MADALGO CENTER FOR MASSIVE DATA ALGORITHMICS

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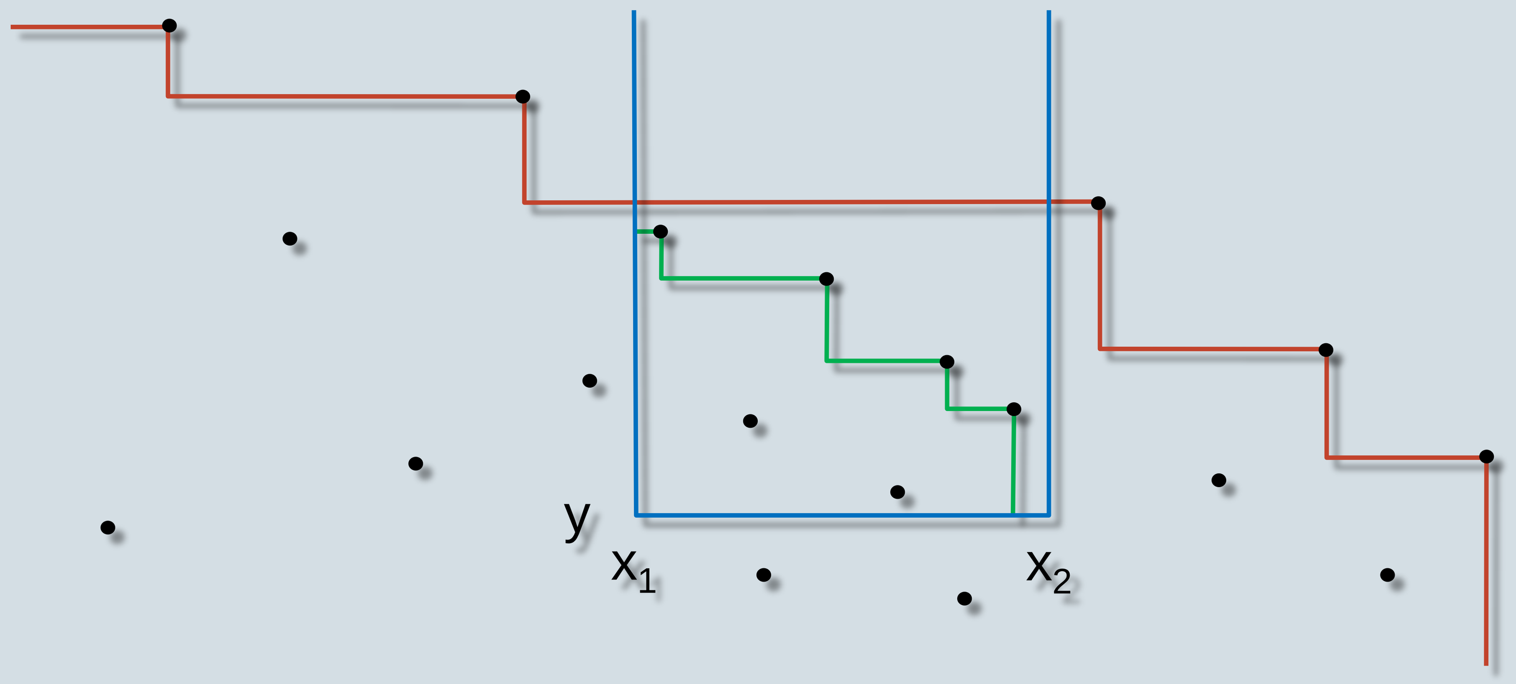
I/O-Efficient Dynamic Planar Range Skyline Queries

Problem Definition

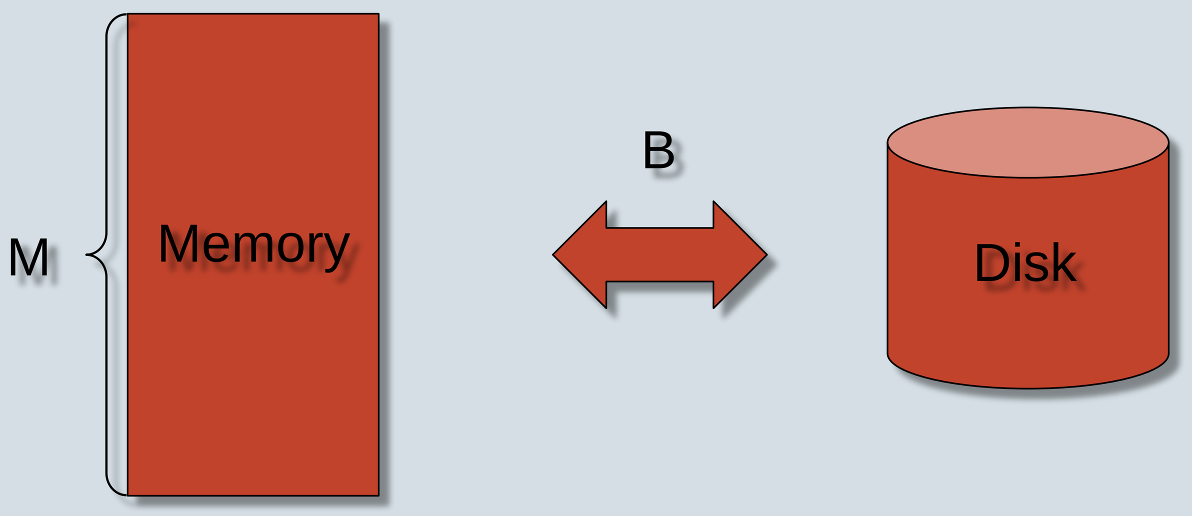
We study the problem of maintaining a planar point set $P \subseteq R^2$ under the modifications of insertion and deletion. Given two points p,q $\in R^2$ we say that p *dominates* q iff $x(p) \geq x(q) \wedge y(p) \geq y(q)$, i.e. all coordinates of p are larger than those of q. The maximal points of $S \subseteq P$ are all the points of S that are not dominated by any point in S.



We want to support 3-sided orthogonal skyline reporting queries for the point set P, i.e. reporting all maximal points in S∩P where $S=[x_1;x_2] \times [y;\infty]$.

The I/O Model

In the I/O model we have a memory capable of holding M elements and a disk of infinite size. When we read or write to the disk we can read or write B elements at a time, and the cost is 1 I/O, the cost is the same if we read or write less than B elements at a time, hence we want to batch reads and writes together.



Previous / Our Results

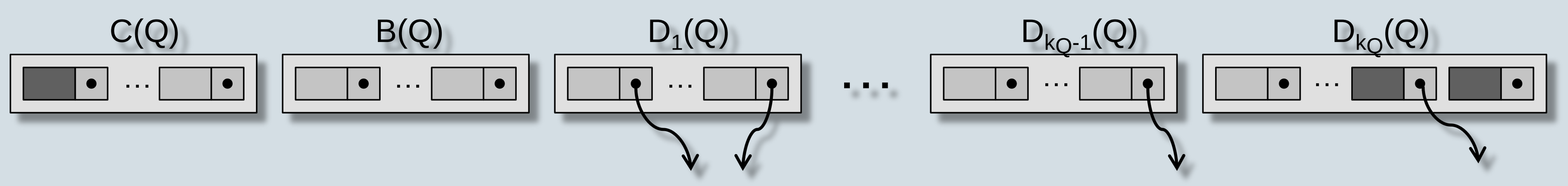
Our result is the first dynamic orthogonal skyline reporting data structure in the I/O model, all previous results are either not fully dynamic or are in a different model.

Reference	Model	Insert / Delete	Query
[BT 2011]	Pointer Machine	O(log n)	O(log n + t)
[BT 2011]	RAM	O(log n / log log n)	O(log n / log log n + t)
[KTT 2012]	I/O	$O(\log_{2B^\varepsilon} n)$	$O(\log_{2B^\varepsilon} n + t/B^{1-\varepsilon})$

I/O-CPQA

The I/O efficient Catanable Priority Queue with Attrition (I/O-CPQA) supports the following operations in O(1) I/O's worst-case and O(1/B) I/O's amortized:

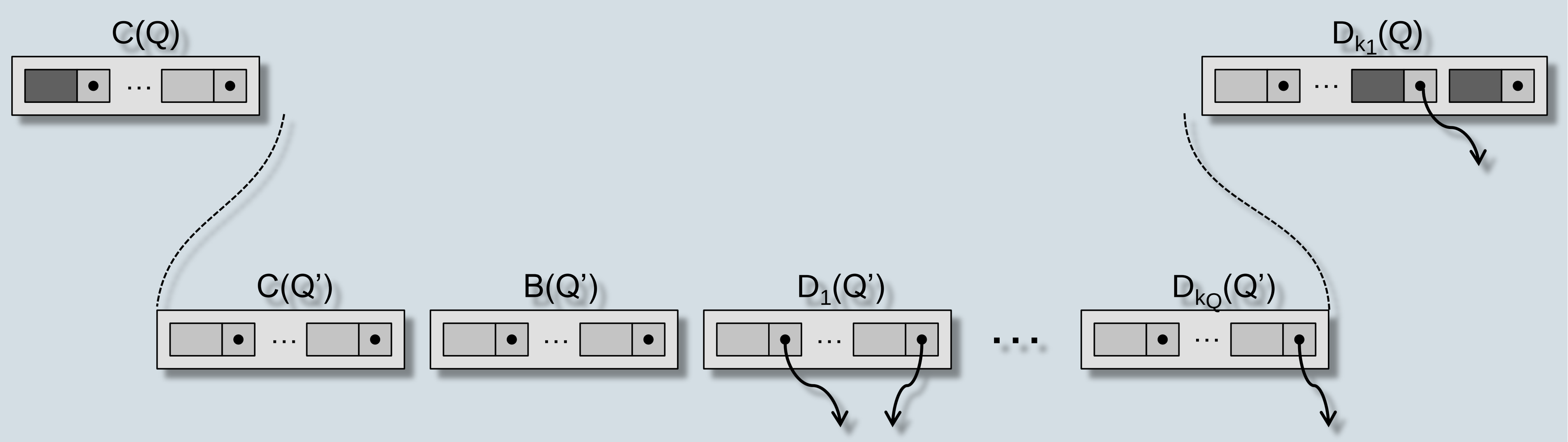
- InsertAndAttrite(Q,e) – appends e to the end of Q and deletes all attrited elements in Q that are smaller than e, i.e. returns Q'={e' ∈Q | e' < e} U {e}.
- DeleteMin(Q) – deletes the minimum element e = min(Q) from Q and returns e and Q\{e}.
- ConcatenateAndAttrite(Q_1,Q_2) – Deletes all attrited elements in Q_1 that are smaller than e=min(Q_2) and prepends the non-attrited elements onto Q_2, i.e. returns Q'={e' ∈Q_1 | e' < e} U Q_2.



A *record* consists of a buffer of [b;4b] elements and a pointer to another I/O-CPQA Q where max(b) < min(Q). An I/O-CPQA Q consists of $2+k_Q$ functional dequeus C,B,D_1, …, D_{kQ} of records. All records in the dequeue fulfills max(p) < min(q) where record p precedes record q in the same dequeue. Also max(C) < min(B) < min(D_1) and min(D_1) is the smallest element in all of the dirty dequeues D_i. The essential size invariant to guarantee the bounds is:

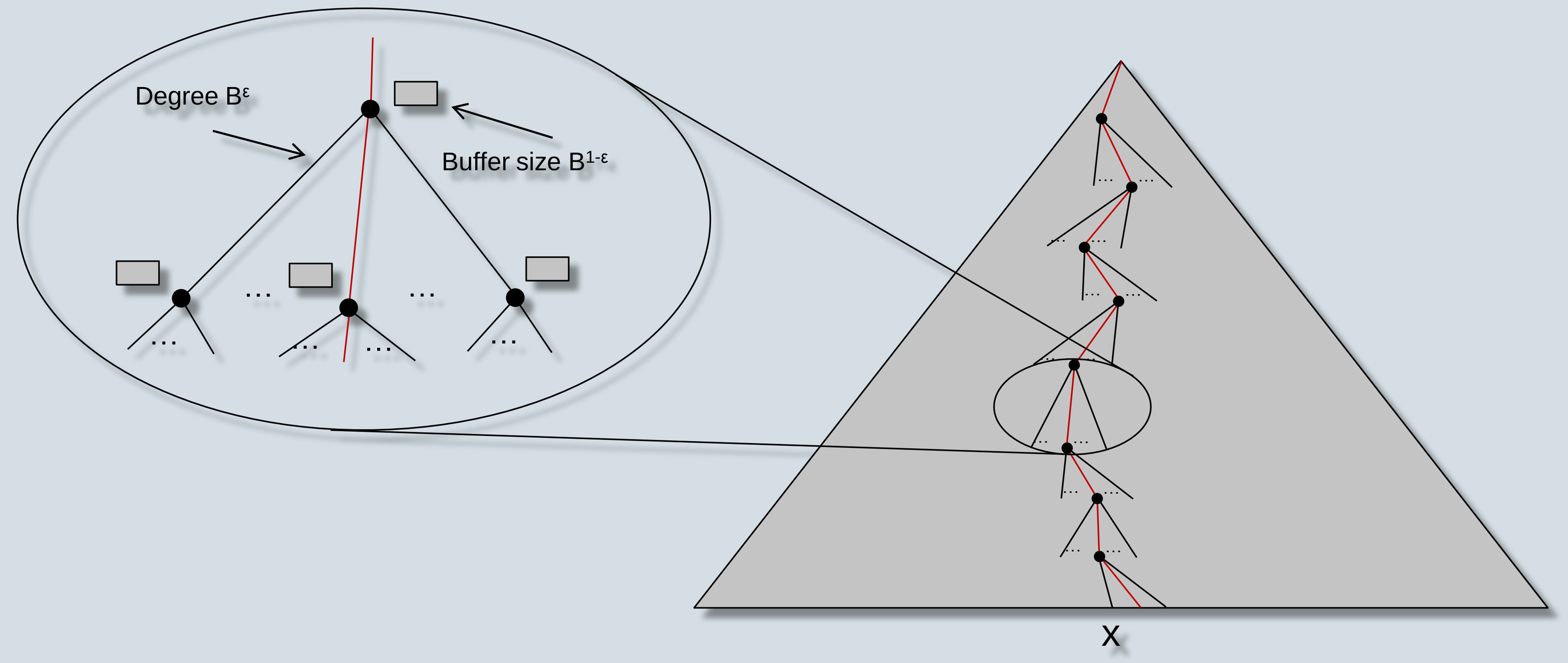
$$|C(Q)| \geq \sum_{i=1}^{k_Q} |D_i(Q)| + k_Q - 1 \qquad (*)$$

When we delete the minimum element from the I/O-CPQA we delete from C, if this violates (*) then we take records out of B and put them into C, else if k_Q>1 we merge D_{kQ-1} and D_{kQ} else we put the first record v of D_1 into C and we put the I/O-CPQA Q' that v points to into Q, see the figure belew.

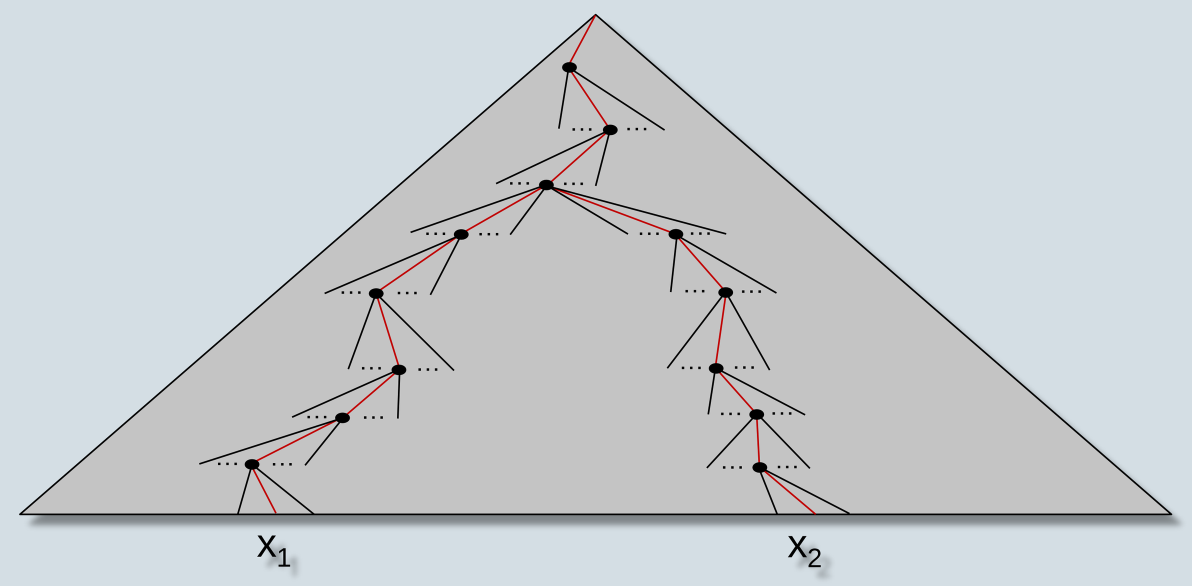


2D I/O Maxima

We build a $(2B^\varepsilon;4B^\varepsilon)$-tree with each leaf being an I/O-CPQA of $B^{1-\varepsilon}$ elements and each internal node stores the concatenation of its childrens I/O-CPQAs. This takes O(n) space since the I/O-CPQAs of the internal nodes will only use O(1) blocks of space as they are functional.



When we receive an update we find the position in the tree and discards all I/O-CPQAs on the same path and reconstruct the path bottom up.



When we do a 3-sided range query for $[x_1;x_2] \times [y;\infty]$ we concatenate the $O(B^\varepsilon \log_{2B^\varepsilon} n)$ I/O-CPQAs in $O(\log_{2B^\varepsilon} n)$ I/Os and then call DeleteMin on the resulting I/O-CPQA until the minimum element m returned is m < y, this then takes $O(t/B^{1-\varepsilon})$ I/Os.

Future Work

It is still an open problem whatever it is possible to obtain bounds of $O(\log_B n)$ for updates and $O(\log_B n + t/B)$ for queries, or it is possible to show a lower bound. The ε in our bounds comes because we need to load B^ε I/O-CPQAs and concatenate them in O(1) I/Os, which we are only able to do if the buffer size of each record is $B^{1-\varepsilon}$.

References

[KTT 2012] Kejlberg-Rasmussen, Tsakalidis, Tsichlas – *I/O-Efficient Dynamic Planar Range Skyline Queries.* Submitted to SODA 2013.

[BT 2011] Brodal, Tsakalidis – *Dynamic Planar Range Maxima Queries.* ALP 2011.