### **Backdoors in Cryptography**

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### Cryptography

# Computational hardness

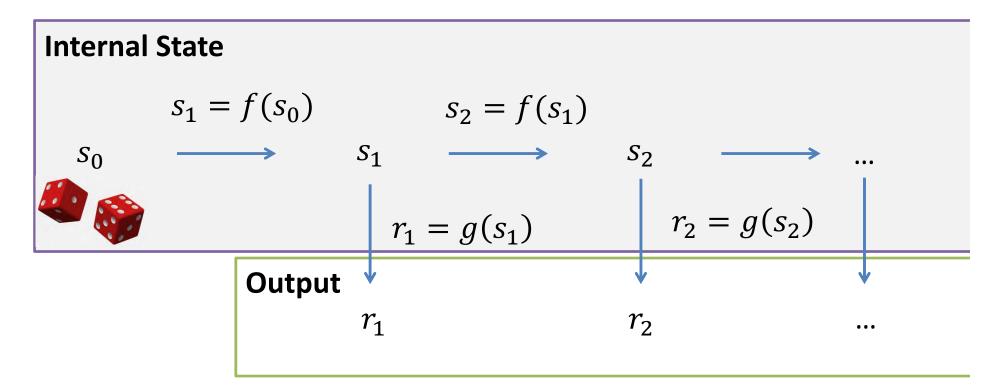


Unpredictability





### Pseudorandom generator



"Looks like truly random outputs"

### Discrete Logarithm Problem

• Logarithm: find x such that  $3^x = 81$ 

Discrete logarithm problem

 $3^x \mod 47 = 34$ 

Easy!
Just try them all!

### Discrete Logarithm Problem

• Logarithm: find *x* such that

$$3^{x} = 81$$

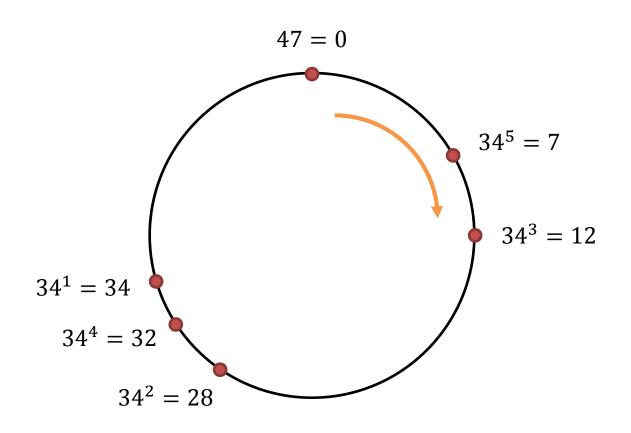
• Discrete logarithm problem

$$3^x \mod 4 \dots 7 = 3 \dots 4$$

1000 digits (3000 bits)

Hard!

### Computing mod 47



$$34^2 = 1156 = 47 \cdot 24 + 28$$
  $34^2 \mod 47 = 28$ 

### Dual\_EC (Simplified)

Two hardwired parameters

Initial (truly) random state

$$S_0$$

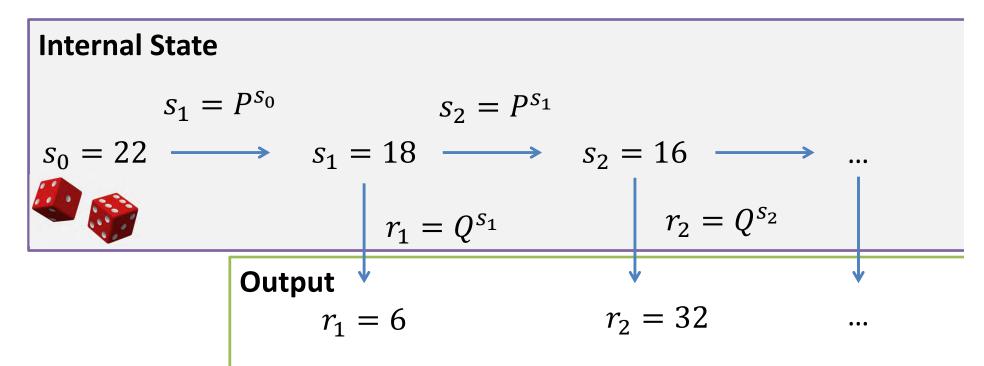
Compute next state

$$s_{i+1} = P^{s_i} \bmod N$$

Compute output

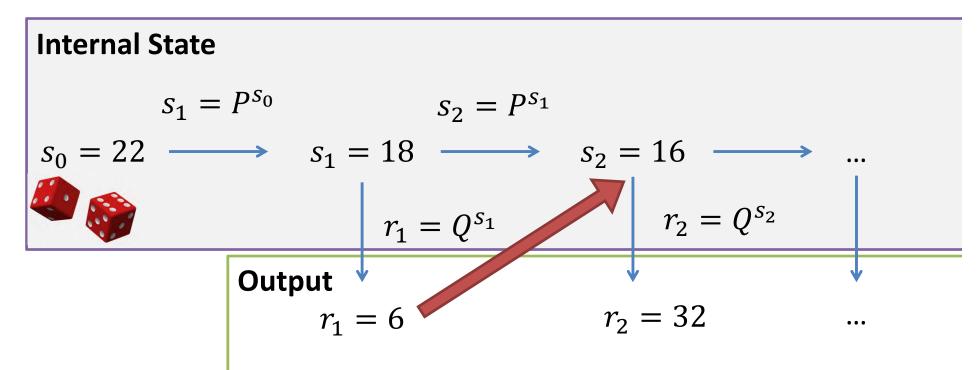
$$r_i = Q^{s_i} \mod N$$

# Dual\_EC (Simplified)



Q=3

# DUAL\_EC Backdoor(Simplified)



Given x such that

$$P = Q^{x}$$

Can recover internal state

$$s_2 = r_1^x$$

# DUAL\_EC Backdoor (Simplified)

#### The user

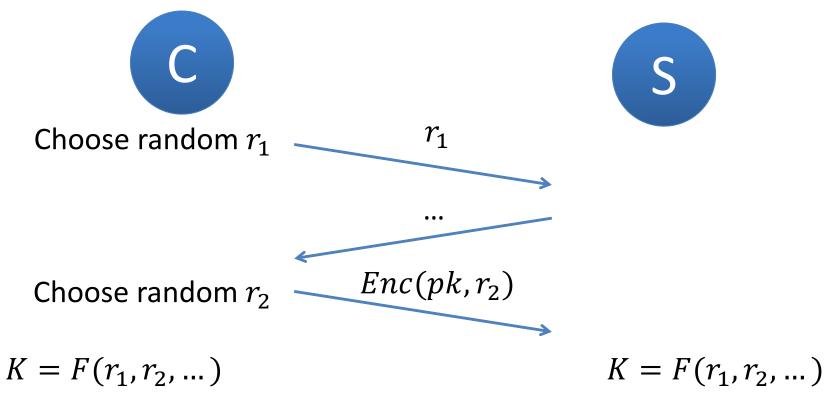
- Two parameters (P, Q)
- Compute next state  $s_{i+1} = P^{s_i} \mod N$
- Compute next output  $r_i = Q^{s_i} \mod N$

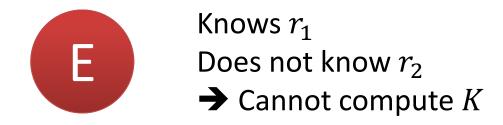
#### The attacker

- Keep x such that  $P = Q^x \mod N$
- Observe any output  $r_i$
- Compute next state  $s_{i+1} = r_i^x \mod N$
- Predict all future outputs!

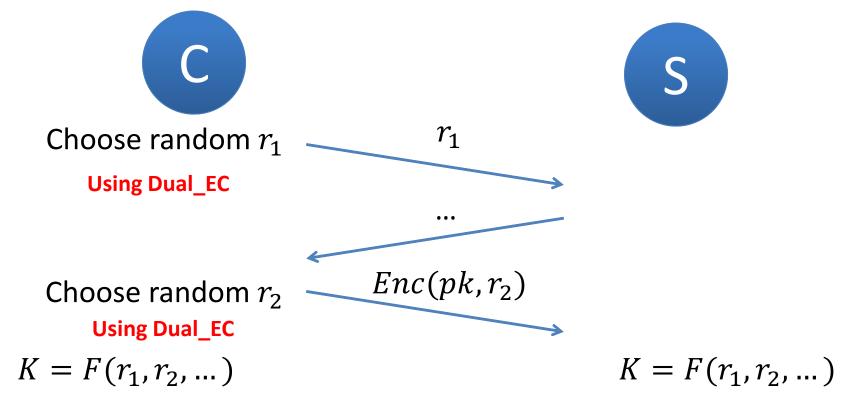
$$s_{i+1} = P^{s_i} = (Q^x)^{s_i} = (Q^{s_i})^x = r_i^x \mod N$$

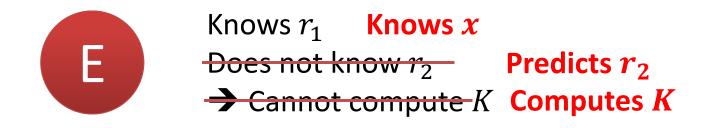
# HTTPS (Simplified)





### HTTPS with Backdoor (Simplified)





 Unless you choose your own parameters, DUAL\_EC is a really bad PRNG.

• If you use parameters (P,Q) chosen by an adversary, you grant the adversary the power to eavesdrop virtually any secure connection you can establish over the internet.

But this would never happen. Right?

Dual\_EC: a brief history

- Dual\_EC standardized by NIST with fixed parameters P,Q
- The standard is criticized at CRYPTO 2007

### Conclusion

- WHAT WE ARE NOT SAYING:
   NIST intentionally put a back door in this PRNG
- WHAT WE ARE SAYING:
   The prediction resistance of this PRNG (as presented in NIST SP800-90) is dependent on solving one instance of the elliptic curve discrete log problem.

(And we do not know if the algorithm designer knew this before hand.)

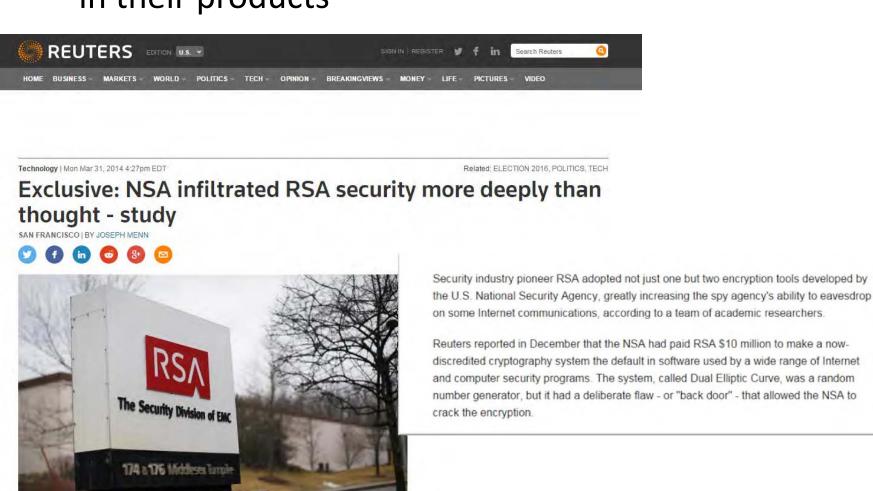
### Snowden reveals existence of Bullrun program

- (TS//SI//REL TO USA, FVEY) Insert vulnerabilities into commercial encryption systems, IT systems, networks, and endpoint communications devices used by targets.
- (TS//SI//REL TO USA, FVEY) Collect target network data and metadata via cooperative network carriers and/or increased control over core networks.
- (TS//SI//RELTO USA, FVEY) Leverage commercial capabilities to remotely deliver or receive information to and from target endpoints.
- (TS//SI//REL TO USA, FVEY) Exploit foreign trusted computing platforms and technologies.
- (TS//SI//REL TO USA, FVEY) Influence policies, standards and specification for commercial public key technologies.
- (TS//SI//REL TO USA, FVEY) Make specific and aggressive investments to facilitate the development of a robust exploitation capability against Next-Generation Wireless (NGW) communications.





 RSA accused of accepting 10M\$ to adopt Dual EC in their products



A sign marks the entrance to RAS's facility in Bedford, Massachusetts March 28, 2014.

RELITERS/BRIAN SNYDER

on some Internet communications, according to a team of academic researchers. Reuters reported in December that the NSA had paid RSA \$10 million to make a now-

discredited cryptography system the default in software used by a wide range of Internet and computer security programs. The system, called Dual Elliptic Curve, was a random number generator, but it had a deliberate flaw - or "back door" - that allowed the NSA to

### NIST investigates

```
Subject: [Fwd: RE: Minding our Ps and Qs in Dual_EC]
Date: Wednesday, October 27, 2004 at 12:09:25 PM Eastern Daylight Time
From: John Kelsey
To: larry.basham@nist.gov
----- Original Message
Subject: RE: Minding our Ps and Qs in Dual_EC
From: "Don Johnson" <DJohnson@cygnacom.com>
Date: Wed, October 27, 200411:42 am
To: "John Kel sey" < john. kel sey@ni st. gov>
John,
P=G.
Q is (in essence) the public key for some random private key.
It could also be generated like another canonical G, but NSA kyboshed
this idea, and I was not allowed to publicly discuss it, just in case you
may think of going there.
Don B. Johnson
----Original Message----
From: John Kelsey fmailto:john.kelsey@nist.gov]
Sent: Wednesday, October 27, 200411:17 AM
To: Don Johnson
Subject: Minding our Ps and Qs in Dual_EC
Do you know where Q comes from in Dual_EC_DRBG?
Thanks,
-John
```

 Juniper (who had been using Dual\_EC but with their own P,Q), discovers that in 2012 someone hacked their servers and changed the constants.



#### 2016

"We cannot build a backdoor that only works for a particular type of government, or only in the presence of a particular court order."

