Online Labeling Problem

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Labeling Problem Description

Input:

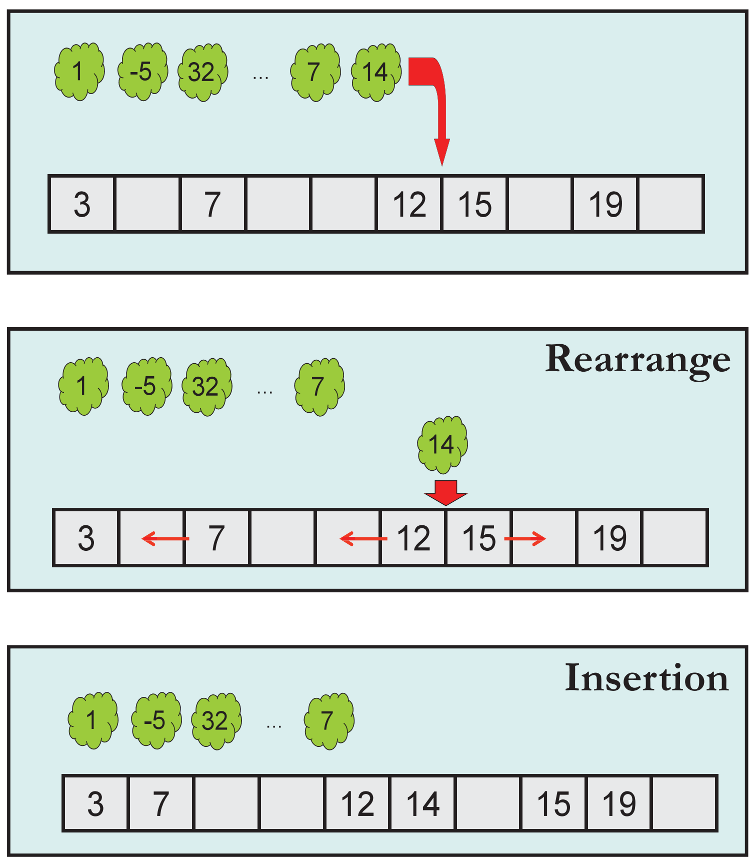
- A stream of n numbers, revealed on-line.
- An array of size $m>n$, initially empty.

Objective:

- Insert the numbers one by one into the array, while maintaining them in correct order after each insertion.
- Rearranging items may be needed.
- We want to minimize the total number of item moves.

Example:

Inserting 14 into the sorted array.



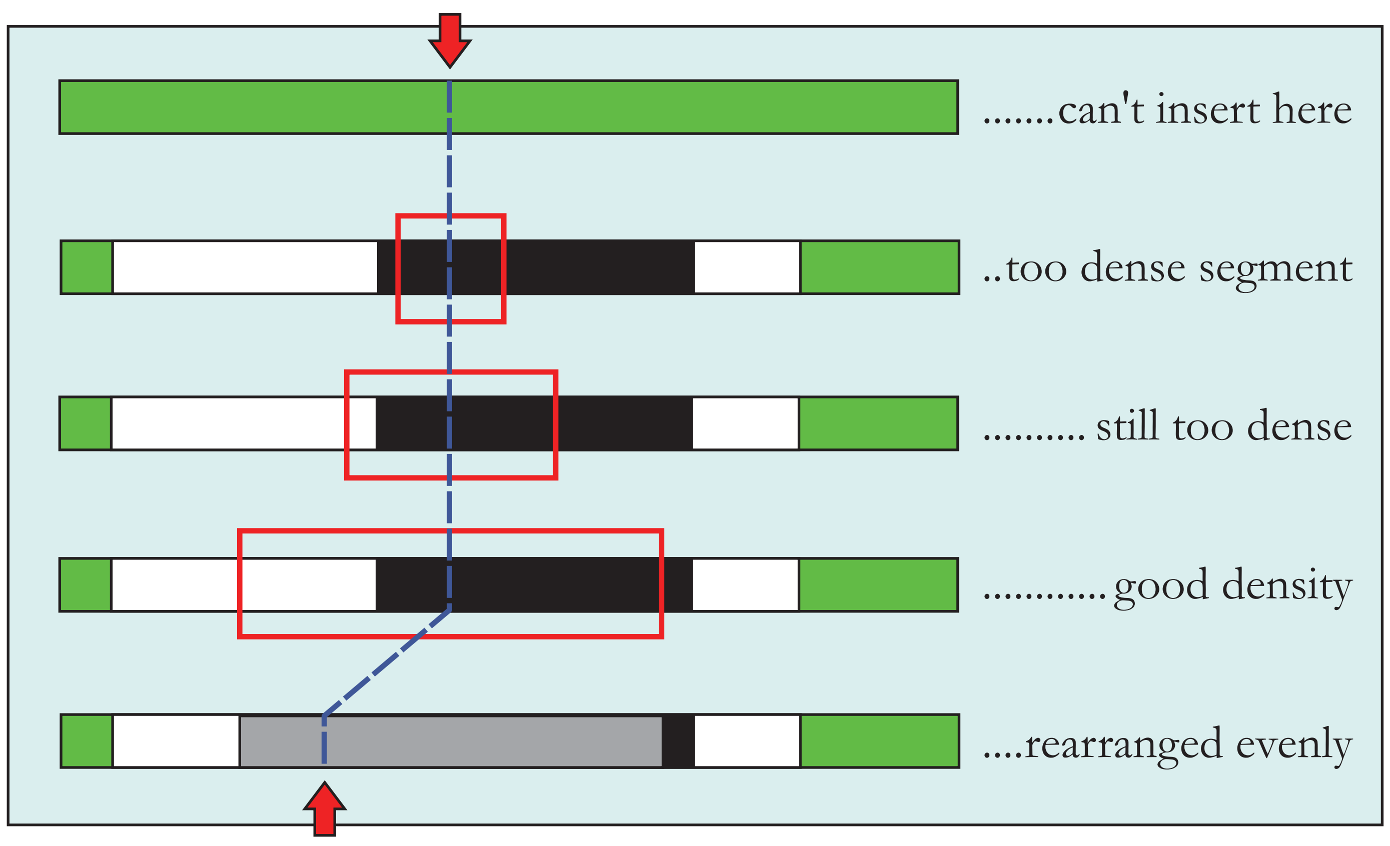
Applications

- Dynamic linear order maintenance. [Bender et al. '02]
  - ↳ Orthogonal graph drawing. [Biedl, Kant '98]
  - ↳ Persistent data structure timestamps. [Driscoll, Sarnak, Sleator, Tarjan '89]
- Cache oblivious dictionaries based on B-trees. [Bender, Demain, Farach-Colton '05]
- Approximate indexed lists, item label used as the index. [Andersson, Petersson '98]
- Distributed controllers. [Emek, Korman '11]

Upper Bound Ideas

- If possible, insert into the middle of free space at the correct position.
- Otherwise find the smallest *acceptable* segment and evenly rearrange all items within. A segment is acceptable iff it contains the insertion point and has a correct density.
- Deletions can be handled by periodically rebuilding the whole array.

Example:



Insert Time Lower Bounds

- Optimal algorithms are mostly known since 80's.
- Our recent work gives **tight** asymptotic lower bounds, building on ideas from Dietz, Seiferas, and Zhang.
- All bounds are amortized, some known upper bounds are even worst-case.

Insertion Time	Array Size
$\Theta\left(\log^2 n \cdot \log \frac{n}{f(n)}\right)$	$m = n + f(n),\ f(n) \geq 1$
Upper bound: [Zhang '93]	
Lower bound: [Bulánek, Koucký, Saks STOC '12]	
$\Theta\left(\log^2 n\right)$	$m = cn,\ c > 1$
Upper bound: [Itai, Konheim, Rodeh '81]; [Itai, Katriel '07] simplified; [Willard '92] worst-case	
Lower bound: [Bulánek, Koucký, Saks STOC '12]	
$\Theta\left(\frac{\log^2 n}{\log f(n)}\right)$	$m = n \cdot f(n),\ f(n) = o(n)$
Upper bound: [Itai, Konheim, Rodeh '81]	
Lower bound: [Babka, Bulánek, Čunát, Koucký, Saks manuscript '12]	
$\Theta\left(\frac{\log n}{\log f(n)}\right)$	$m = n^{f(n)},\ f(n) > 1$
Upper bound: [Bulánek, Koucký, Saks STOC '12]	
Lower bound: [Babka, Bulánek, Čunát, Koucký, Saks ESA'12]	

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╦ Supported by SVV project nr. 265 314 and by projects of Grant Agency of Charles University nr. 265 111 and 344 711.

╬ Partially supported by the Sino-Danish Center CTIC grant 61061130540, by GAČR P202/10/0854, GA AV ČR grant IAA100190902, Center of Excellence CE-ITI (P202/12/G061 of GA ČR), and RVO 67985840.

╩ Also supported in part by NSF under grant CCF-0832787.