Summer School 2014
Learning at Scale

Machine Learning is…

…the branch of engineering that develops technology for automated inference

--- Cosma Shalizi

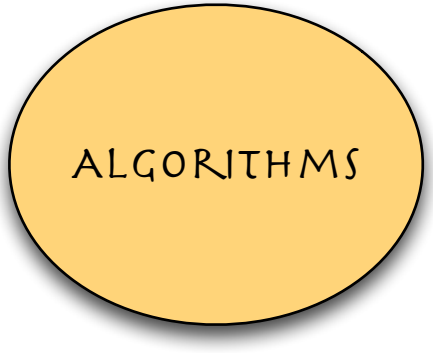
It combines…

Machine Learning is…

…the branch of engineering that develops technology for automated inference

--- Cosma Shalizi

It combines…

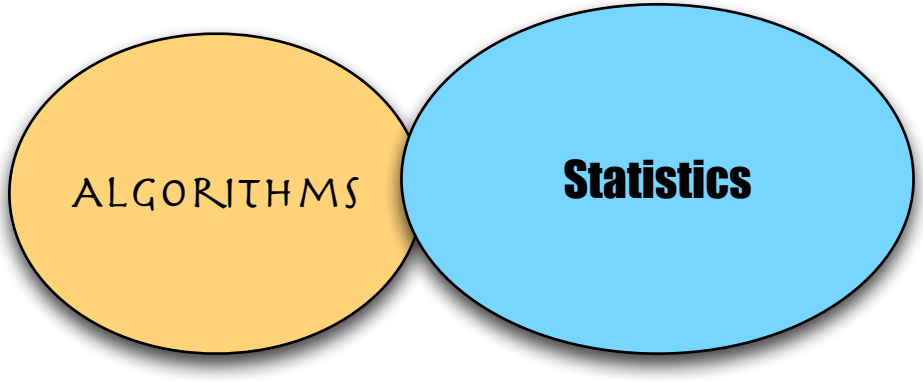


Machine Learning is…

…the branch of engineering that develops technology for automated inference

--- Cosma Shalizi

It combines…

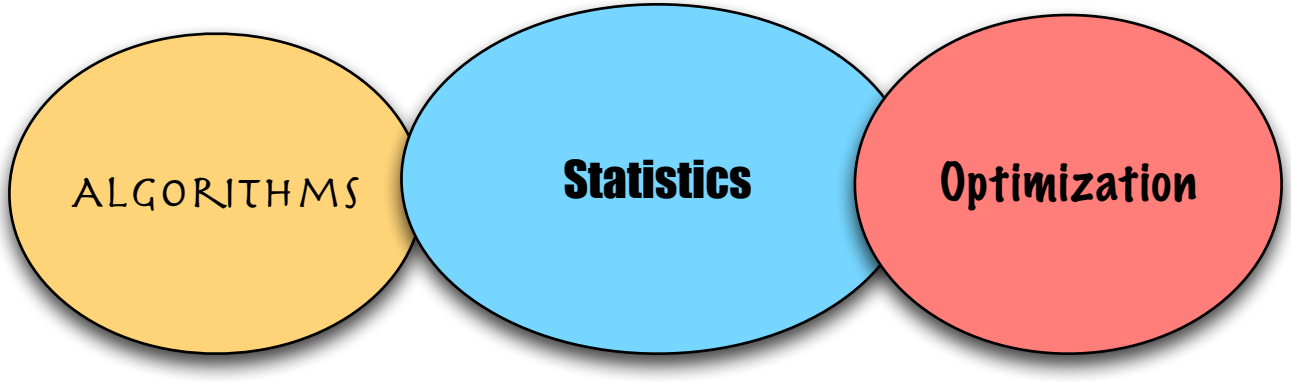


Machine Learning is…

…the branch of engineering that develops technology for automated inference

--- Cosma Shalizi

It combines…

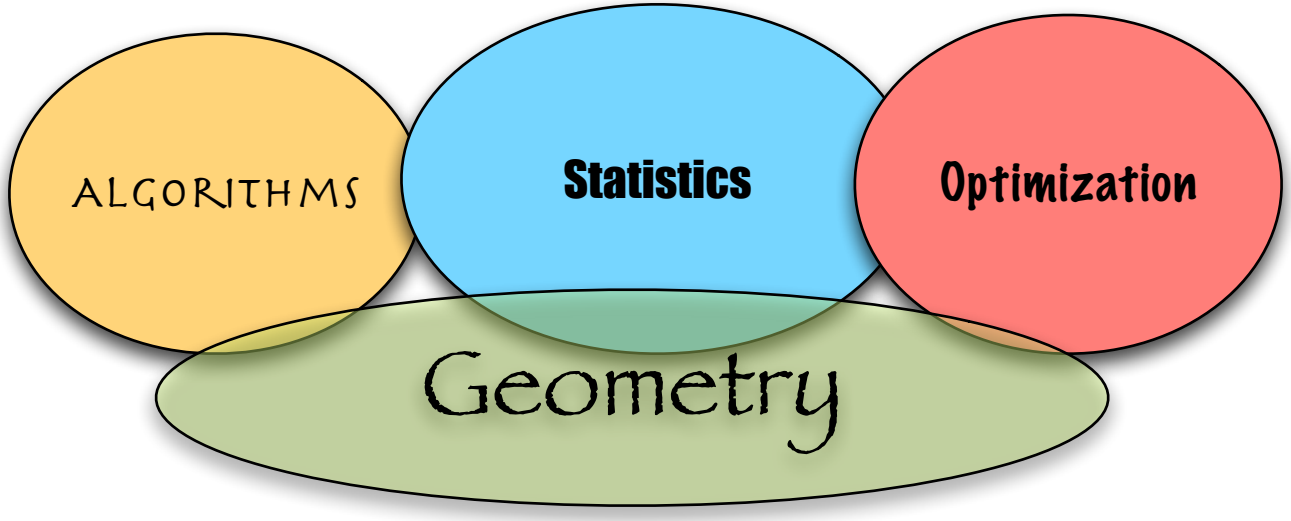


Machine Learning is…

…the branch of engineering that develops technology for automated inference

--- Cosma Shalizi

It combines…



Flavors of Learning

Supervised Learning

(Binary) classification

Given: $\{(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \ldots\}$ drawn from some source

$$\mathbf{x_i} \in \mathbb{R}^d \qquad y_i \in \{+1, -1\}$$

Find a function $f : \mathbb{R}^d \mapsto \{+1, -1\}$ such that

f captures the relationship between $\mathbf{x}$ and y

$$\forall i, f(x_i) = y_i$$

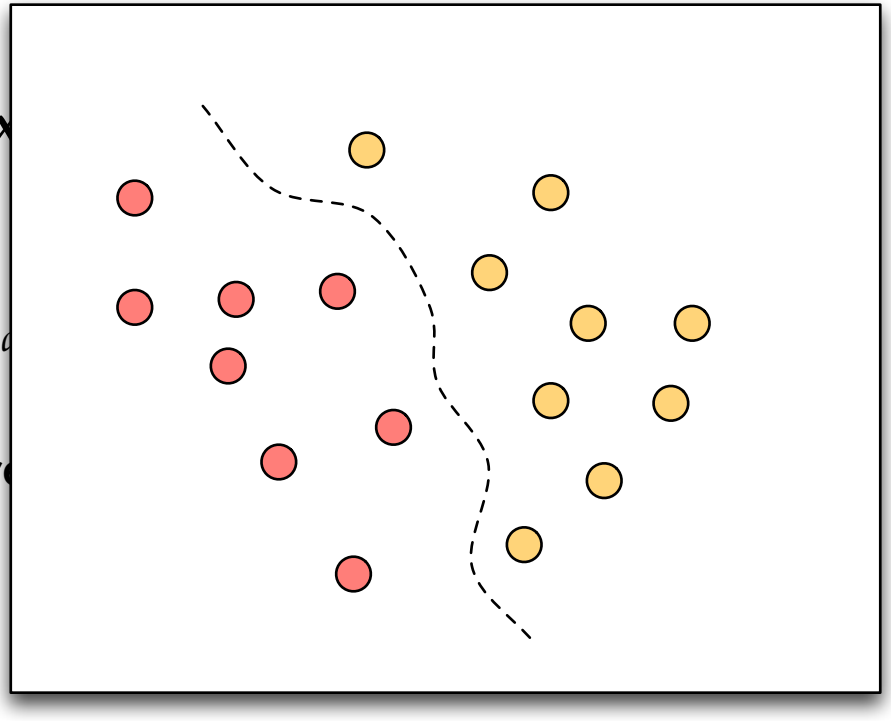
Supervised Learning

(Binary) classification

Given: $\{(\mathbf{x}_1, y_1), (\mathbf{x}$

Find a function $f : \mathbb{R}^$

f captur



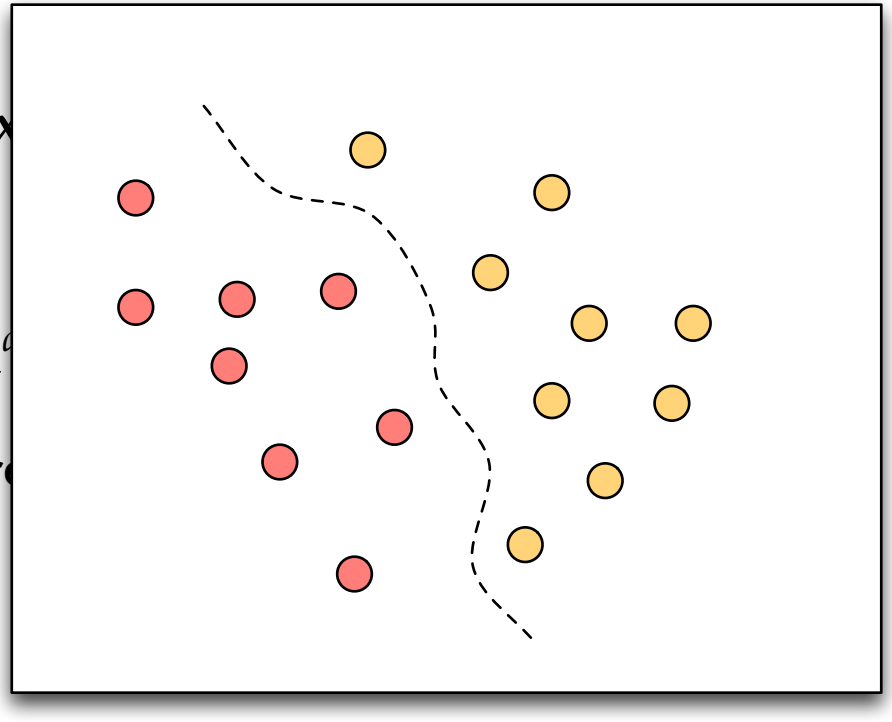
Supervised Learning

(Binary) classification

Given: $\{(\mathbf{x}_1, y_1), (\mathbf{x}$

Find a function $f : \mathbb{R}^$

f captur



- Spam: Testing if email is spam or not
- Sentiment analysis: is a product review positive or negative

Supervised Learning

Regression

Given: $\{(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \ldots\}$ drawn from some source

$$\mathbf{x_i} \in \mathbb{R}^d \quad y_i \in \mathbb{R}$$

Find a function $f : \mathbb{R}^d \mapsto \mathbb{R}$

such that f captures the relationship between $\mathbf{x}$ and y

$$\forall i, f(x_i) = y_i$$

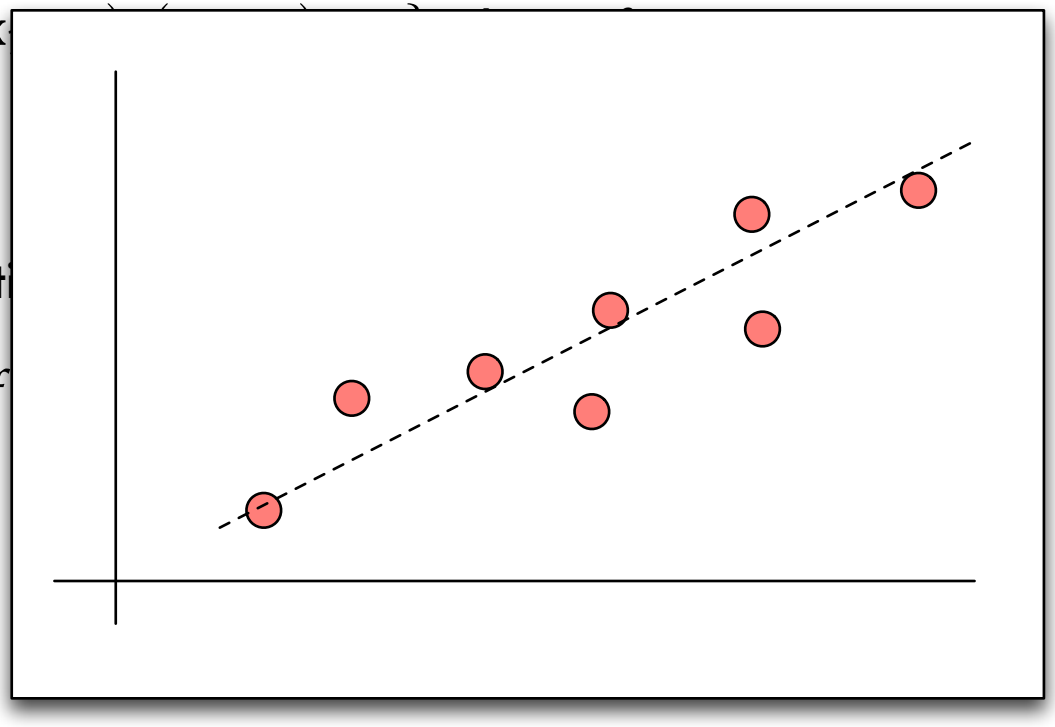
Supervised Learning

Regression

Given: $\{(x$

Find a functi

such that f



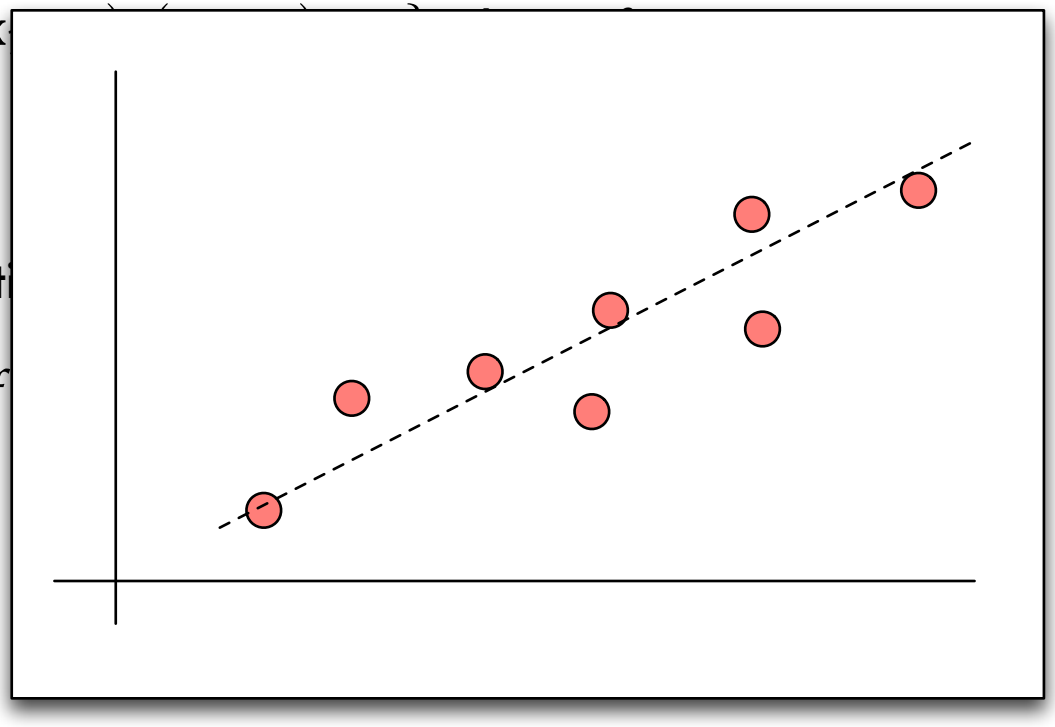
Supervised Learning

Regression

Given: $\{(x$

Find a functi

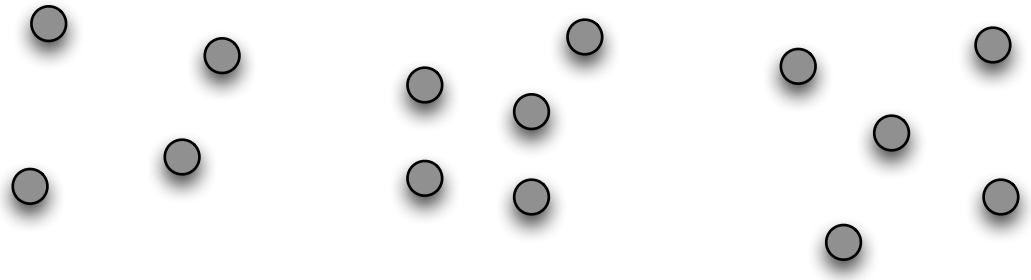
such that f



- Predictions: Stock market price as function of financial specs
- Relationship between dosage and effectiveness

Unsupervised Learning

Clustering



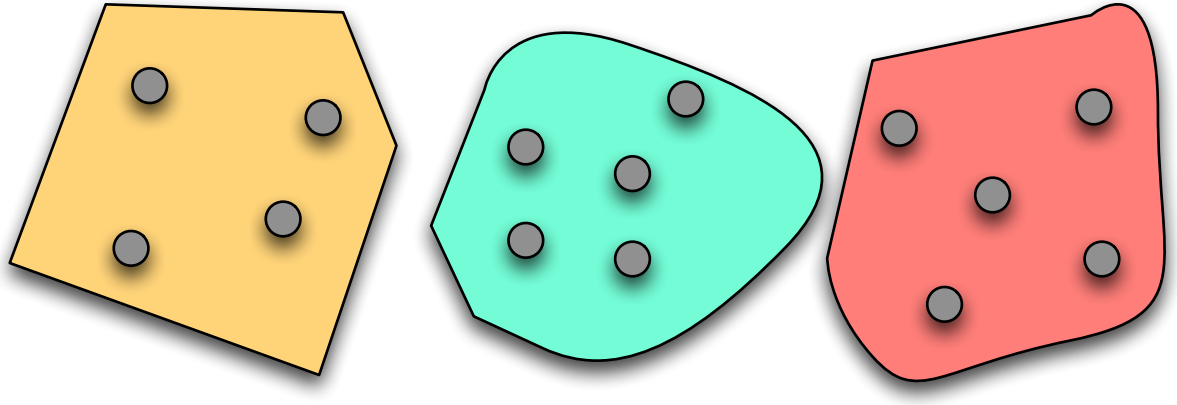
Given a collection of objects, find a way to "group" them into similar pieces

Learning a function $f : \mathbb{R}^d \mapsto \{1, 2, \ldots, k\}$

But we don't have any examples of the "correct" answer !
Clustering is closely related to classification with multiple classes

Unsupervised Learning

Clustering



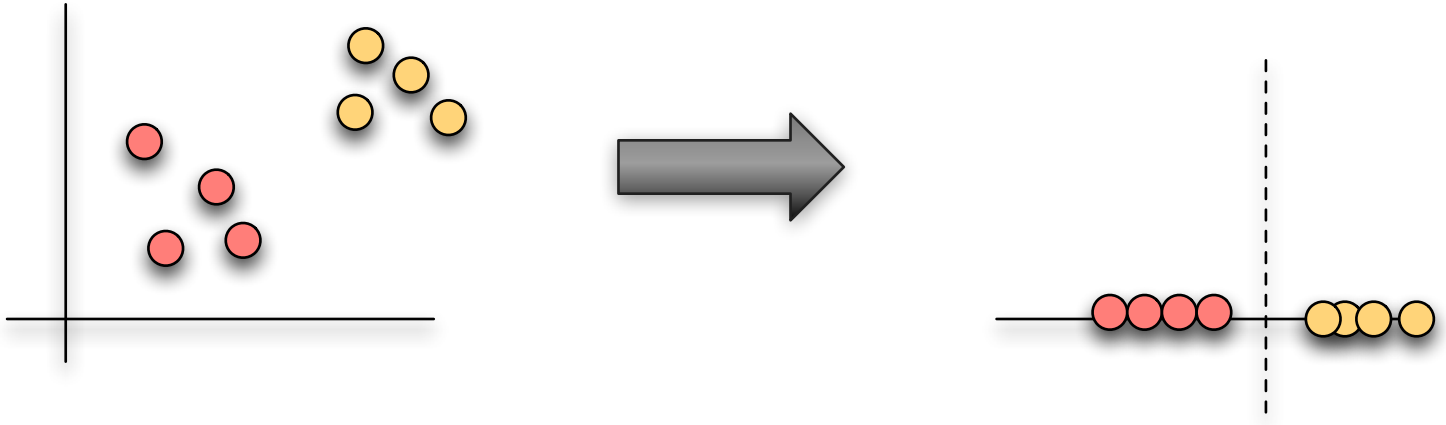
Given a collection of objects, find a way to "group" them into similar pieces

Learning a function $f : \mathbb{R}^d \mapsto \{1, 2, \ldots, k\}$

But we don't have any examples of the "correct" answer !
Clustering is closely related to classification with multiple classes

Unsupervised Learning

Dimensionality Reduction (or Feature Learning)



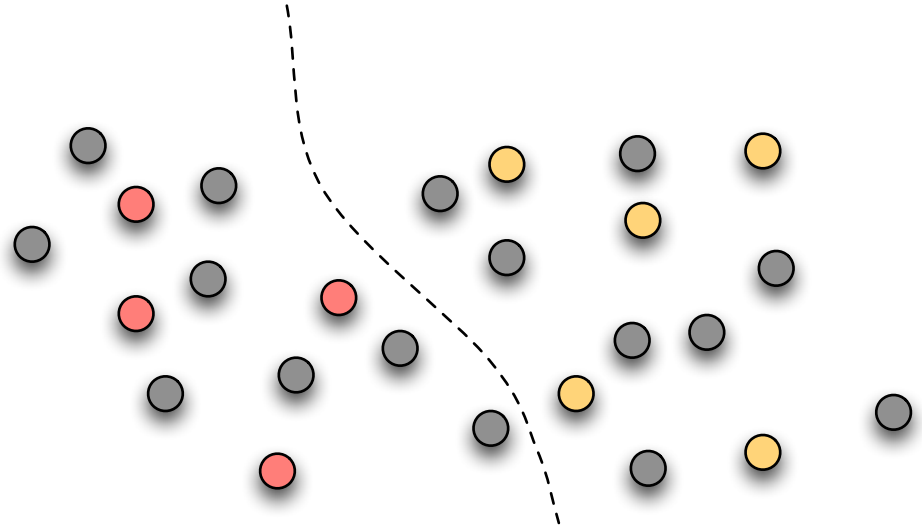
Given objects in $\mathbb{R}^d$ find a mapping $A : \mathbb{R}^d \mapsto \mathbb{R}^k, k \ll d$

that preserves the "structure" of the objects

- Find "relevant" dimensions for a task
- Reduce dimensionality to manage complexity of algorithms

Mixing and Matching

Semi-supervised learning: labelled and unlabeled data

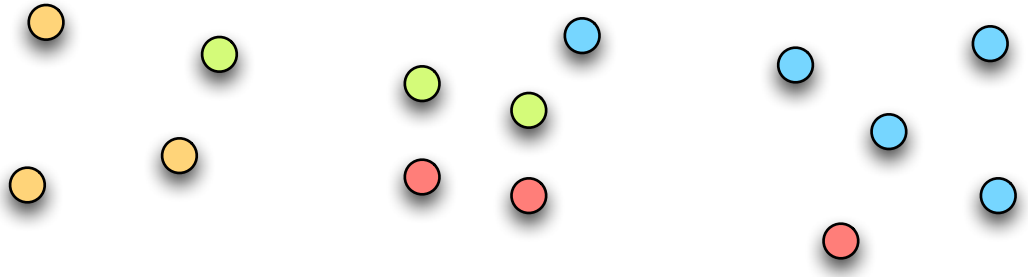


Find a classifier that separates the labeled points and separates the unlabeled points "well"

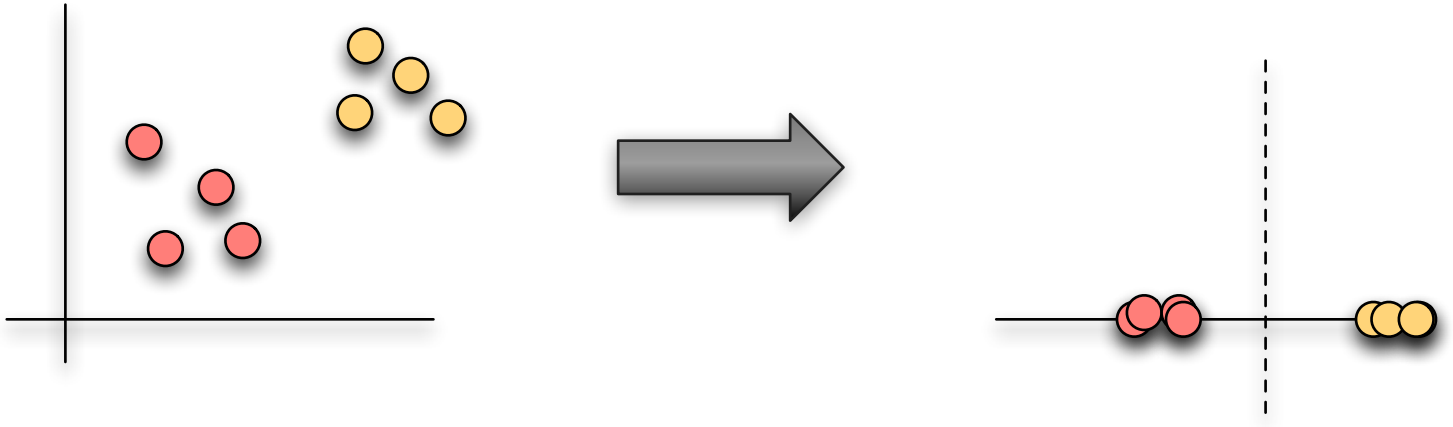
Often have lots of unlabeled data and only a little labeled data to guide efforts

Mixing and Matching

Supervised clustering = multiclass classification



Supervised dimensionality reduction = (linear) discriminant analysis



Understanding vs Predicting

$\{(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \ldots\}$ is drawn from distribution $p(\mathbf{X}, Y)$

Generative learning: Learn the distribution $p(\mathbf{X}, Y)$

"What controls the rise and fall of the tides ?"

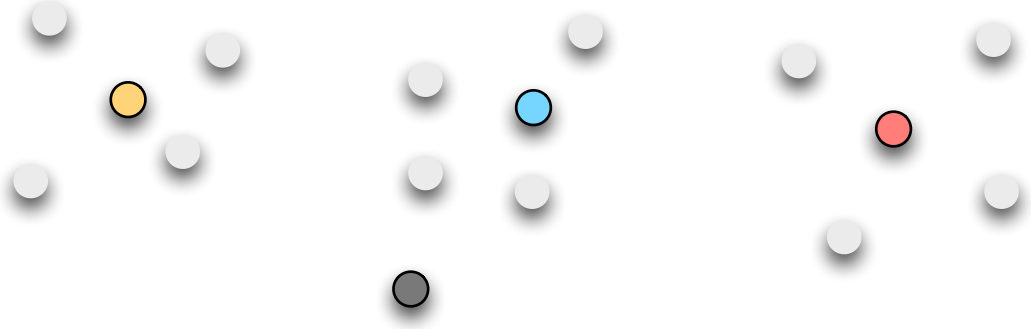
Discriminative learning: Learn the conditional distribution $p(Y \mid \mathbf{X})$

"Will there be a high tide tomorrow evening ?"

$$p(Y \mid \mathbf{X}) = \frac{p(\mathbf{X}, Y)}{p(\mathbf{X})}$$

Understanding vs Predicting

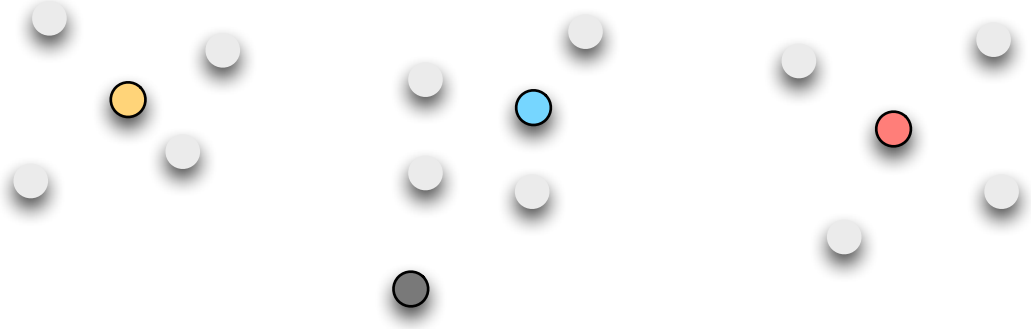
Discriminative clustering: predict the cluster of a new point



$$p(\bigcirc|\bigcirc) = \exp(-\|\bigcirc - \bigcirc\|^2)$$

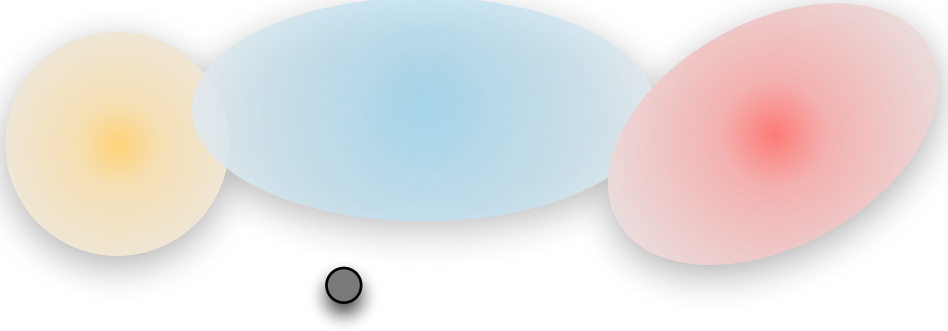
Understanding vs Predicting

Discriminative clustering: predict the cluster of a new point



$$p(\bigcirc|\bigcirc) = \exp(-\|\bigcirc - \bigcirc\|^2)$$

Generative clustering: mixture density estimation



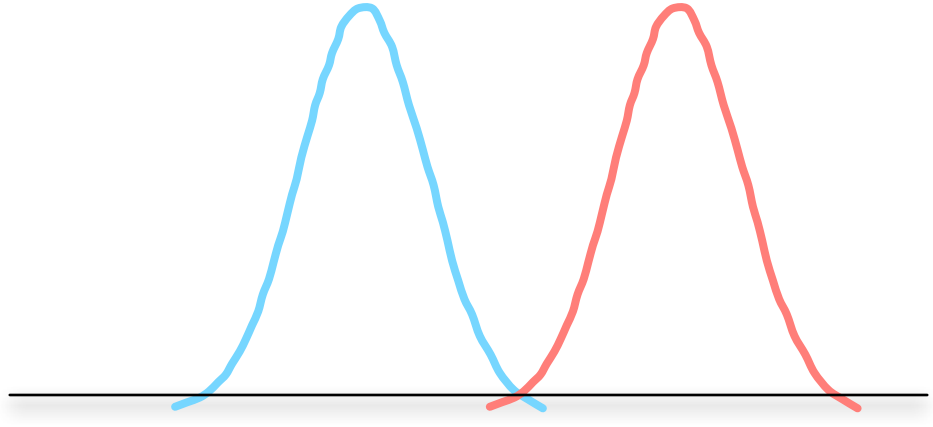
Parameters

Parametric Learning:

- Define a space of models parametrized by fixed number of parameters
- Find model that best fits the data (by searching over parameters)

Parametric binary classification:

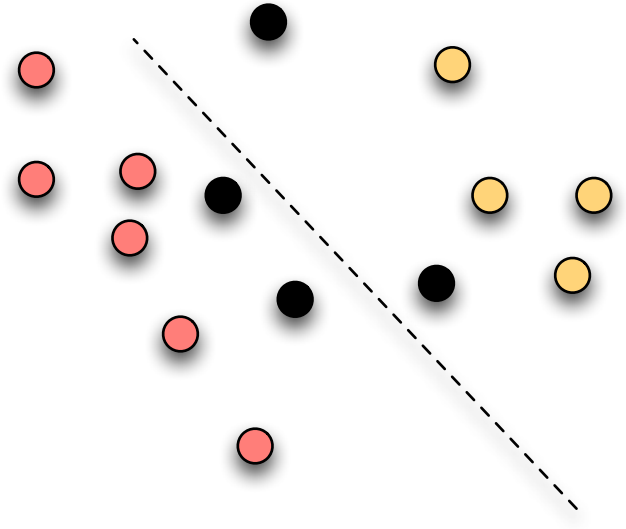
- Model: $(\mu_1, \Sigma_1, \mu_2, \Sigma_2)$ $p(\mathbf{x}) \propto \exp(-(\mathbf{x}-\mu)^\top \Sigma(\mathbf{x}-\mu))$
- Maximize likelihood of any model
- $d^2 + 2d$ parameters



Parameters

Non-parametric Learning:

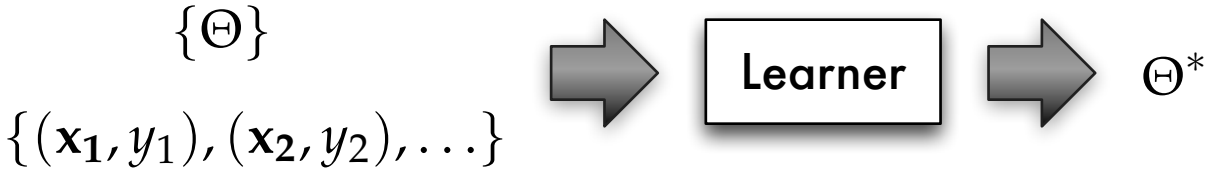
- Define a space of models that can grow in size with data.
- Find model that best fits the data
- "Non-parametric" means "Not-fixed", not "none" !



4 "support points" define the resulting classifier.

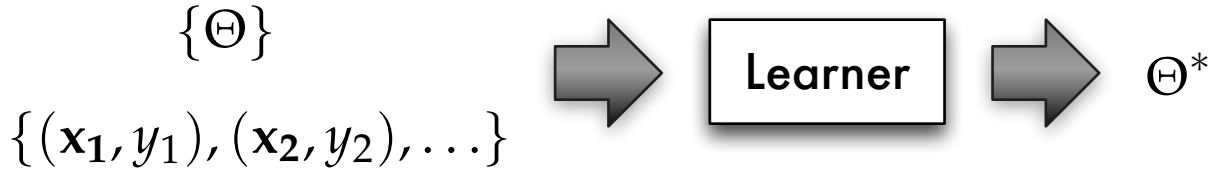
Bayesian Learning

Non-Bayesian (parametric) learning:

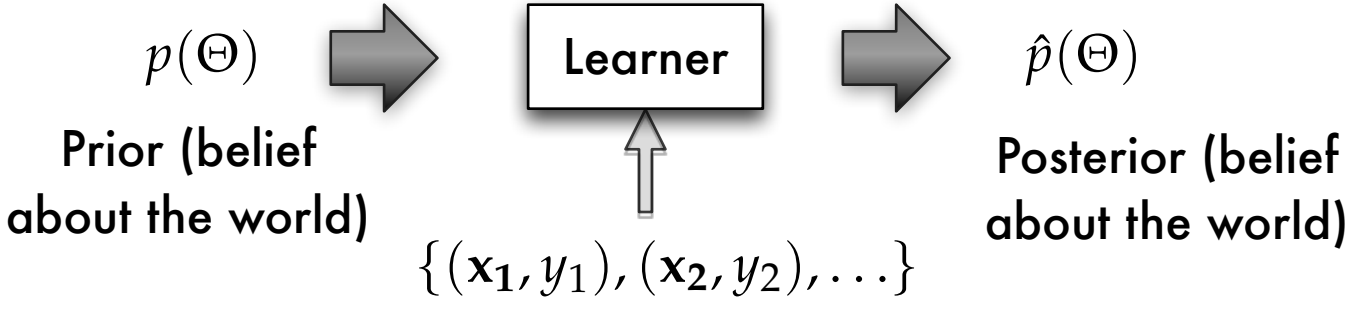


Bayesian Learning

Non-Bayesian (parametric) learning:



Bayesian learning:



Θ^* is a point estimate.

$\hat{p}(\Theta)$ is a distribution over possible worlds

Bayesian Learning

You know you're talking to a Bayesian if...

What's your prior ?

Conjugate priors

Maximum a posteriori (MAP)

Many Learning Frameworks

- Online Learning: must make prediction as soon as item arrives
- Active Learning: I can get labels for data, but it's expensive.
- Multi-task Learning: I'm learning different tasks, but they're related so maybe the tasks can learn from each other.
- Transfer Learning: I can learn well in one domain: can I transfer this
- knowledge into a different domain ?
- Ensemble Learning: I have bad learners, but together they're decent

The Mechanics of Learning

Loss Functions

Given: $\{(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \ldots\}$ drawn from some source

$$\mathbf{x_i} \in \mathbb{R}^d \qquad y_i \in \{+1, -1\}$$

Find a function $f : \mathbb{R}^d \mapsto \{+1, -1\}$

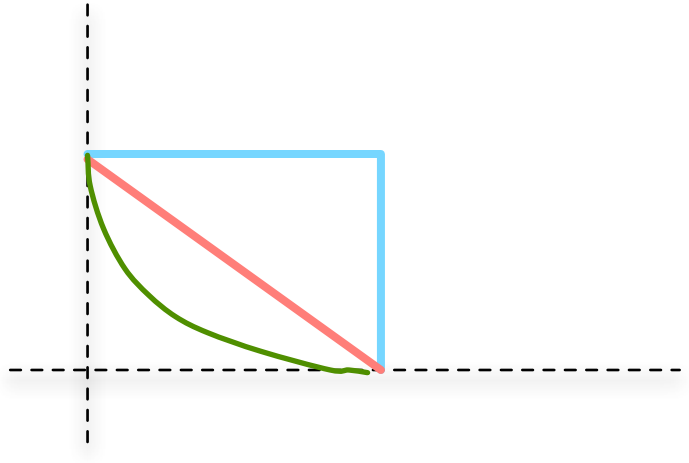
such that f captures the relationship between $\mathbf{x}$ and y

Loss functions $L(f((x), y))$ measure the quality of f:

0-1 loss: $\mathbf{1}_{f(\mathbf{x}) \neq y}$

Hinge loss: $\max(0, 1 - y \cdot f(\mathbf{x}))$

Square loss: $(y - f(\mathbf{x}))^2$



Estimating Risk

Once we have a loss function, we can quantify how good a predictor is:

$$R(f) = E_{\mathbf{x},y}[L(f(\mathbf{x}), y)] = \int p(\mathbf{x}, y) L(f(\mathbf{x}), y)$$

and find a good predictor:

$$f^* = \arg\min_{f \in \mathcal{F}} R(f)$$

But we don't usually know what the data distribution is, so we can't solve the minimization !

Empirical Risk Minimization

Assume the given data is drawn from the source distribution.
Replace

$$R(f) = E_{\mathbf{x},y}[L(f(\mathbf{x}), y)] = \int p(\mathbf{x}, y) L(f(\mathbf{x}), y)$$

by the empirical mean:

$$\hat{R}(f) = \frac{1}{n} \sum_i L(f(\mathbf{x}_i, y_i)$$

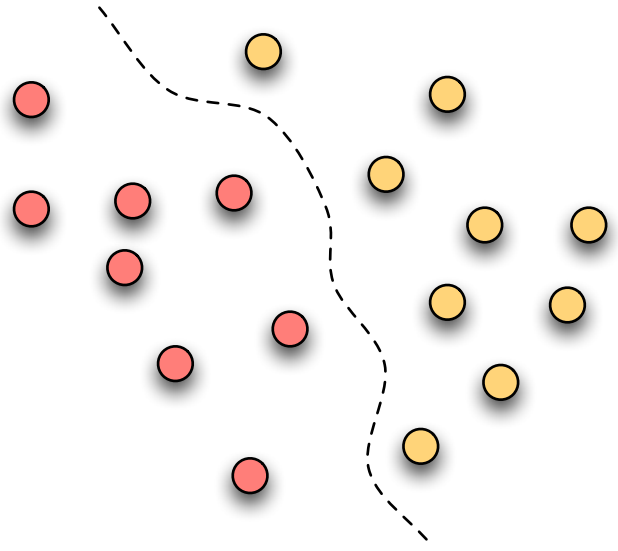
with the hope that the estimate is unbiased and converges:

$$E[\hat{R}(f)] = R(f), \quad \hat{R}(f) \to R(f)$$

But now we have a "normal" optimization:

$$\min_{f \in \mathcal{F}} \frac{1}{n} \sum_i L(f(\mathbf{x}_i, y_i)$$

Overfitting and Regularization



The problem with optimizing over the data is that you can over-fit to your samples. (low *bias*)

This is bad because then your predictive power goes down (high *variance*) and you can't *generalize*

Complex models (with more parameters) can overfit. Penalize them !

$$\min_{f \in \mathcal{F}} \frac{1}{n} \sum_i L(f(\mathbf{x}_i, y_i) + \boxed{c(f)}$$

model complexity term

This is called *regularization*.

Generalization

How many samples of the data do you need for the empirically optimized answer to get close to the true answer ?

If your function space is "well behaved" (not too wiggly), then you don't need too many samples.

Well behaved:

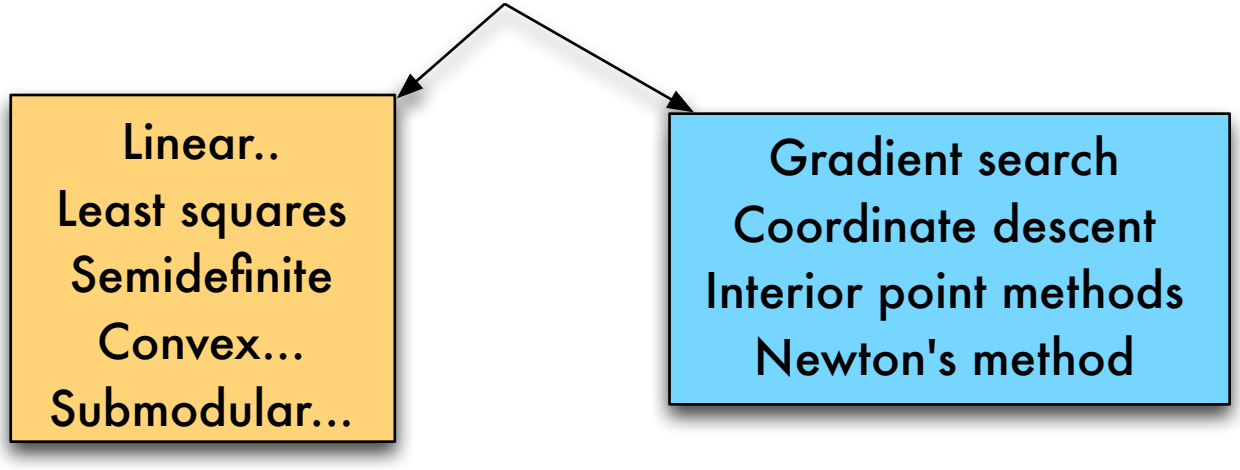
- VC dimension is small
- Rademacher complexity is small
- Fat shattering dimension is small
- ... and others.

All of this assumes that you sample from the real distribution...

Overview... so far...

1) Choose a learning task (classification, clustering, regression, ...)
2) Pick a convenient loss function
3) Sample a sufficient number of points from a source
4) Build an optimization using the data, the loss function, and any regularizers
5) OPTIMIZE !!!
6) Use learned model on new data to predict.
7) (if you're doing online learning, repeat)

ML = Design choices + careful optimization



Representations

The Computational Geometry prayer:
Let P be a set of points in the plane. Amen.

Representations

The Computational Geometry prayer:
Let P be a set of points in the plane. Amen.

Let G be a graph with n vertices and m edges

Representations

The Computational Geometry prayer:
Let P be a set of points in the plane. Amen.

Let G be a graph with n vertices and m edges

Let S be a set of elements drawn from a universe U

Representations

The Computational Geometry prayer:
Let P be a set of points in the plane. Amen.

Let G be a graph with n vertices and m edges

Let S be a set of elements drawn from a universe U

Let M be an m X n matrix of reals.

Representations

The Computational Geometry prayer:
Let P be a set of points in the plane. Amen.

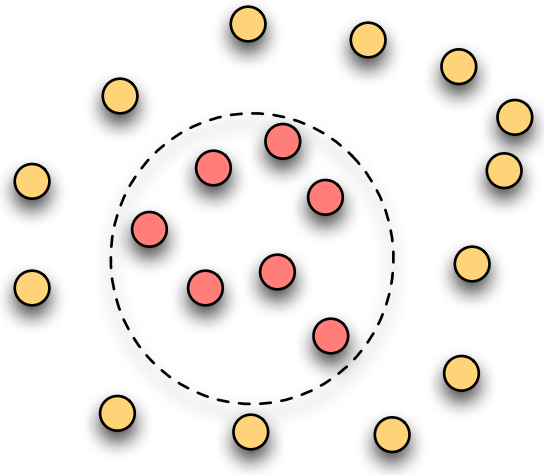
Let G be a graph with n vertices and m edges

Let S be a set of elements drawn from a universe U

Let M be an m X n matrix of reals.

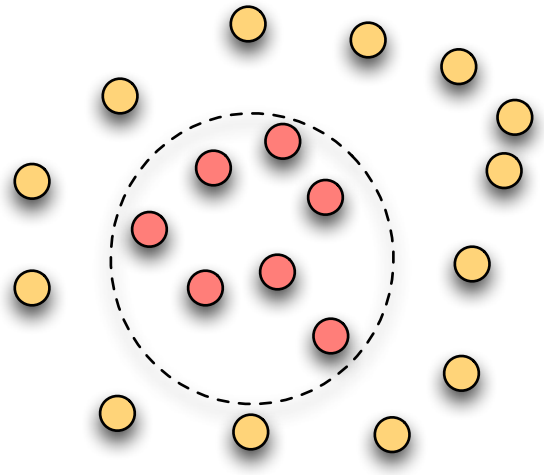
But in learning, we don't have a "natural" representation.
We have to CHOOSE one.

Representations help algorithms



Learning a circle separating classes can be tricky

Representations help algorithms

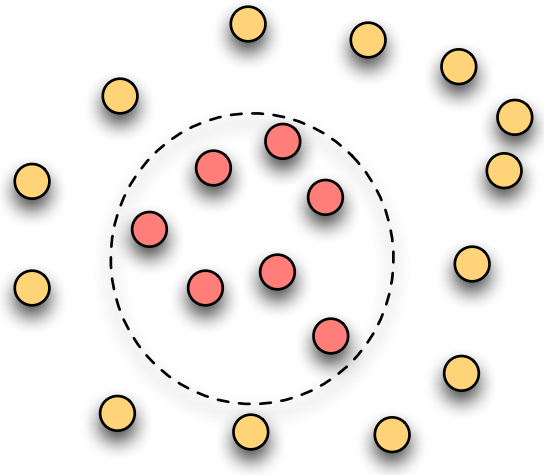


Learning a circle separating classes can be tricky

If we change the representation

$$\ell : (x, y) \mapsto (x, y, x^2 + y^2)$$

Representations help algorithms

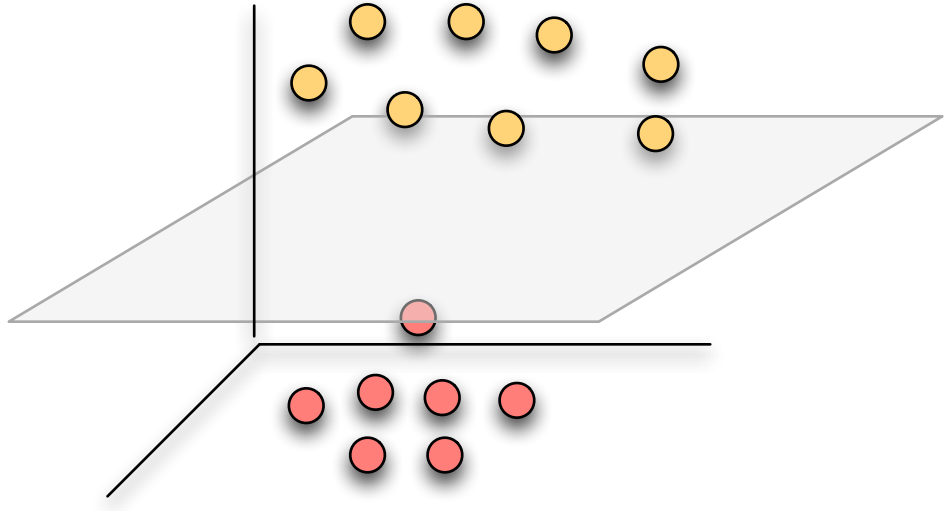


Learning a circle separating classes can be tricky

If we change the representation

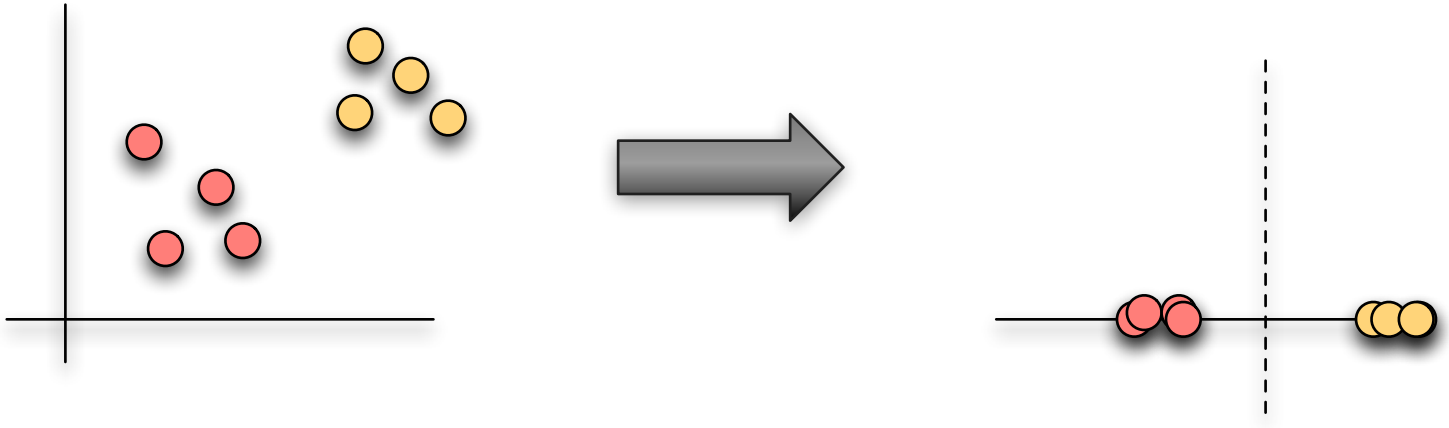
$$\ell : (x, y) \mapsto (x, y, x^2 + y^2)$$

Circle separation becomes linear separation !

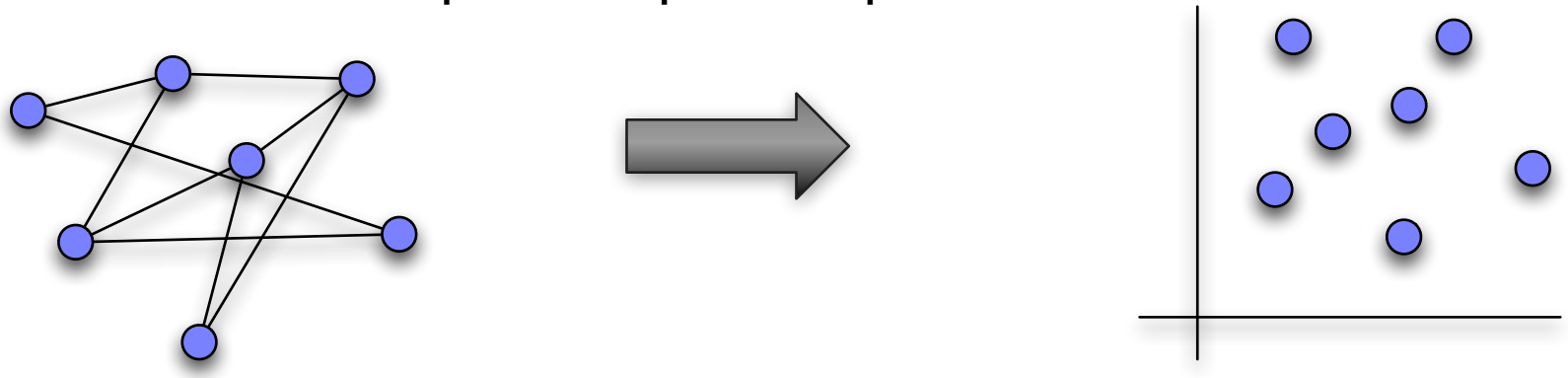


Constructing a representation

Supervision guides the representation



Unsupervised spectral representation



And many other kinds...

A new overview of learning

Build a model of data

Choose a learning task

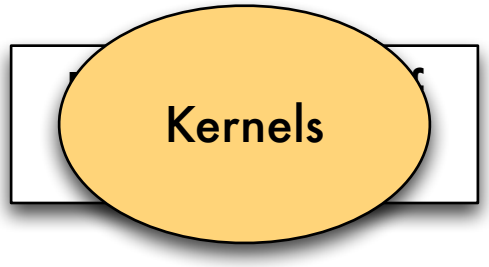
Shape your learner (loss, regularizer)

Choose samples from a source

Optimize !

Predict !

A new overview of learning



Choose a learning task

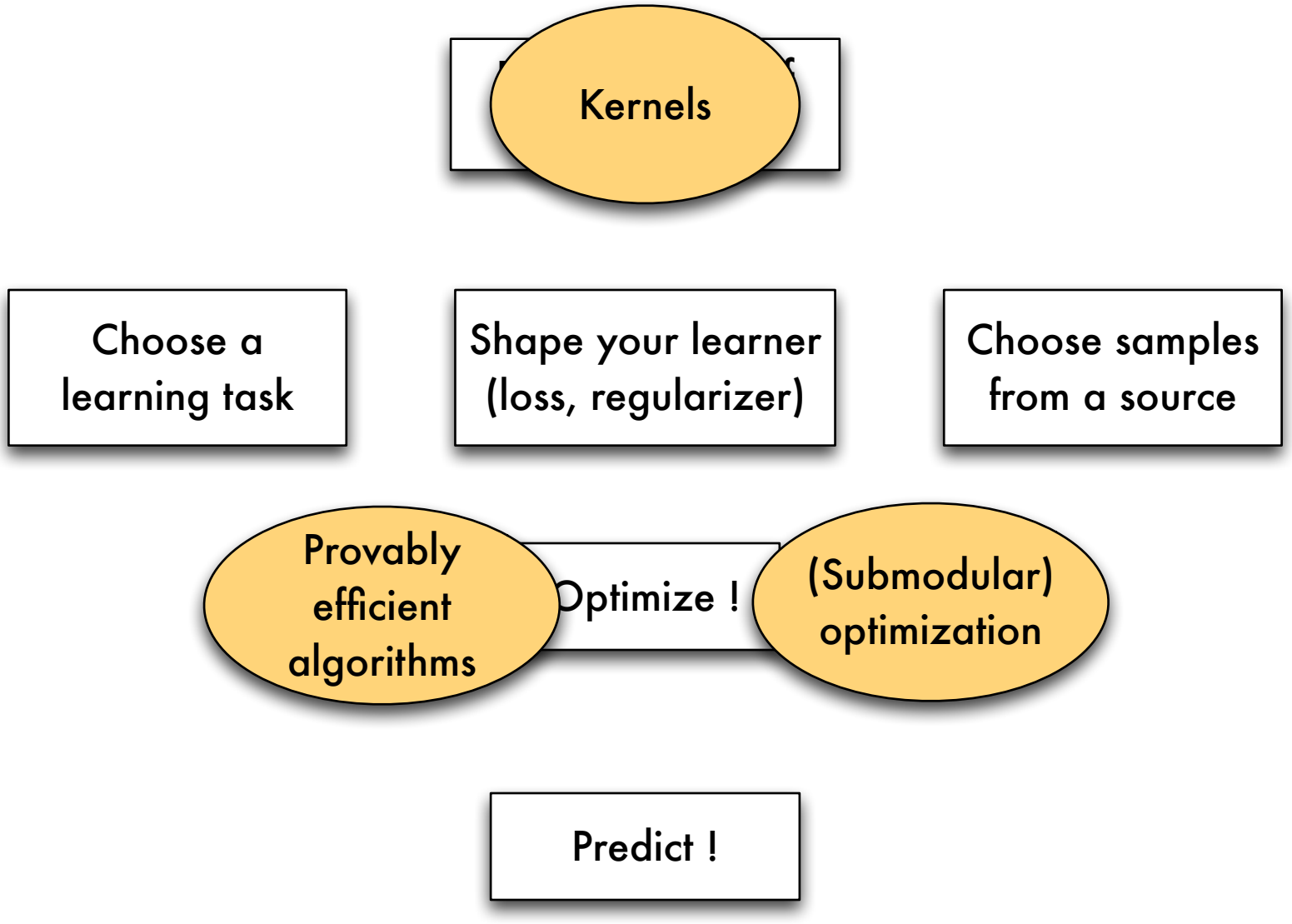
Shape your learner (loss, regularizer)

Choose samples from a source

Optimize !

Predict !

A new overview of learning



A new overview of learning