Verifiably Truthful Mechanisms

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Mechanism $M(\theta_1, \ldots, \theta_n)$
Mechanism $M$ is **strategyproof** if $u_i(M(\theta_i, \theta_{-i})) \geq u_i(M(\theta'_i, \theta_{-i}))$, for each agent $i$ and profile $\theta'_i$.

When do agents follow the protocol?
- If strategyproof, etc
Optimal mechanisms can get really hairy in richer settings (with and without money)

To be or not to be truthful?

When is a mechanism “simple” enough to be implemented in “reality”?
Goal: Design truthful mechanisms for which truthfulness can be verified efficiently.
Three Step Approach

I. Specifying the structure of mechanisms

II. Constructing a verification algorithm

III. Measuring the quality of the mechanisms
Case Study

Facility Location Problem: Mayor plans to open a new library

Inquire inhabitants about preferred locations
Facility Location

Decide location based on the reports
Facility Location

Agents $N = \{1, \ldots, n\}$; outcome space: real line

Input of agent $i$: bliss point $x_i$

Output: Location of the facility
Objective Functions

Social cost = sum of distances to the facility
Objective Functions

**Maximum cost** = max distance of any agent from the facility
Deterministic Mechanisms: Structure

Mechanisms: decision trees of bounded size

Internal nodes: input comparisons

Locations in the leaves
(= convex combinations of inputs)
Dictatorship
Average

\[
\frac{x_1 + x_2 + \ldots + x_n}{n}
\]
Decision Trees

Dictatorship

\[ x_1 \]

Average

\[ \frac{x_1 + x_2 + \ldots + x_n}{n} \]

Median

\[ x_1 \geq x_2 \]

\[ x_2 \geq x_3 \]

\[ x_1 \geq x_3 \]

\[ x_2 \]

\[ x_3 \]

\[ x_1 \]

\[ x_3 \]

\[ x_1 \]
Deterministic Mechanisms: Verifier

**Input:** tree mechanism $T$

**Output:** decide if $T$ is truthful

For all $i \in \{1, \ldots, n\}$:

For every two leaves $L, L'$:

// Find deviation of $i$ from $L$ to $L'$

Solve $LP$ in $x_1, \ldots, x_n, x'_i$

If solution exists Then:

Return **False**

Return **True**

Verifier runtime: polynomial in $n$ and $|T|$
Deterministic Mechanisms: Bounds for Social Cost

**Theorem:** Deterministic decision trees of polynomial size approximate the social cost within a factor of $\Theta(n/\log(n))$

- Pick the median of the first $\log(n)$ agents
Deterministic Mechanisms: Bounds for Max Cost

**Theorem:** Deterministic decision trees of polynomial size approximate the max cost within a factor of 2.

- Pick any dictator
- The average is optimal but not truthful
Randomized Mechanisms: Structure

Tree $T_i$ : parameterized by some set of agents $Z_i$ selected according to a distribution $D_i$
Input: Randomized mechanism \( M \)
Output: decide if \( M \) is universally truthful

For each tree \( T_i \):

Run deterministic verifier for some binding of the agents in \( T_i \)

Verifier runtime: polynomial in \( n \) and \( \sum_{i=1,k} |T_i| \)
Randomized Mechanisms: Bounds for Social Cost

Randomized Mechanism:

Select $K$ agents from $N = \{1, \ldots, n\}$ at random without replacement
- Output the median of the sampled set

$K = n$: Median mechanism

... 

$K = 1$: Random dictatorship
Randomized Mechanisms: Bounds for Social Cost

**Theorem:** For every $n$ and $0 < \varepsilon < 1/10$, there exists a universally truthful randomized decision tree of polynomial size in $n$ that approximates the social cost to a factor of $1 + \varepsilon$.

- Sample a subset $S$ of $O(\ln(n/\varepsilon) / \varepsilon^2)$ agents and output the median of $S$
Randomized Mechanisms: Bounds for Max Cost

**Theorem:** For each $\epsilon > 0$, there exists no universally truthful mechanism that approximates the max cost within a factor of $2 - \epsilon$.

- Strict separation from truthfulness in expectation (where the optimal ratio is 3/2)
Discussion

Truthfulness in expectation

Many settings of interest

- Other measures for capturing properties of economic systems?
- Can the mechanisms be learned?