Et eller andet datalogi…

Voronoi Diagrammer

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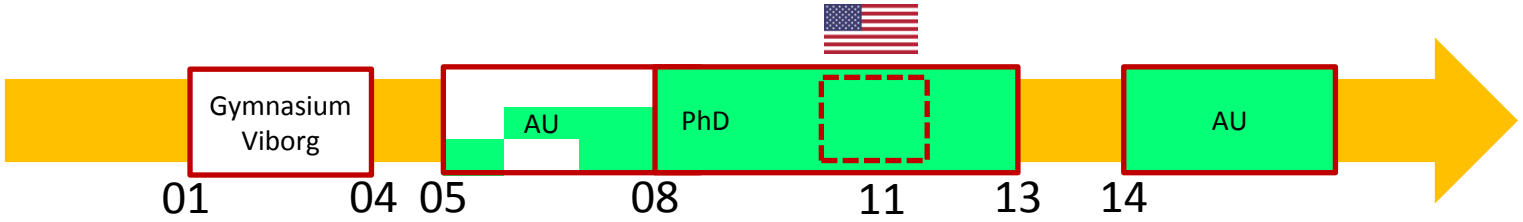
- Ph.d. Datalogi, Aarhus Universitet (2005-2013)
- Ansat ved Institut for Datalogi (2014-)
- Forskning og undervisning: Algoritmik



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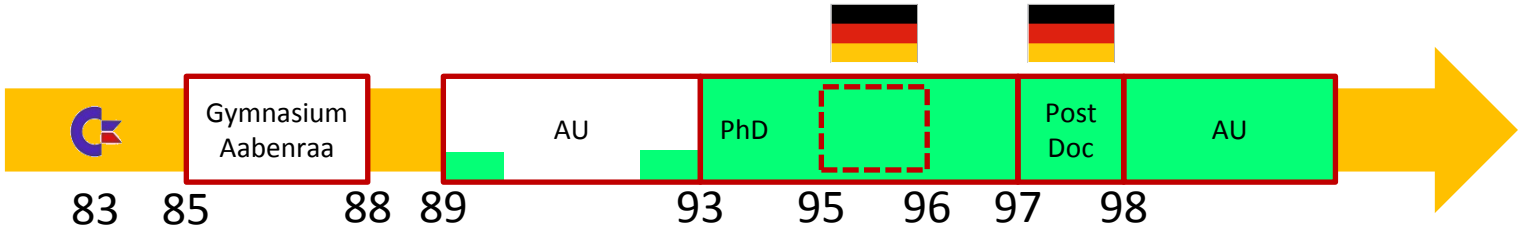
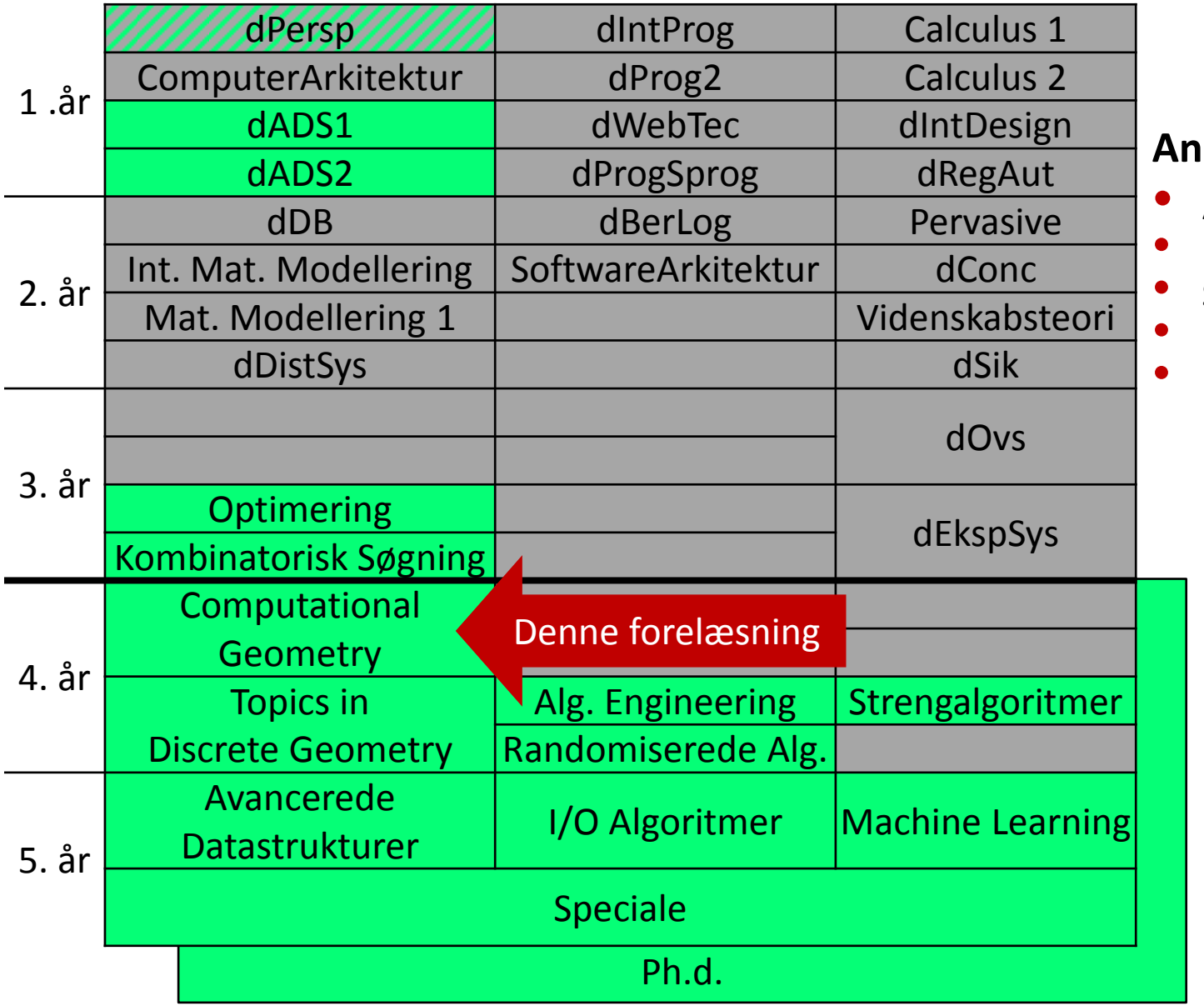
- Ph.d. Datalogi, Aarhus Universitet (1989-1997)
- Ansat ved Institut for Datalogi (1998-)
- Forskning og undervisning: Algoritmik



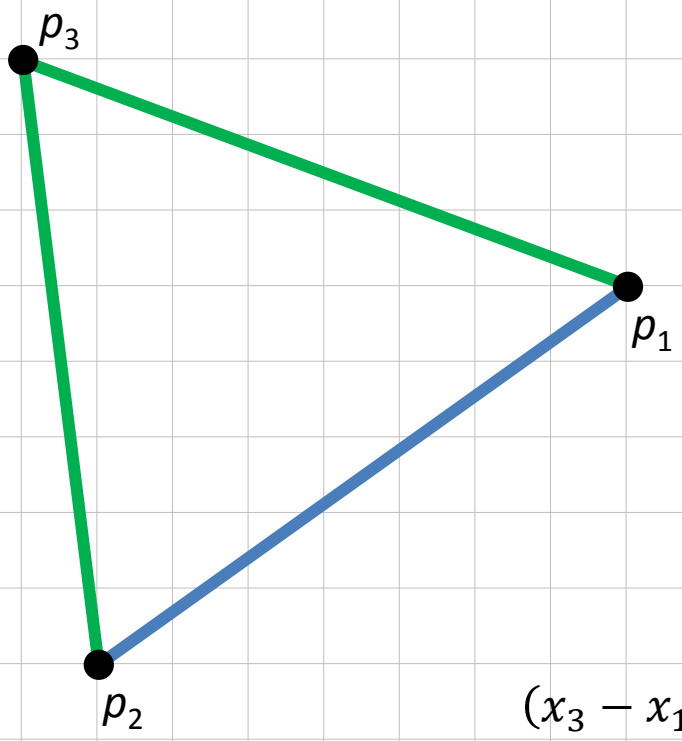
Algoritmik på Datalogi



Andre algoritmikkurser

- Algoritmer i bioinformatik
- Dynamiske algoritmer
- Spilteori
- Kompleksitetsteori
- …

Punkter og Linier



irrational

$$\text{Dist}(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\text{Dist}(p_3, p_1) < \text{Dist}(p_3, p_2) \quad ?$$

$$\Updownarrow$$

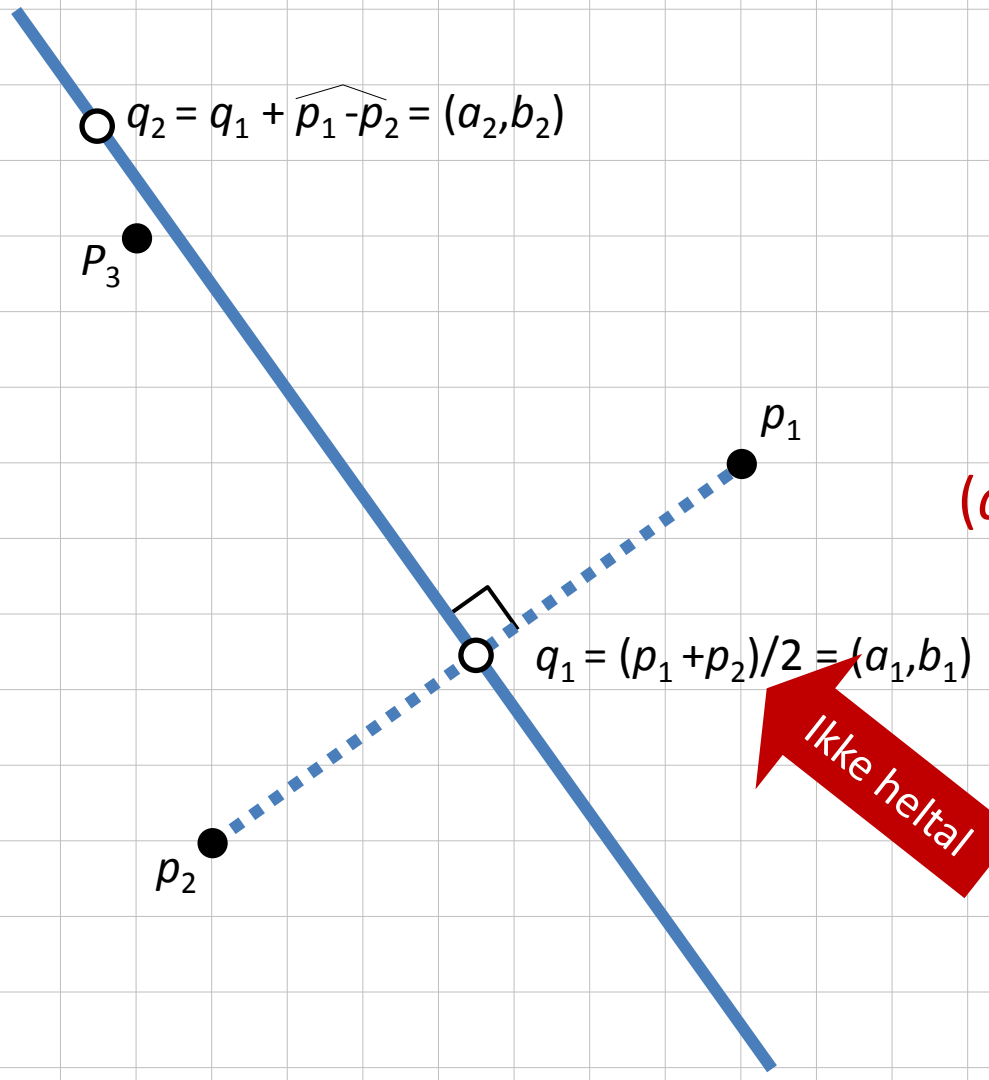
$$(x_3 - x_1)^2 + (y_3 - y_1)^2 < (x_3 - x_2)^2 + (y_3 - y_2)^2$$

overløb
$\leq 2 \cdot 4294967295^2$

1) Undgå kvadratrødder
2) Vurder størrelsen af mellemresultater

koordinater = heltal 0..4294967295

Punkter og Linier



p_3 tættest på p_2

⇕

p_3 til venstre for linien gennem q_1 og q_2

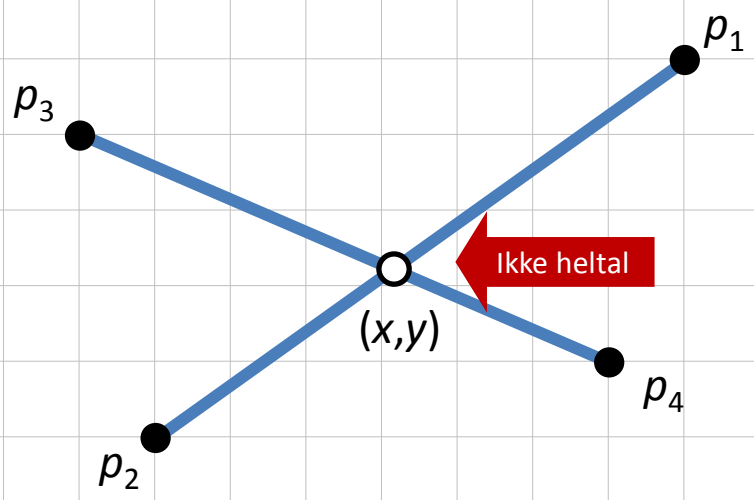
⇕

$(a_1-x_3)\cdot(b_2-y_3) - (b_1-y_3)\cdot(a_2-x_3) > 0$

Gang alle koordinater med 2 for at ungå 1/2

koordinater = heltal 0..4294967295

Punkter og Linier

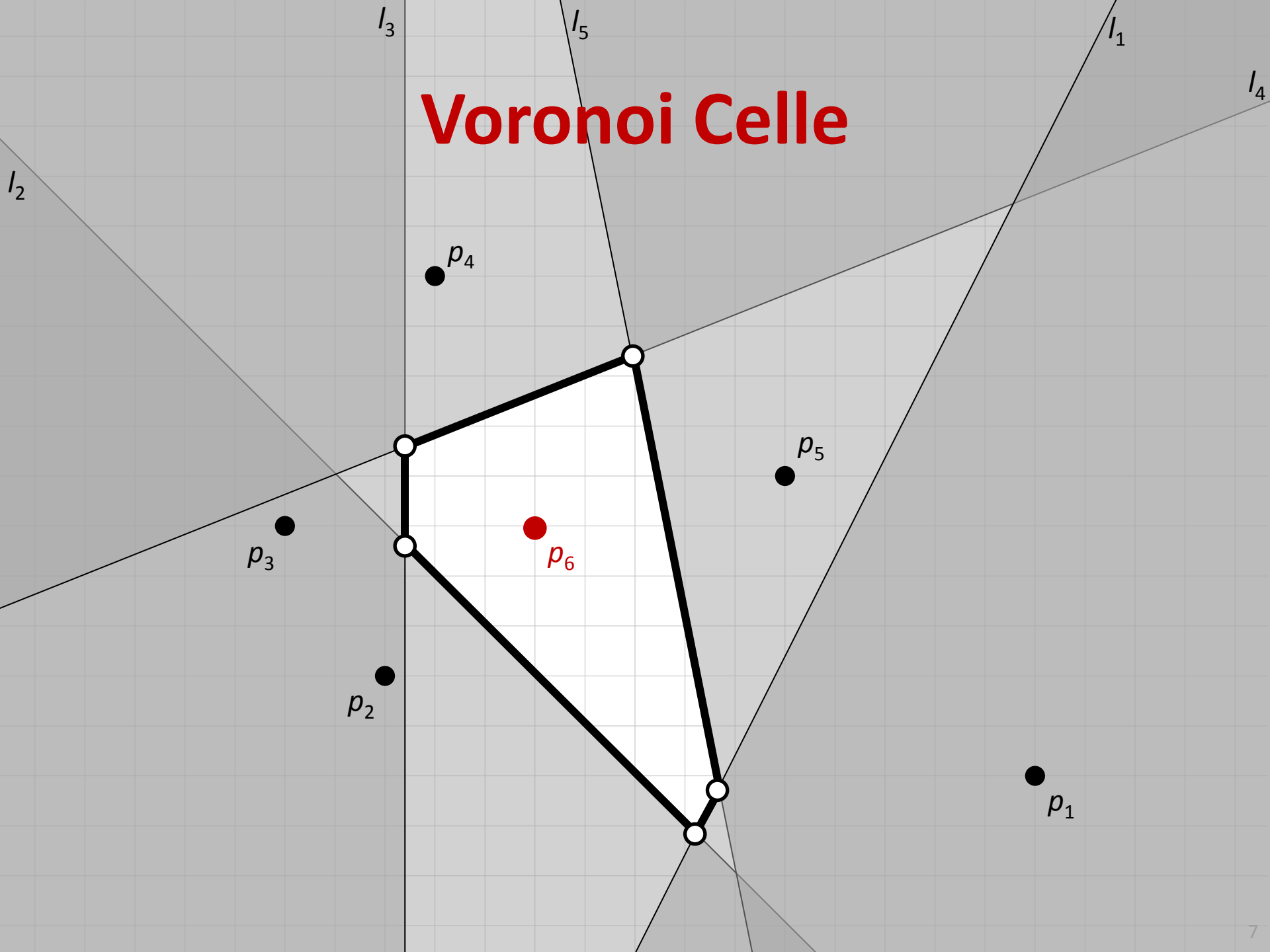


1) Linieskæringer har rationale koordinater
2) Regn med brøkker

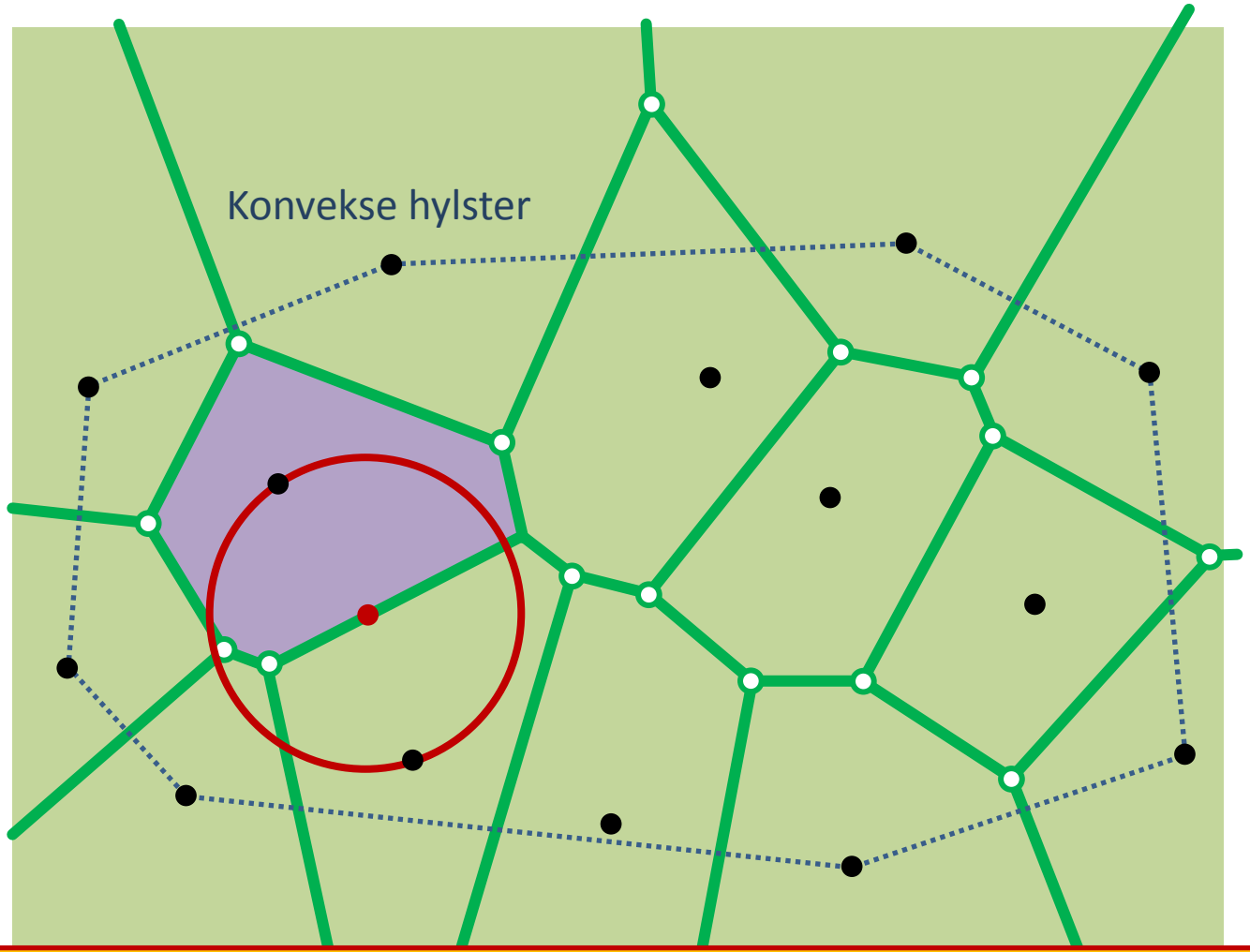
$$x = \frac{(x_1y_2 - y_1x_2)\cdot(x_3 - x_4) - (x_1 - x_2)\cdot(x_3y_4 - y_3x_4)}{(x_1 - x_2)\cdot(y_3 - y_4) - (y_1 - y_2)\cdot(x_3 - x_4)}$$

$$y = \frac{(x_1y_2 - y_1x_2)\cdot(y_3 - y_4) - (y_1 - y_2)\cdot(x_3y_4 - y_3x_4)}{(x_1 - x_2)\cdot(y_3 - y_4) - (y_1 - y_2)\cdot(x_3 - x_4)}$$

koordinater = heltal 0..4294967295

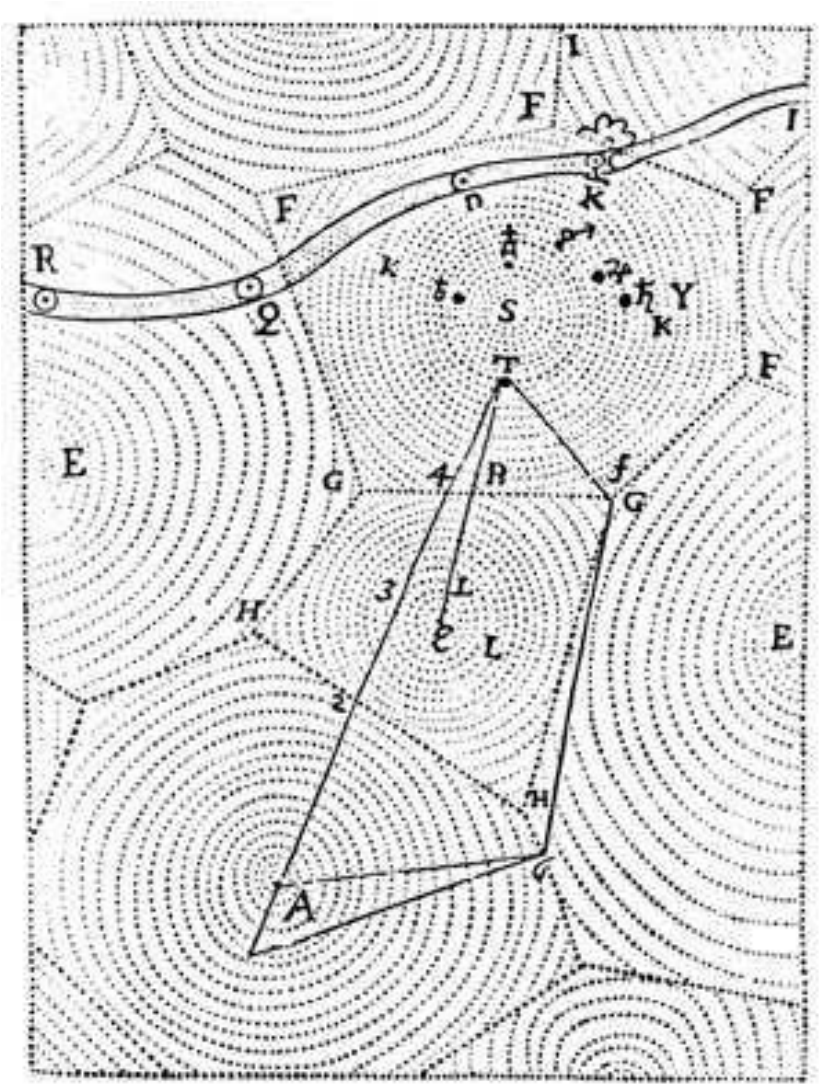

Voronoi Celle
l_1
l_2
l_3
l_4
l_5
p_1
p_2
p_3
p_4
p_5
p_6

Voronoi Diagram

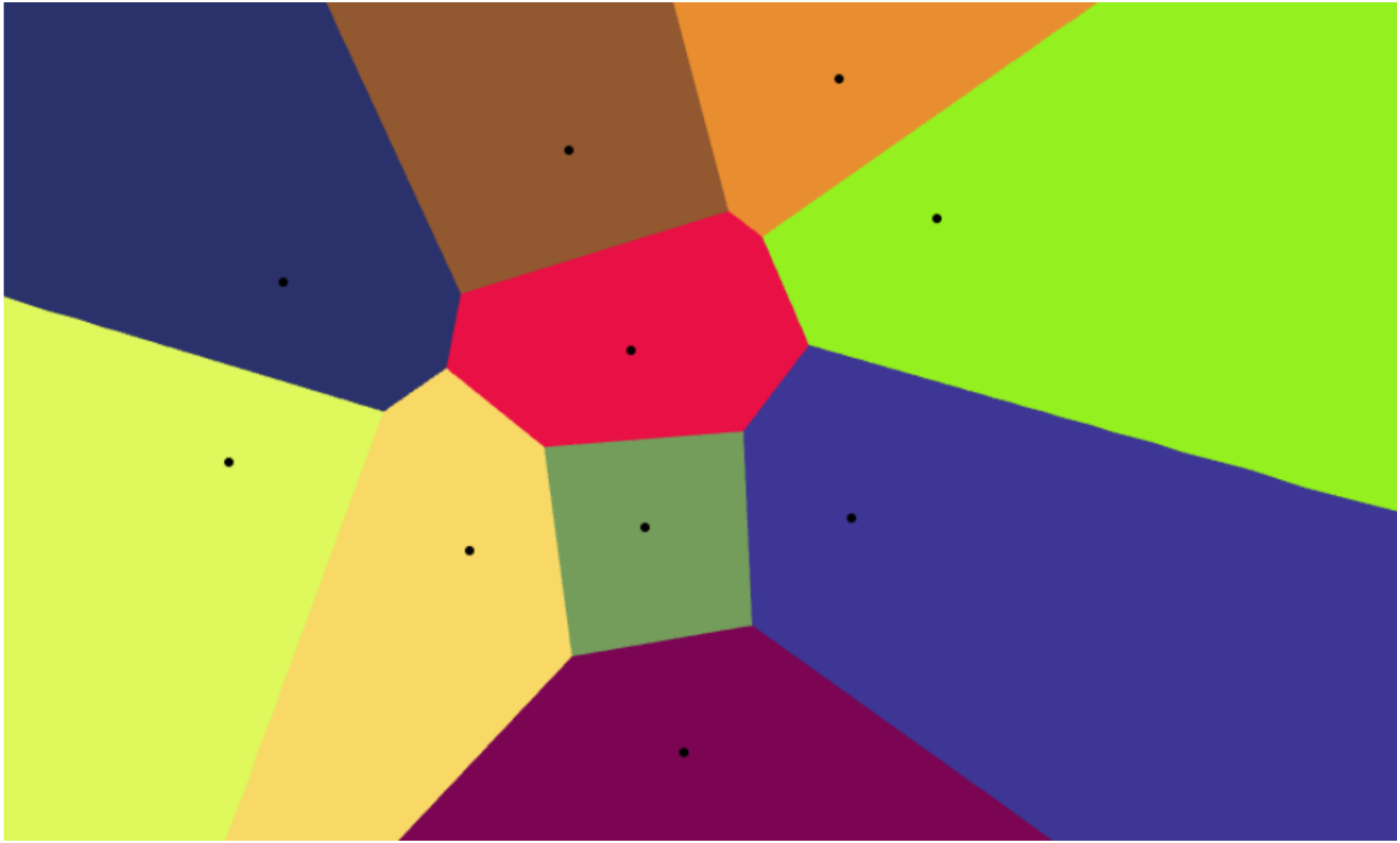


1) Voronoi knuder ≡ centrum for cirkel med tre randpunkter
2) Største tomme cirkel har centrum i en Voronoi knude
3) ”Uendelige” Voronoi kanter ≡ kanter på det konvekse hylster

Descartes 1644

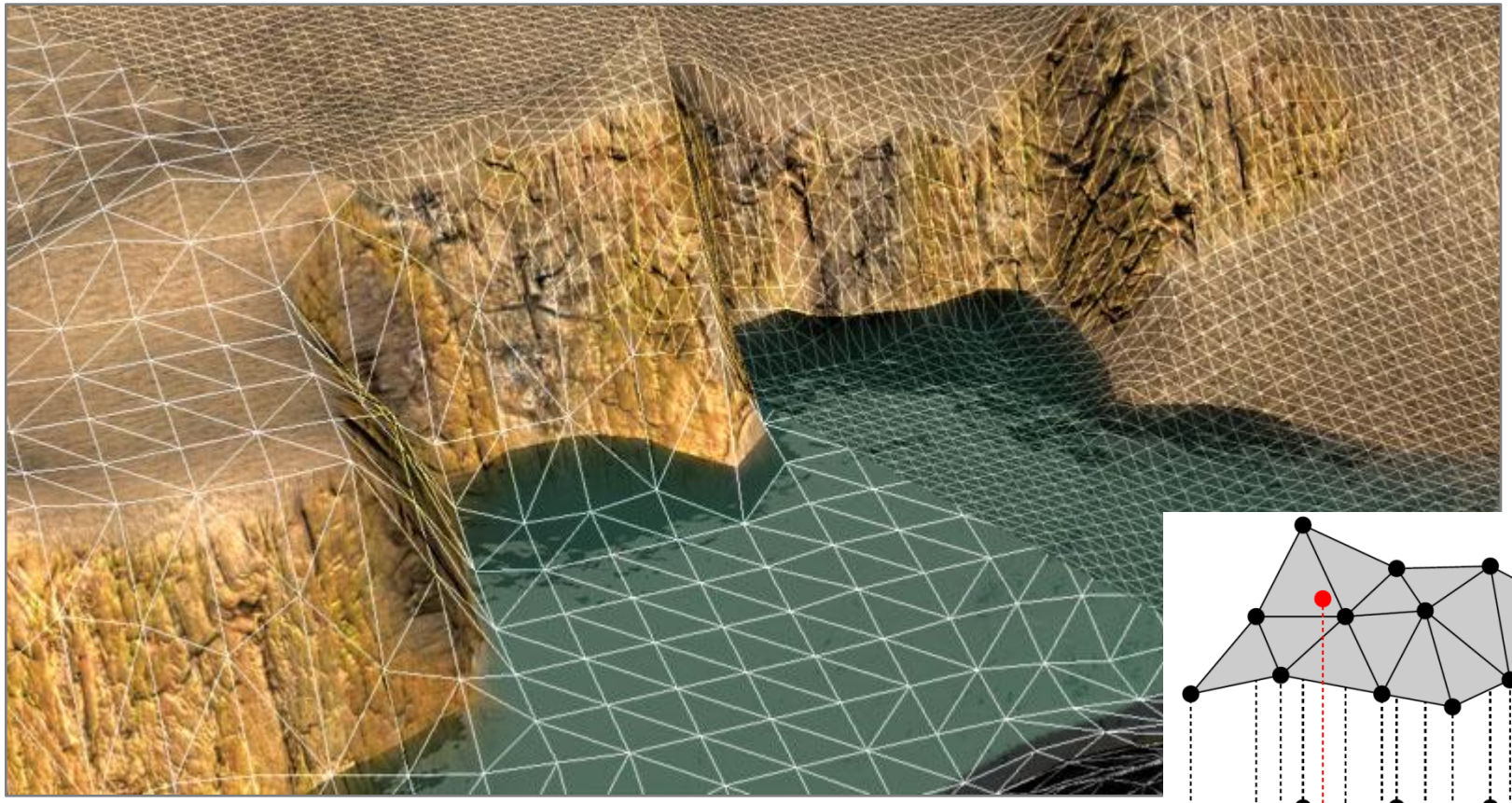


Dirichlet 1850, Voronoi 1908, Boldyrev 1909, …

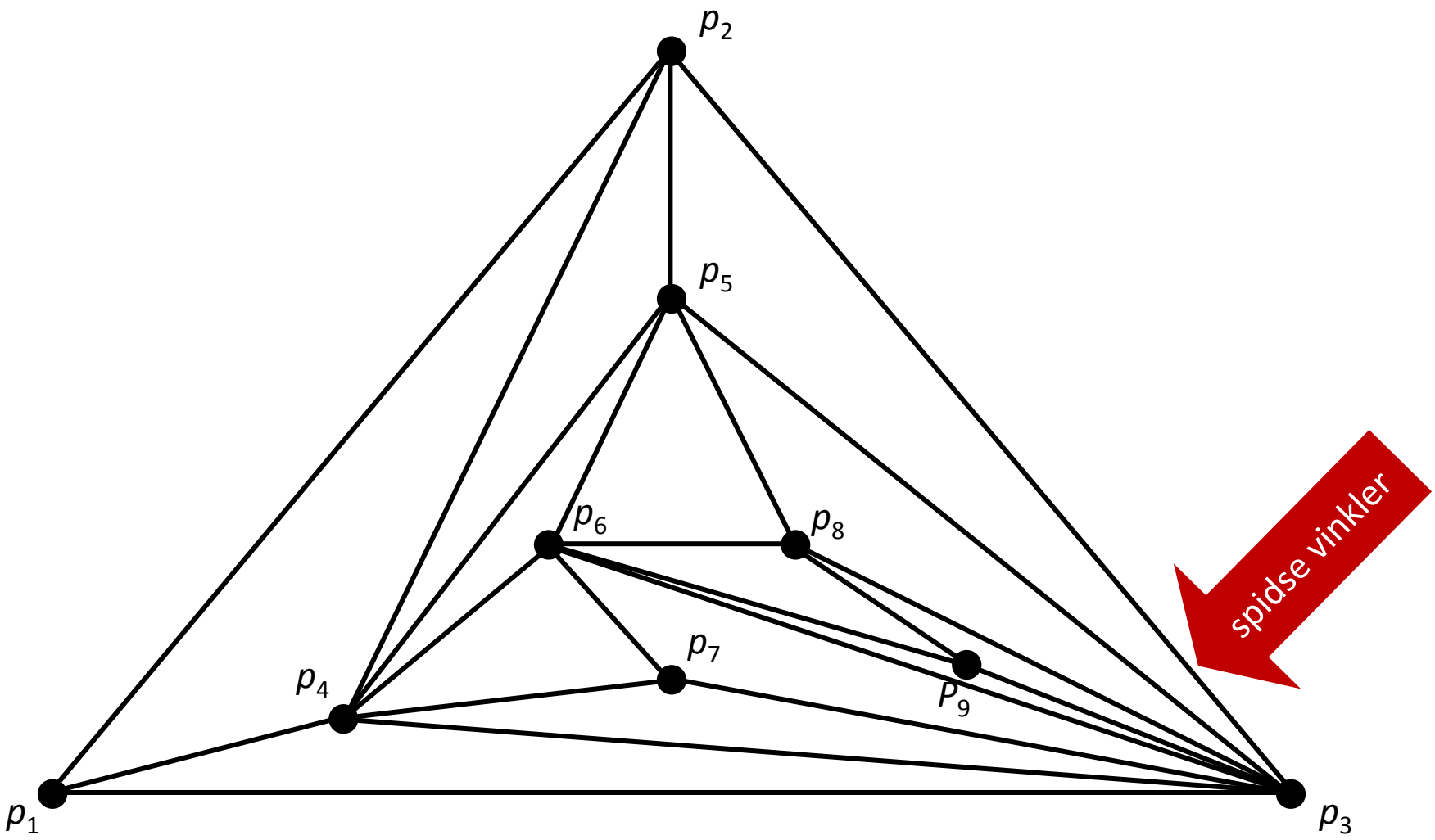


alexbeutel.com/webgl/voronoi.html

Triangulering af Terrain Data



Hvilken Triangulering ?

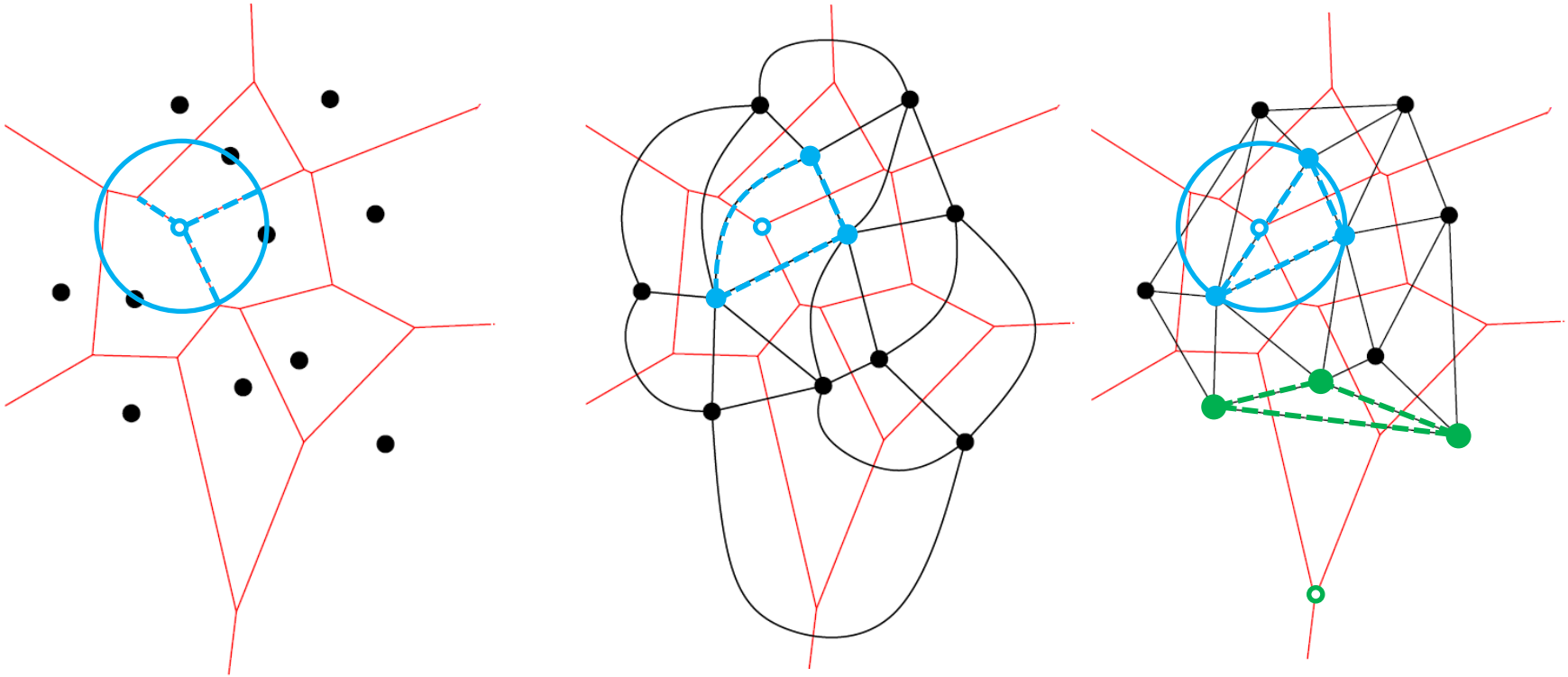


Delauney Triangulering

Voronoi diagram

Dual

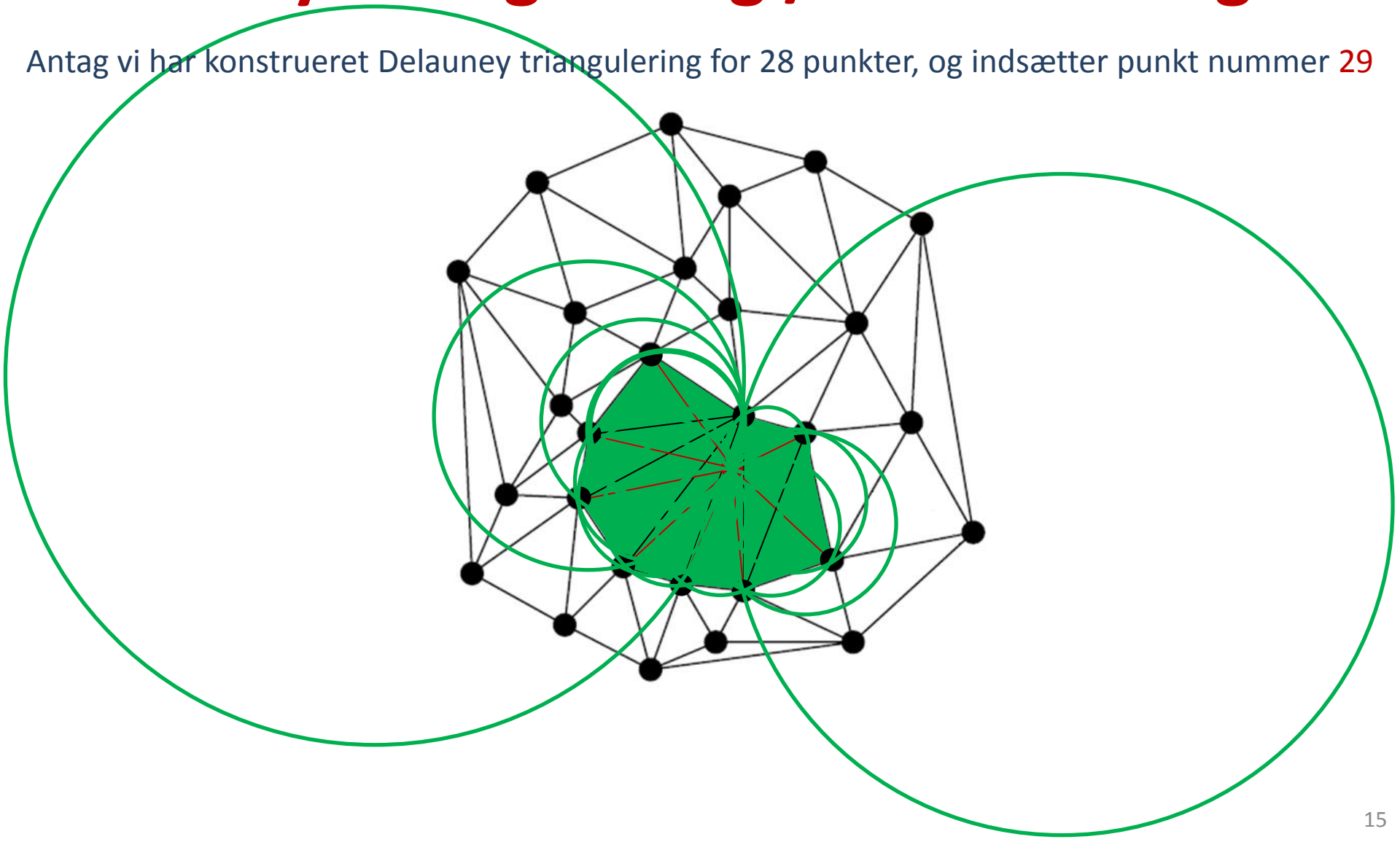
Delauney triangulering



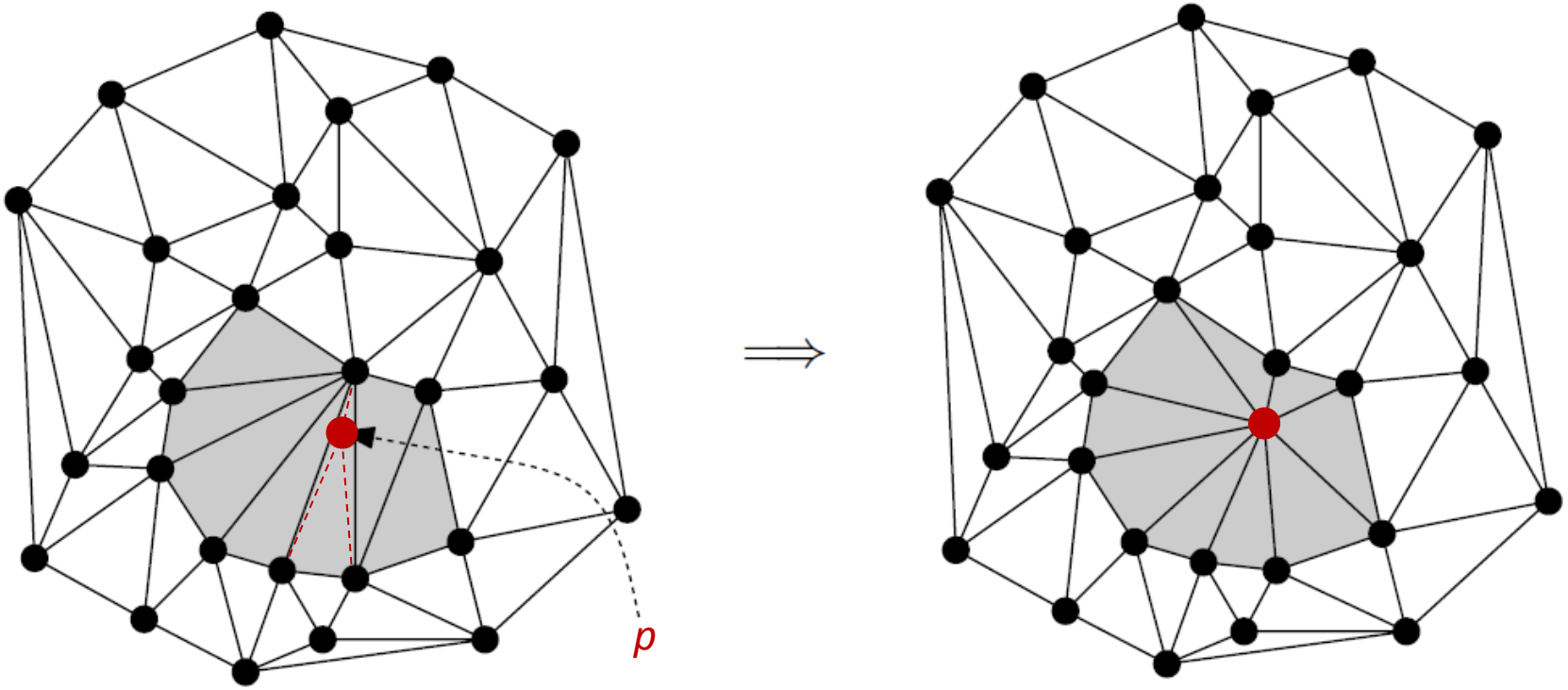
Delauney trianguleringer maximerer mindste vinkel

Inkrementel Konstruktion af Delauney Triangulering / Voronoi Diagram

Antag vi har konstrueret Delauney triangulering for 28 punkter, og indsætter punkt nummer 29



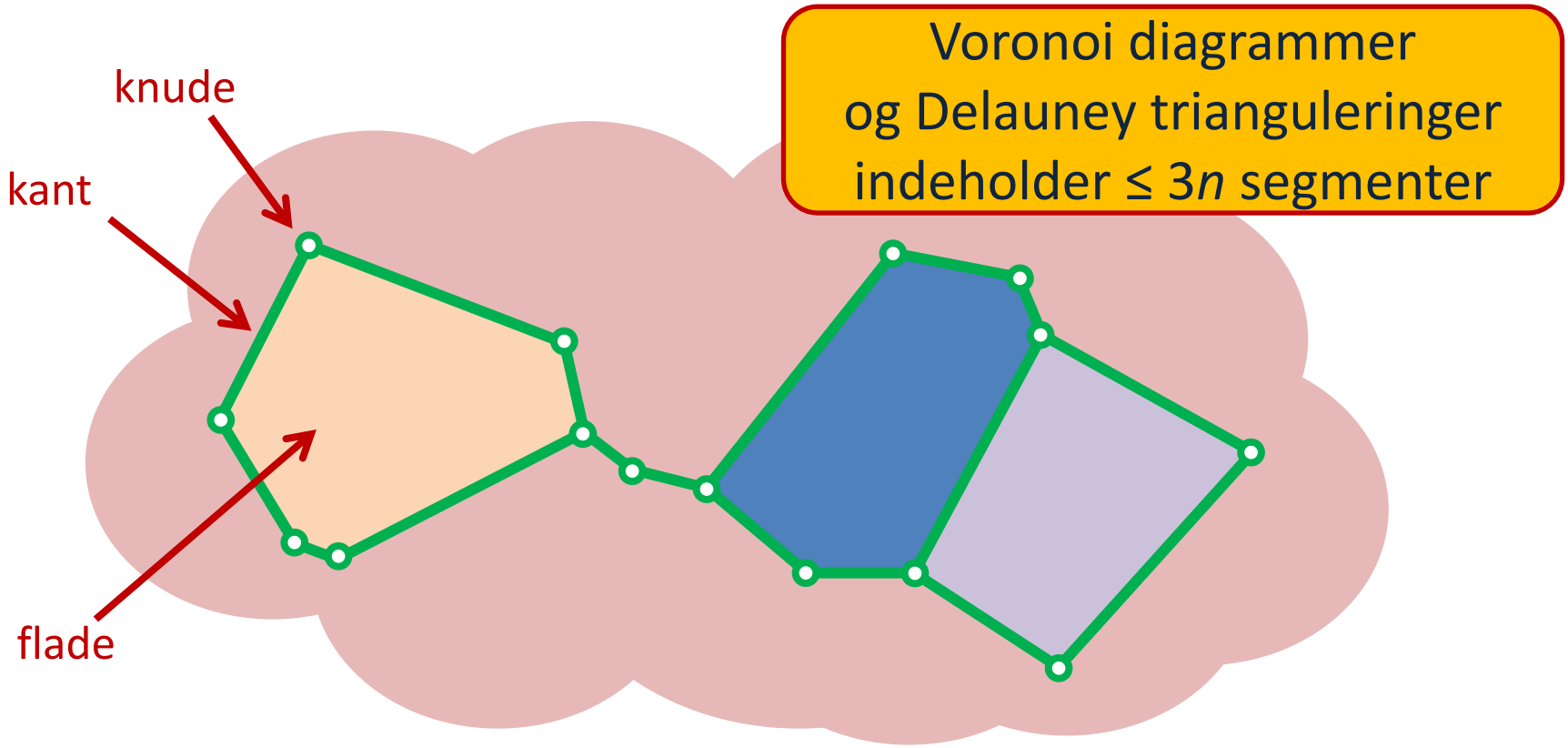
Inkrementel Konstruktion af Delauney Triangulering / Voronoi Diagram



Indsæt punkterne i tilfældig rækkefølge:

1. Find trekanten indeholdene næste punkt *p*
2. Lav kanter til trekantens hjørner
3. ”Flip” kanter i ulovlige trekanter

Euler's Sætning for Plane Grafer

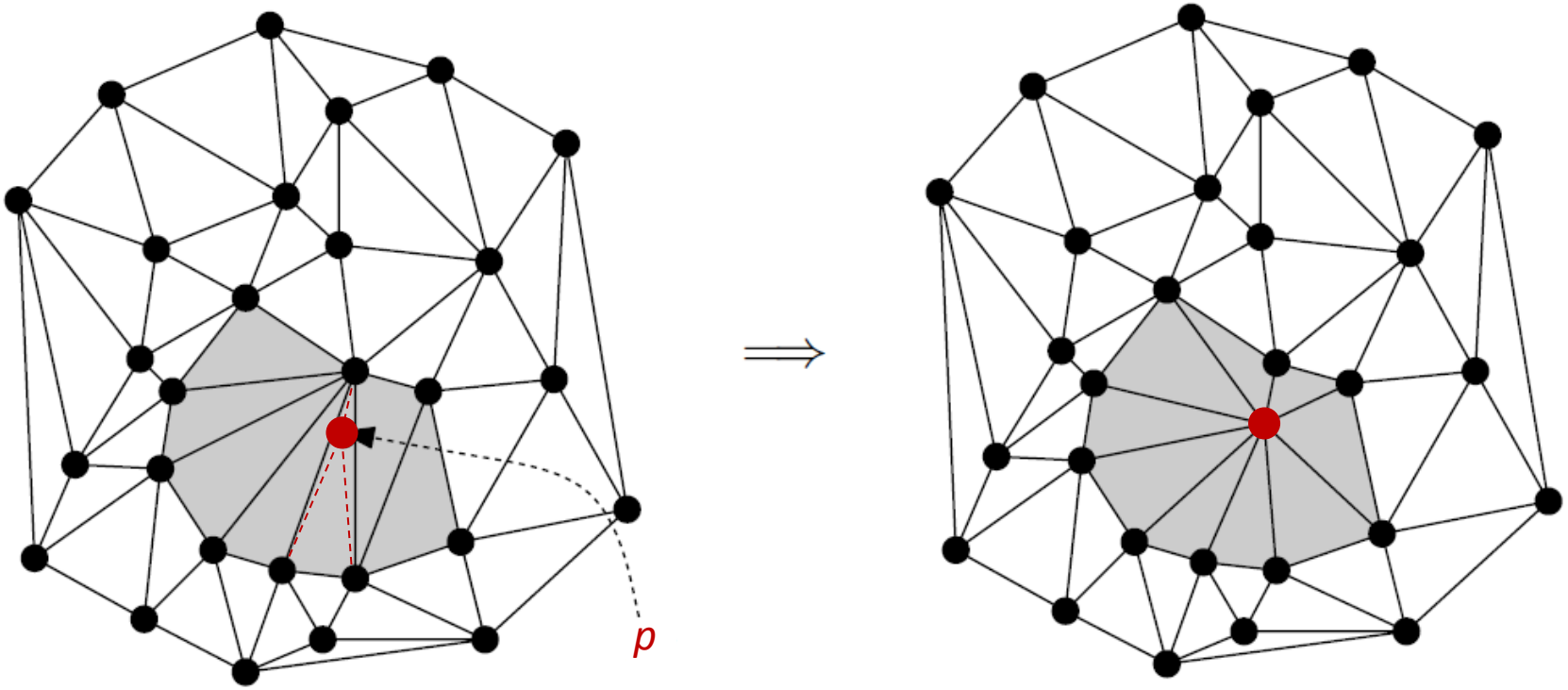


knuder + # flader - # kanter = 2

15 + 4 - 17 = 2

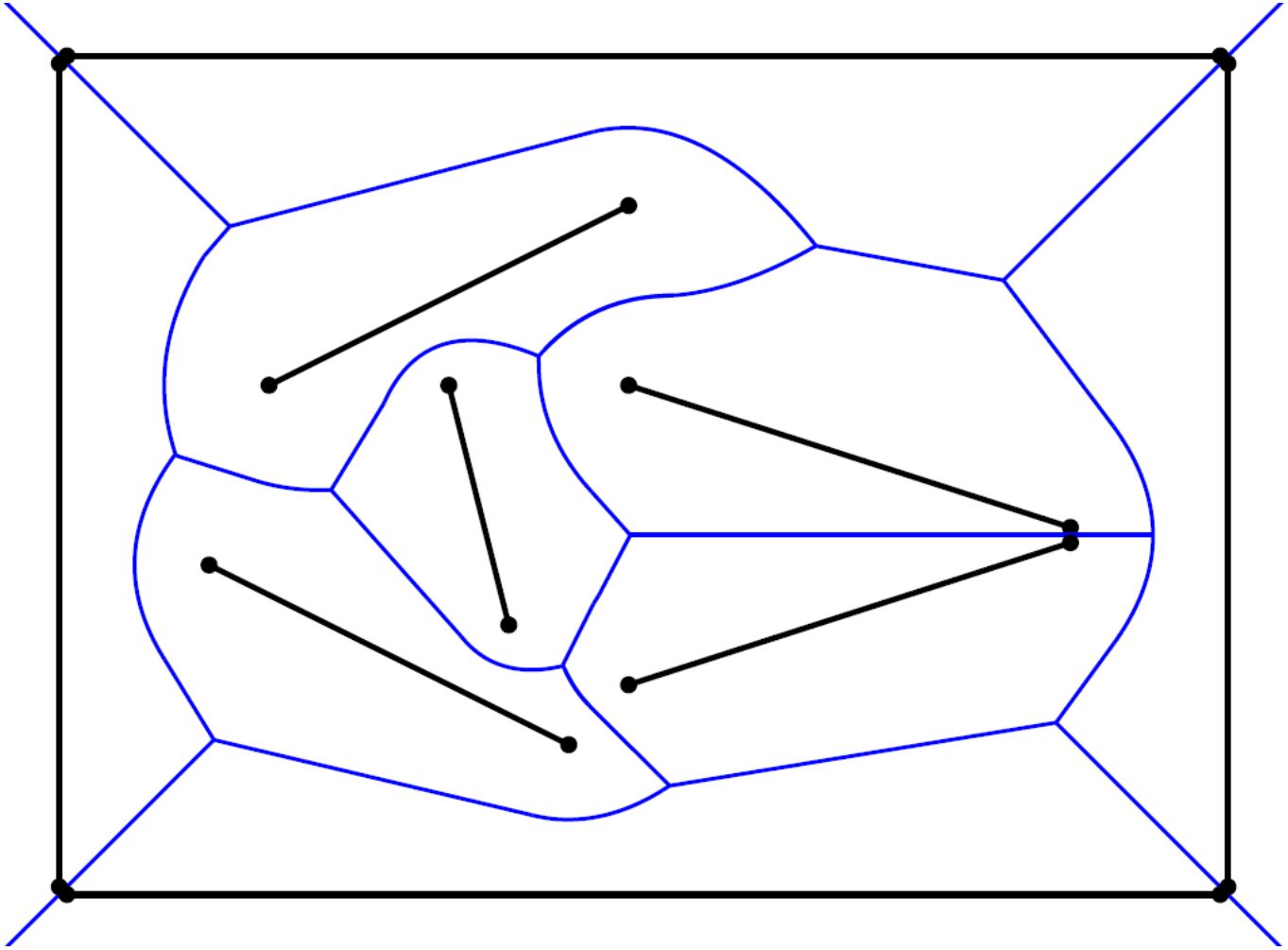
(gælder for sammenhængende grafer der kan tegnes uden krydsende kanter)

Inkrementel Konstruktion af Delauney Triangulering / Voronoi Diagram

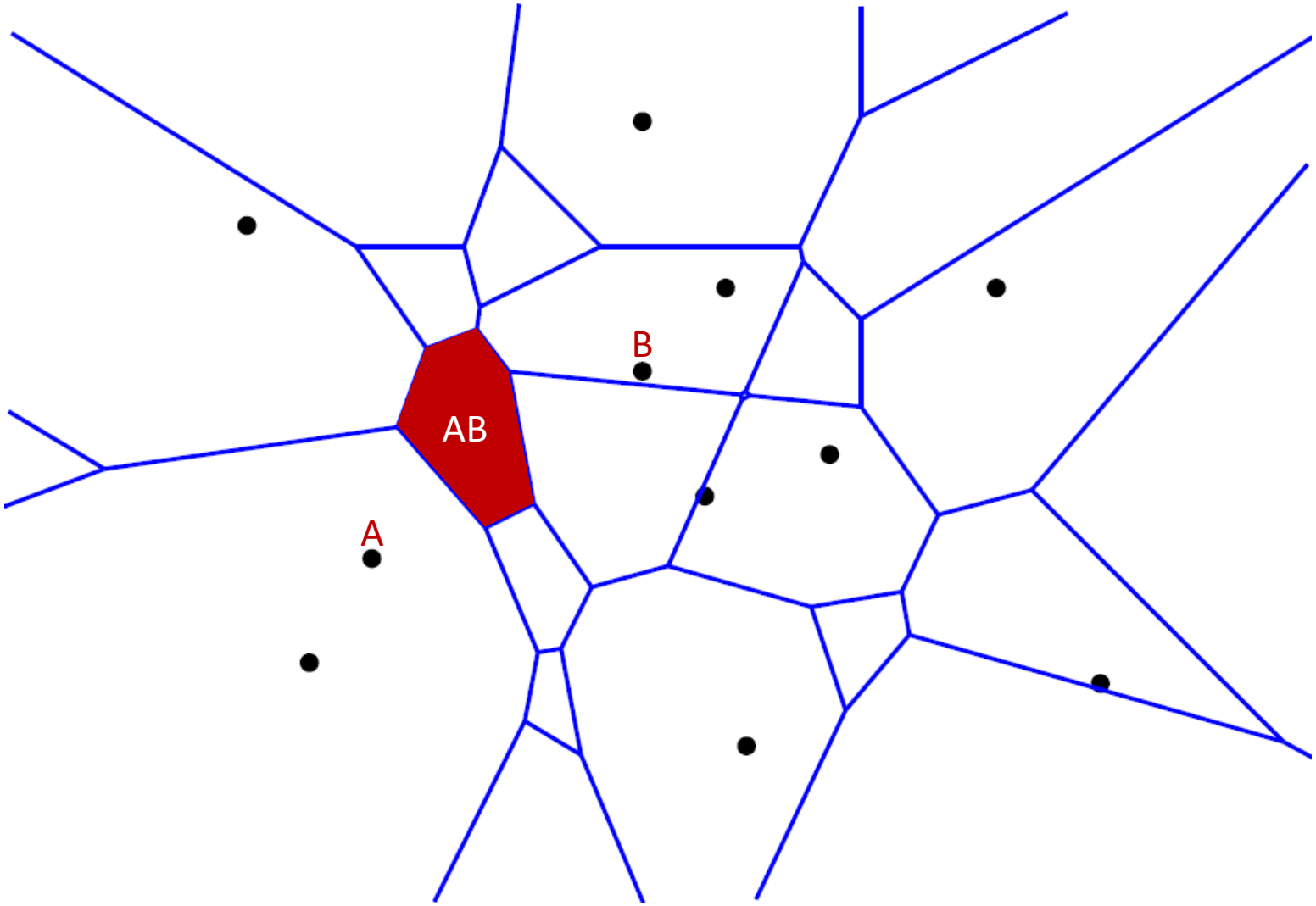


Forventet ≤ 6 ”flips” per indsættelse ⟹ totalt forventet ≤ 6*n* ”flips”

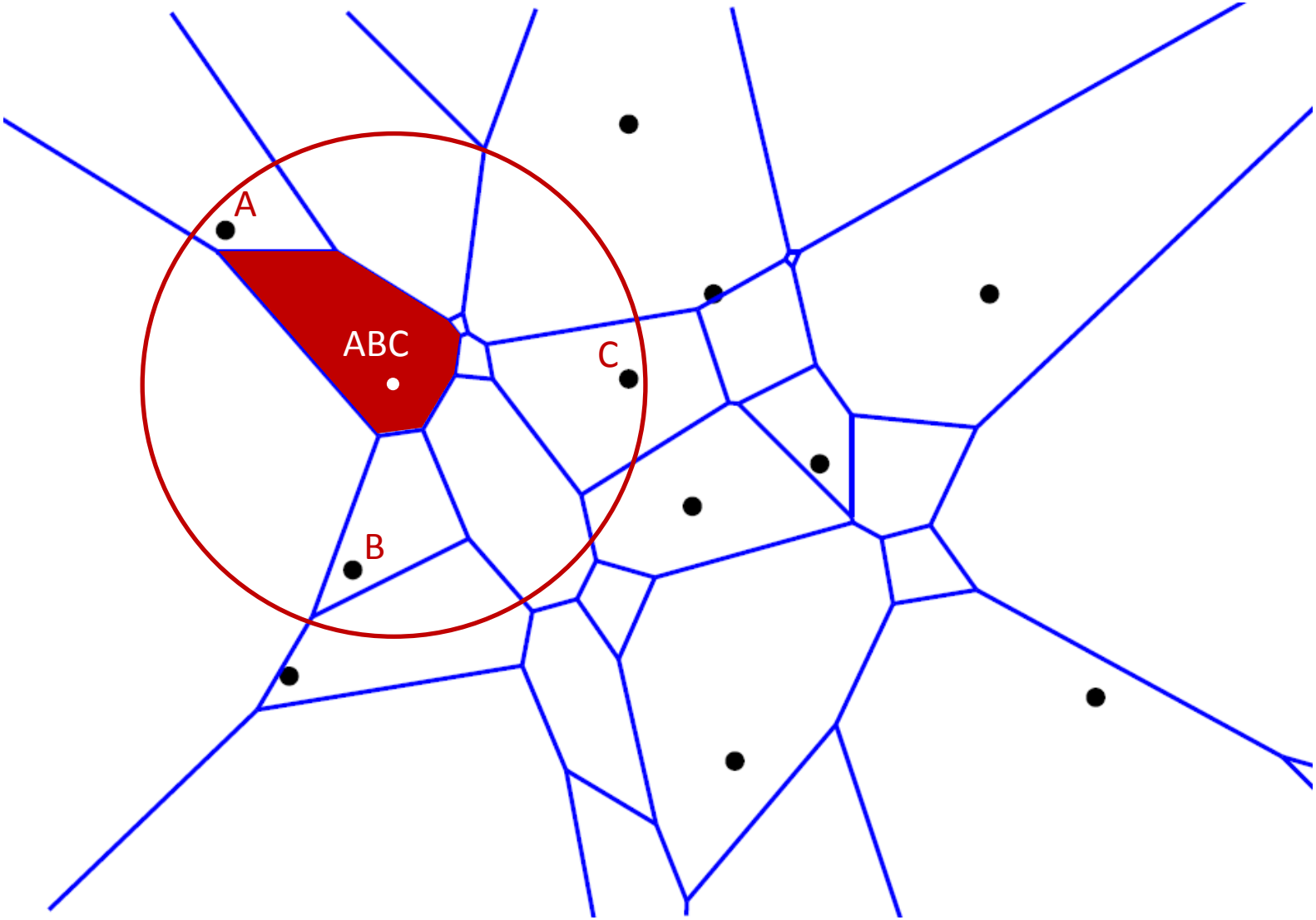
Voronoi Diagram af Linier



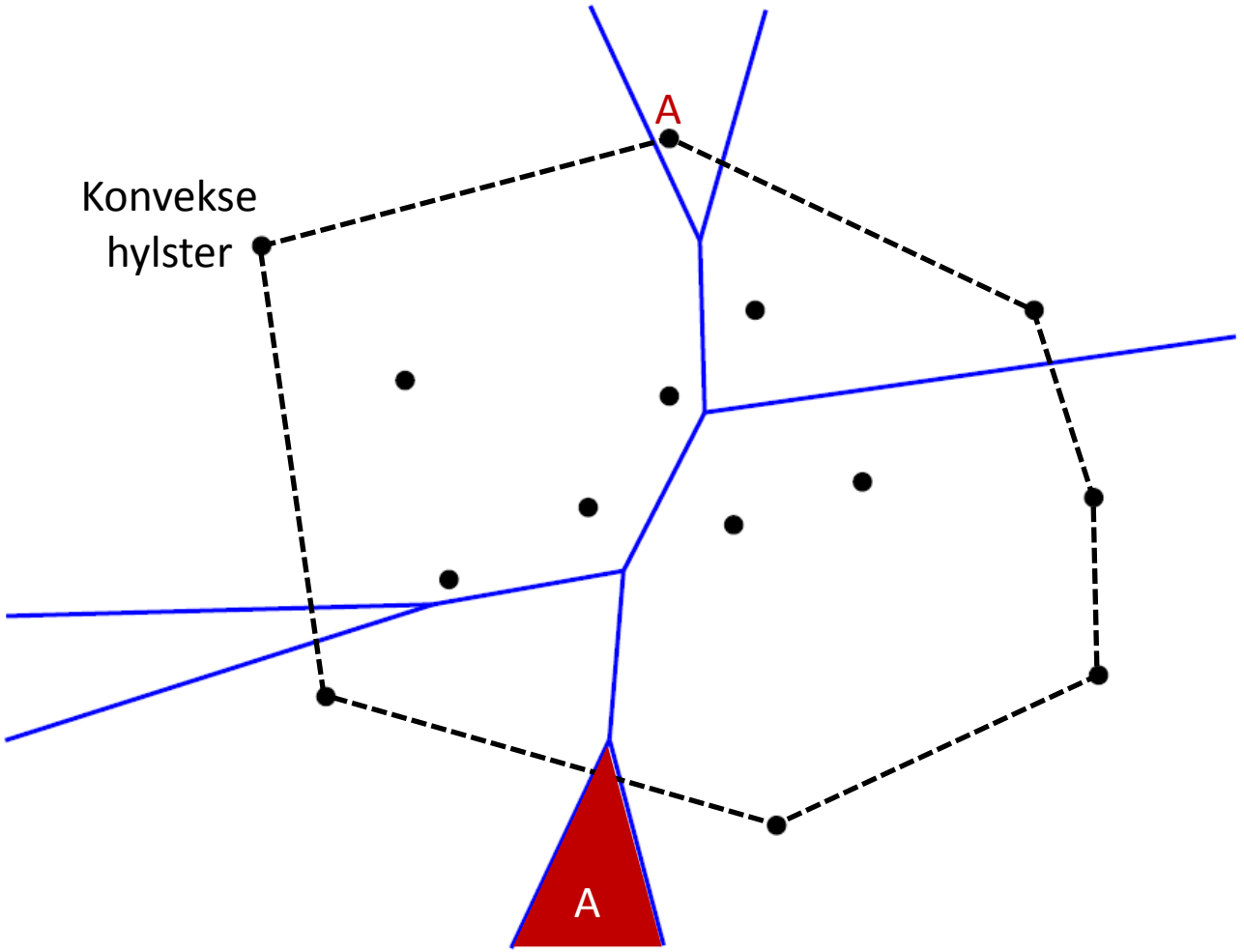
2. ordens Voronoi Diagram



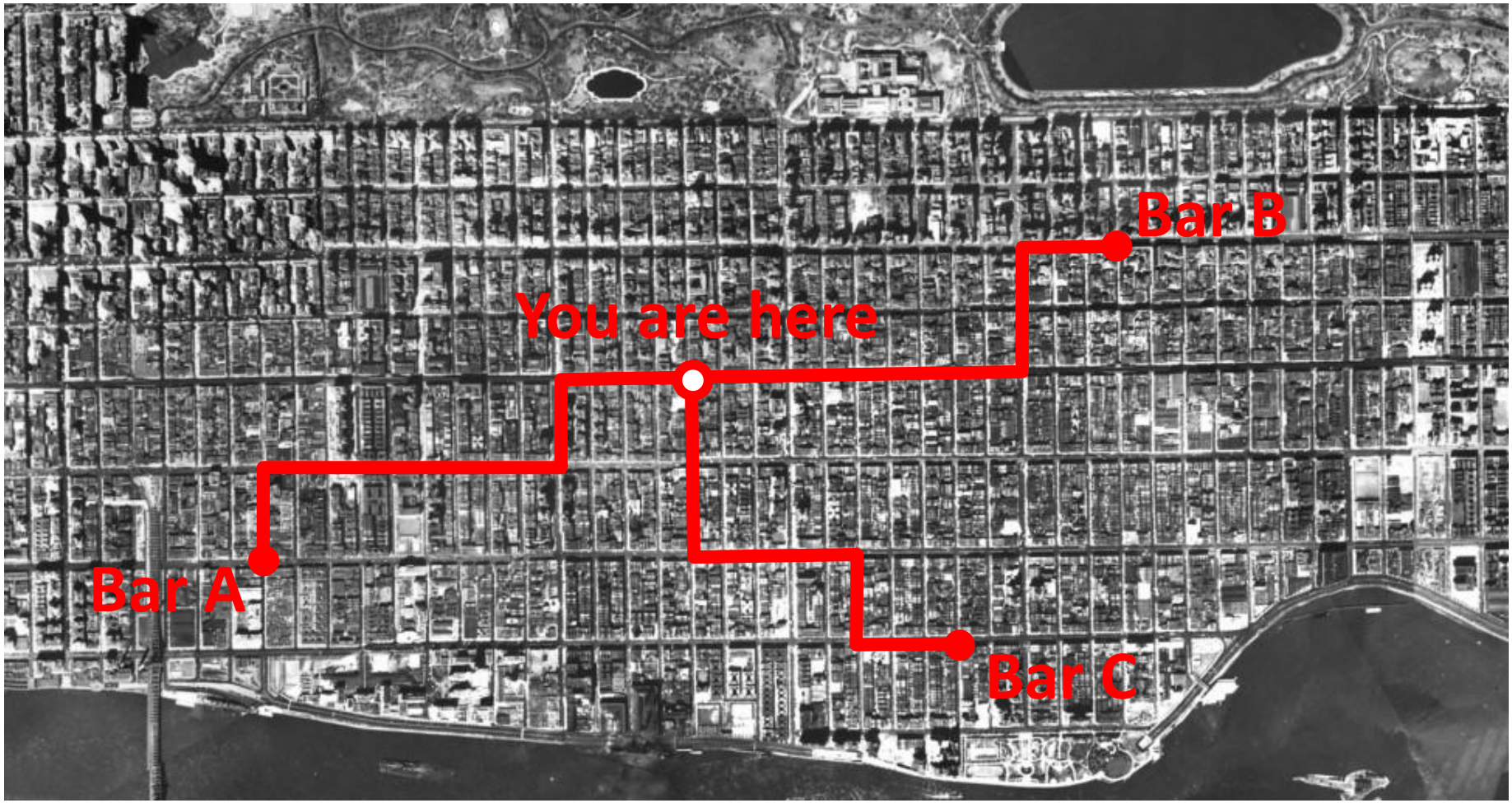
3. ordens Voronoi Diagram



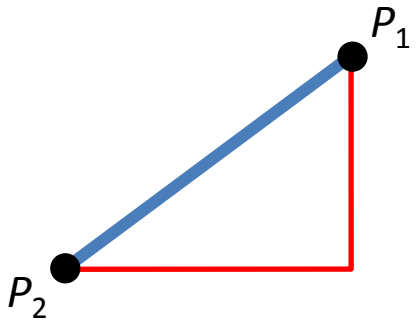
Længst Væk Voronoi Diagram



Manhattan



Afstandsmål

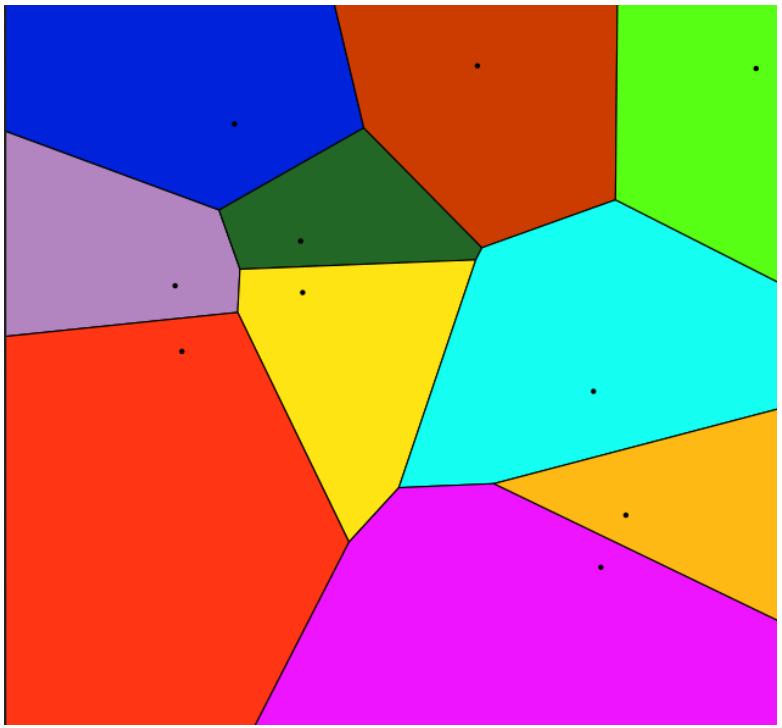


Euklidisk afstand = L_2 afstand

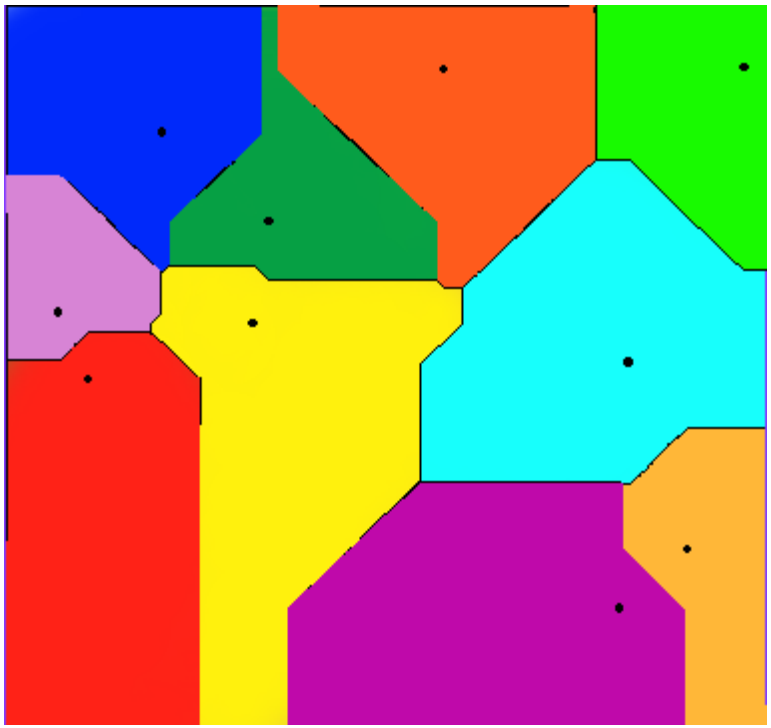
$$\|p_1 - p_2\|_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Manhattan afstand = L_1 afstand

$$\|p_1 - p_2\|_1 = |x_1 - x_2| + |y_1 - y_2|$$

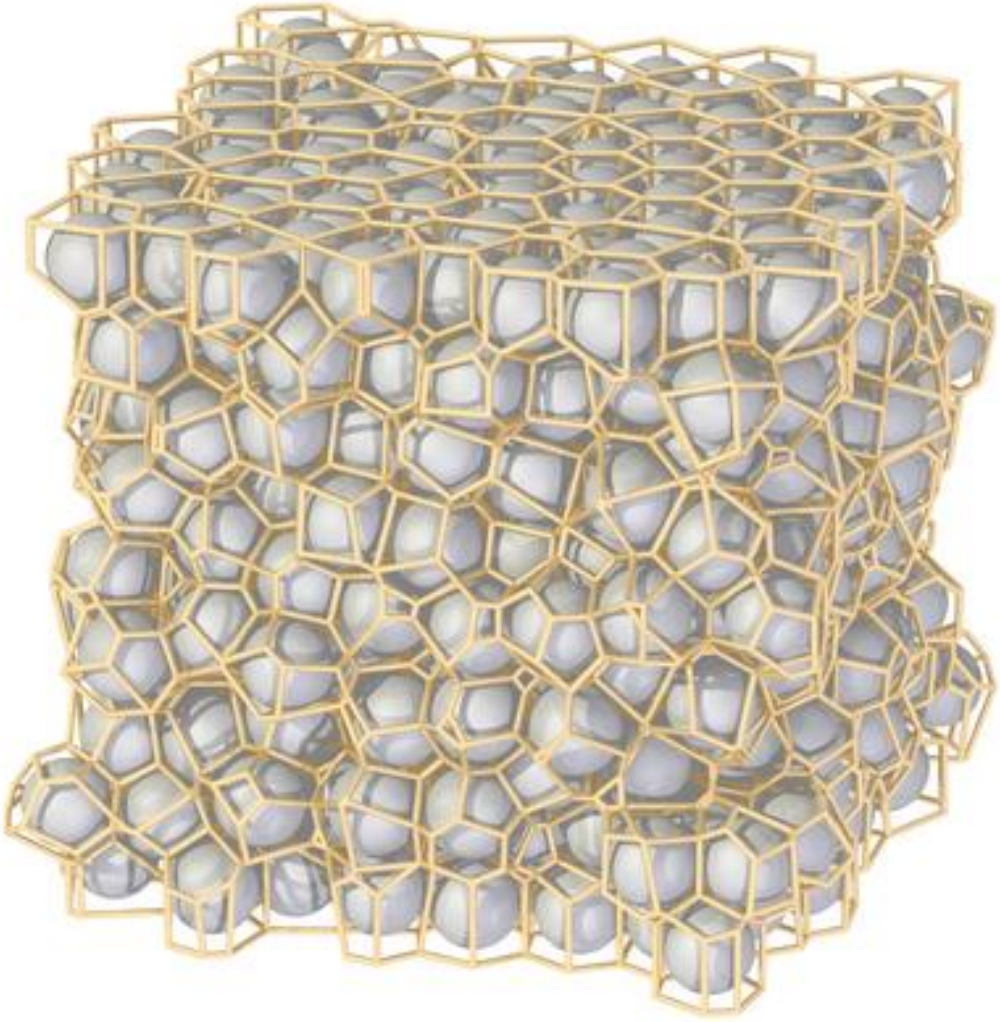


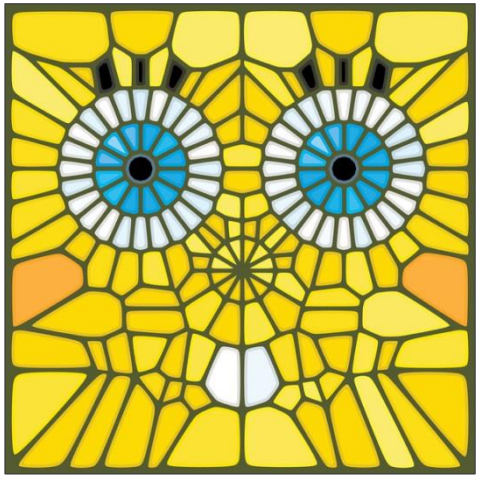
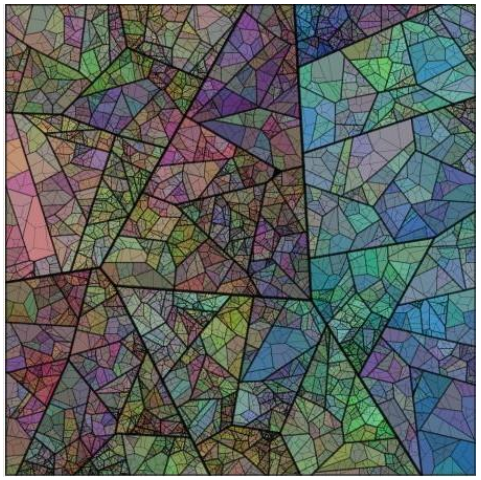
L_2 Voronoi Diagram

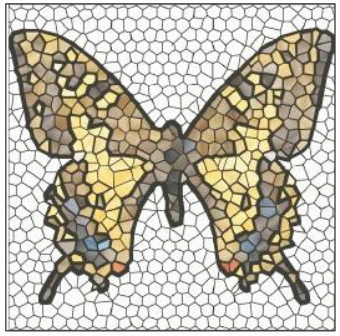


L_1 Voronoi Diagram

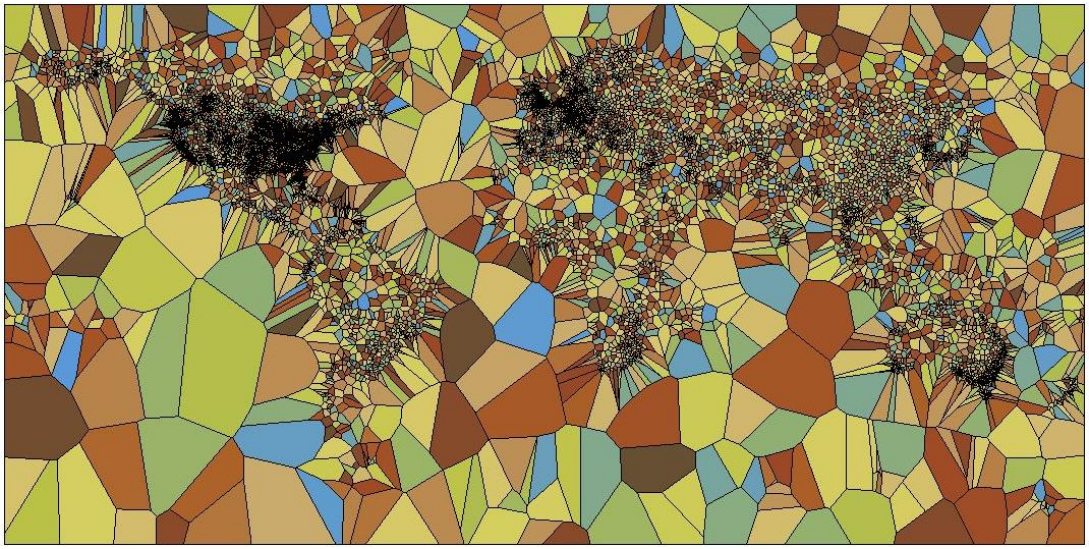
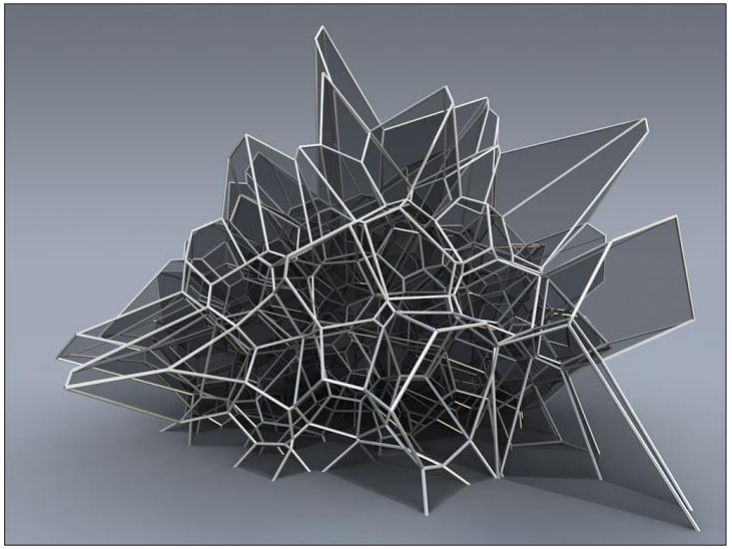
3D Voronoi Diagram



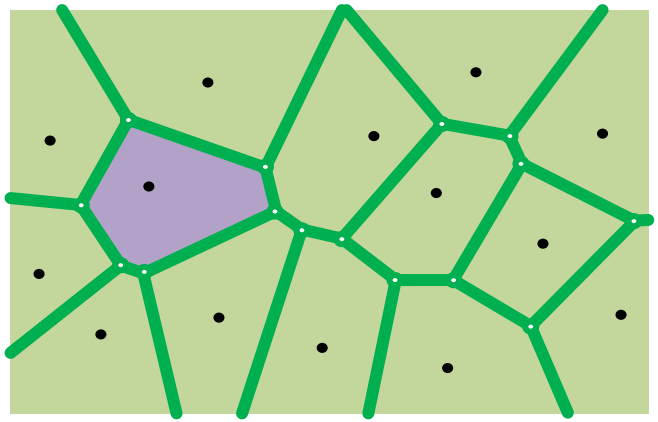



Voronoi Art

Opsummering

- Algoritmik – et datalogisk forskningsområde
- Voronoi diagrammer = eksempel inden for delområdet ”computational geometry”
- Matematiske begreber og bevisførelser essentielle for at kunne arbejde med algoritmik



www.cs.au.dk/~gerth/slides/voronoi14.pdf