

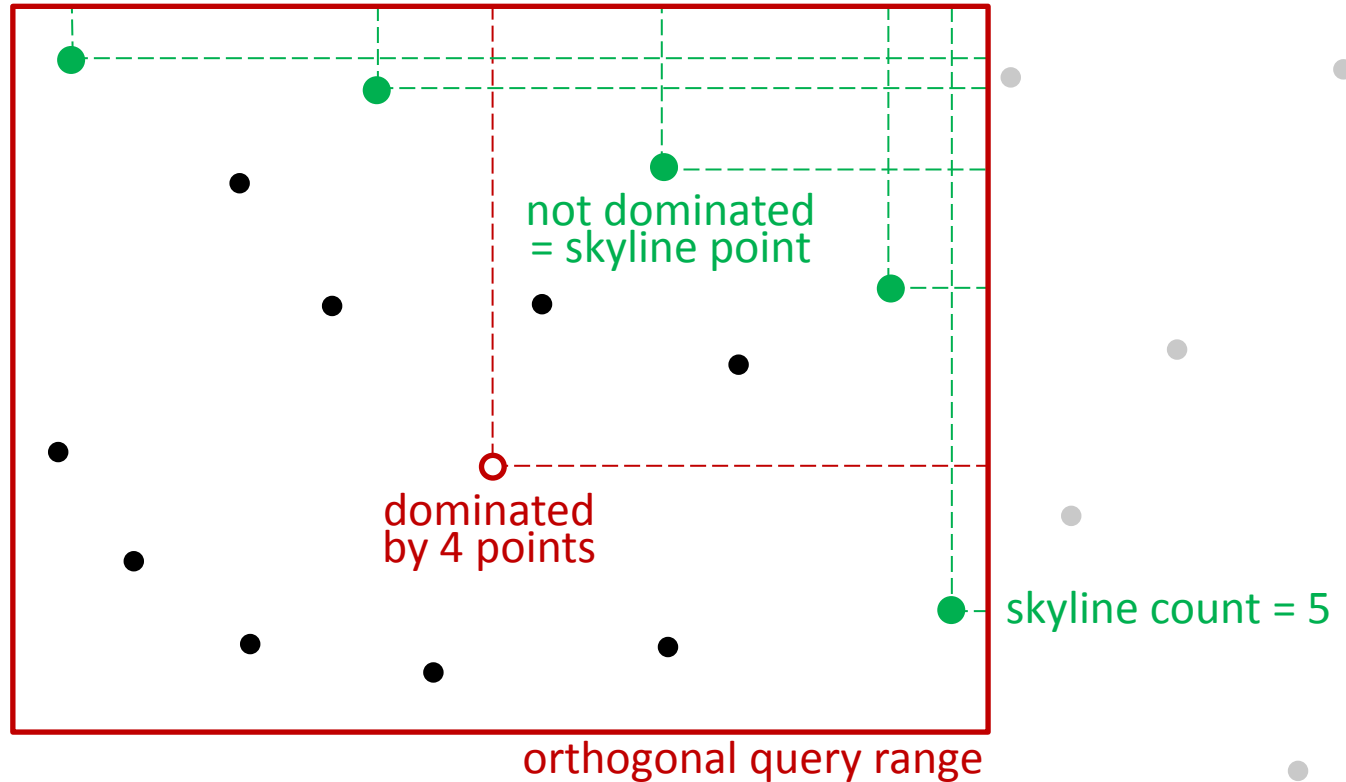
Optimal Planar Orthogonal Skyline Counting Queries

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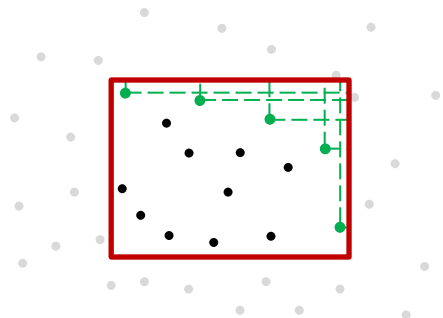
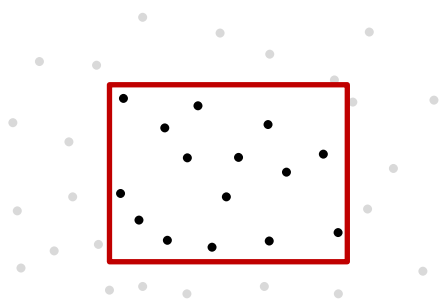
n points
 k output



Assumptions

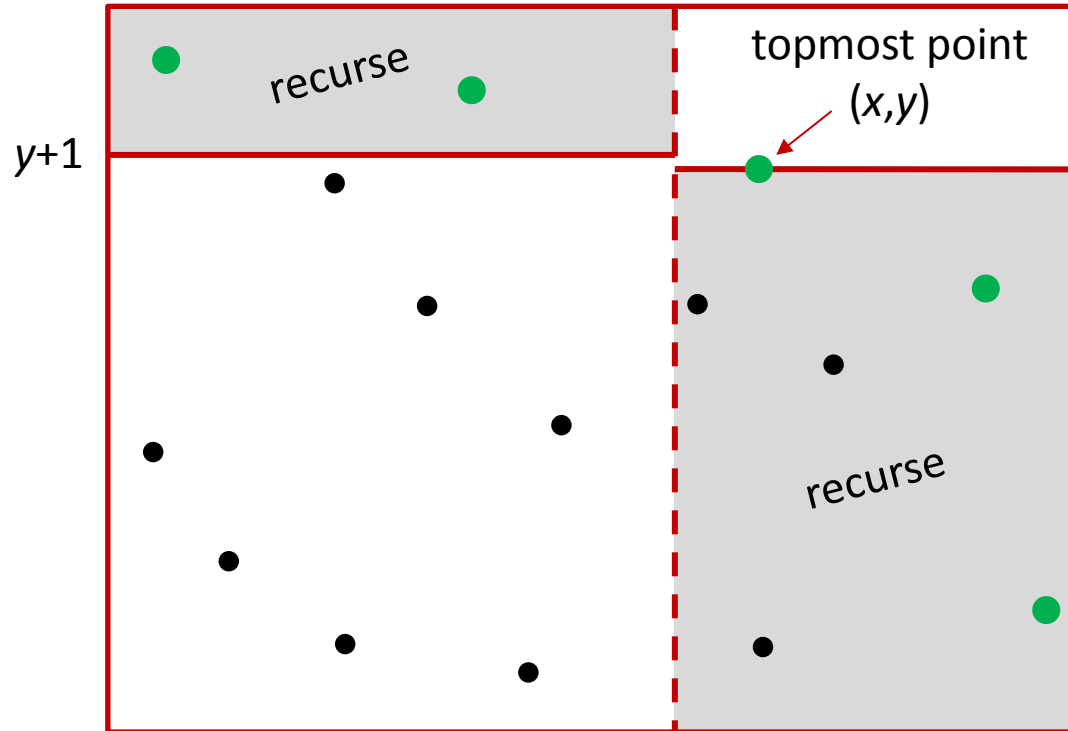
- coordinates $\{0, 1, \dots, n-1\}$
- Unit cost RAM with word size $w = \Omega(\log n)$

Results

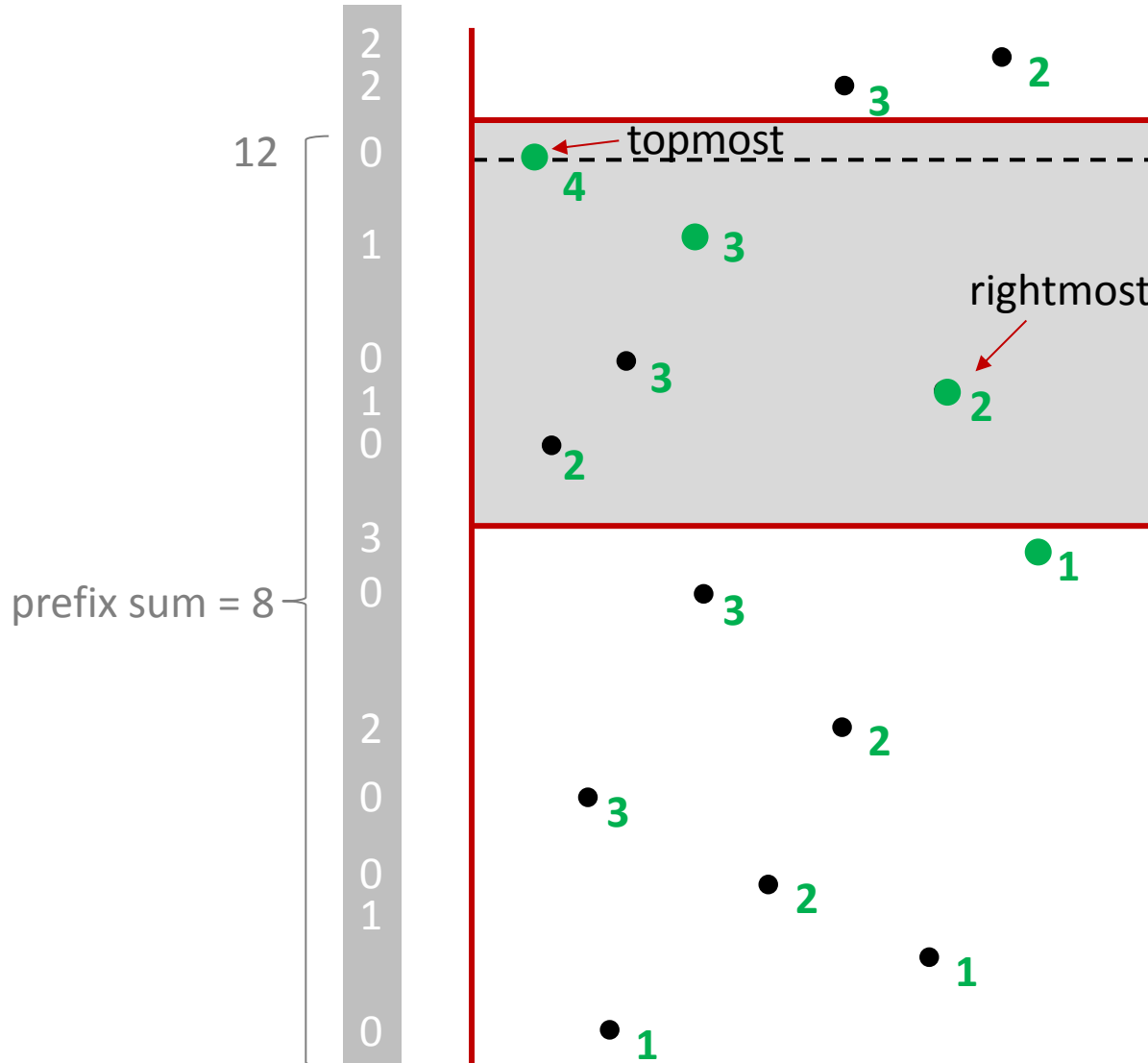


	Orthogonal range			Skyline		
	Space (words)	Query		Space (words)	Query	
Reporting	n	$k \cdot \lg^\epsilon n$	CLP11	n	$k \cdot \lg^\epsilon n$	new
	$n \cdot \lg^\epsilon n$	$k + \lg \lg n$	ABR00	$n \cdot \lg \lg n$	$k \cdot (\lg \lg n)^2$	NN12
	$n \cdot \lg^{O(1)} n \Rightarrow \Omega(k + \lg \lg n)$		PT06	$n \cdot \lg^\epsilon n$	$k \cdot \lg \lg n + \lg n / \lg \lg n$	new
				$n \cdot \lg n / \lg \lg n$	$k \cdot \lg \lg n$	NN12
				$k + \lg n / \lg \lg n$	new	
				$k + \lg n / \lg \lg n$	DGKASK12	
Counting	n	$\lg n / \lg \lg n$	JMS04	$n \cdot \lg n$	$\lg n$	DGKASK12
	$n \cdot \lg^{O(1)} n \Rightarrow$	$\lg n / \lg \lg n$	P07	$n \cdot \lg^3 n / \lg \lg n$	$\lg n / \lg \lg n$	DGS13
				n	$\lg n / \lg \lg n$	new
				$n \cdot \lg^{O(1)} n \Rightarrow$	$\Omega(\lg n / \lg \lg n)$	new

Basic Geometry – Divide and Conquer



Basic Counting – Vertical Slab



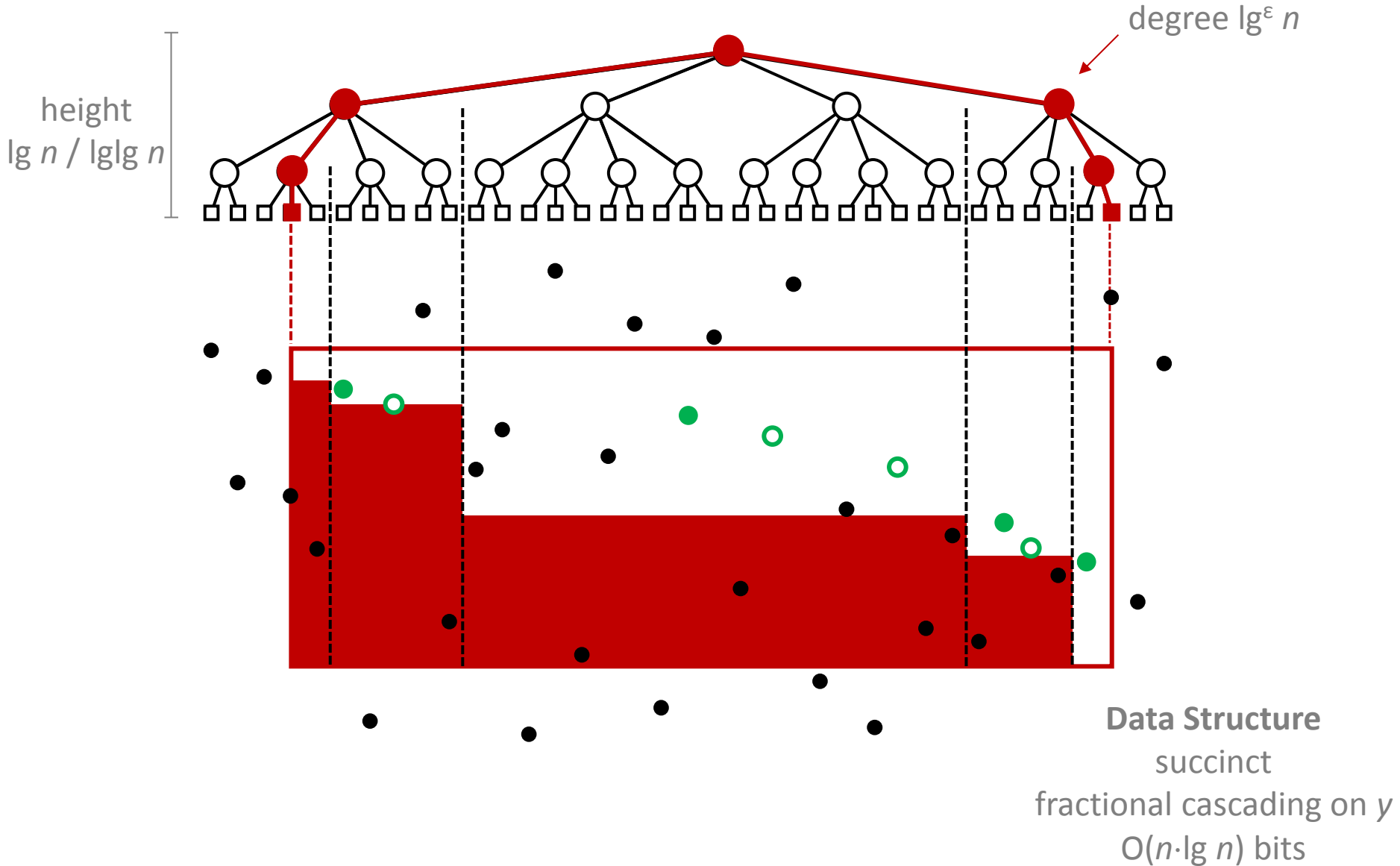
skyline count = 4 - 2 + 1

topmost

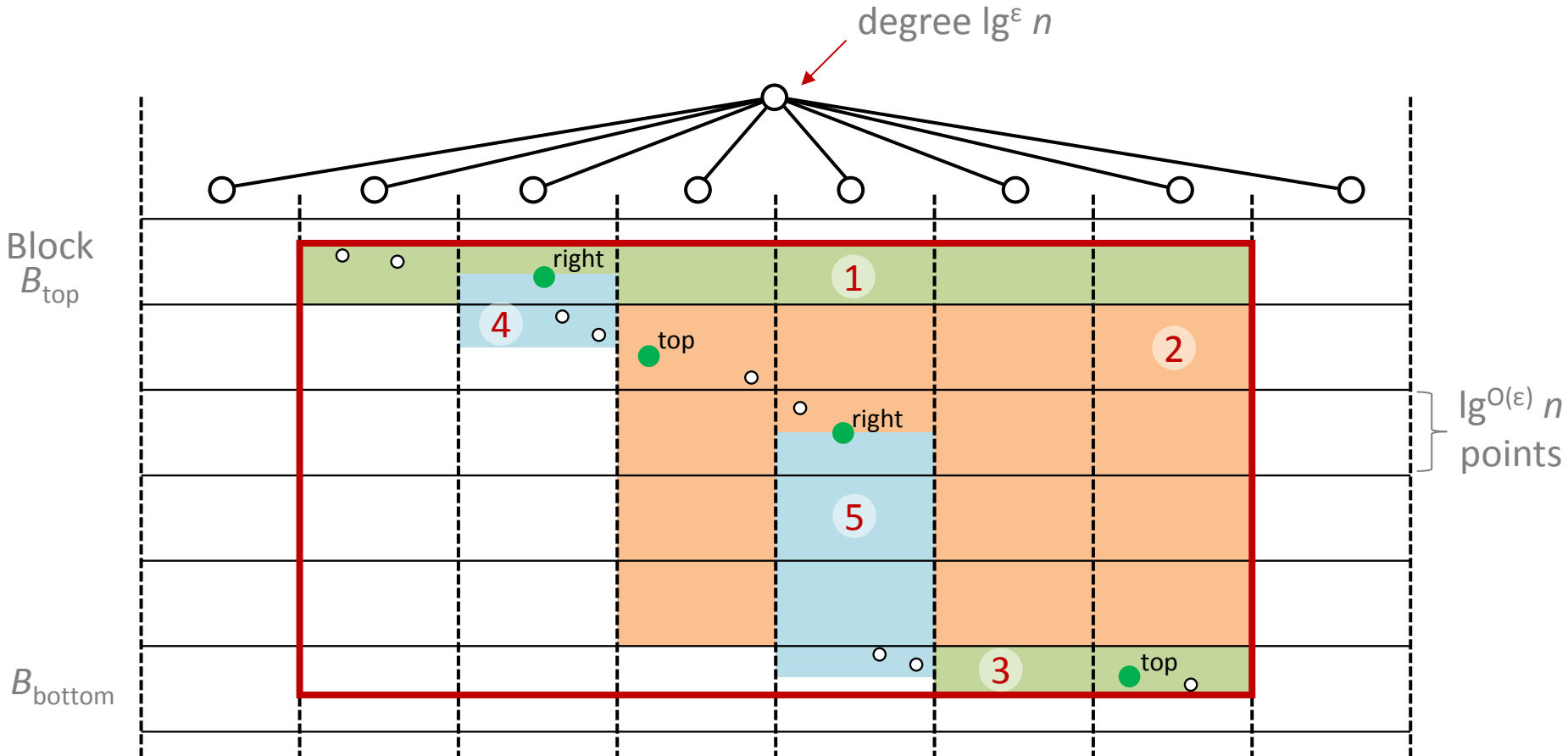
rightmost

Data Structure
succinct
prefix sum
 $O(n)$ bits
+
succinct
range maxima
 $O(n)$ bits

Upper Bound



Upper Bound – Multi-slab



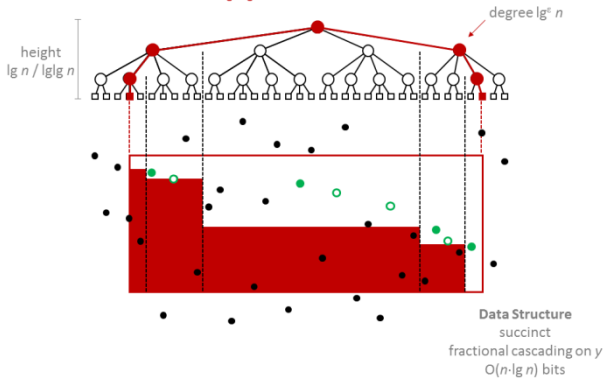
1 + 3 tabulation (blocks have $o(\lg n)$ bit signatures)

4 + 5 single slab queries (succinct prefix sum)

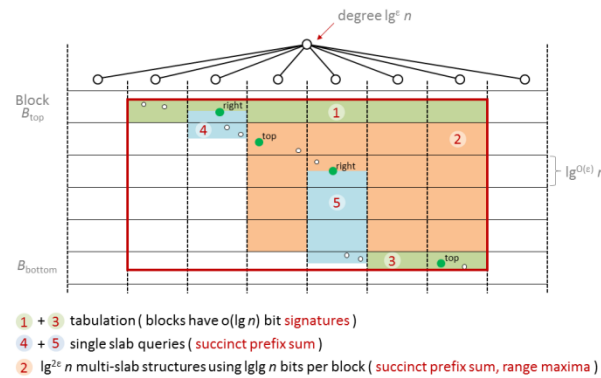
2 $\lg^{2\epsilon} n$ multi-slab structures using $\lg \lg n$ bits per block (succinct prefix sum, range maxima)

Upper Bound – Summary

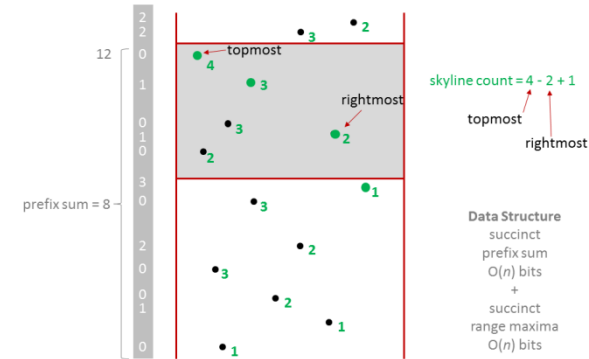
Upper Bound



Upper Bound – Multi-slab



Basic Counting – Vertical Slab



+ succinct stuff ...

$O(\lg n / \lg \lg n)$ orthogonal skyline counting
 Space $O(n)$ words

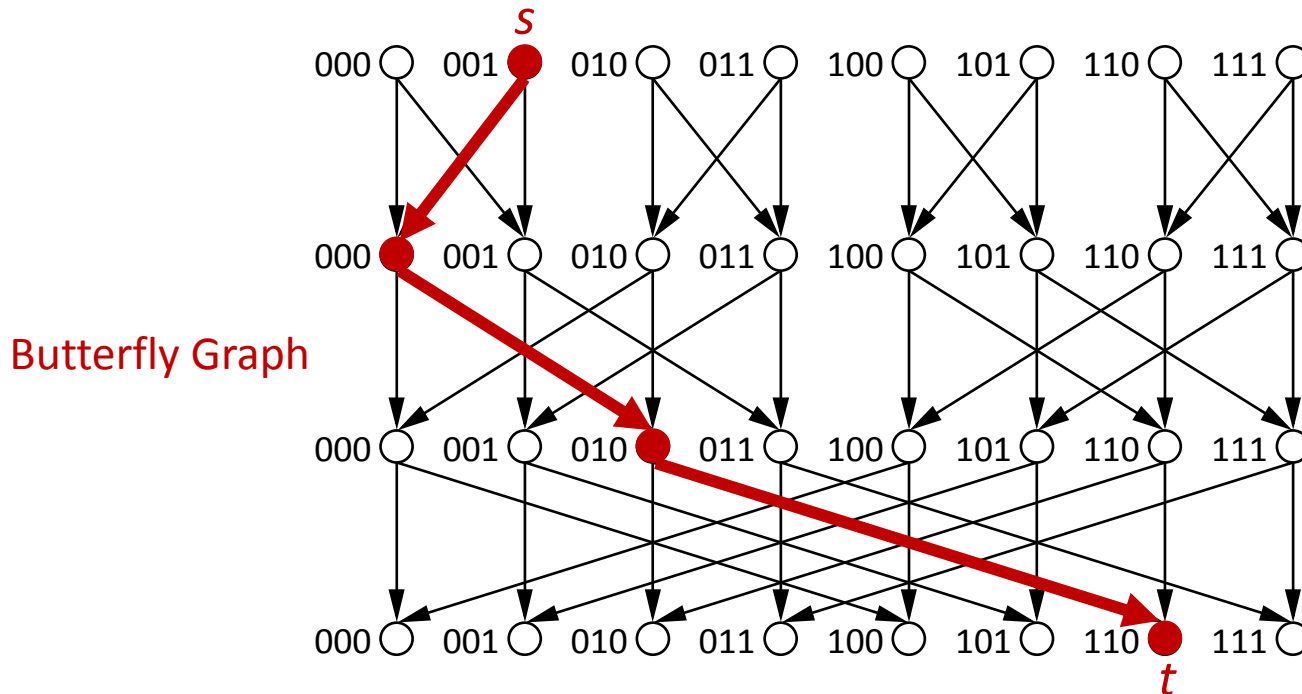
Lower Bound – Skyline Counting

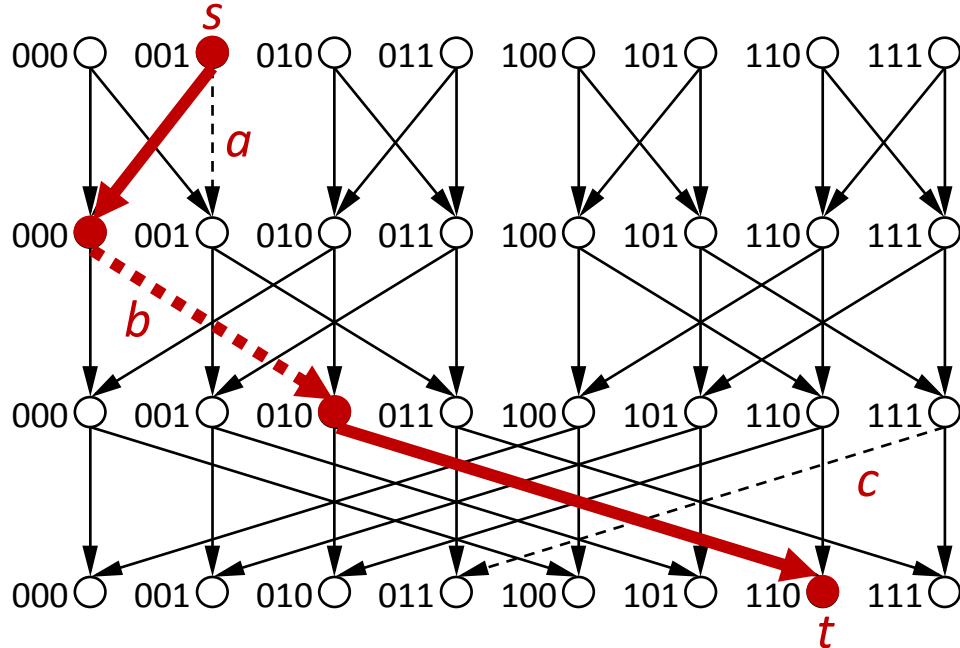
Reduction

$$[-\infty, x] \times [-\infty, y]$$

Reachability in the Butterfly Graph \Rightarrow Skyline Counting

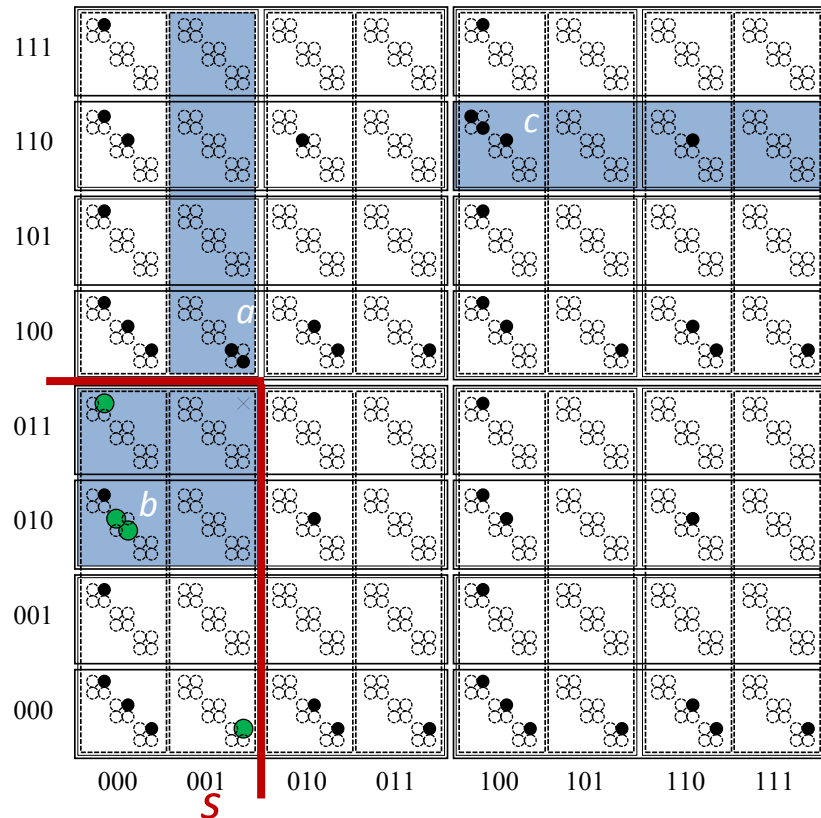
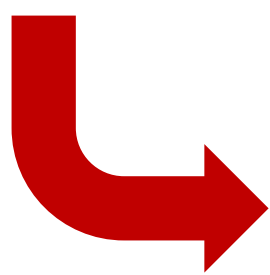
Word size $\lg^{O(1)} n$ bits, space $O(n \cdot \lg^{O(1)} n)$
 $\Rightarrow \Omega(\lg n / \lg \lg n)$ query





Butterfly Graph

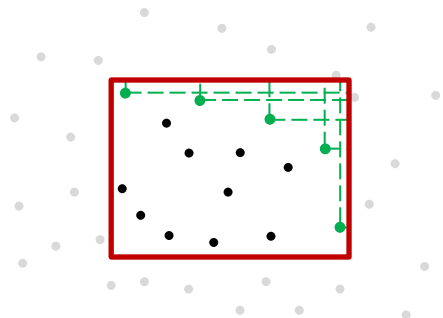
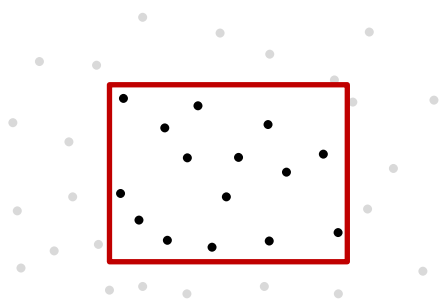
- dashed edges are deleted
- s - t paths are unique



2-sided Skyline Range Counting

- depth of edge \rightarrow aspect ratio of rectangle
- edge = 1 point, deleted edge = 2 points

Results



	Orthogonal range			Skyline		
	Space (words)	Query		Space (words)	Query	
Reporting	n $n \cdot \lg^\epsilon n$ $n \cdot \lg^{O(1)} n \Rightarrow \Omega(k + \lg \lg n)$	$k \cdot \lg^\epsilon n$ $k + \lg \lg n$	CLP11 ABR00 PT06	n $n \cdot \lg \lg n$ $n \cdot \lg^\epsilon n$ $n \cdot \lg n / \lg \lg n$	$k \cdot \lg^\epsilon n$ $k \cdot (\lg \lg n)^2$ $k \cdot \lg \lg n + \lg n / \lg \lg n$ $k \cdot \lg \lg n$ $k + \lg n / \lg \lg n$ $k + \lg n / \lg \lg n$	new NN12 new NN12 new DGKASK12
						improve improve
Counting	n $n \cdot \lg^{O(1)} n \Rightarrow$	$\lg n / \lg \lg n$ $\lg n / \lg \lg n$	JMS04 P07	$n \cdot \lg n$ $n \cdot \lg^3 n / \lg \lg n$ n $n \cdot \lg^{O(1)} n \Rightarrow$	$\lg n$ $\lg n / \lg \lg n$ $\lg n / \lg \lg n$ $\Omega(\lg n / \lg \lg n)$	DGKASK12 DGS13 new new
						improve