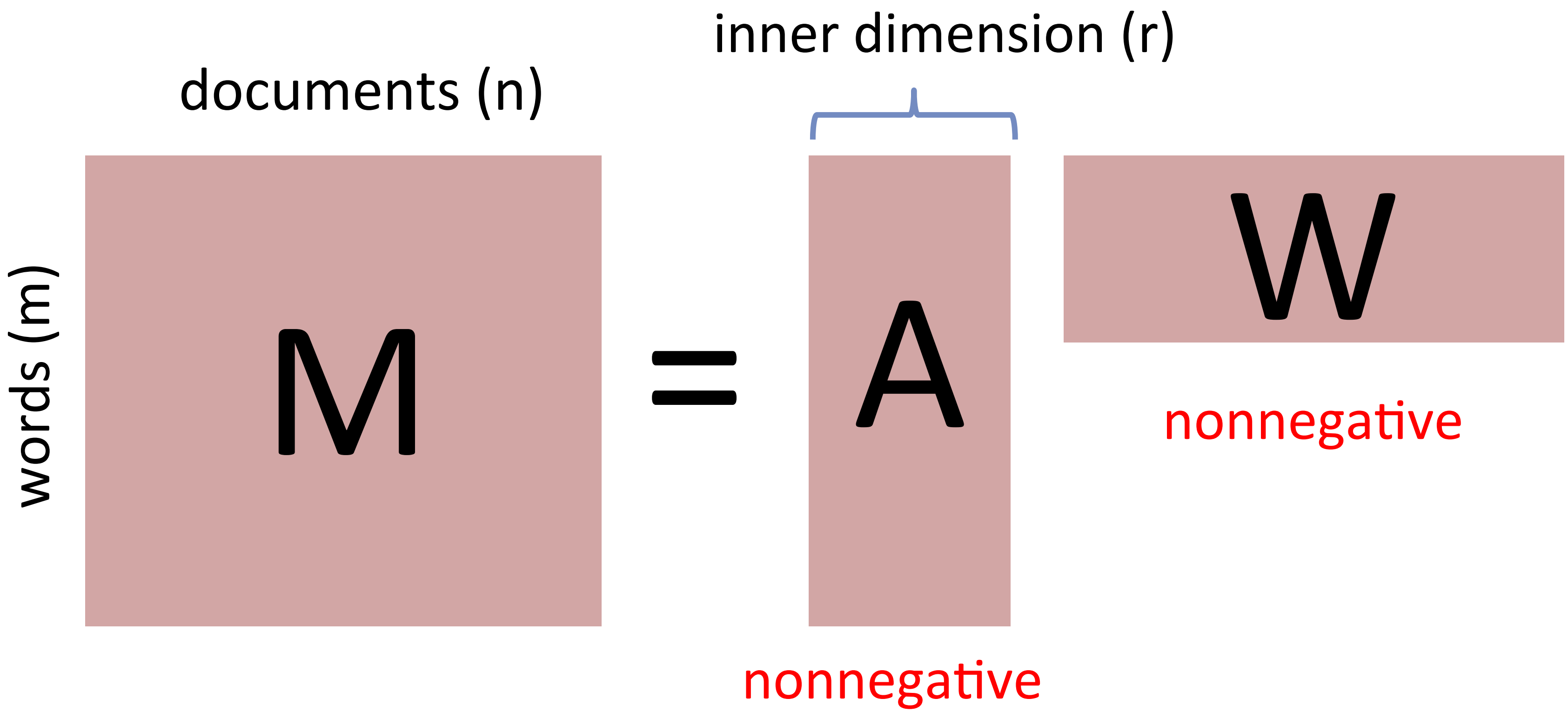

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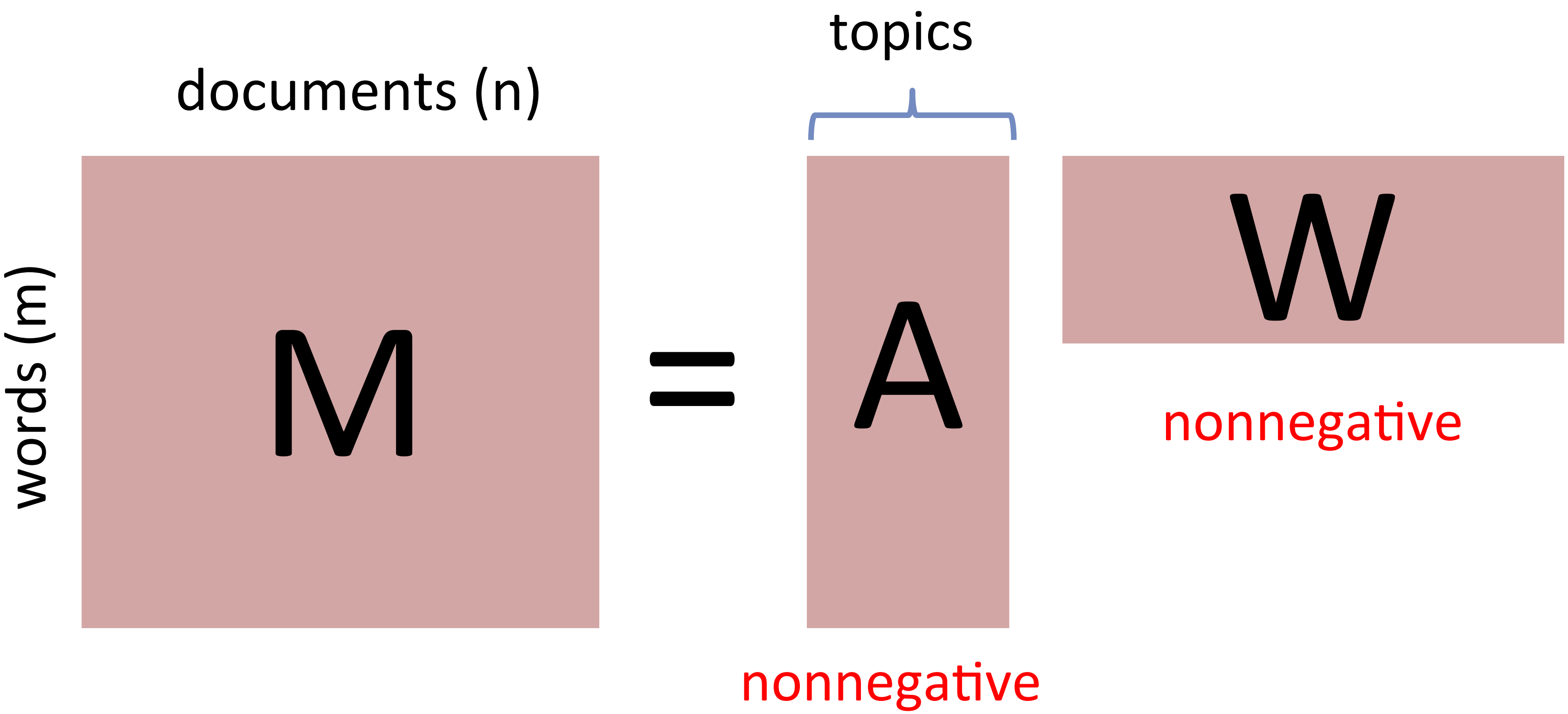

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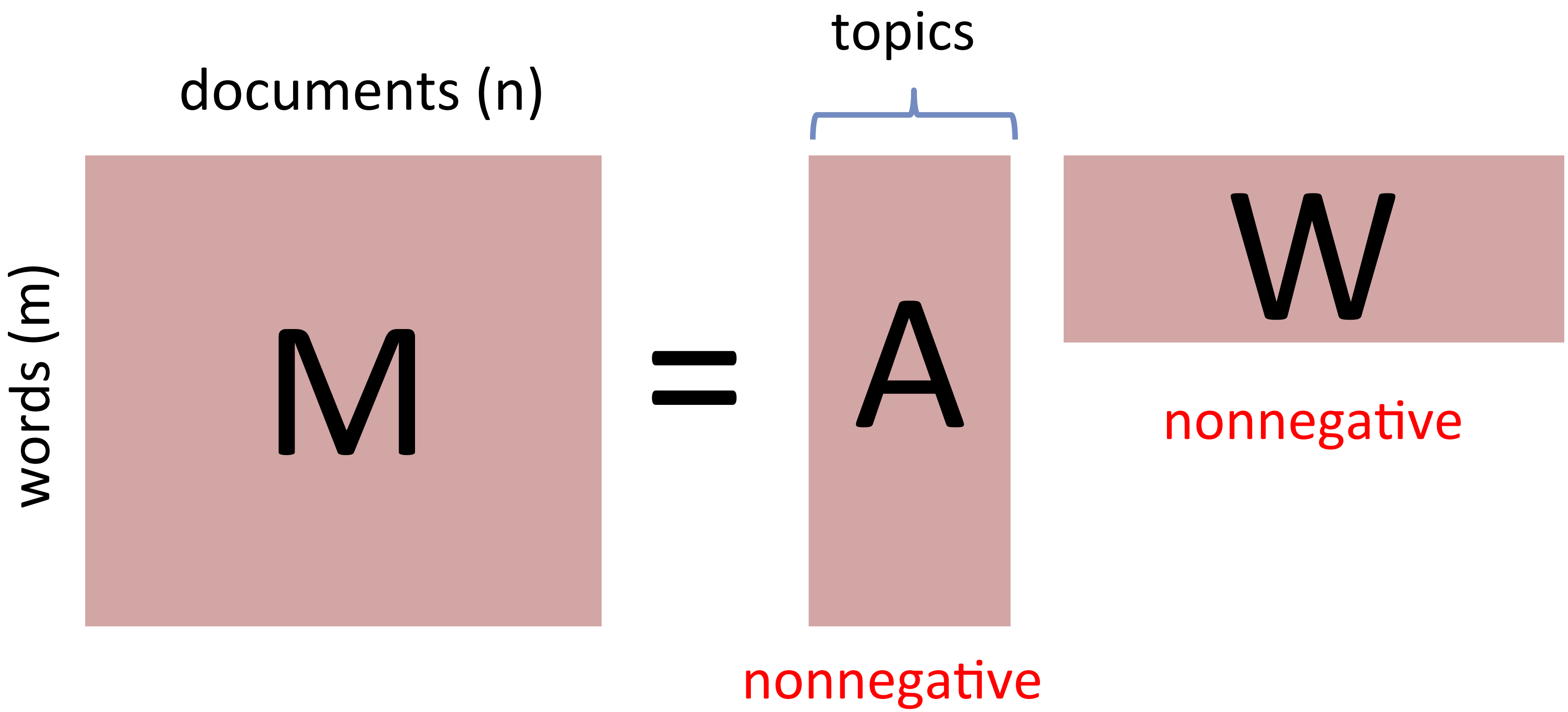

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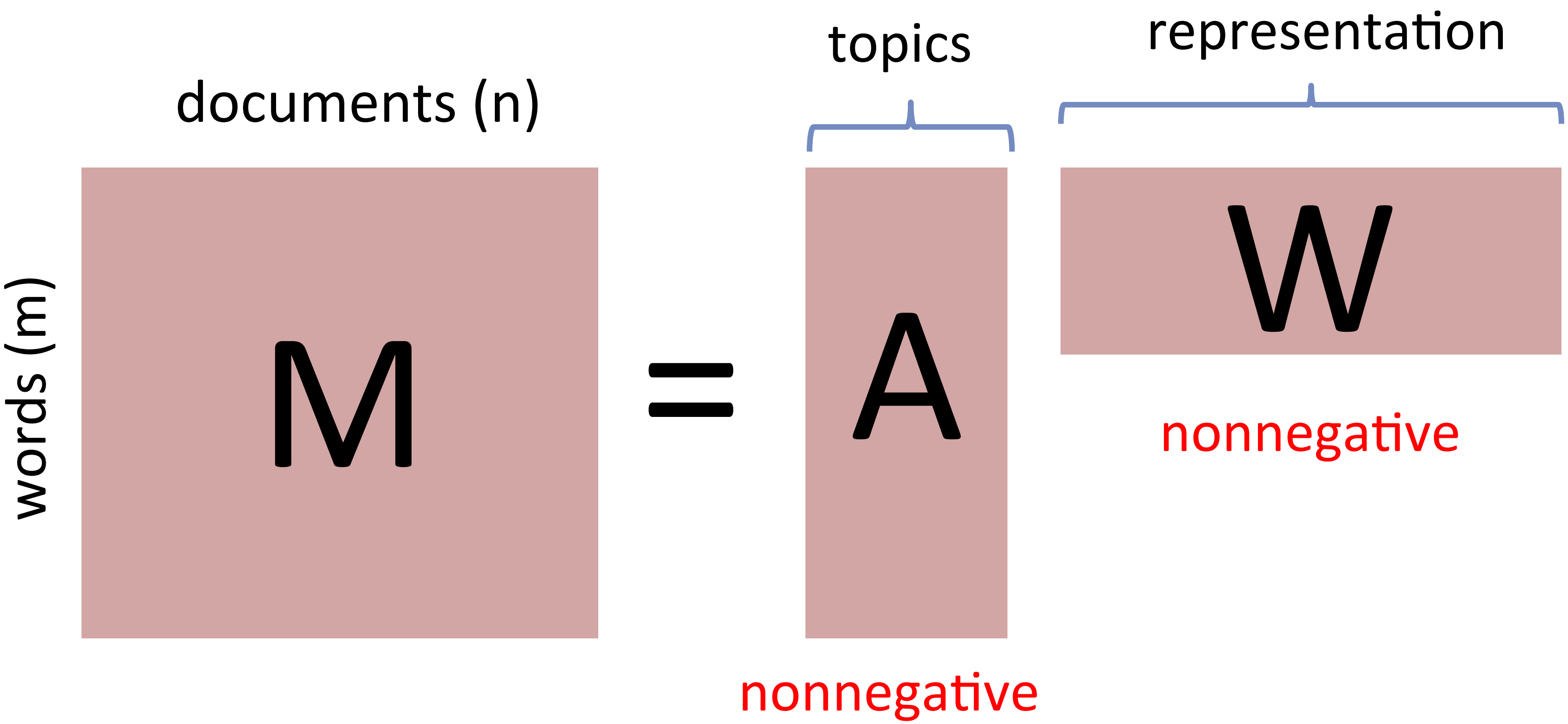

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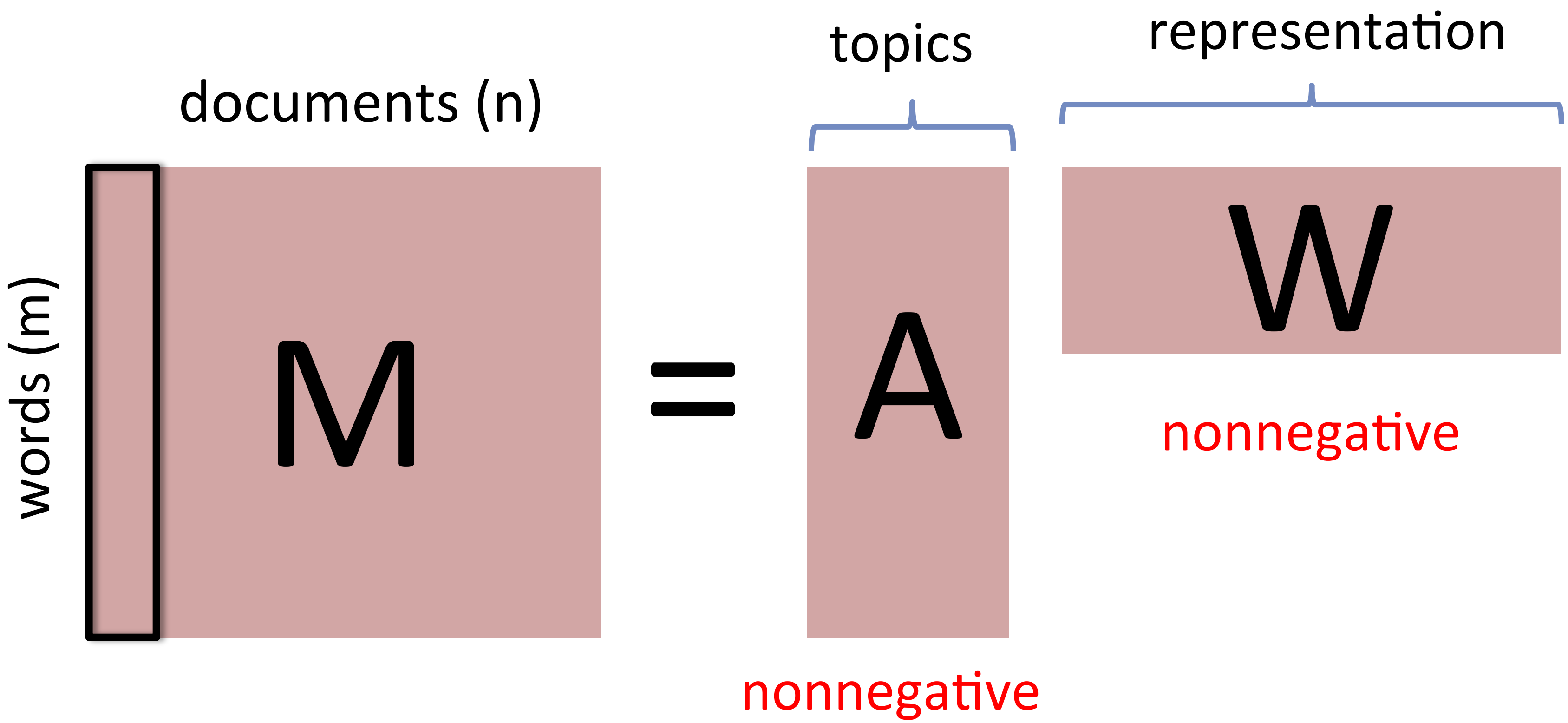

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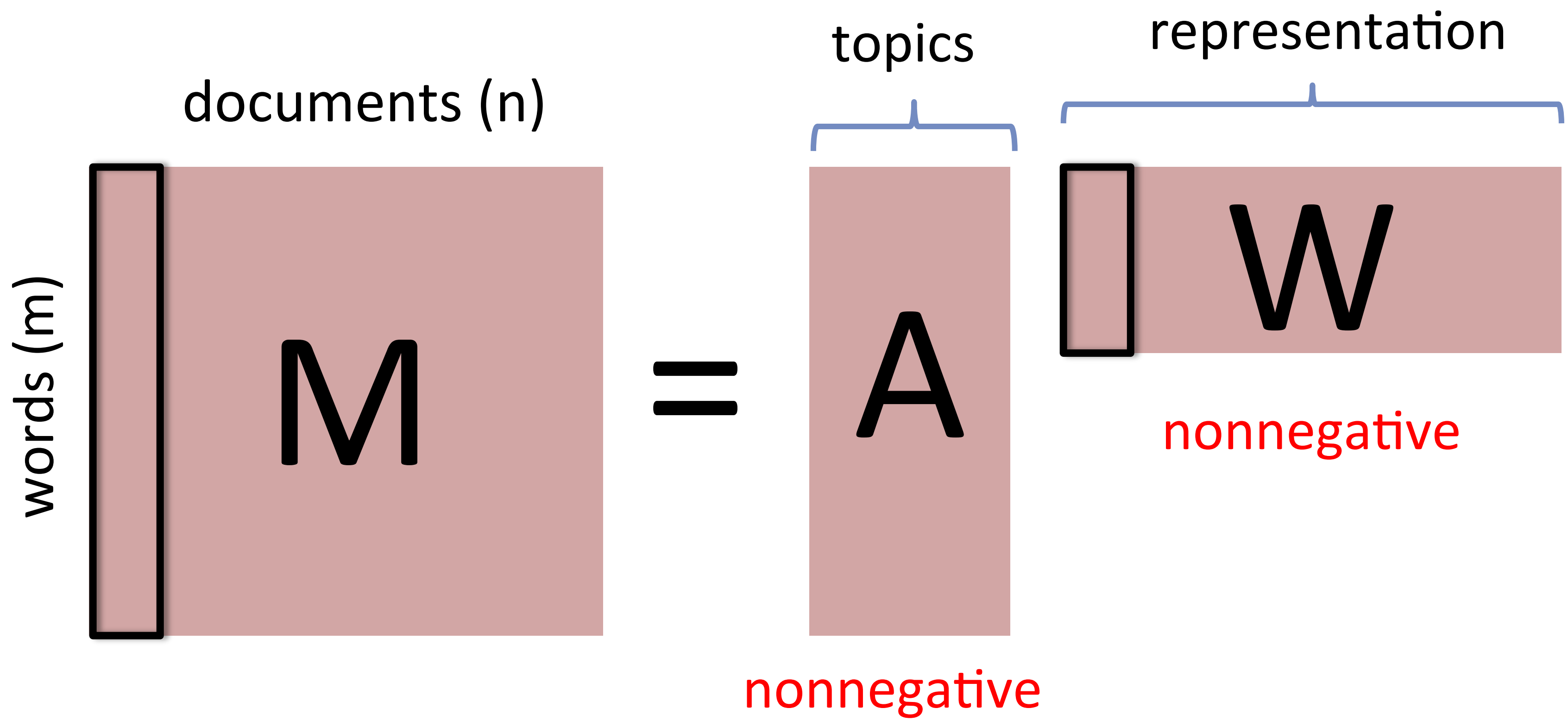

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- Introduced by **[Lee, Seung, ‘99]**
- Goal: extract **latent** relationships in the data
- Applications to text classification, information retrieval, collaborative filtering, etc **[Hofmann ‘99], [Kumar et al ‘98], [Xu et al ‘03], [Kleinberg, Sandler ‘04],…**

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Physical Modeling:

- Introduced by **[Lawton, Sylvestre '71]**
- Applications in chemometrics, environmetrics, economics

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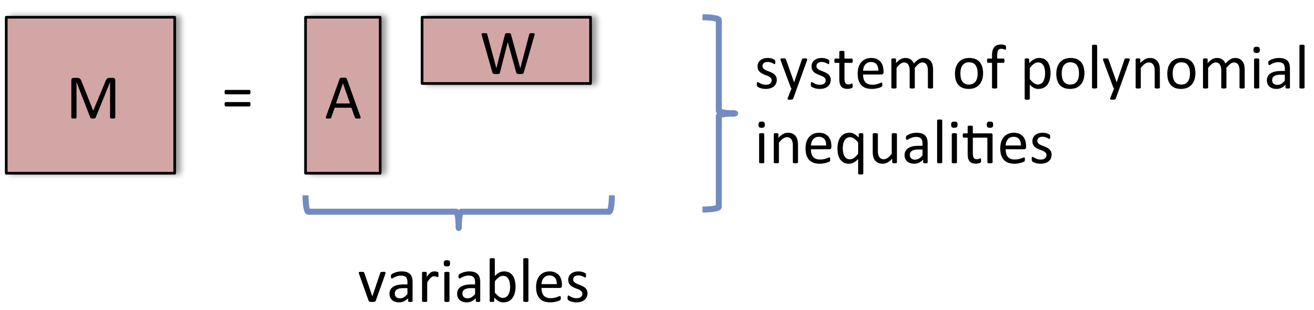

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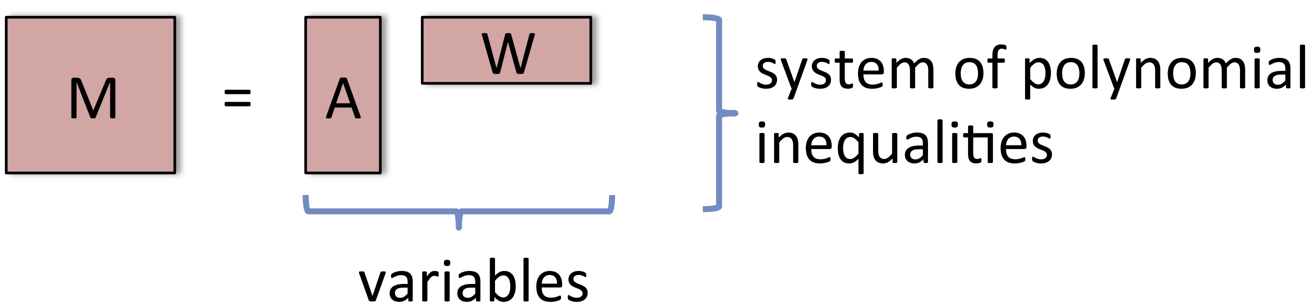

Can we reduce the number of variables from nr+mr to $O(r^2)$?

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Focus of this talk: a natural condition so that a **simple** algorithm **provably** works, **quickly**

SEPARABILITY AND ANCHOR WORDS

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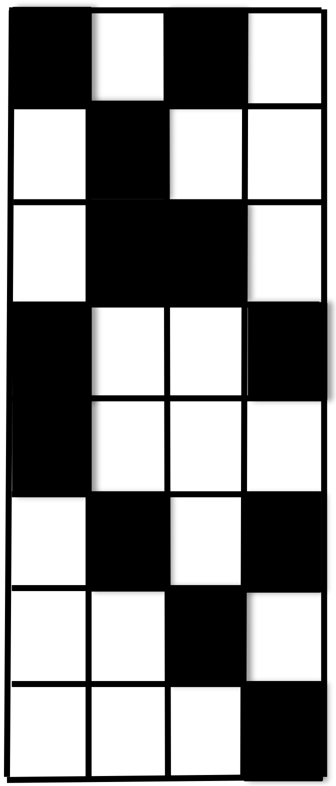
topics (r)

words (m)



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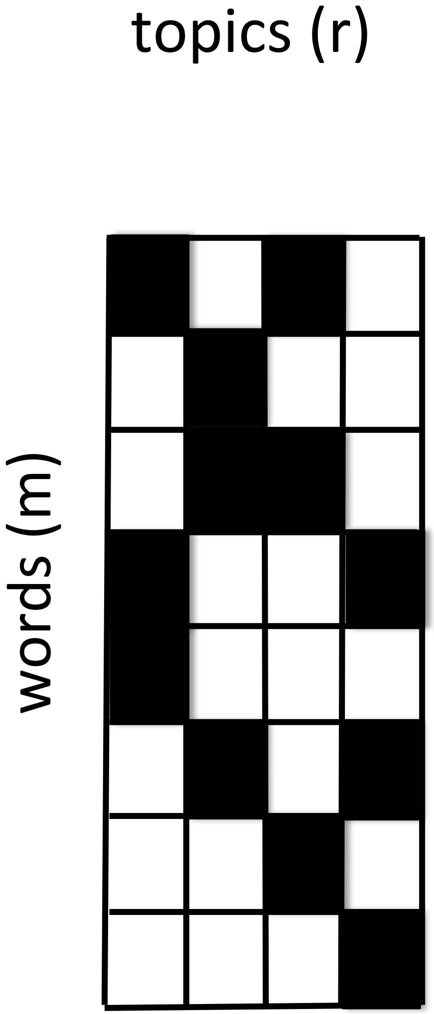

If an **anchor word** occurs then the document is at least partially about the topic

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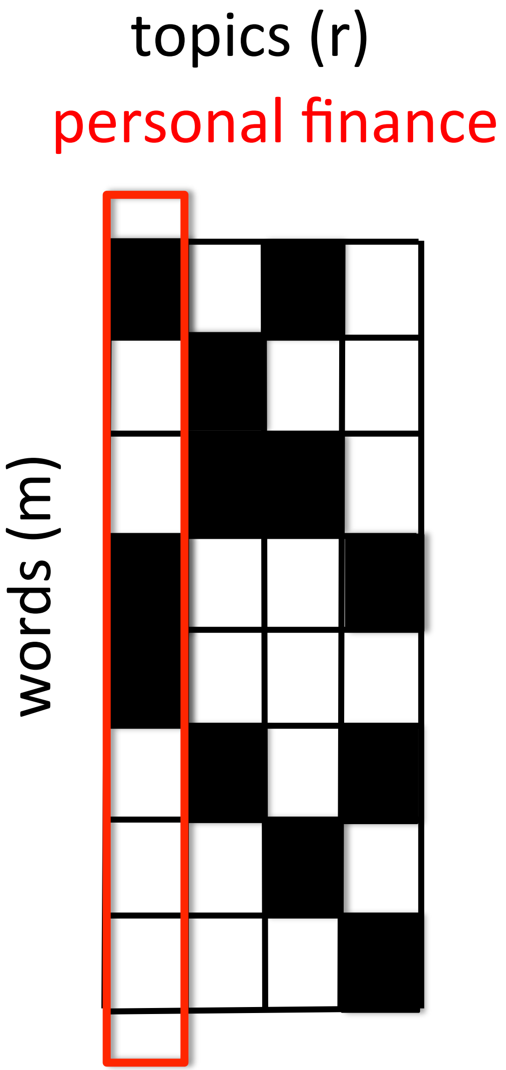

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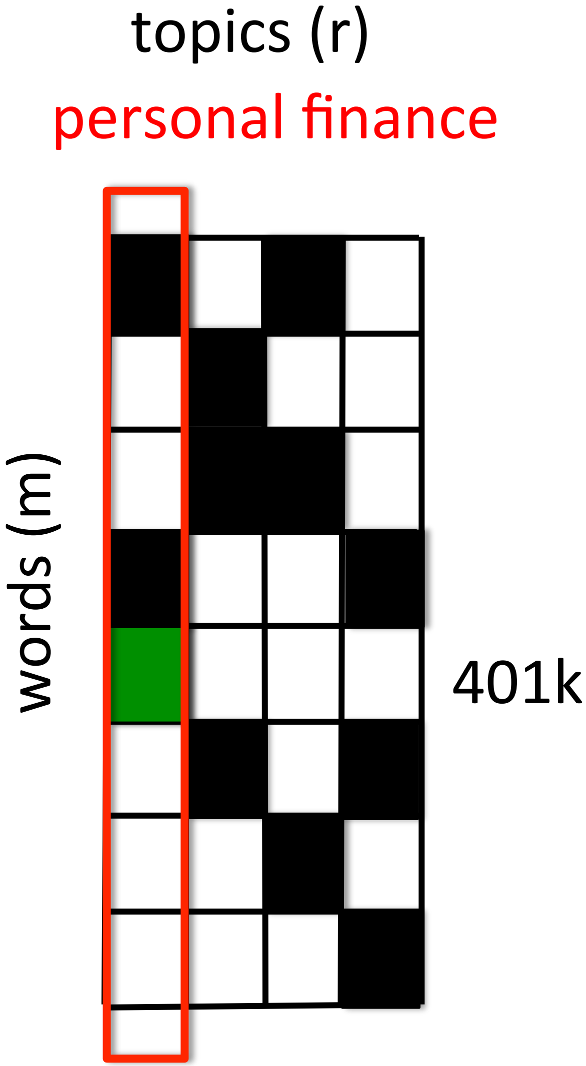

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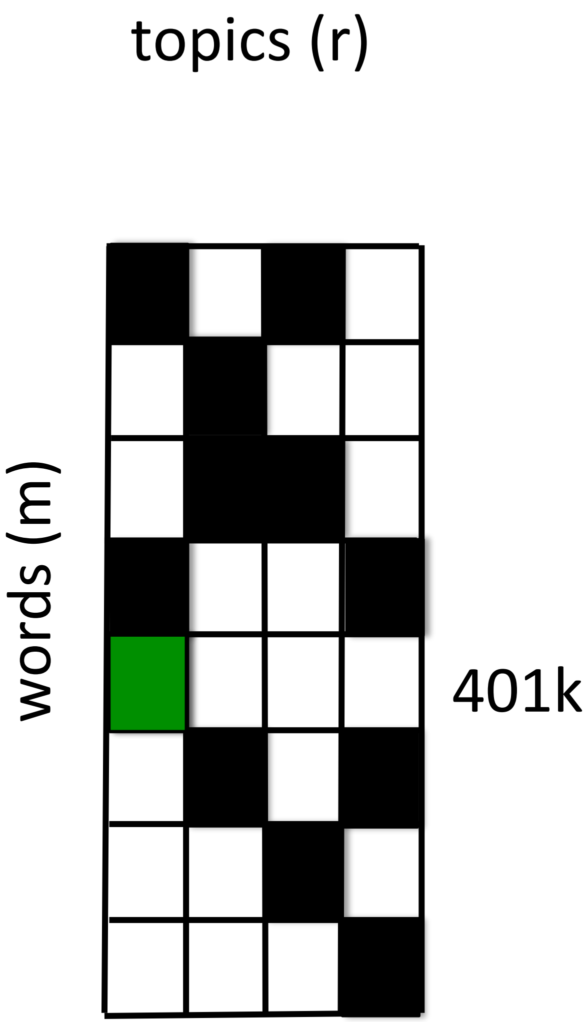

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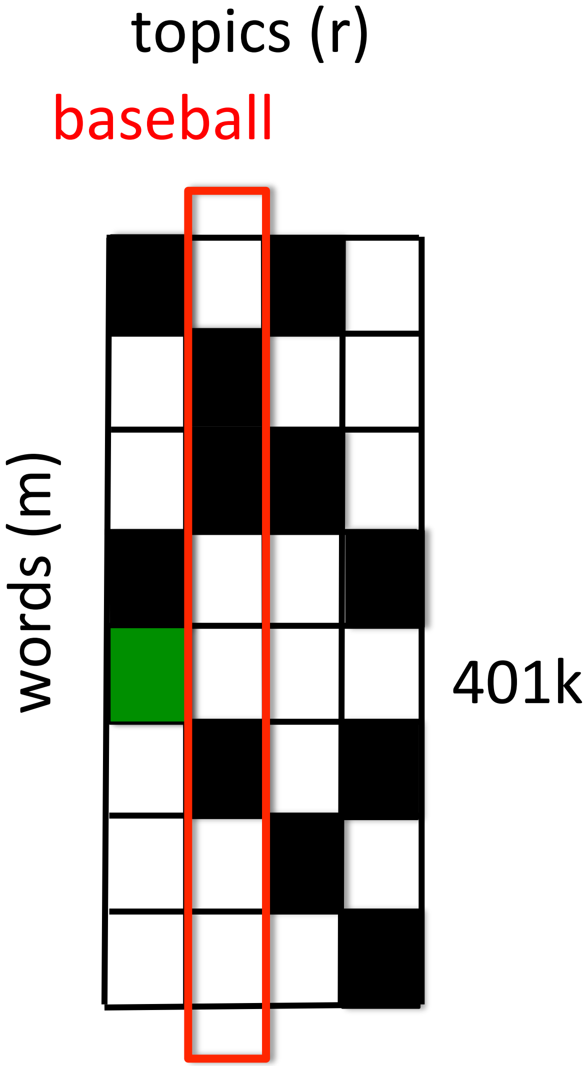

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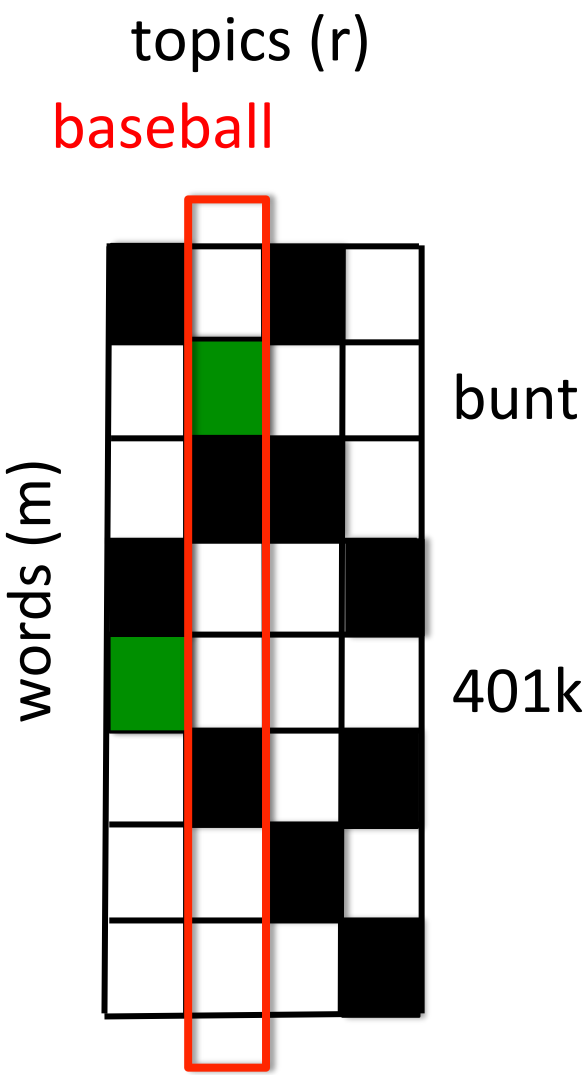

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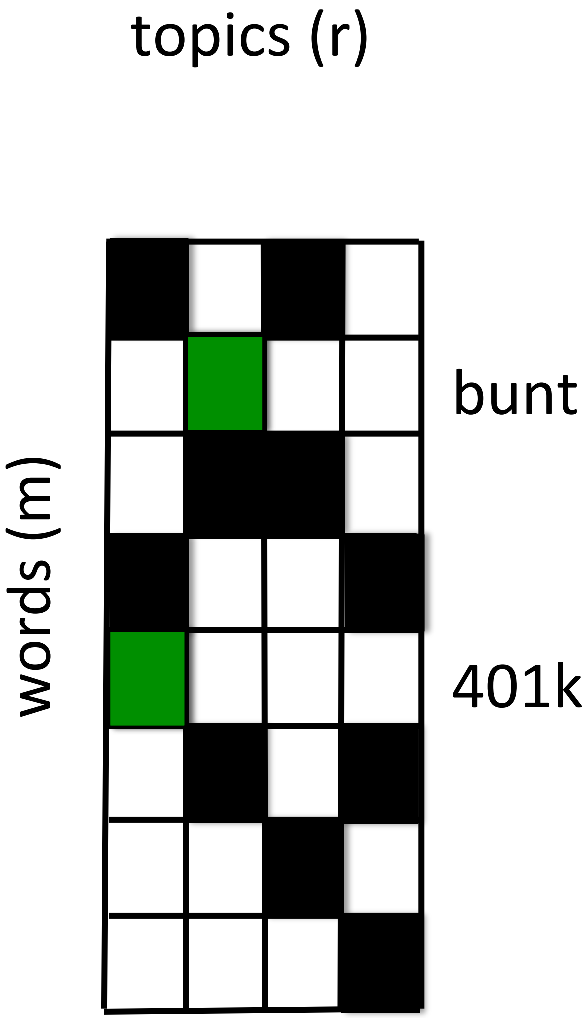

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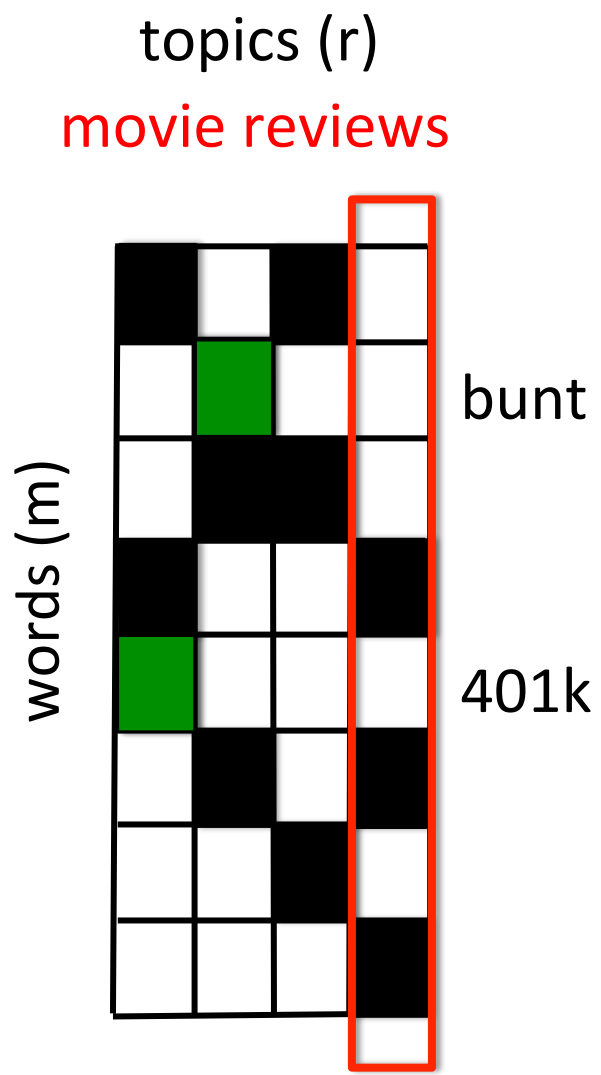

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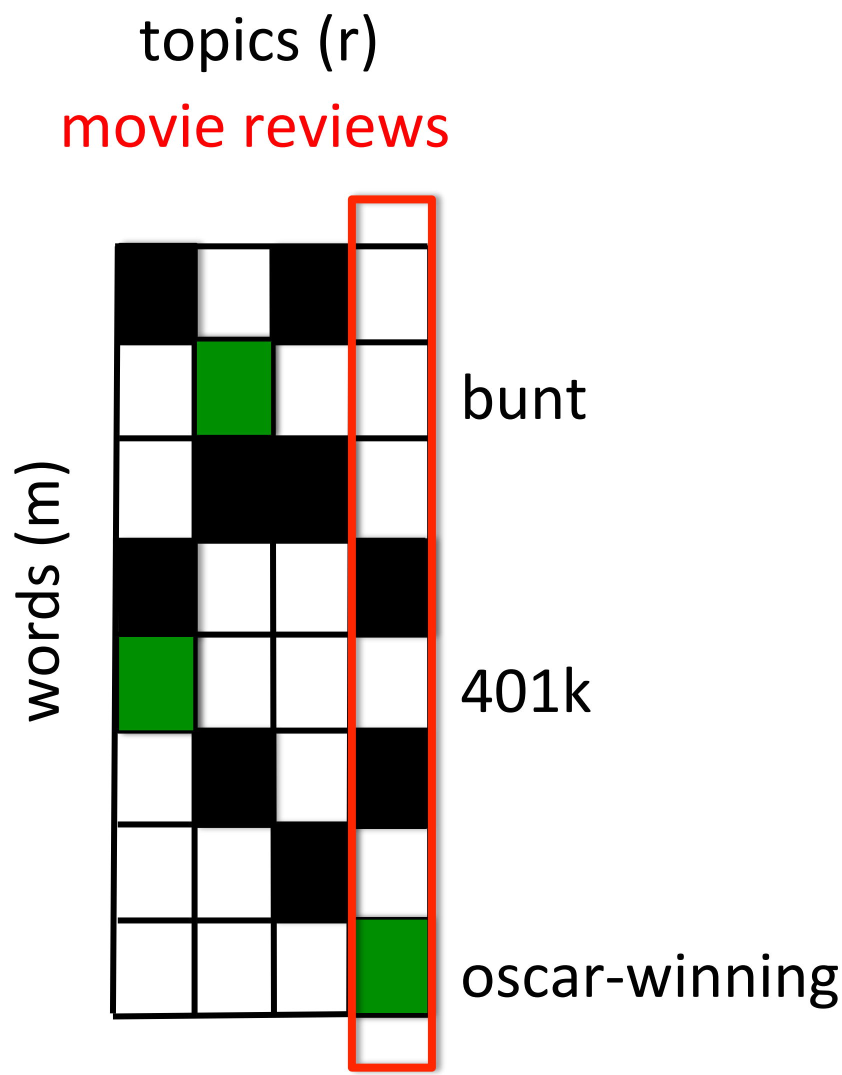

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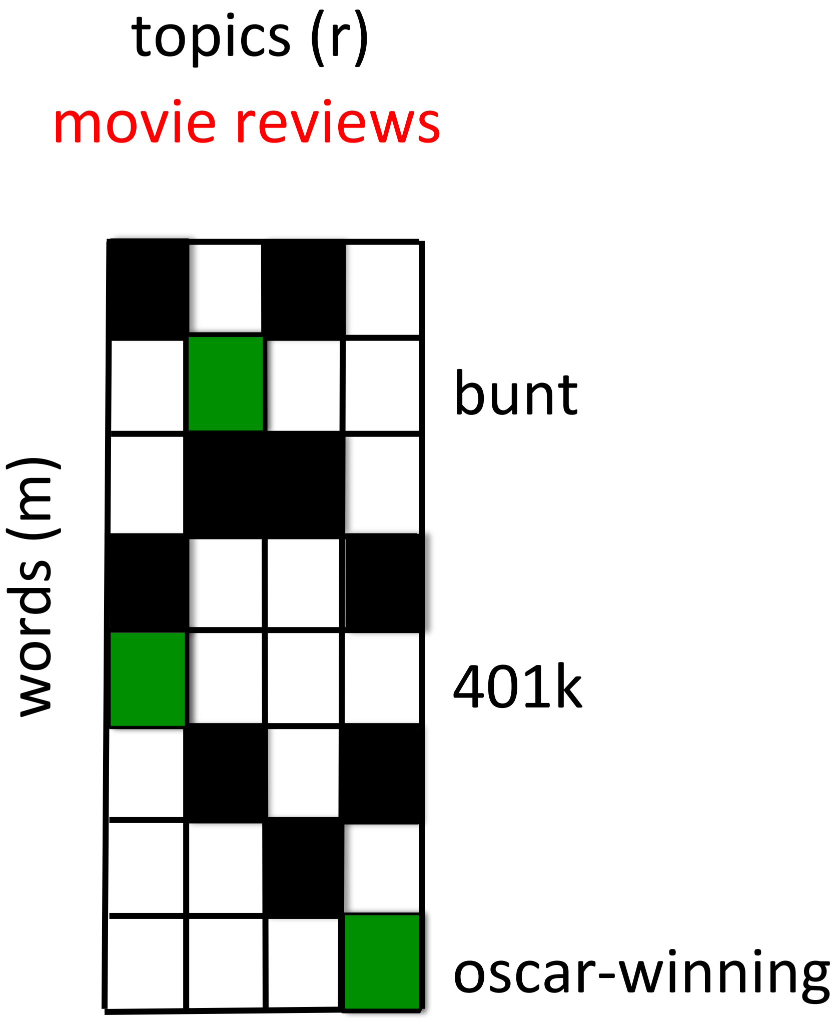

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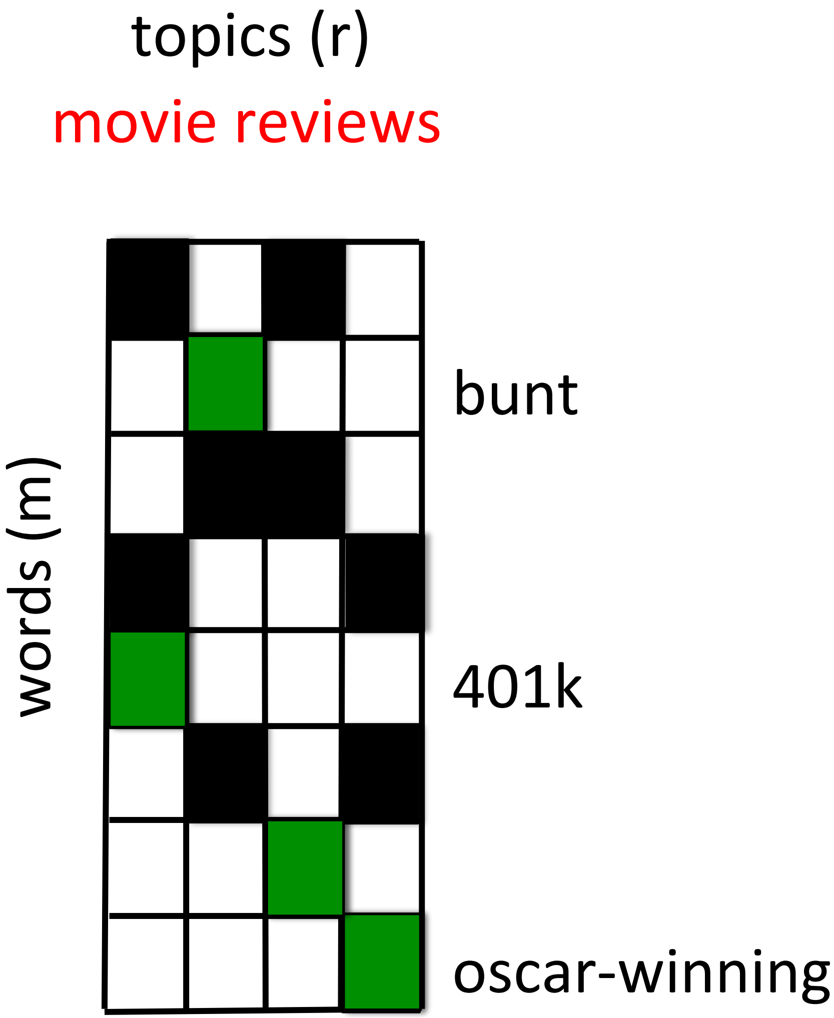

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A is **p-separable** if each topic has an anchor word that occurs with probability ≥ p

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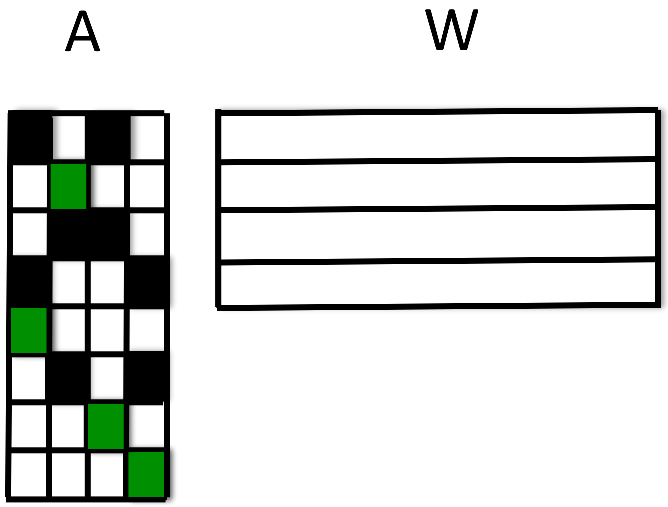
See also **[Anandkumar et al '12], [Rabani et al '12]** that give algorithms based on the method of methods

How do anchor words help?

ANCHOR WORDS $\cong$ VERTICES

ANCHOR WORDS ≅ VERTICES



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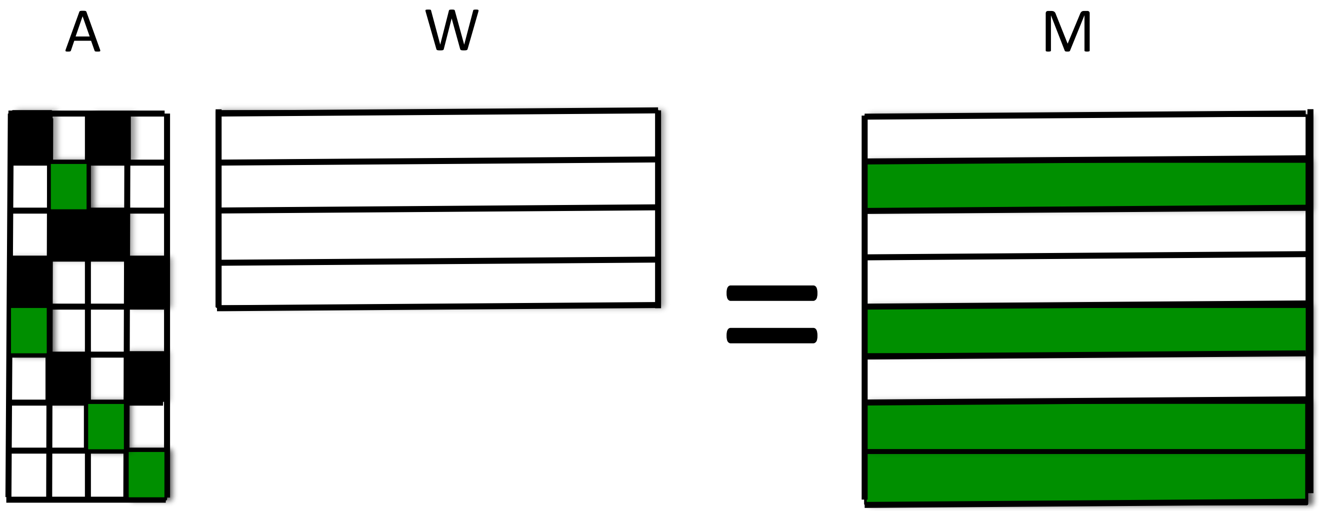

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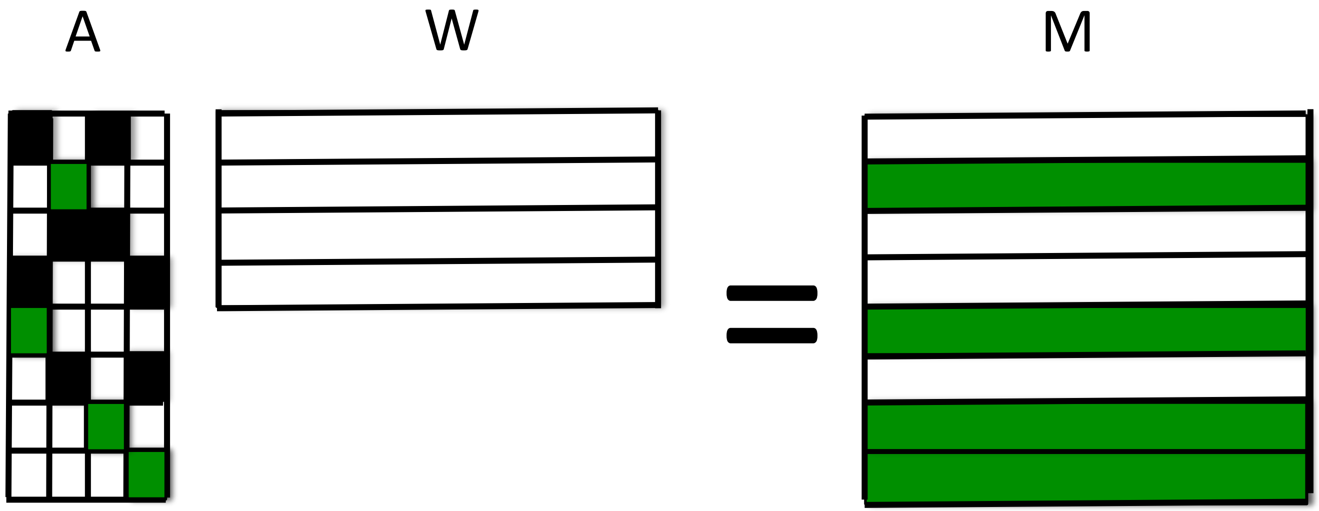

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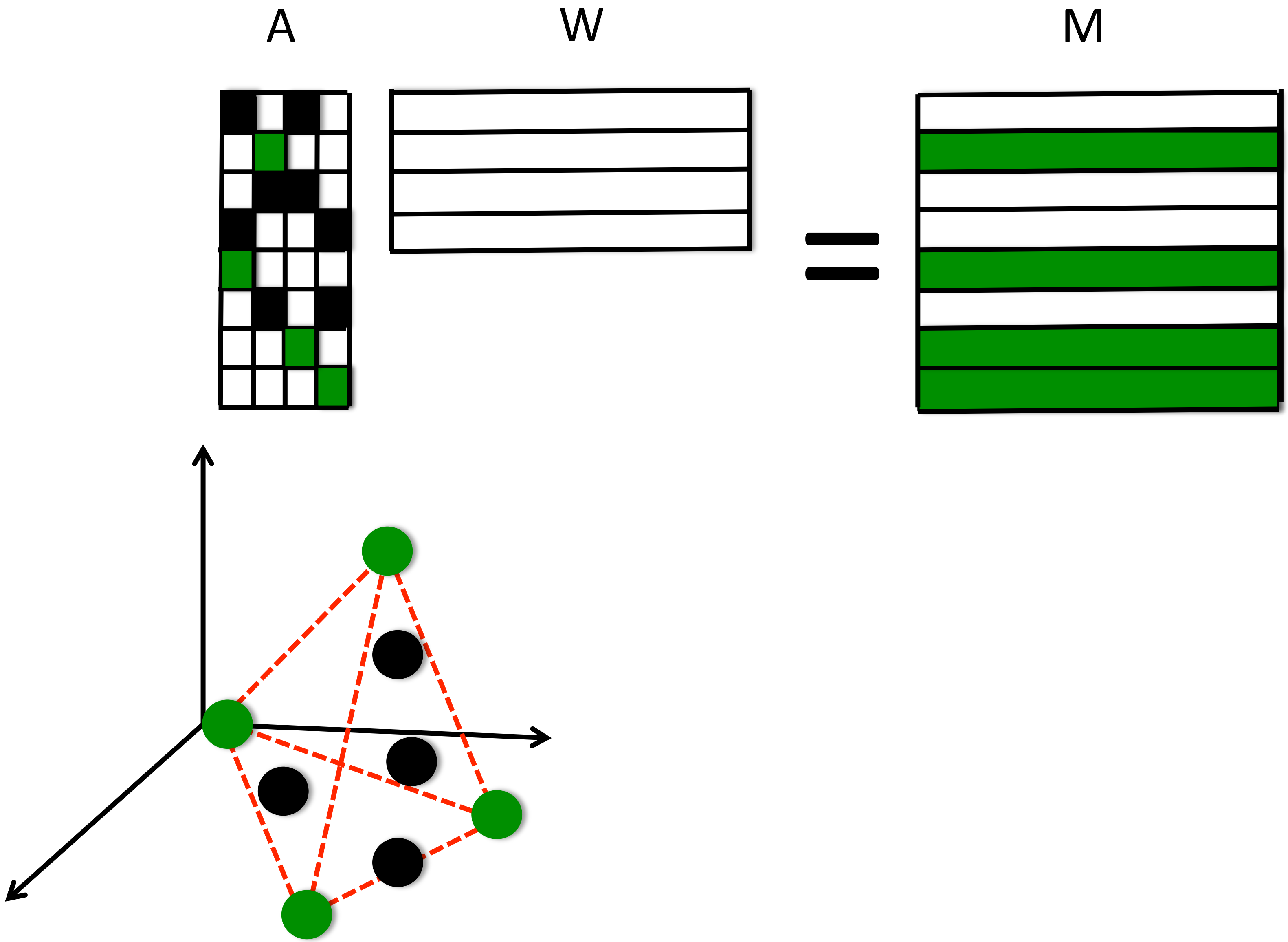

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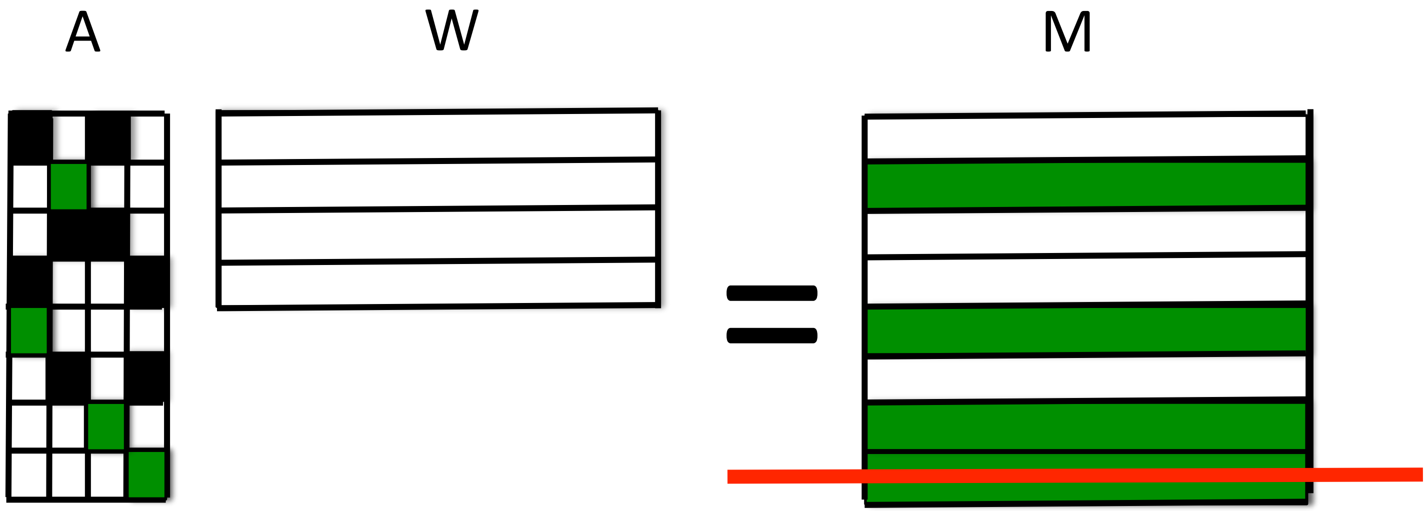

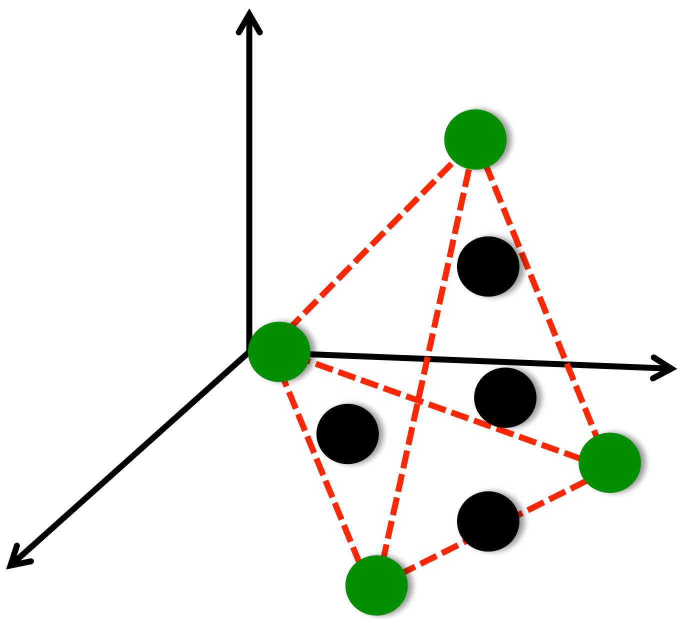

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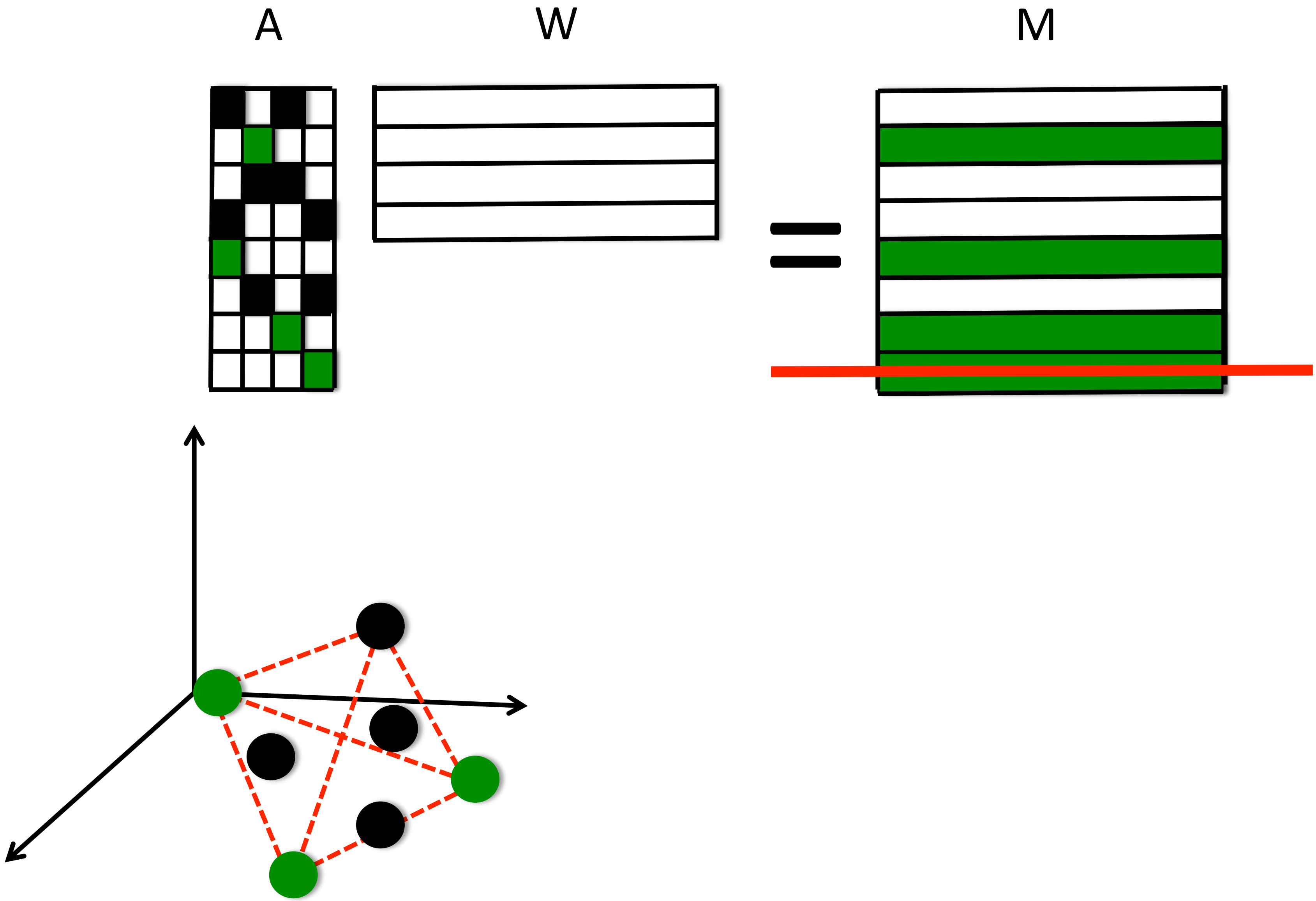

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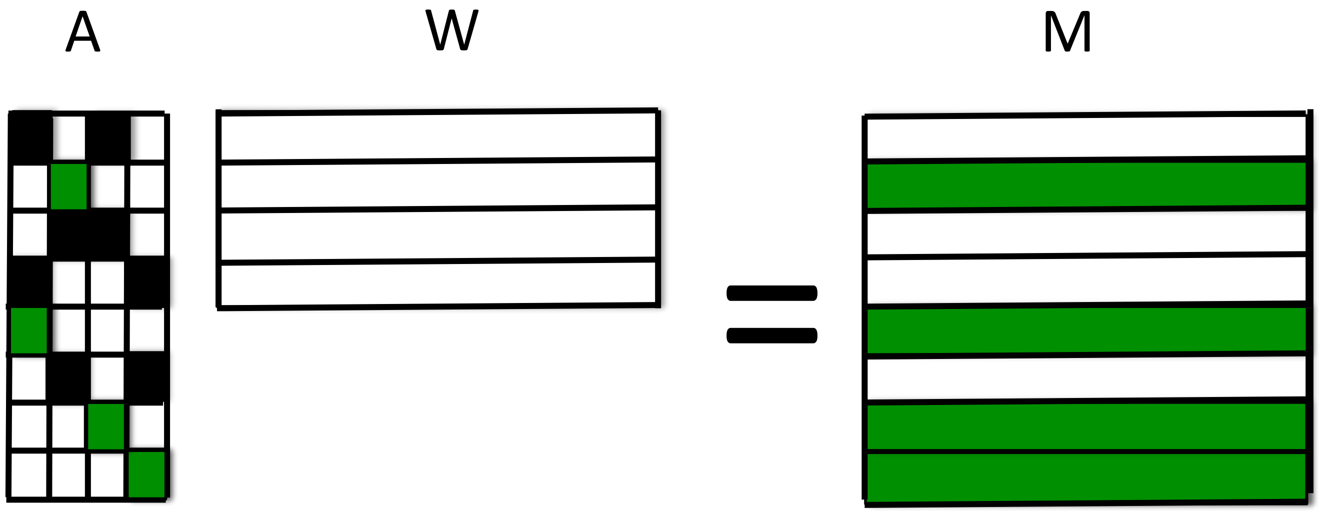

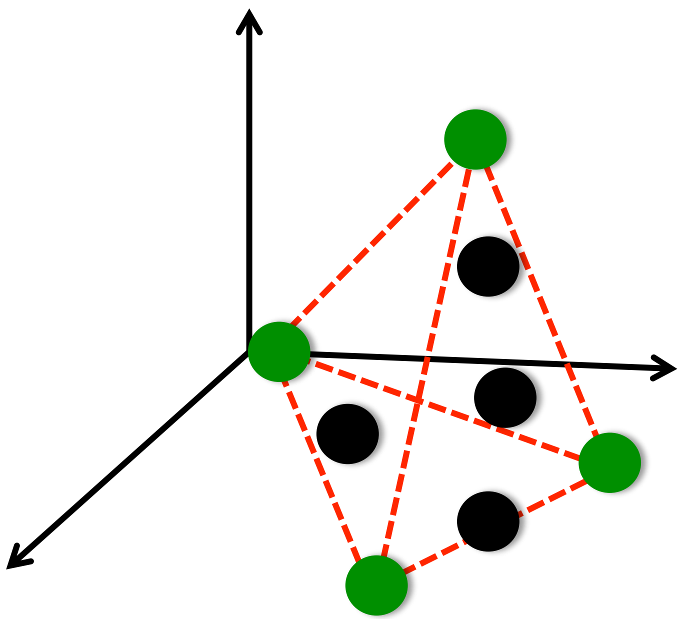

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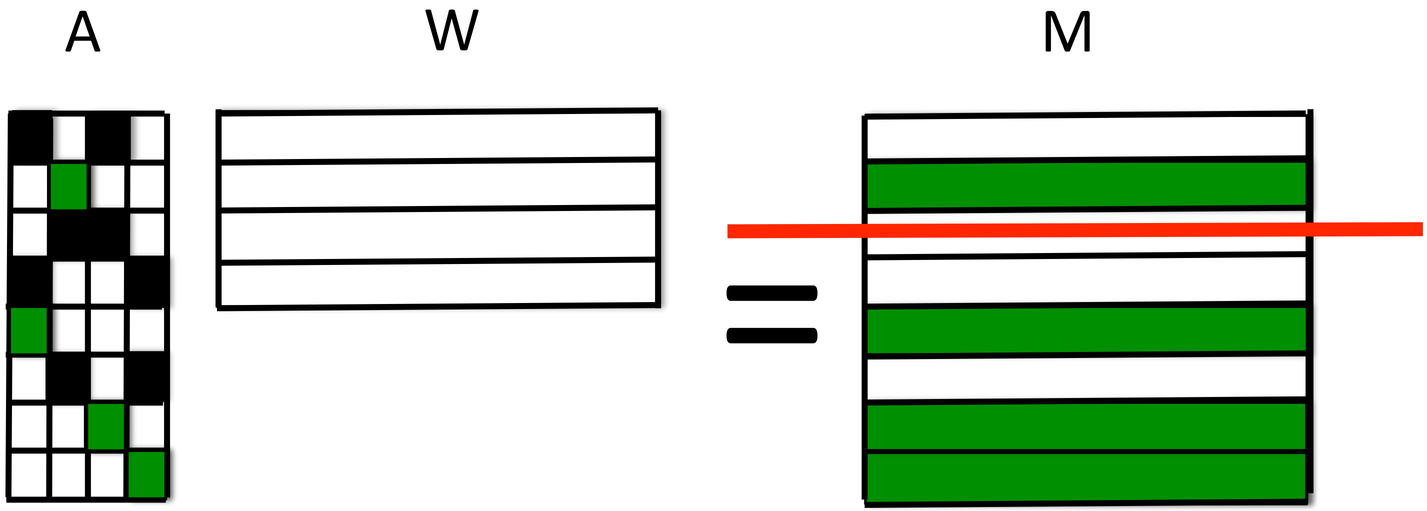

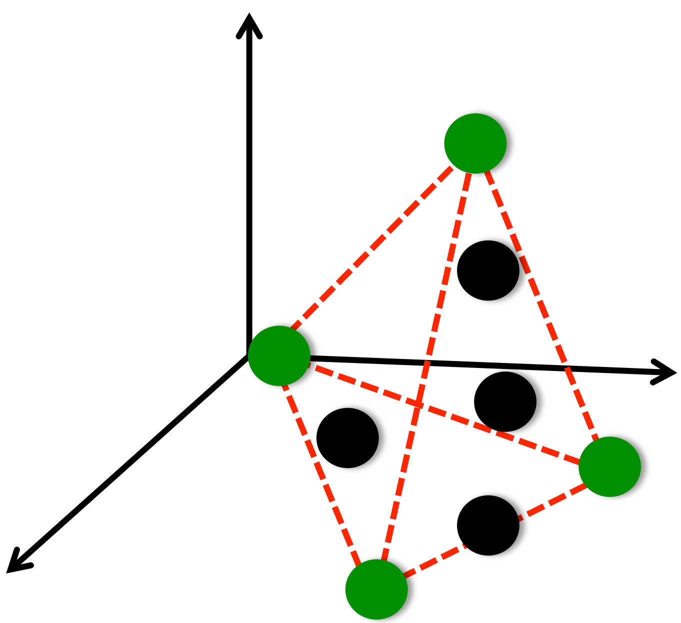

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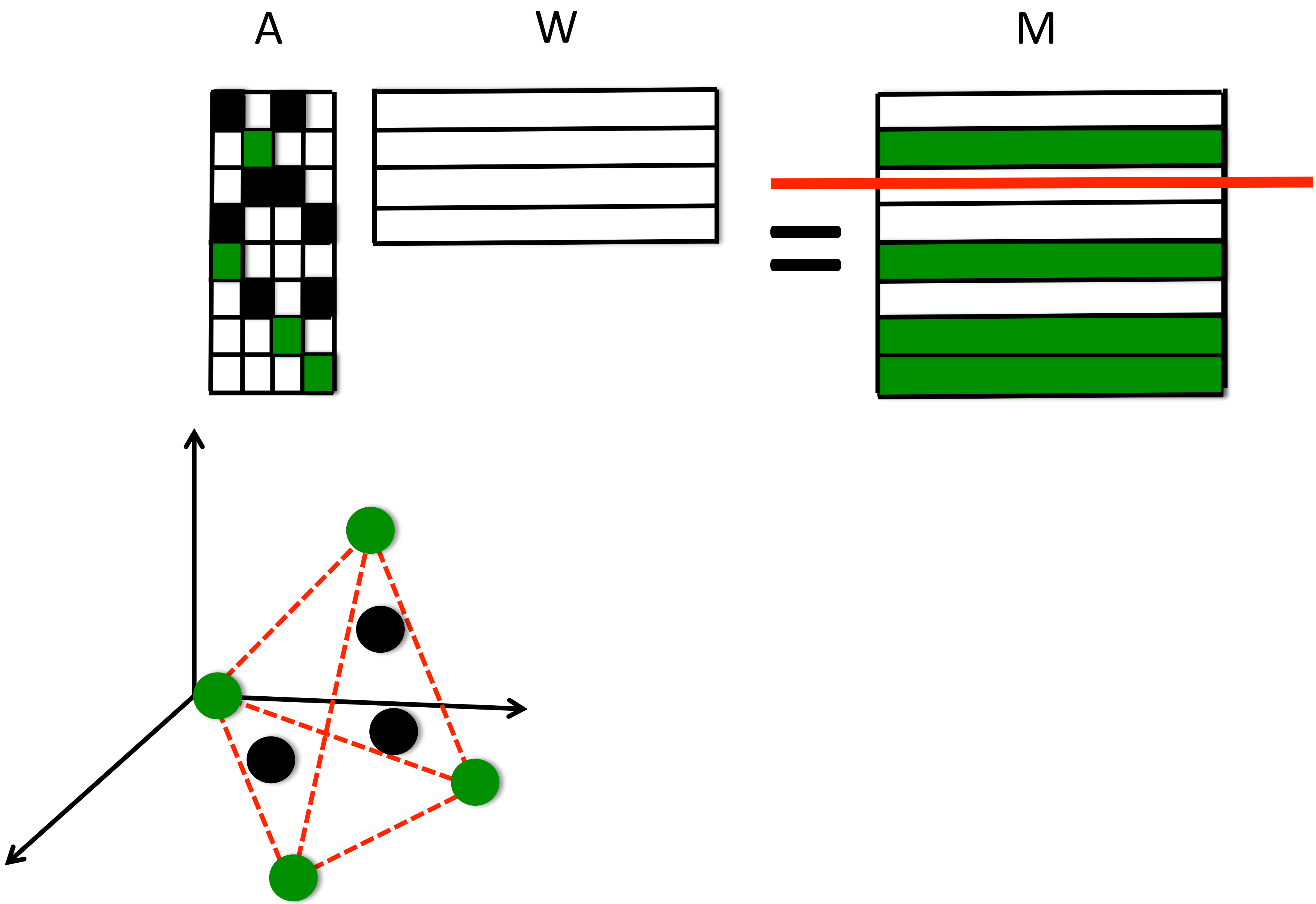

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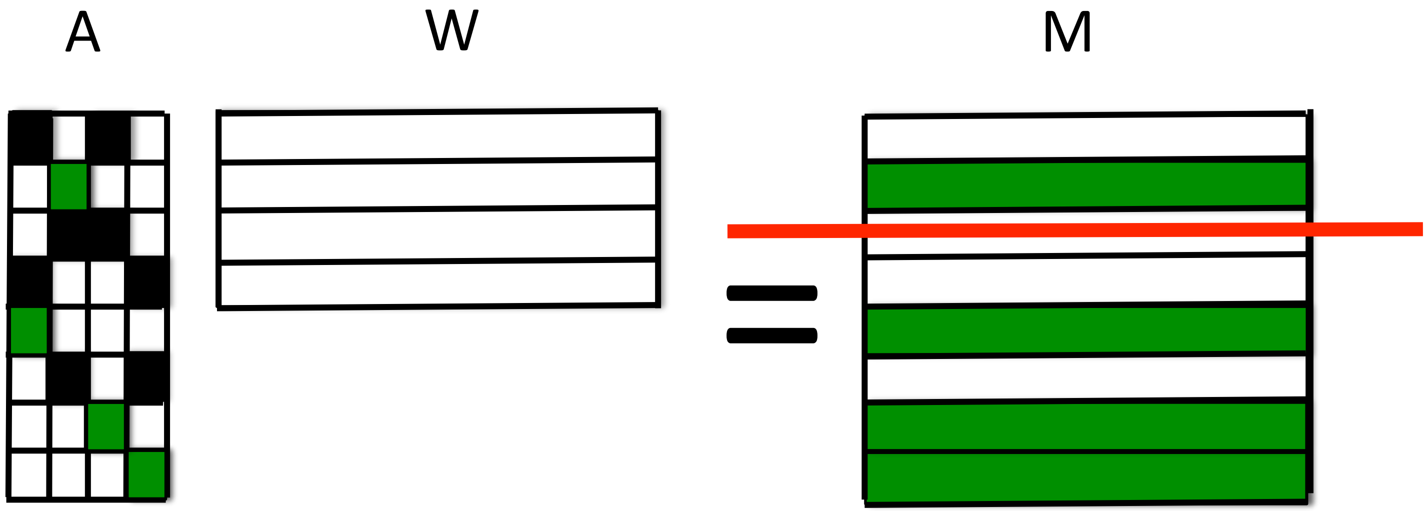

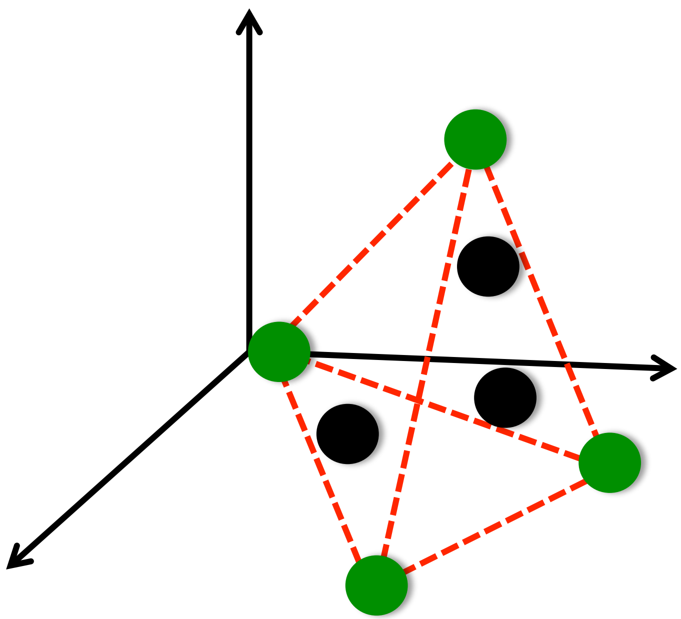

Deleting a word changes the convex hull

⇕

it is an anchor word

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What if we are given a **very sparse** approximation to **M**?

OUTLINE

Are there efficient algorithms to find the topics?

Challenge: We cannot **rigorously** analyze algorithms used in practice! (When do they work? run quickly?)

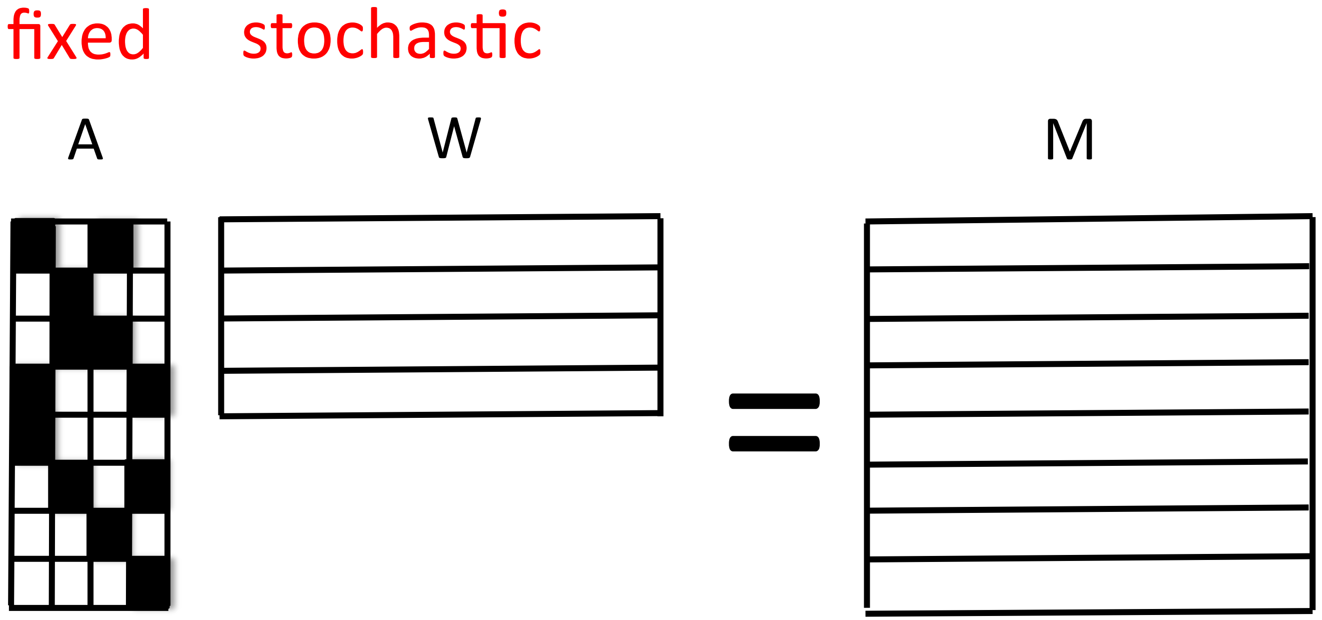
Part I: An Optimization Perspective

- Nonnegative Matrix Factorization
- Separability and Anchor Words
- Algorithms for Separable Instances

Part II: A Bayesian Perspective

- Topic Models (e.g. LDA, CTM, PAM, …)
- Algorithms for Inferring the Topics
- Experimental Results

TOPIC MODELS



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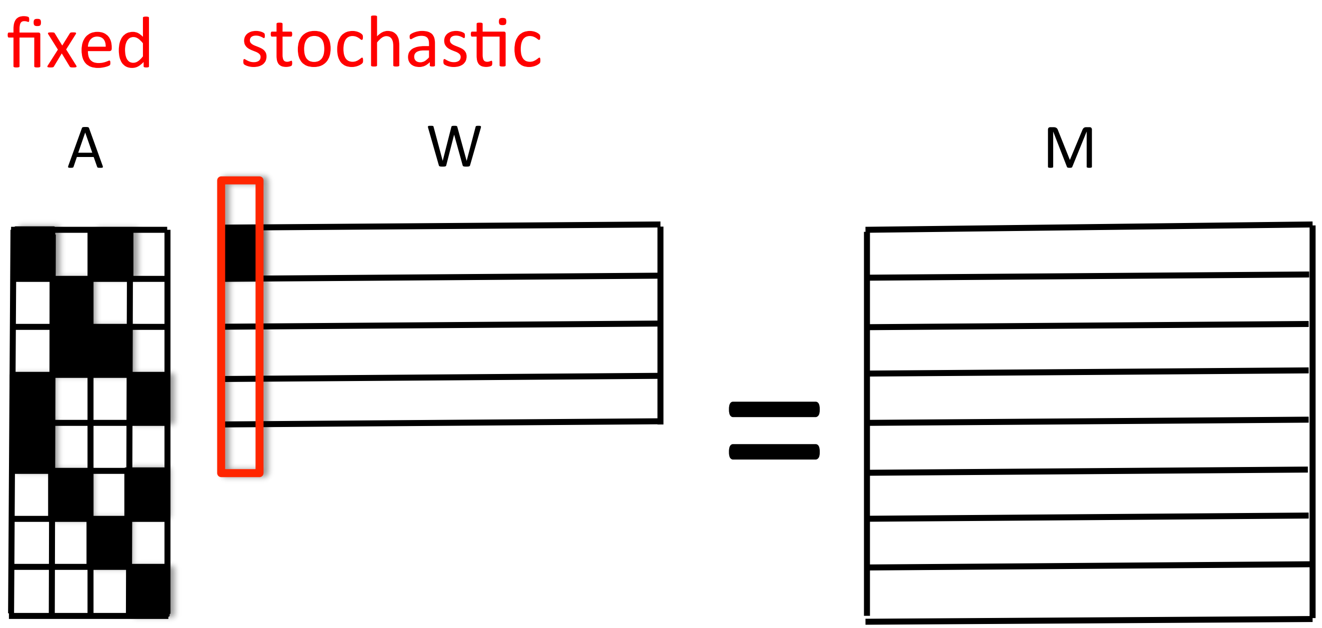

document #1: (1.0, personal finance)

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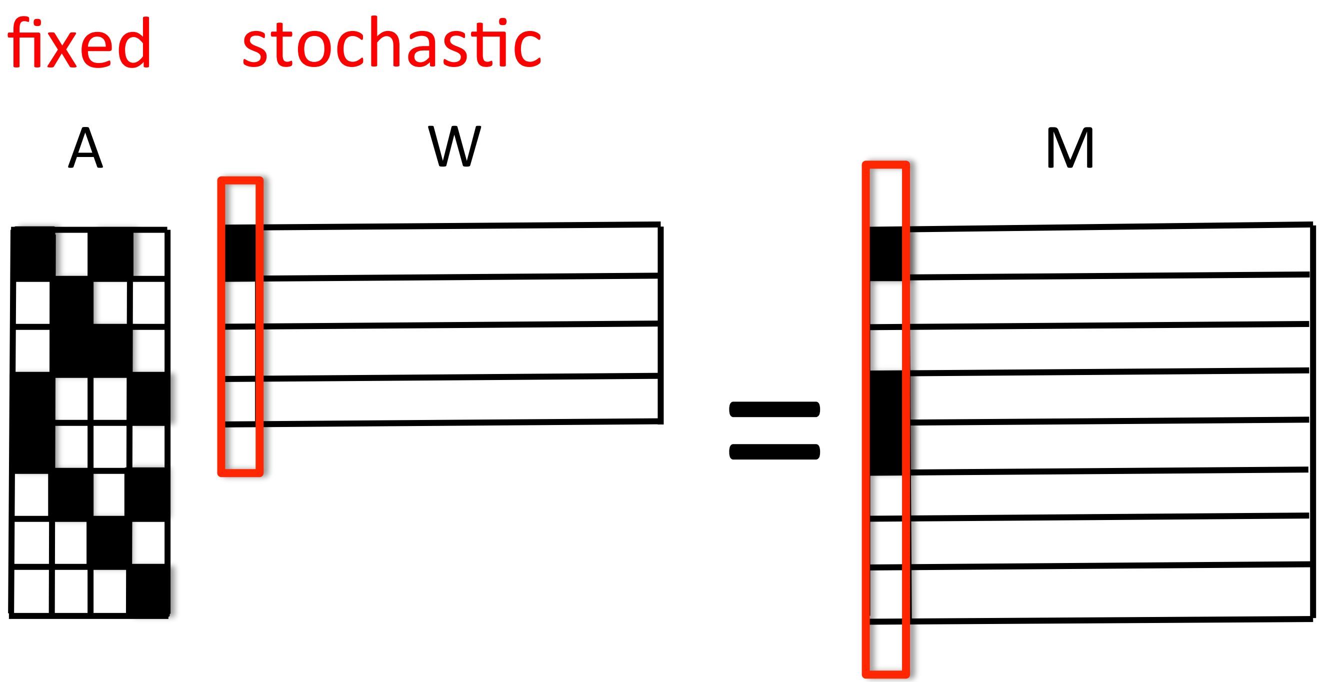

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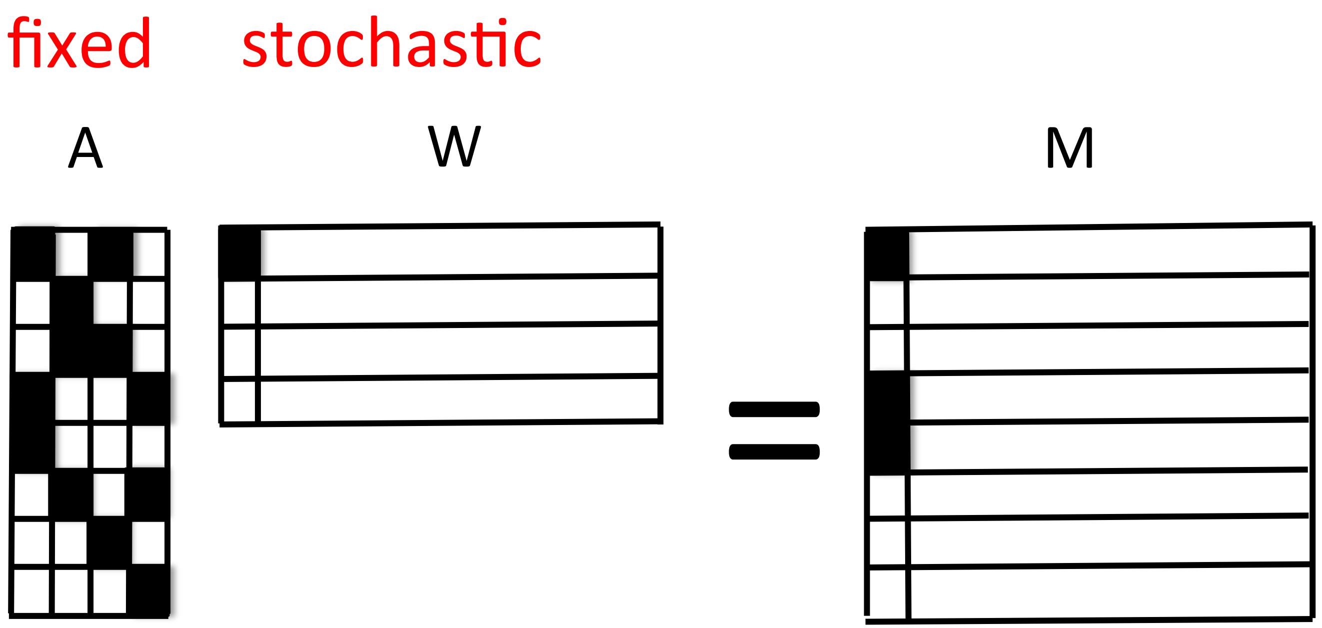

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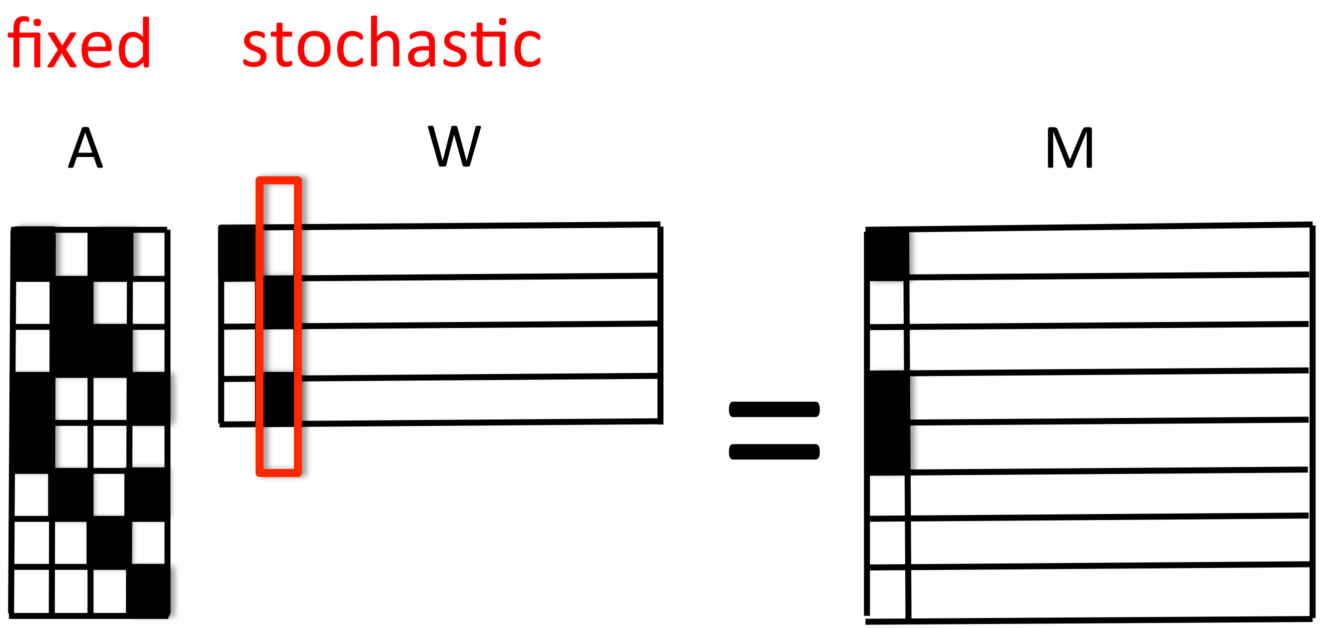

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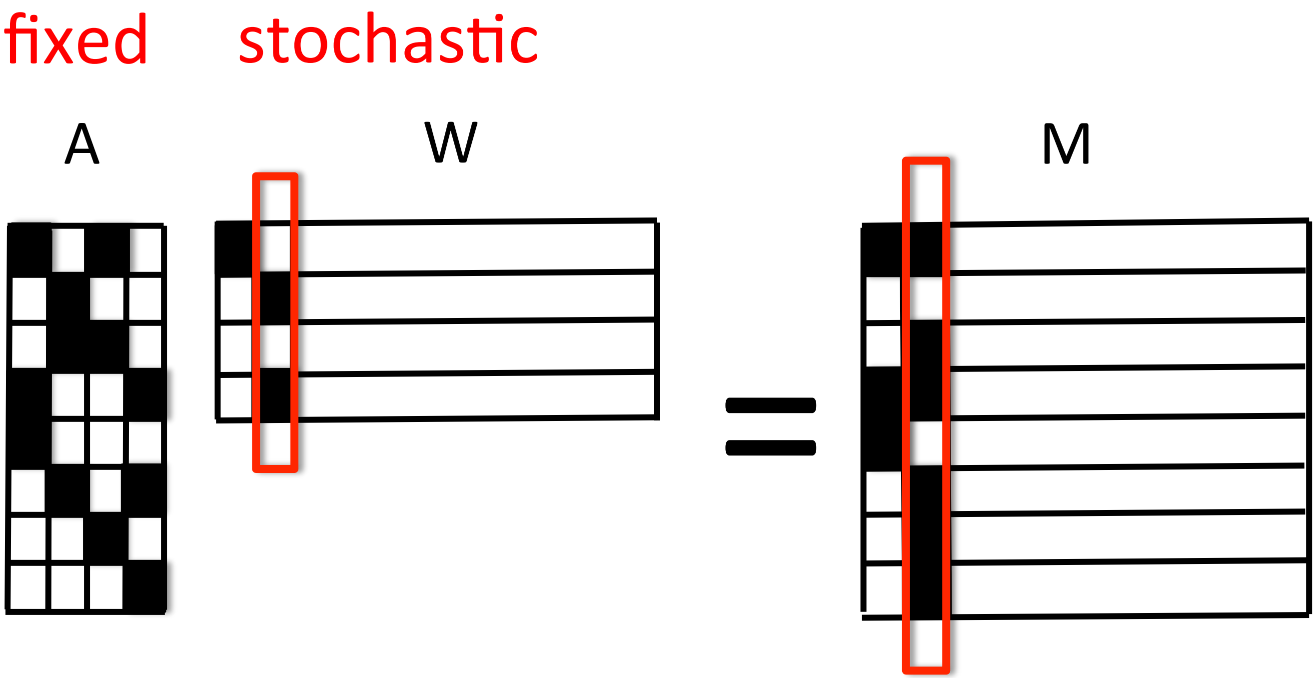

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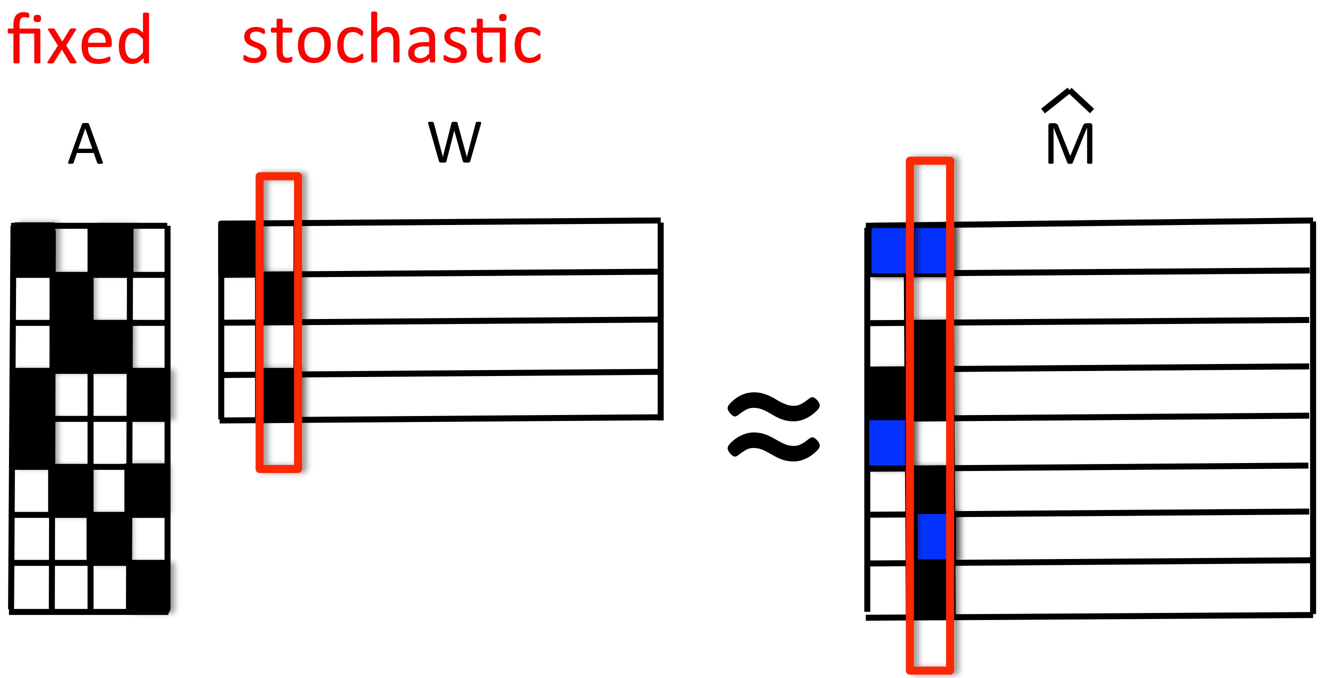

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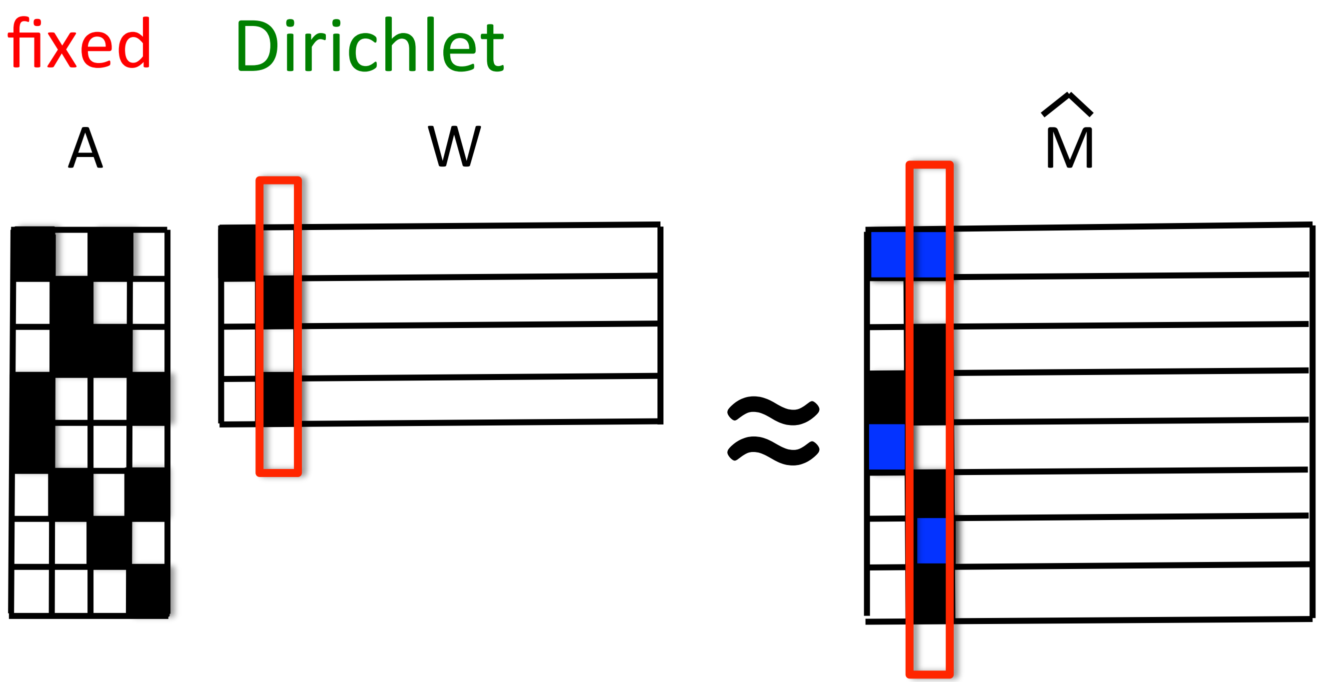
TOPIC MODELS

Latent Dirichlet Allocation (Blei, Ng, Jordan)



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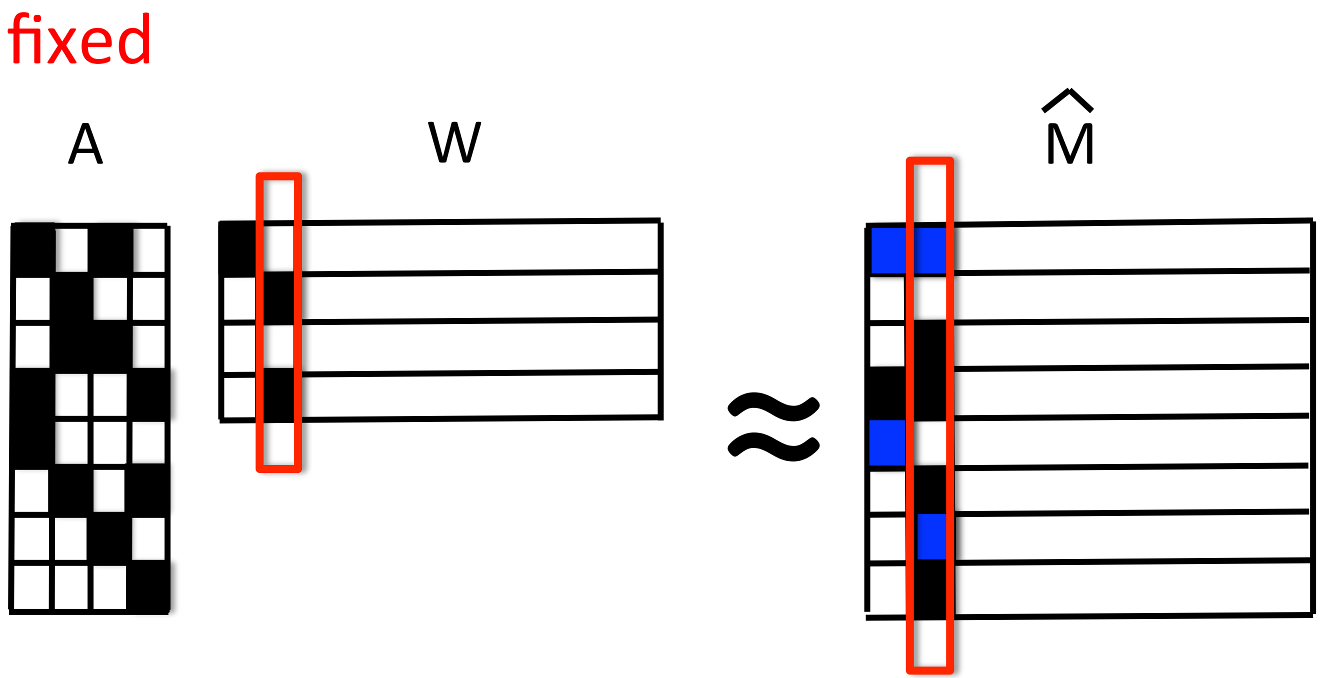

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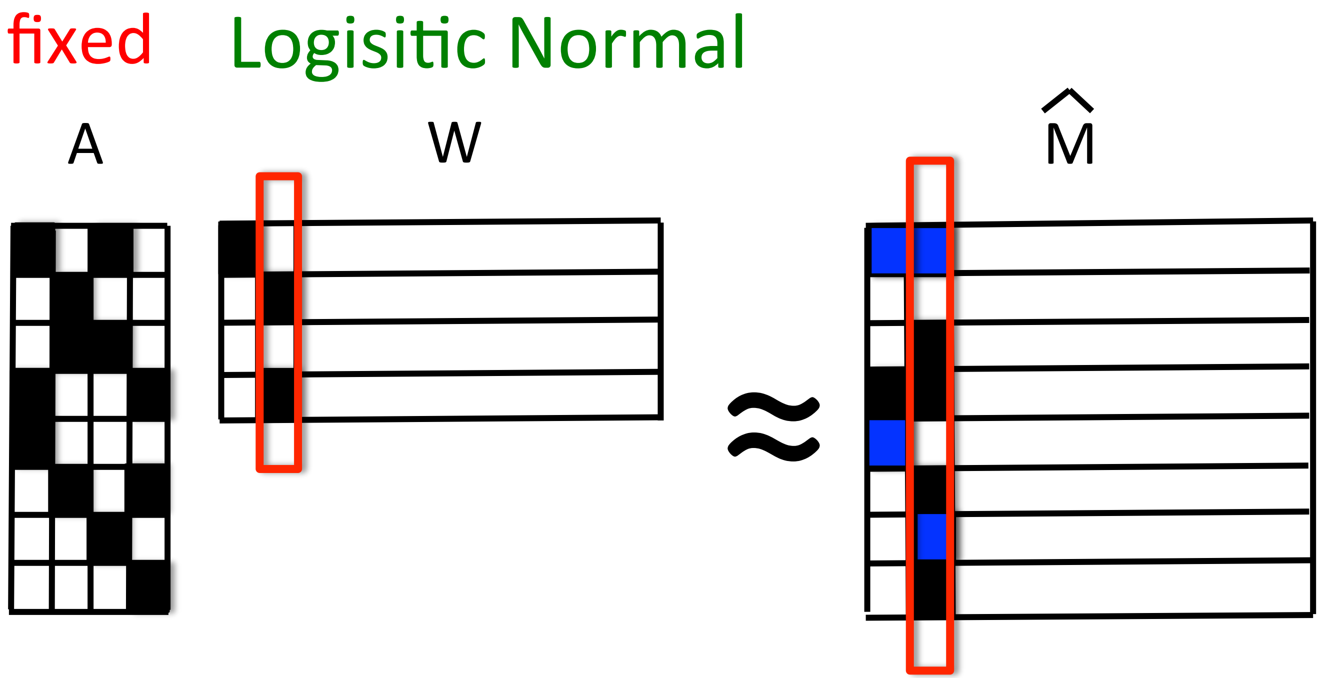
TOPIC MODELS

Correlated Topic Model (Blei, Lafferty)



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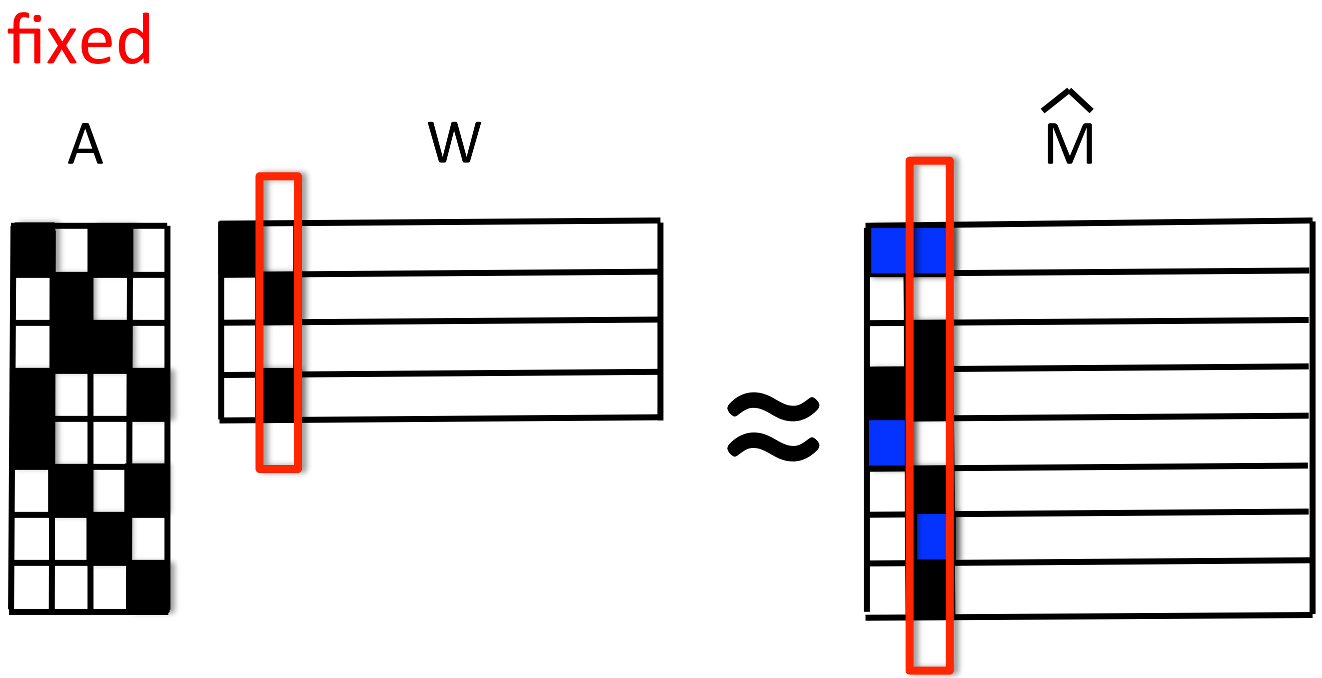

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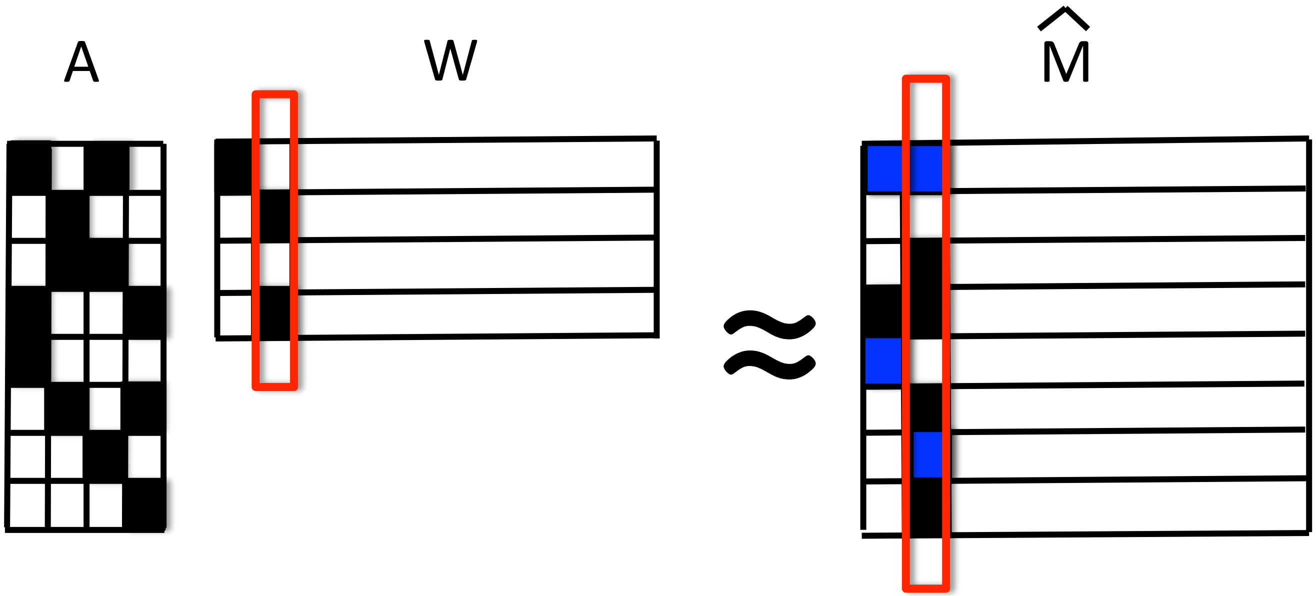
Pachinko Allocation Model (Li, McCallum)

fixed Multilevel DAG



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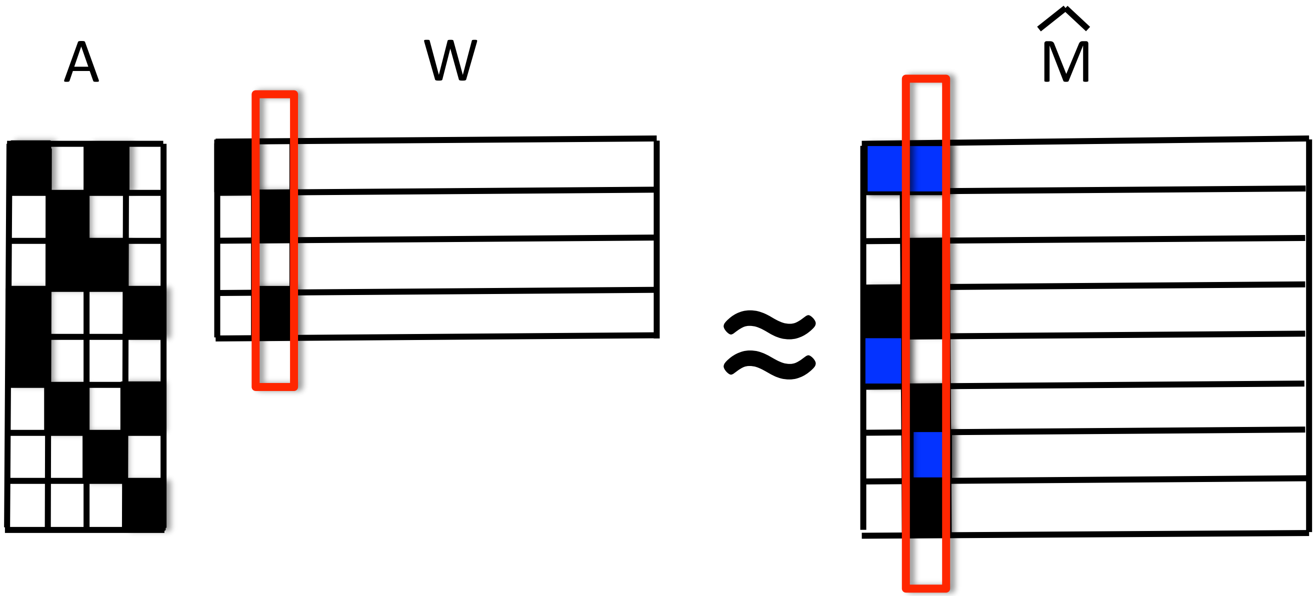

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These models differ only in how **W** is generated

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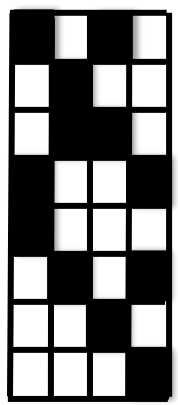

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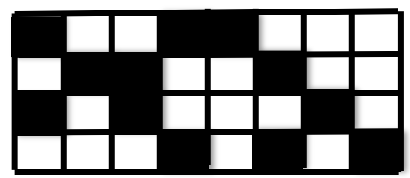
A^T



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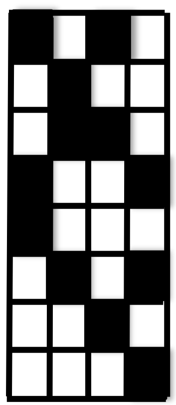
$$\hat{M}\,\hat{M}^T \rightarrow E[M\,M^T]$$

A



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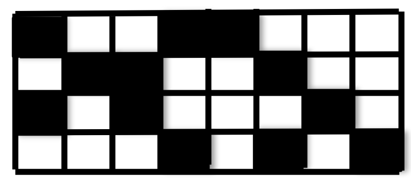

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$$\hat{M}\hat{M}^T \rightarrow E[MM^T] = A\,E[WW^T]\,A^T$$

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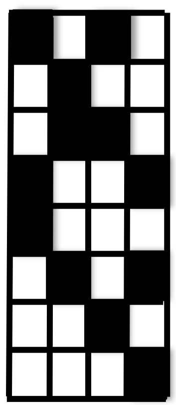

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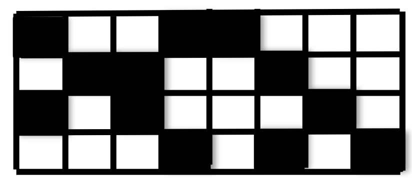

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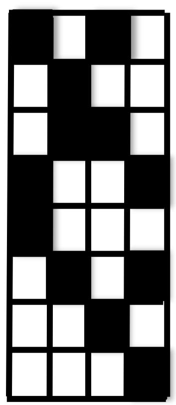
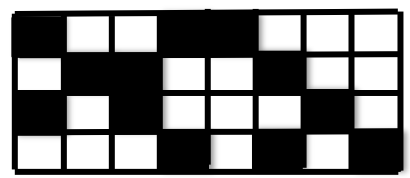


$$\overbrace{W\ \ W^T}^{R}$$

A^T



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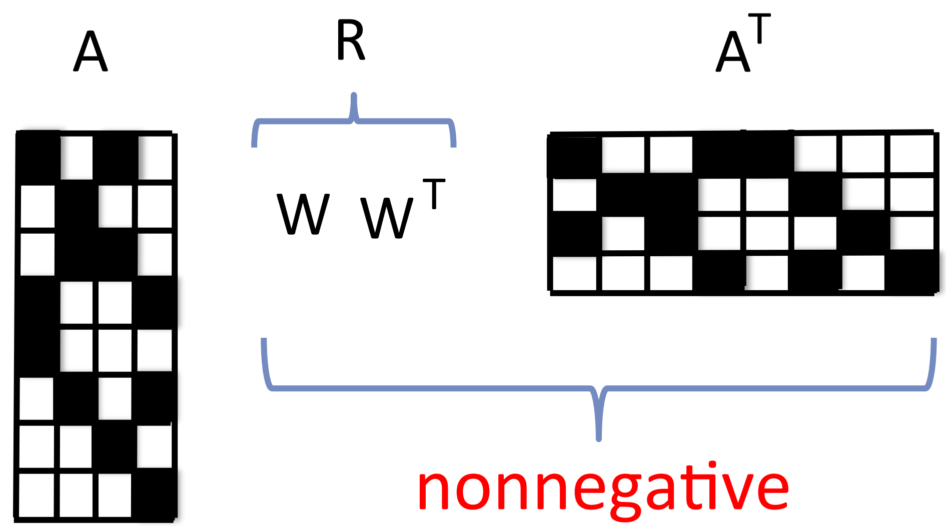

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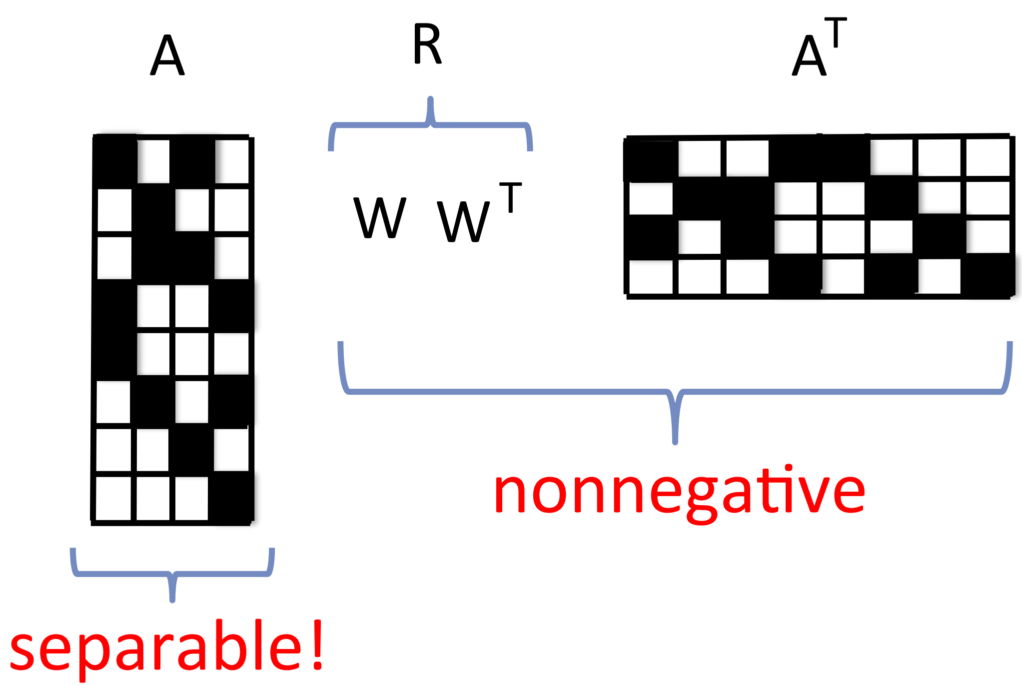

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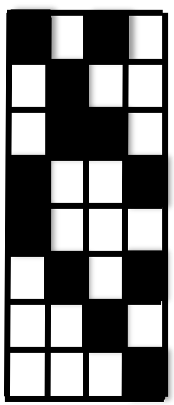
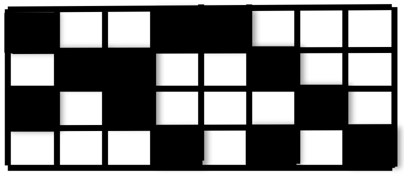


separable!

$$\overbrace{W\;W^T}^{R}$$

A^T



nonnegative

Anchor words are extreme rows of the Gram matrix!

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We can use the anchor words to find Pr[topic|word] for all the other words…

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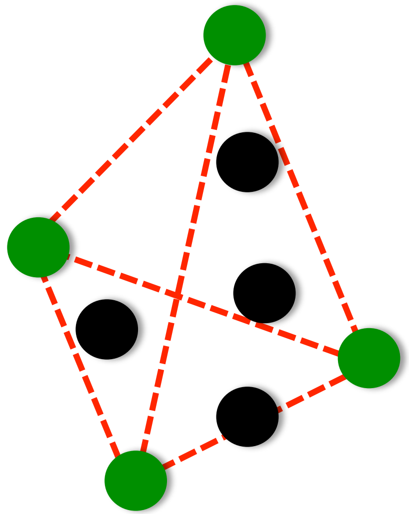
points are now (normalized) rows of $\widehat{M}\widehat{M}^T$



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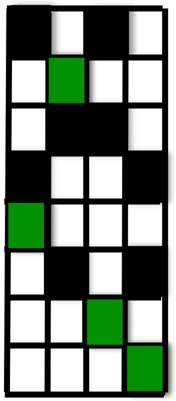

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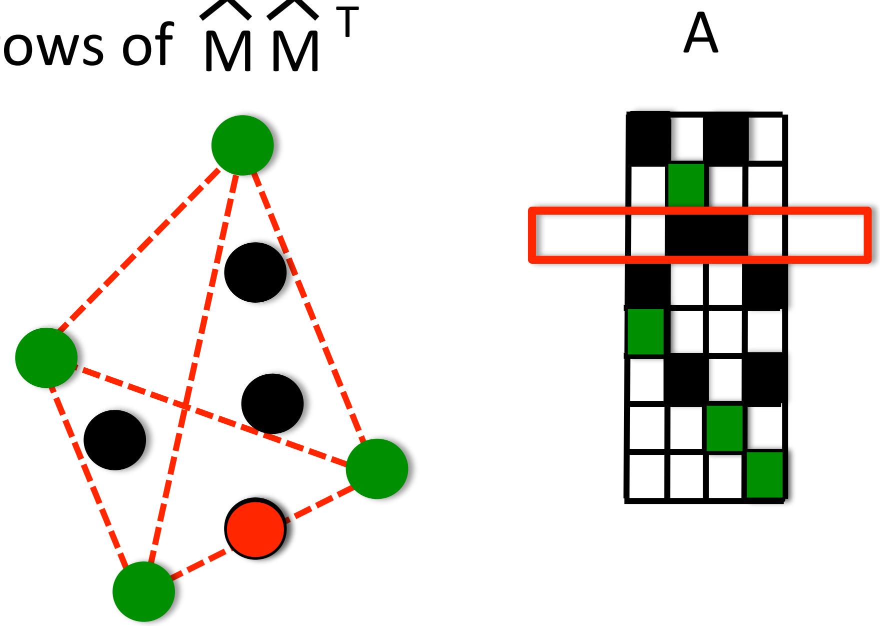

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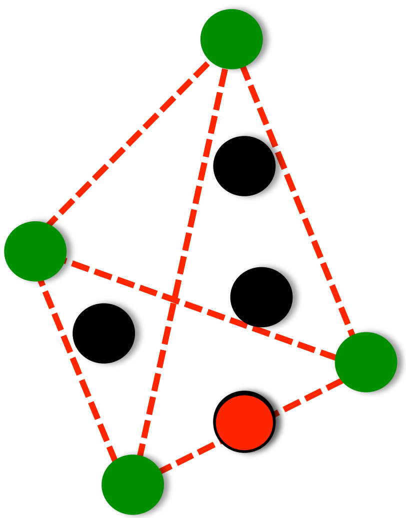

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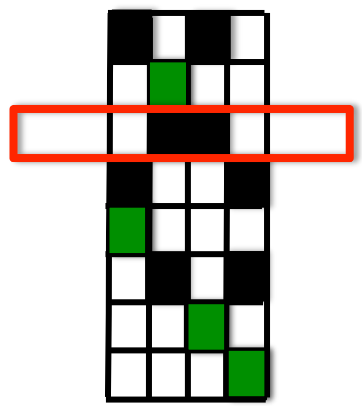

word #3: (0.5, anchor #2); (0.5, anchor #3)

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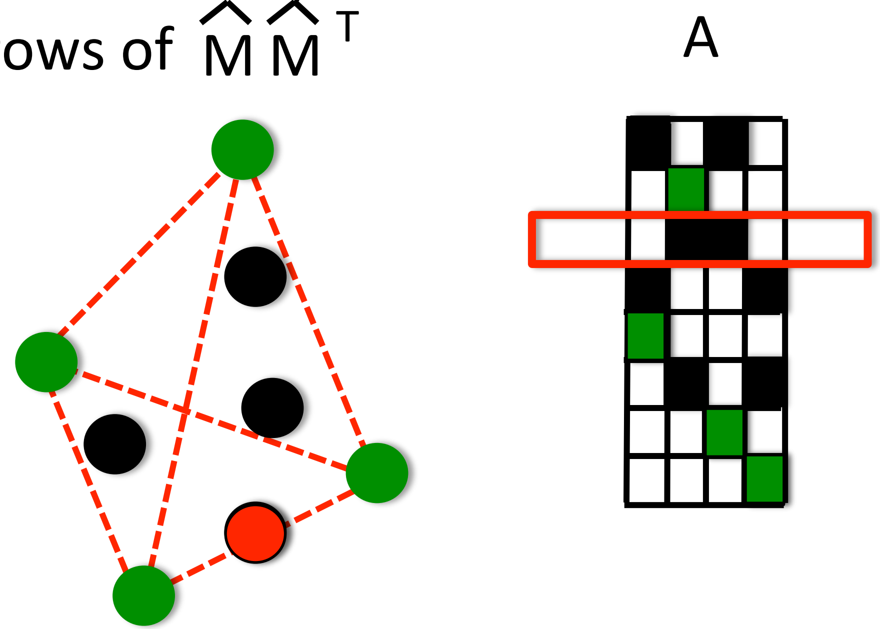

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Pr[topic|word #3]: (0.5, topic #2); (0.5, topic #3)

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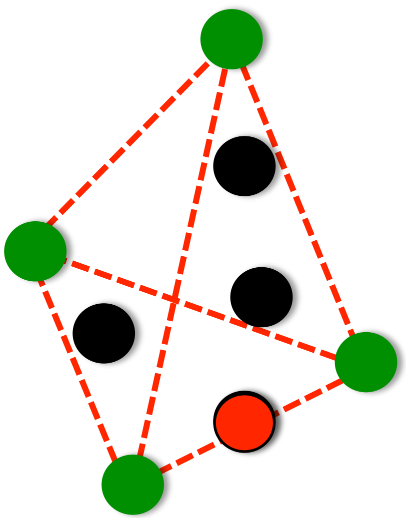

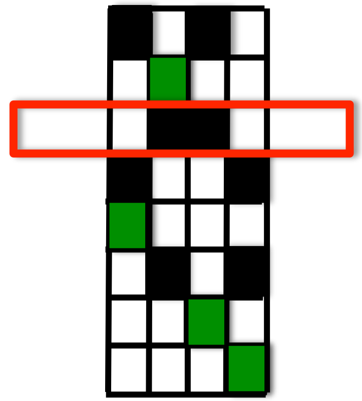

what we have:

Pr[topic|word]

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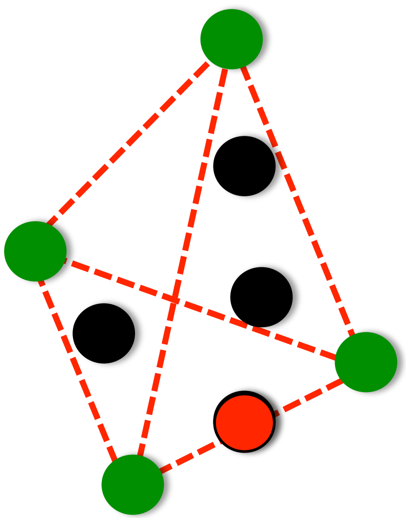
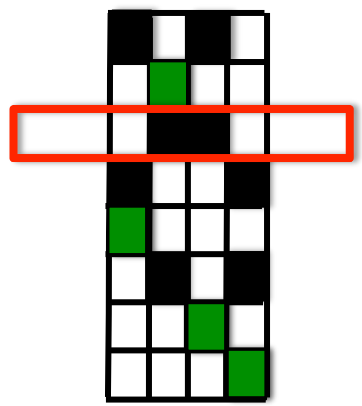


A



what we have:

Pr[topic|word]

what we want:

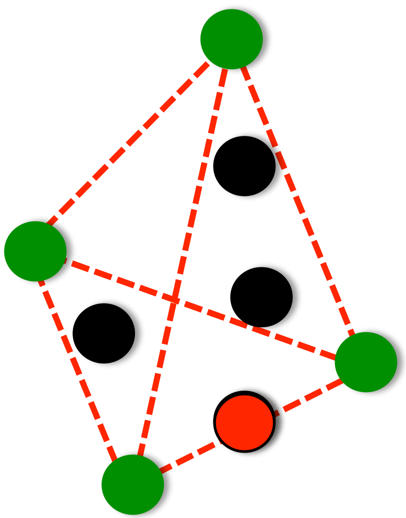
Pr[word|topic]

word #3: (0.5, anchor #2); (0.5, anchor #3)

Pr[topic|word #3]: (0.5, topic #2); (0.5, topic #3)

BAYES RULE (OR HOW TO USE ANCHOR WORDS)

points are now (normalized) rows of $\hat{M}\hat{M}^T$



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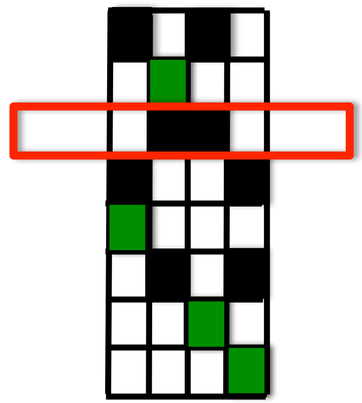

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$\Updownarrow$

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The Topic Model Algorithm:

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This **provably** works for **any** topic model (LDA, CTM, PAM, etc …) provided **A** is separable and **R** is non-singular

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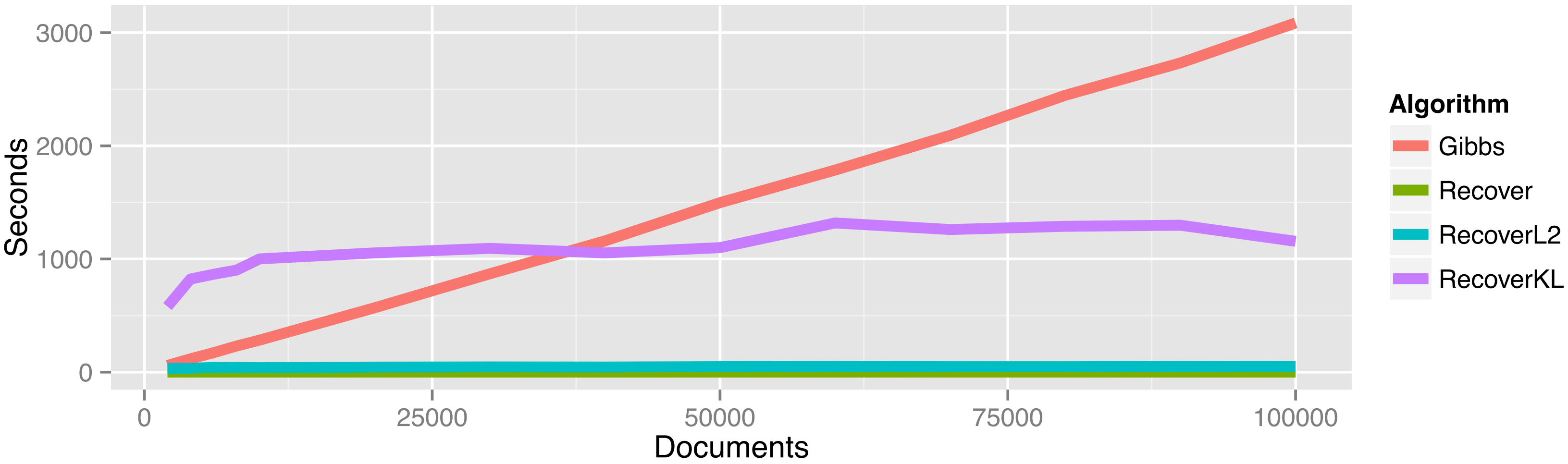

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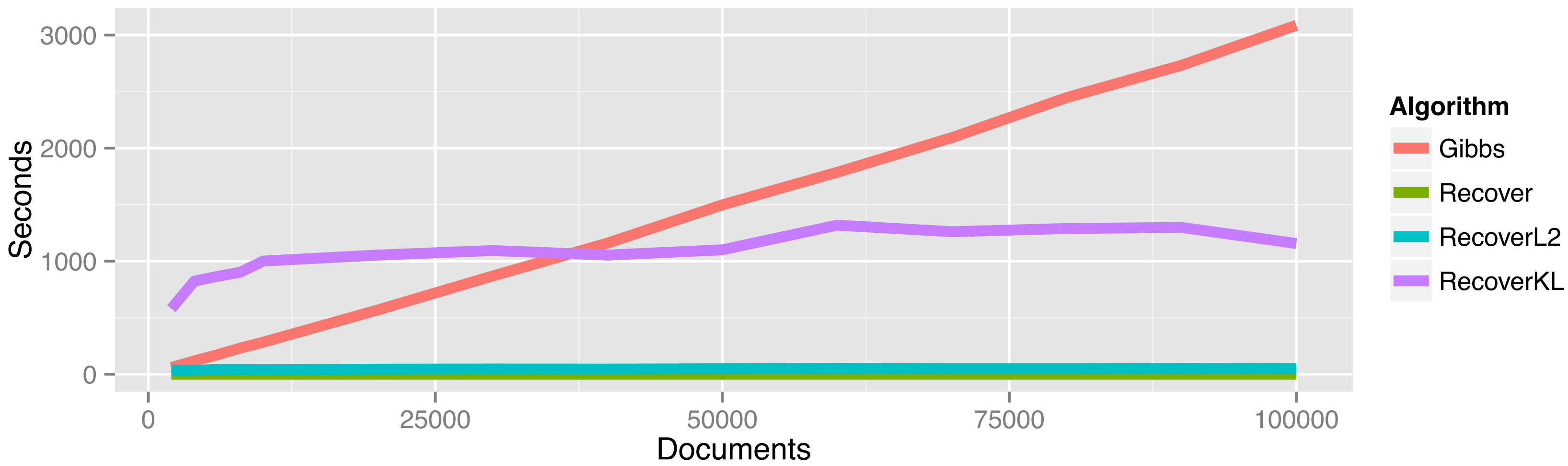

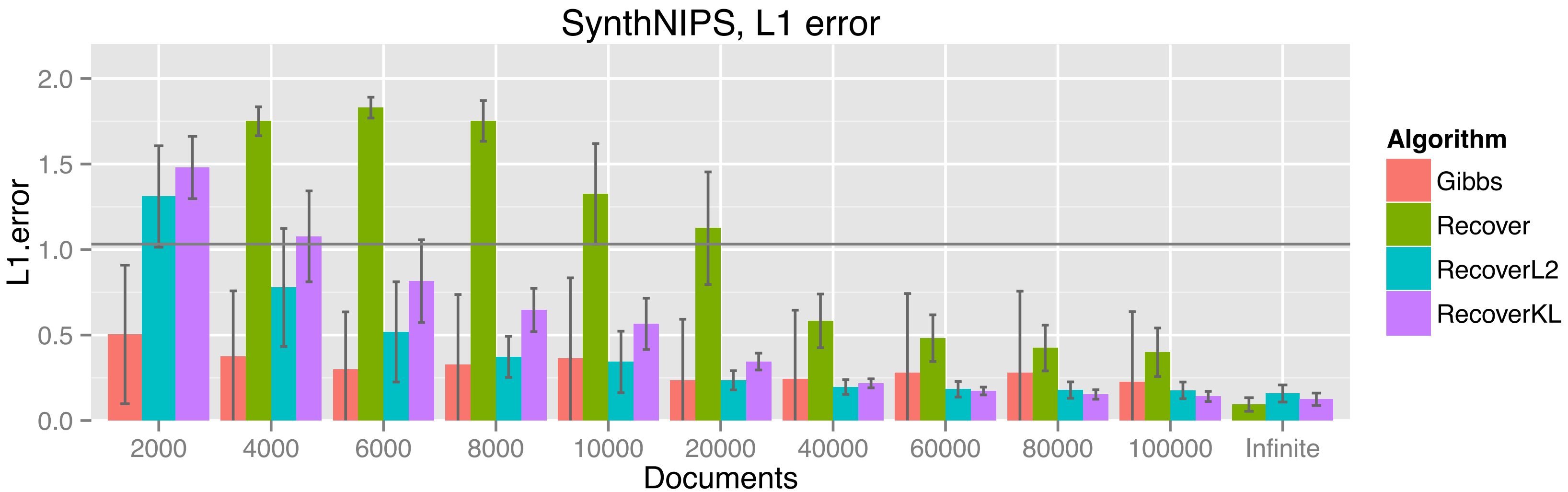

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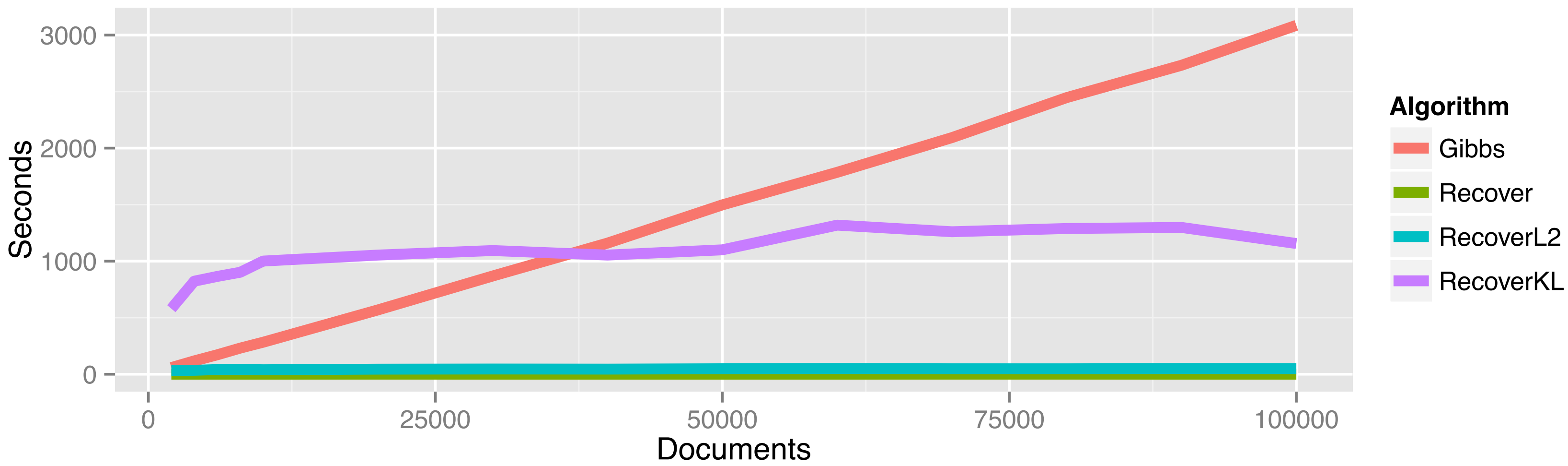

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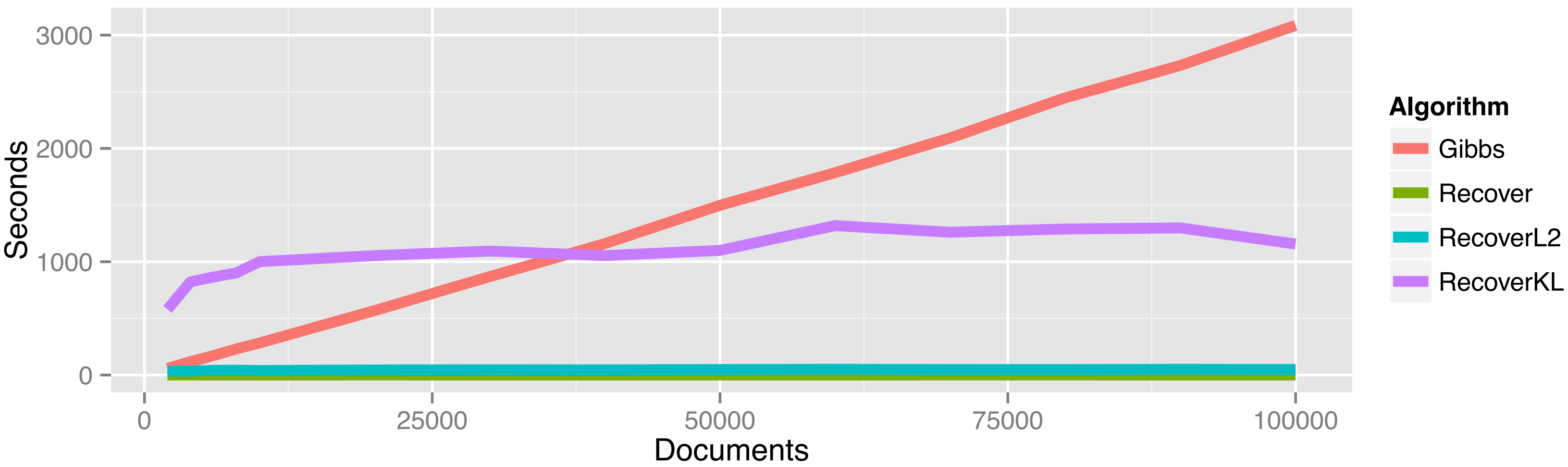

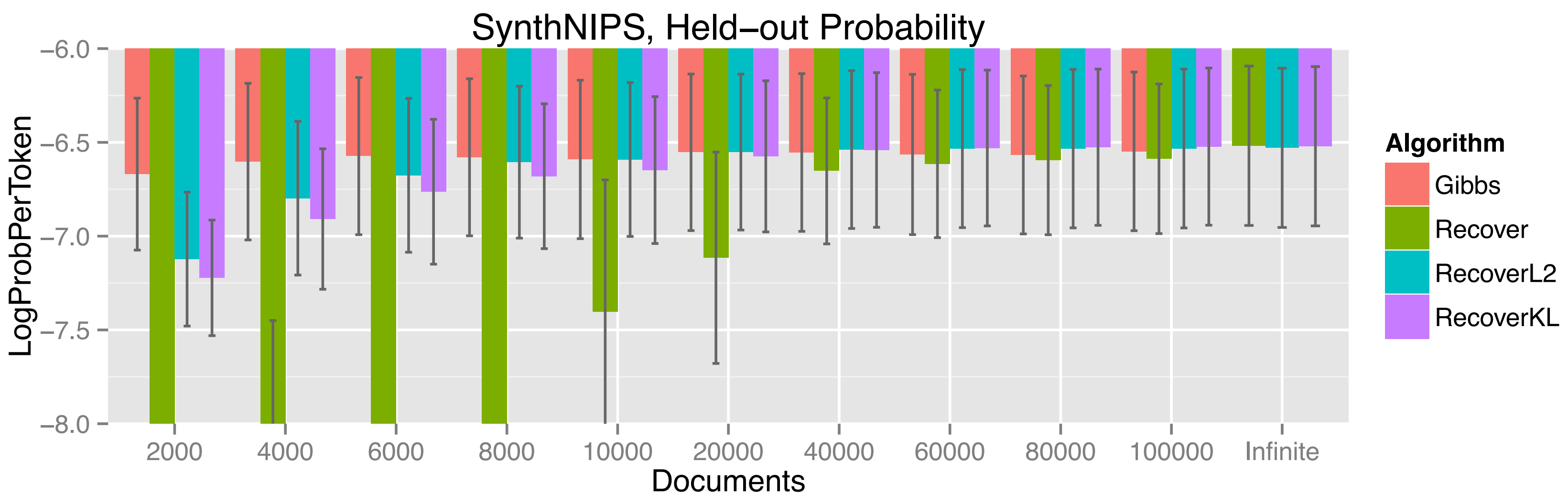

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- real data: UCI collection of 300,000 NYT articles, 10 minutes!

A CONCEPTUAL OVERVIEW

Is Learning Computationally Easy?

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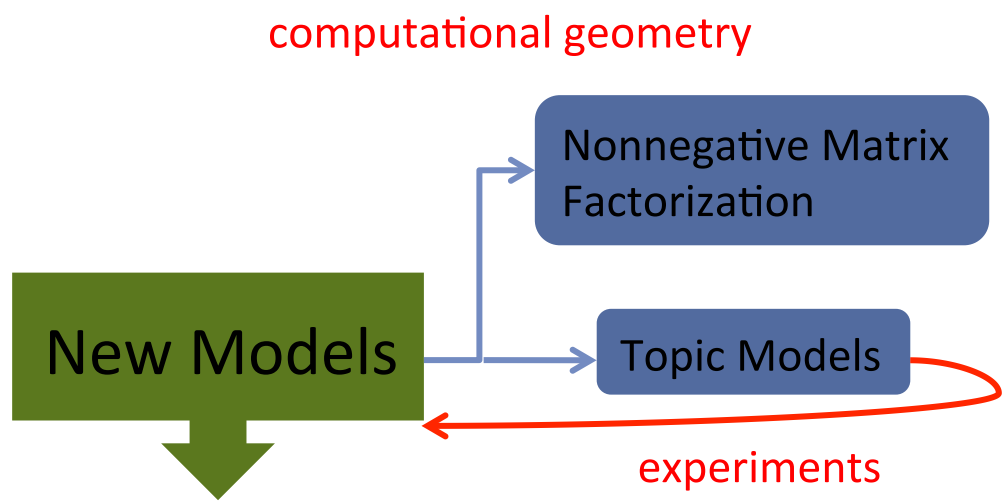

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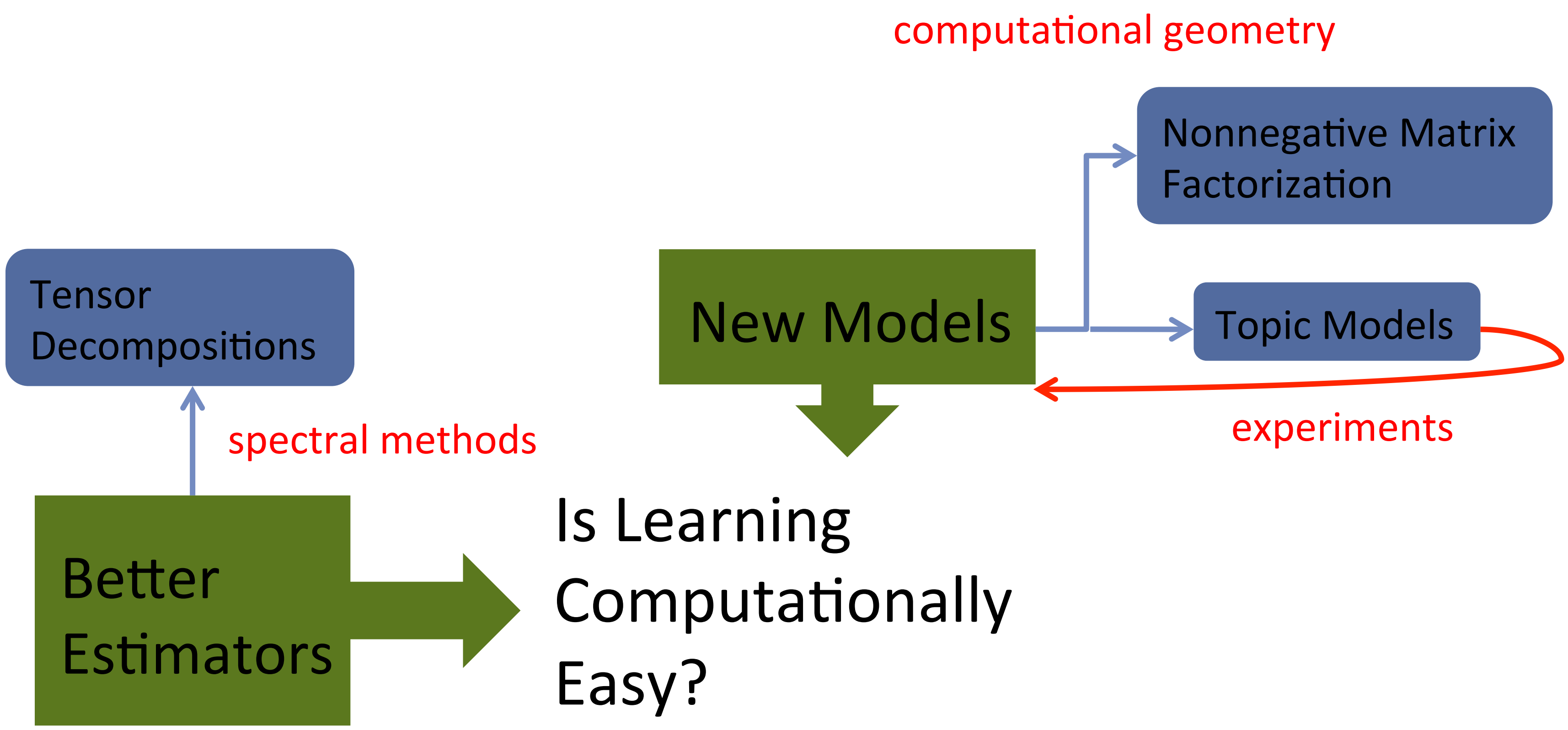

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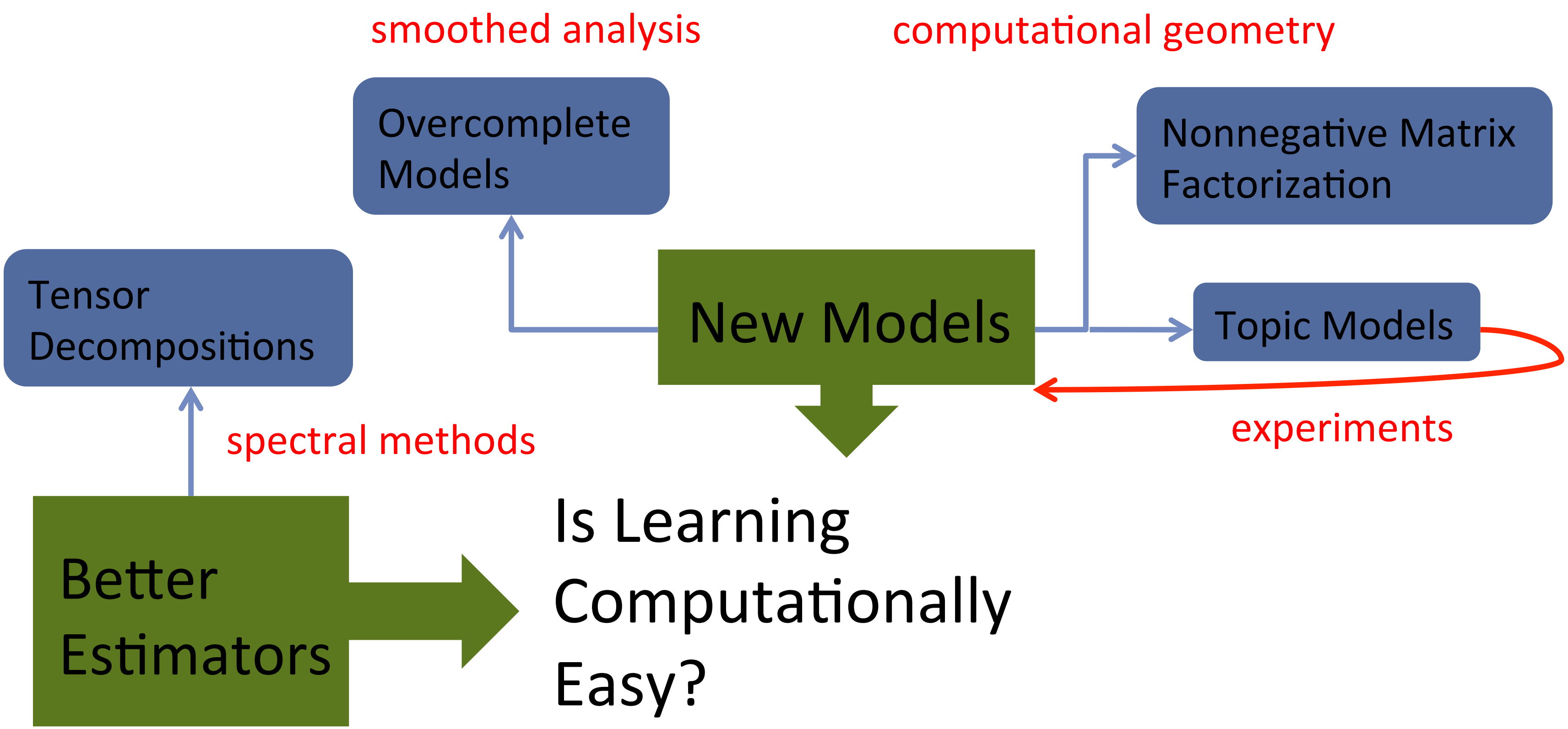

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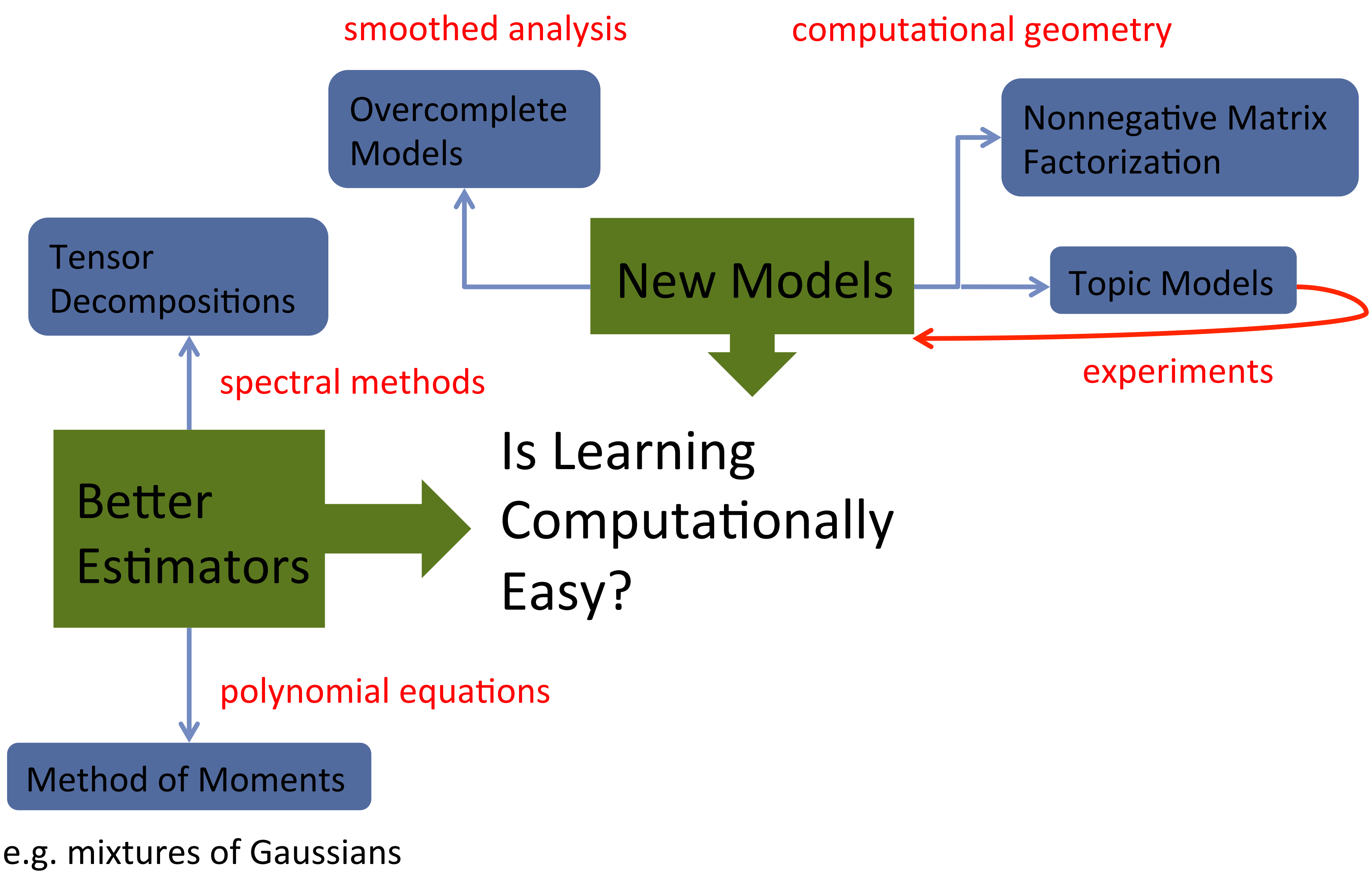

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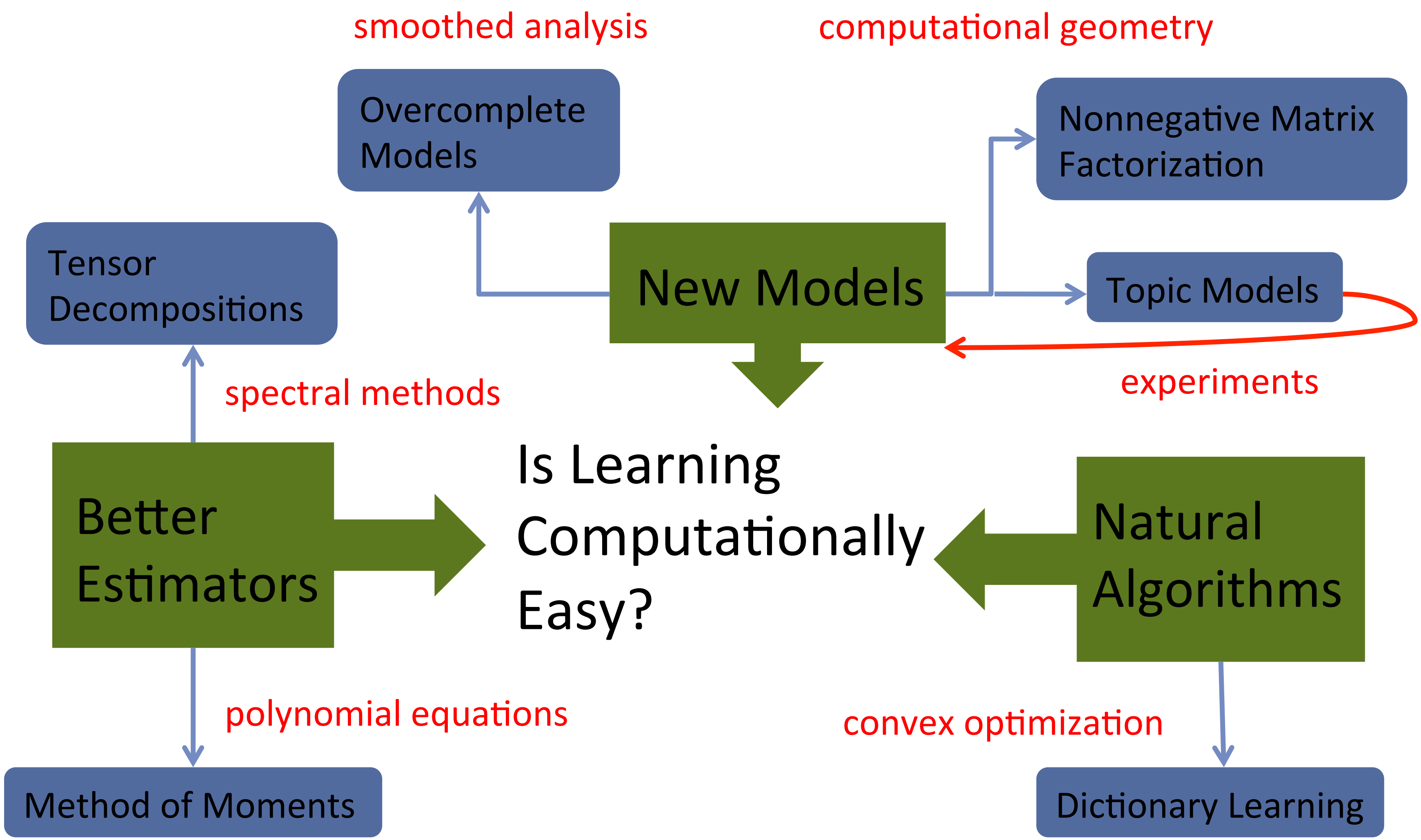

e.g. mixtures of Gaussians

Summary:

- Often optimization problems abstracted from learning are **intractable**!
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Any Questions?

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