Verifiably Truthful Mechanisms

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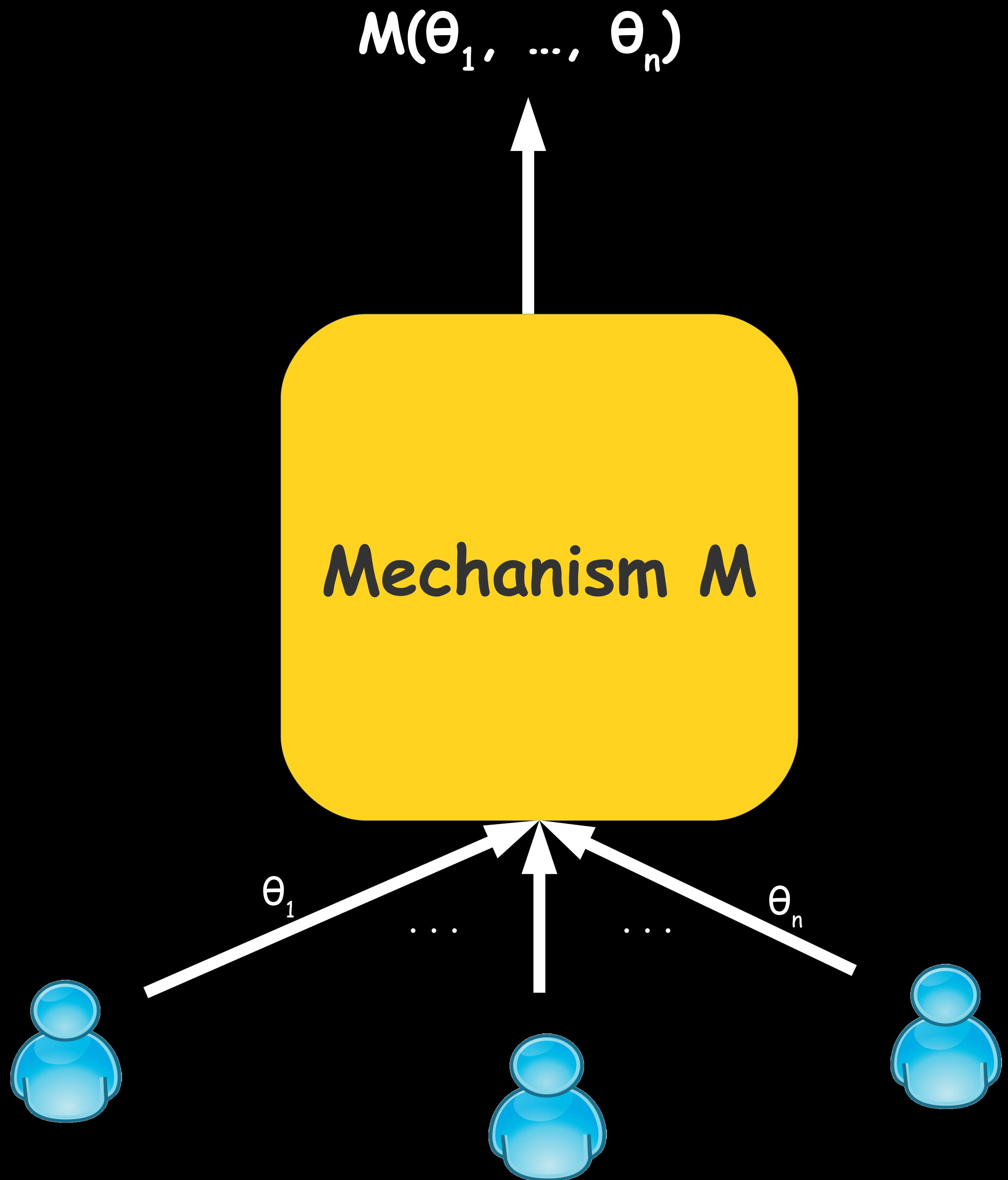
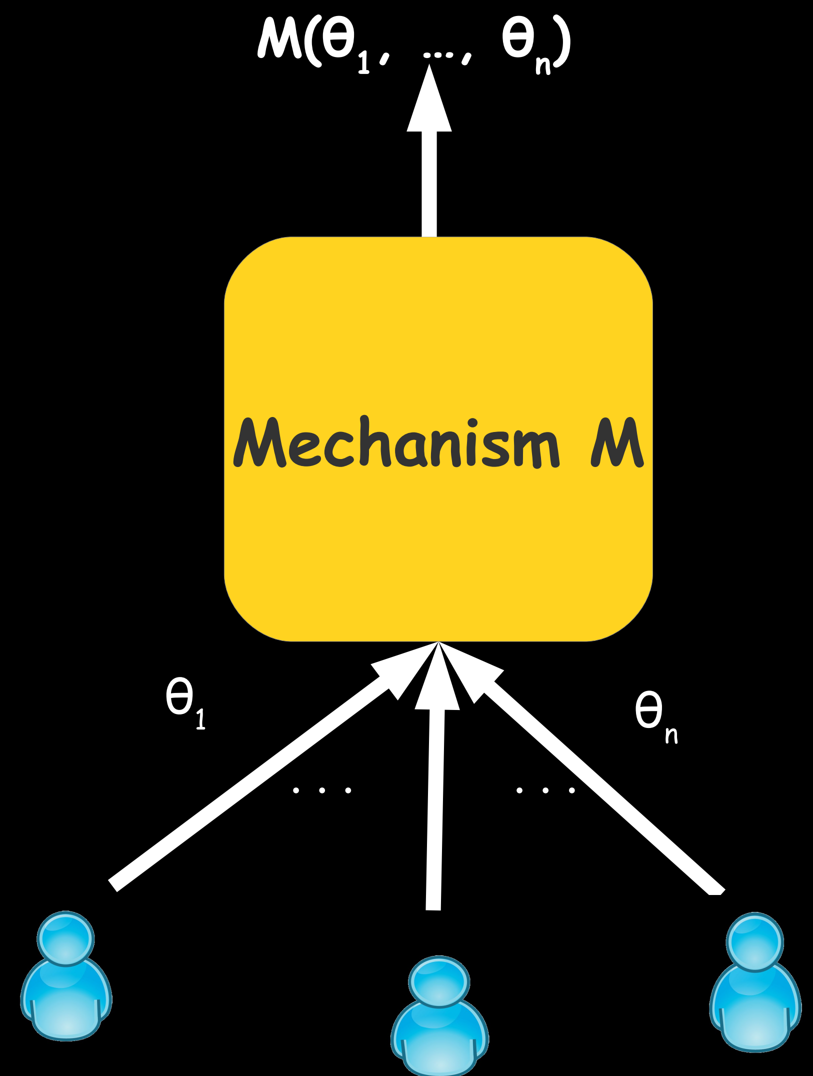


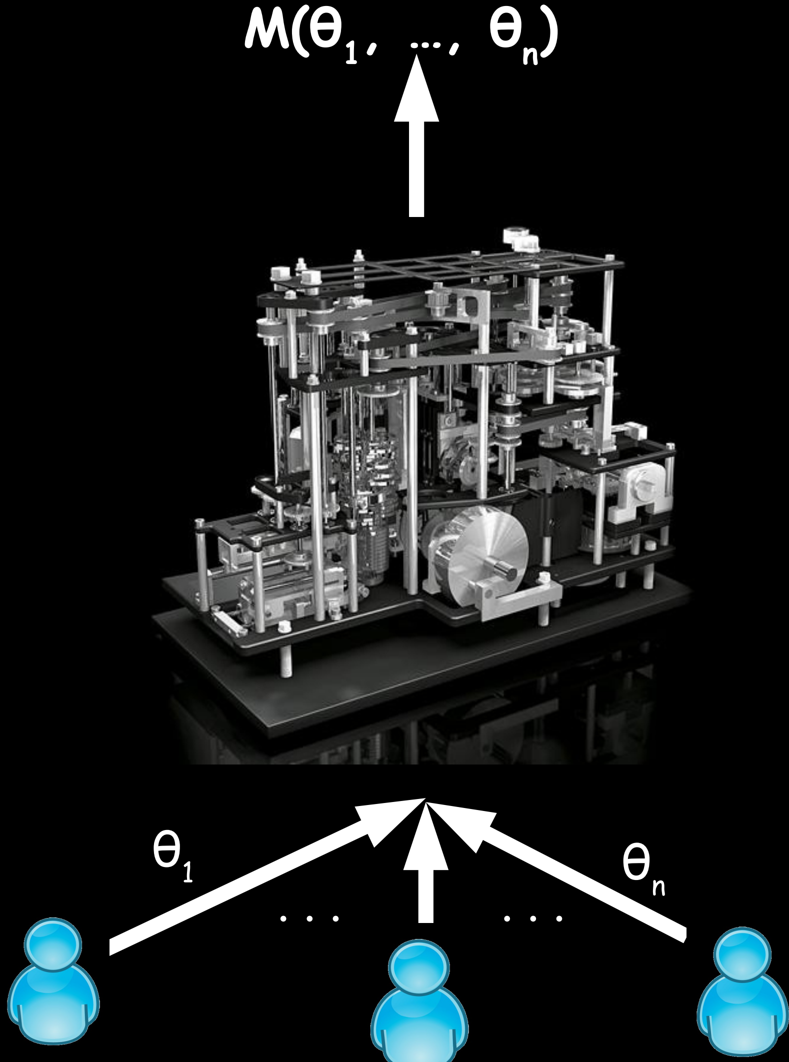
$M(\theta_1, \ldots, \theta_n)$
Mechanism M
θ_1
. . .
. . .
θ_n

Mechanism M is **strategyproof** if $u_i(M(\theta_i, \theta_{-i})) \geq u_i(M(\theta_i', \theta_{-i}))$, for each agent i and profile θ_i'

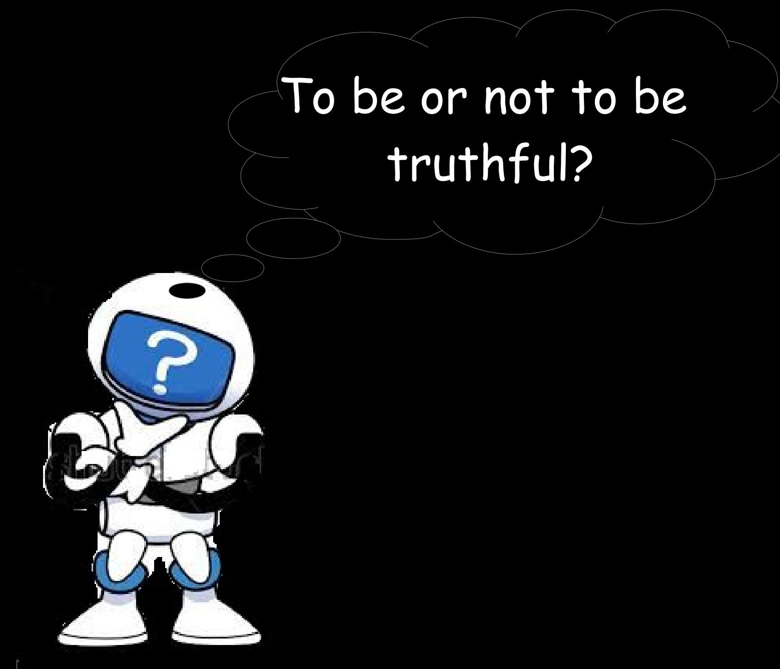


When do agents follow the protocol?

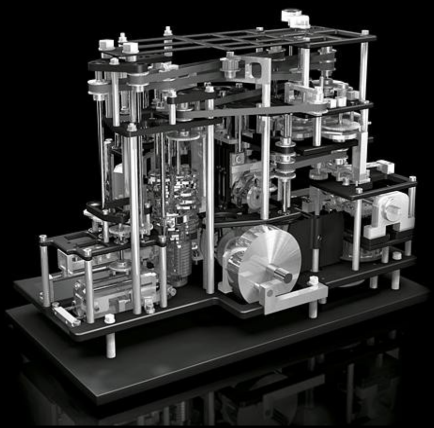
- **If strategyproof, etc**



Optimal mechanisms can get really hairy in richer settings (with and without money)



When is a mechanism "simple" enough to be implemented in "reality"?



Goal : Design truthful mechanisms for which truthfulness can be verified efficiently

Three Step Approach

I. Specifying the structure of mechanisms

II. Constructing a verification algorithm

III. Measuring the quality of the mechanisms

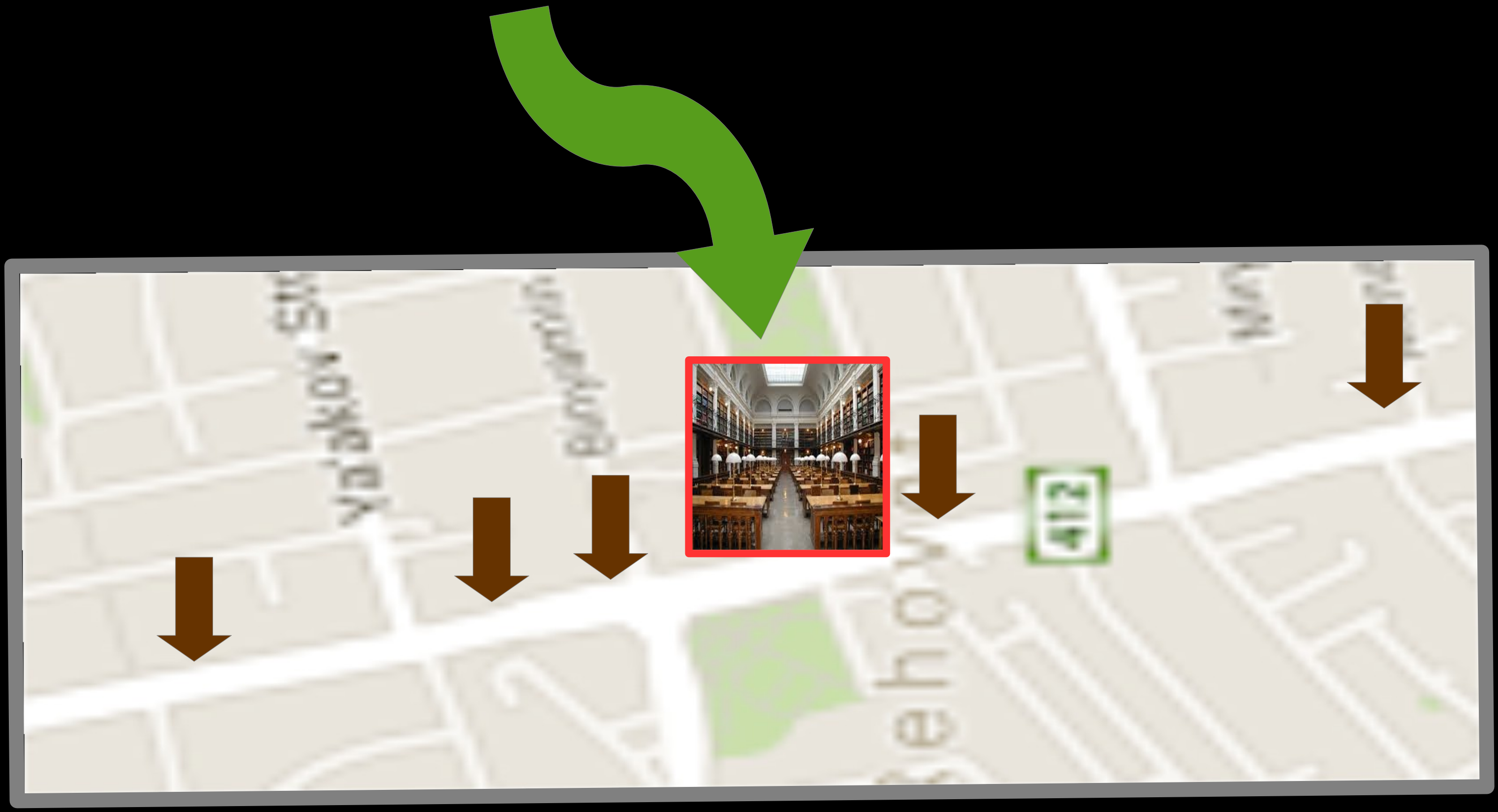
Case Study

Facility Location Problem: Mayor plans to open a new library

Inquire inhabitants about preferred locations

Facility Location

Decide location based on the reports



Facility Location

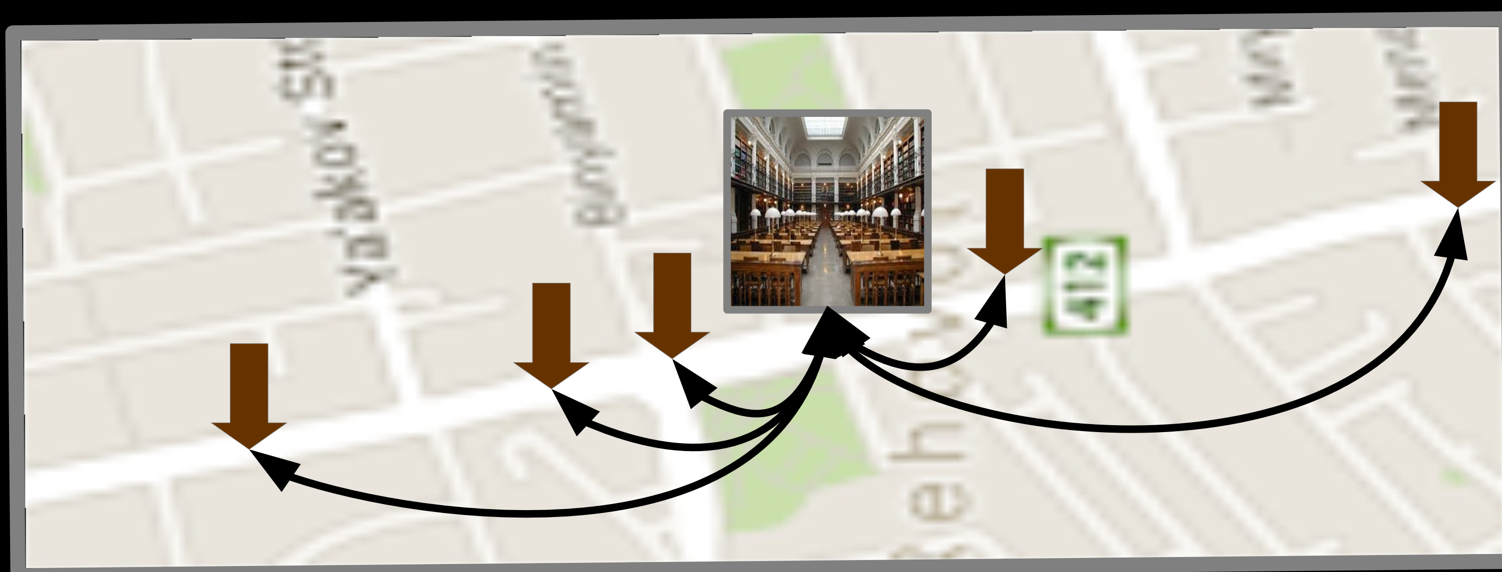
Agents N = {1, …, n} ; outcome space : real line

Input of agent i : bliss point x_i

Output: Location of the facility

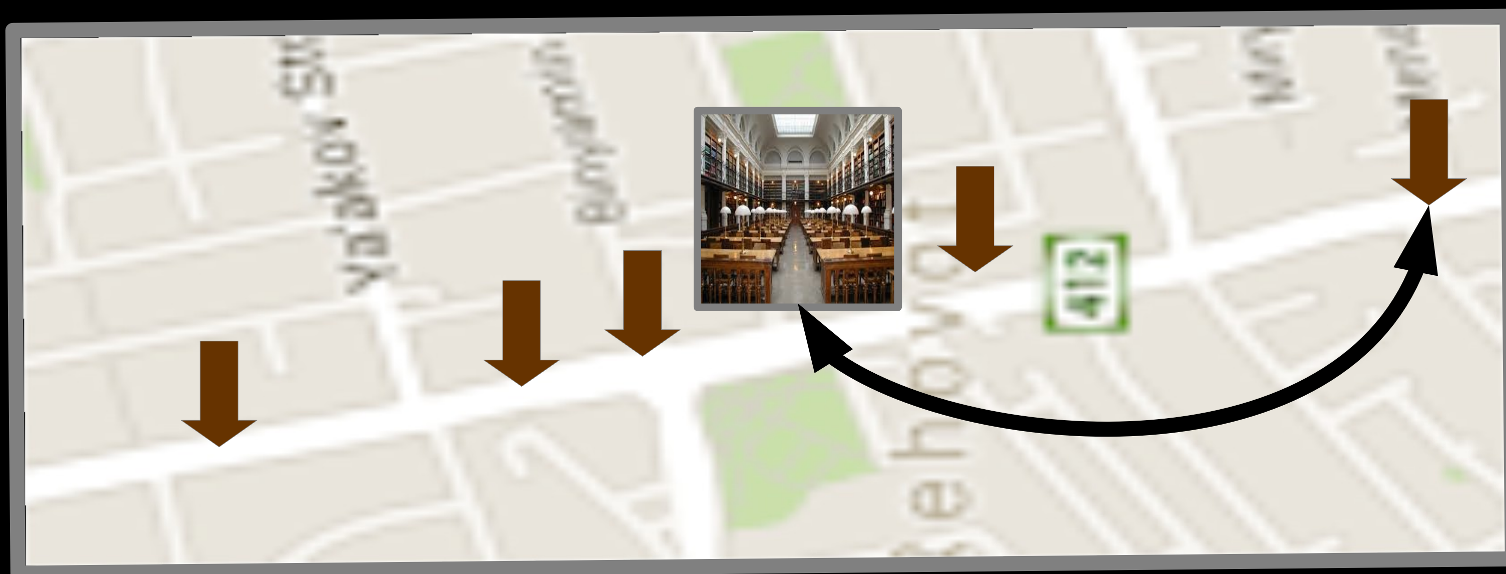
Objective Functions

Social cost = sum of distances to the facility



Objective Functions

Maximum cost = max distance of any agent from the facility

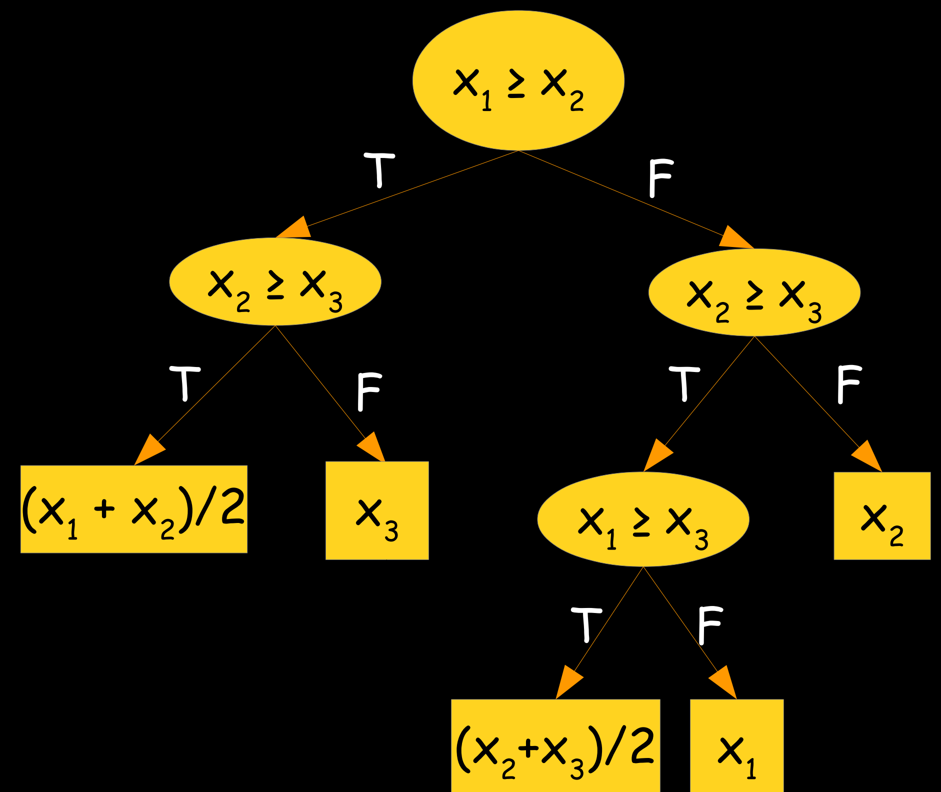


Deterministic Mechanisms: Structure

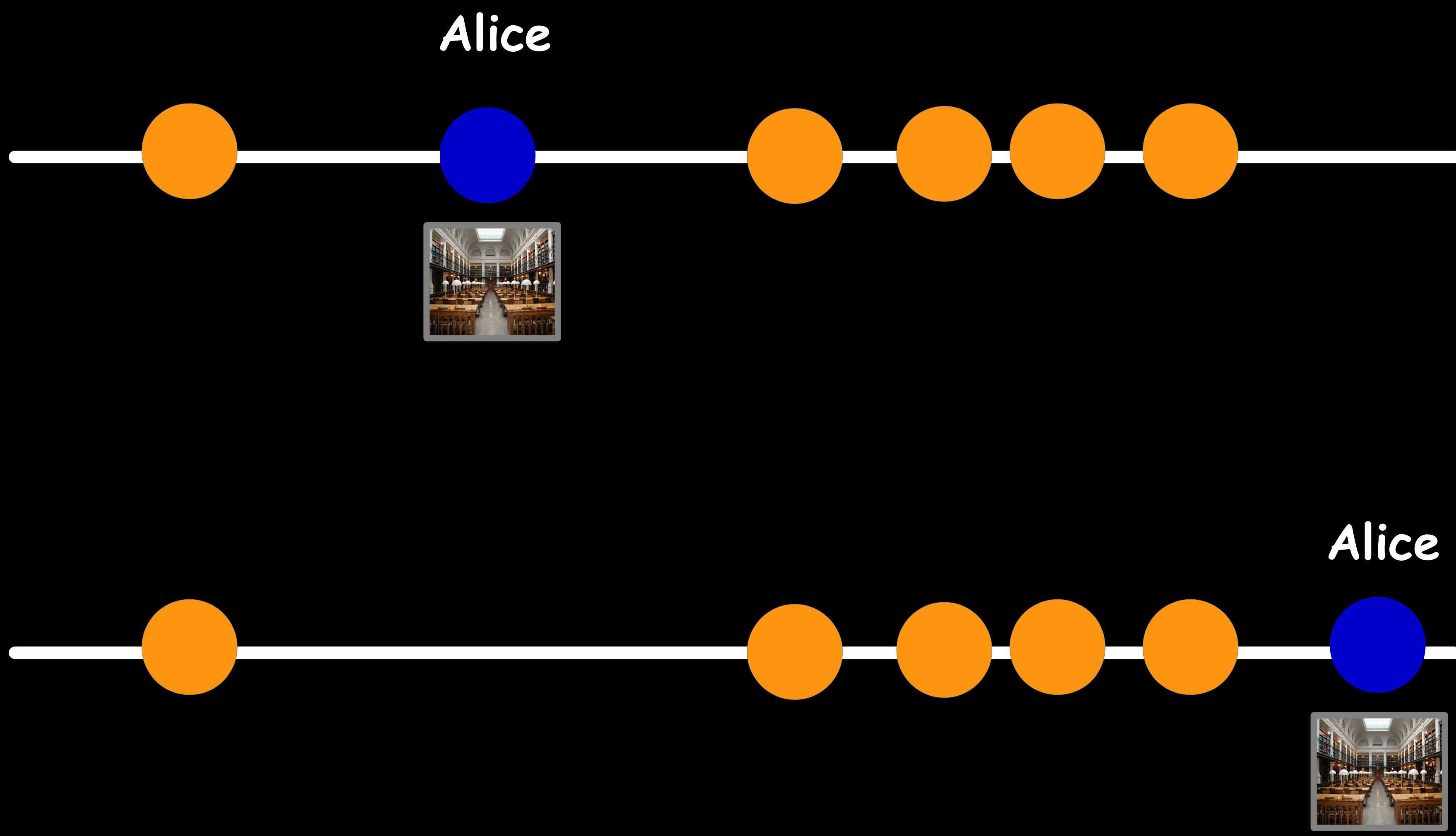
Mechanisms : decision trees of bounded size

Internal nodes: input comparisons

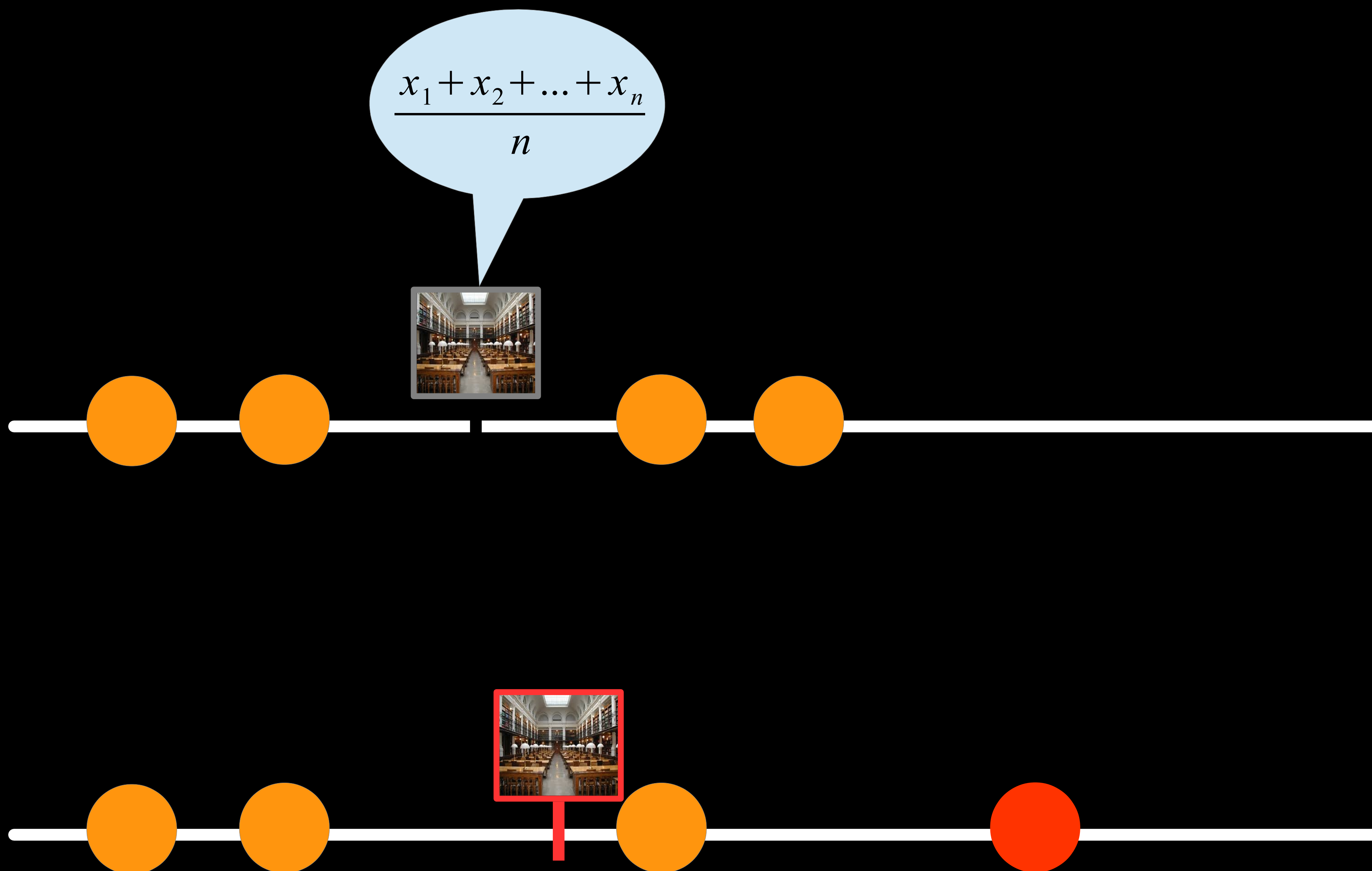
Locations in the leaves
(convex combinations of inputs)



Dictatorship



Average



Decision Trees

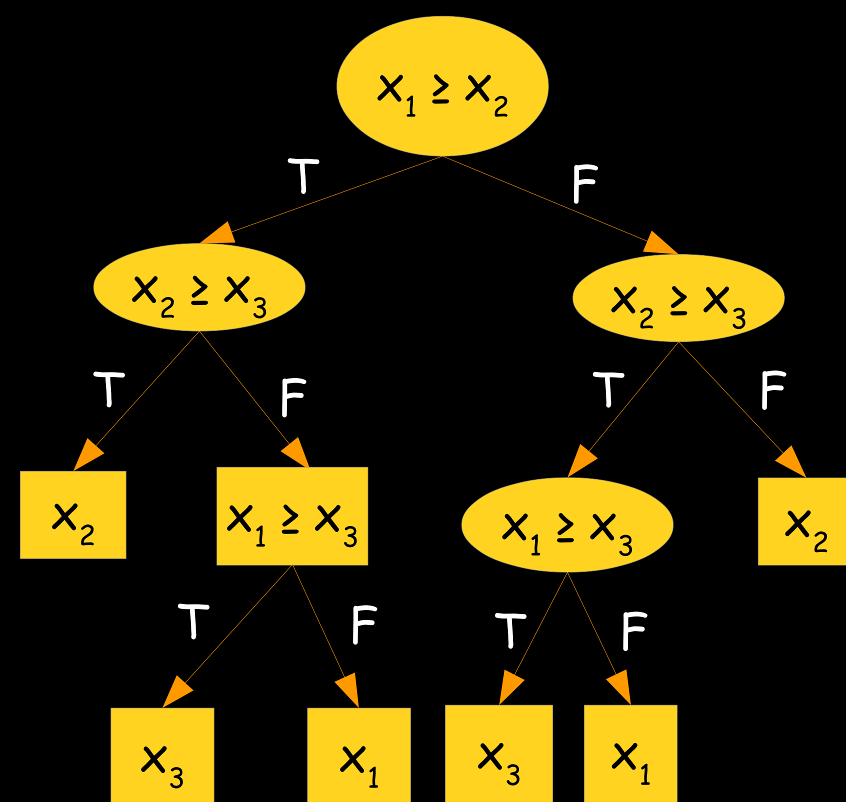
Dictatorship

x_1

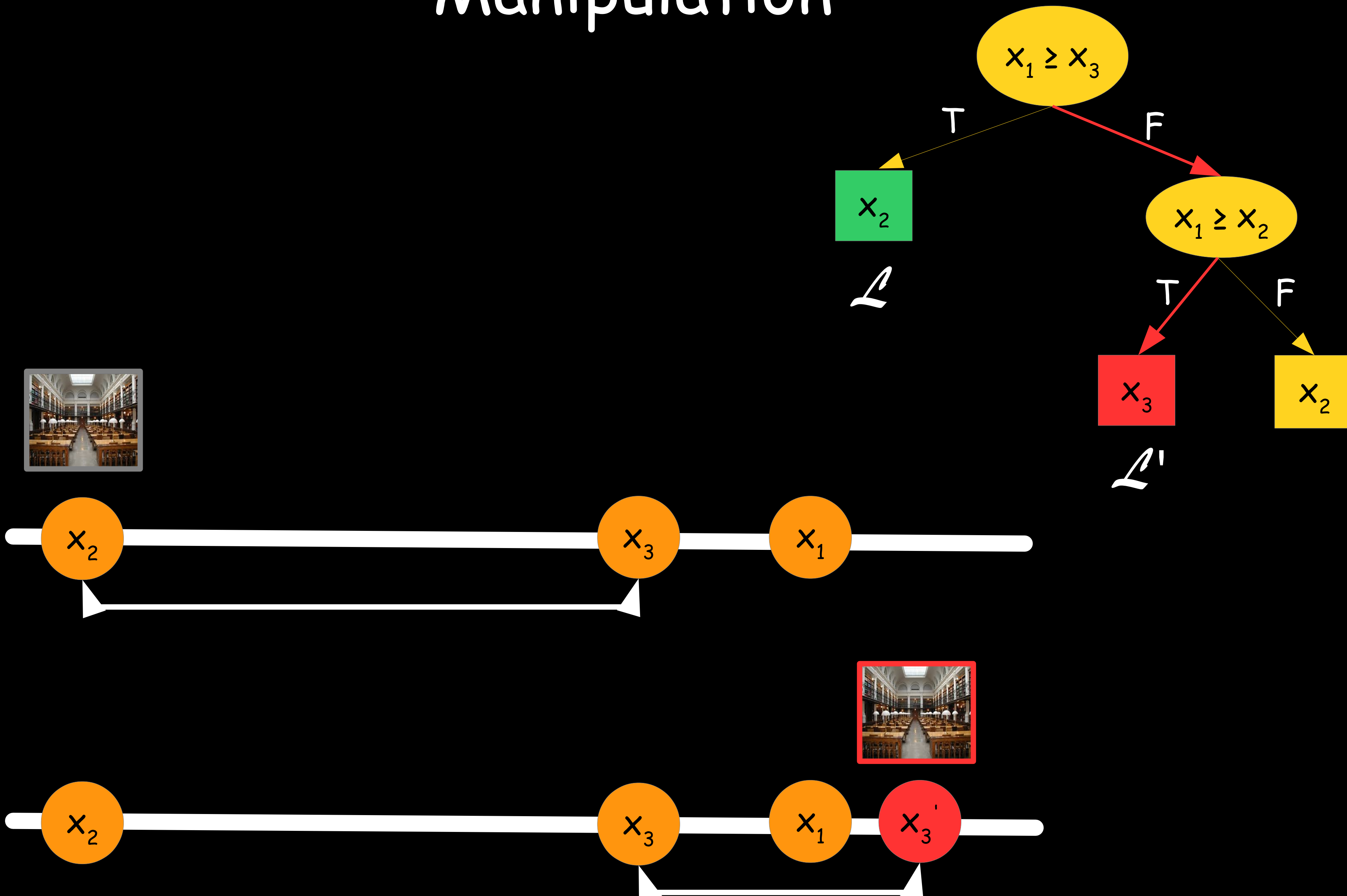
Average

$$\frac{x_1+x_2+\ldots+x_n}{n}$$

Median



Manipulation



Deterministic Mechanisms : Verifier

Input: *tree mechanism T*

Output : decide if T is truthful

For all i ∈ {1, …, n}:

For every two leaves *L*, *L'*:

// Find deviation of i from L to L'

Solve *LP* in $x_1, \dots, x_n, x_i'$

If solution exists Then:

Return **False**

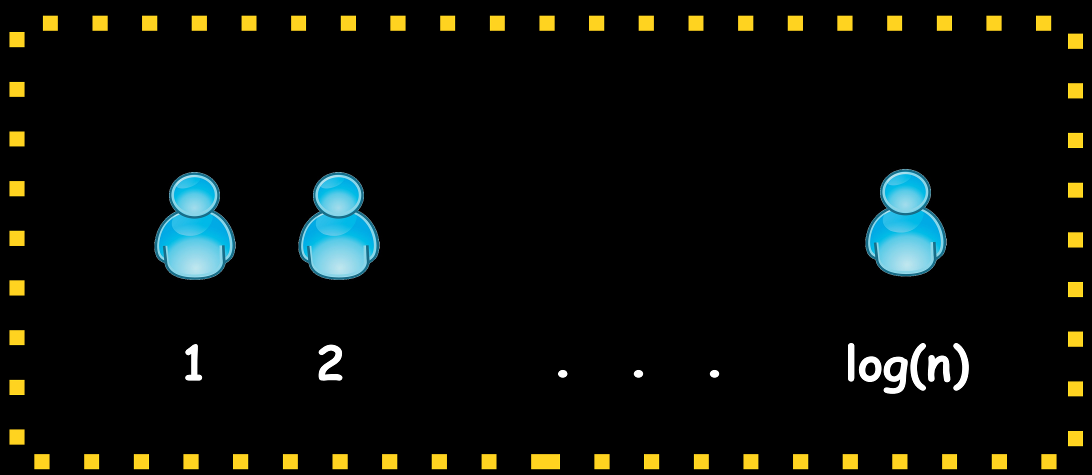
Return **True**

Verifier runtime: polynomial in n and |T|

Deterministic Mechanisms : Bounds for Social Cost

Theorem: Deterministic decision trees of polynomial size approximate the **social cost** within a factor of $\Theta(n/\log(n))$

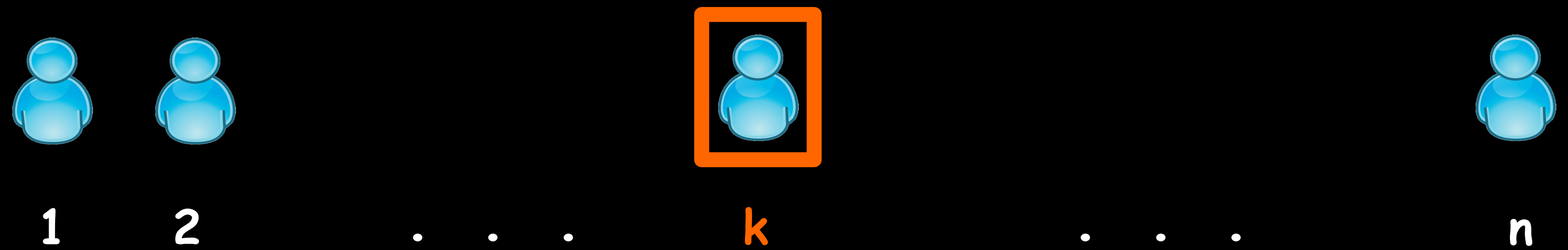
- Pick the median of the first log(n) agents



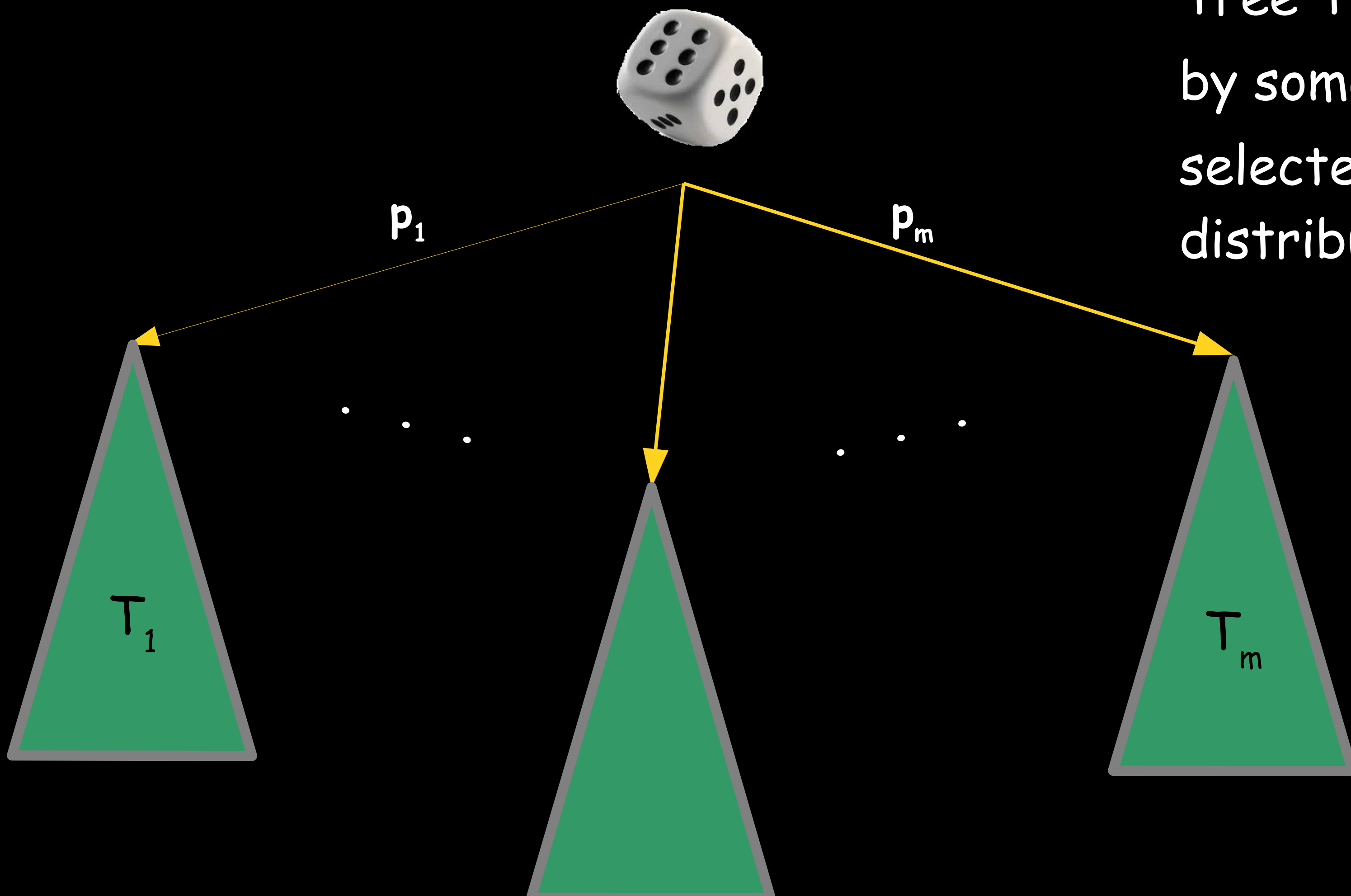
Deterministic Mechanisms : Bounds for Max Cost

Theorem: Deterministic decision trees of polynomial size approximate the **max cost** within a factor of 2.

- Pick any dictator
- The average is optimal but not truthful



Randomized Mechanisms: Structure



Tree T_i : parameterized by some set of agents Z_i selected according to a distribution D_i

Randomized Mechanisms : Verifier

Input: Randomized mechanism M
Output: decide if M is universally truthful

For each tree T_i :

Run deterministic verifier for some binding of the agents in T_i

Verifier runtime : polynomial in n and $\Sigma_{i=1,k} |T_i|$

Randomized Mechanisms : Bounds for Social Cost

Randomized Mechanism:

Select K agents from N = {1, …, n} at random without replacement

- Output the median of the sampled set

K = n : Median mechanism

. . .

K = 1 : Random dictatorship

Randomized Mechanisms : Bounds for Social Cost

Theorem: For every n and $0 < \varepsilon < 1/10$, there exists a universally truthful randomized decision tree of polynomial size in n that approximates the social cost to a factor of $1+\varepsilon$.

- Sample a subset S of $O(\ln(n/\varepsilon) / \varepsilon^2)$ agents and output the median of S

Randomized Mechanisms : Bounds for Max Cost

Theorem: For each $\varepsilon > 0$, there exists no universally truthful mechanism that approximates the max cost within a factor of $2 - \varepsilon$.

- Strict separation from truthfulness in expectation (where the optimal ratio is 3/2)

Discussion

Truthfulness in expectation

Many settings of interest

- Other measures for capturing properties of economic systems?
- Can the mechanisms be learned?