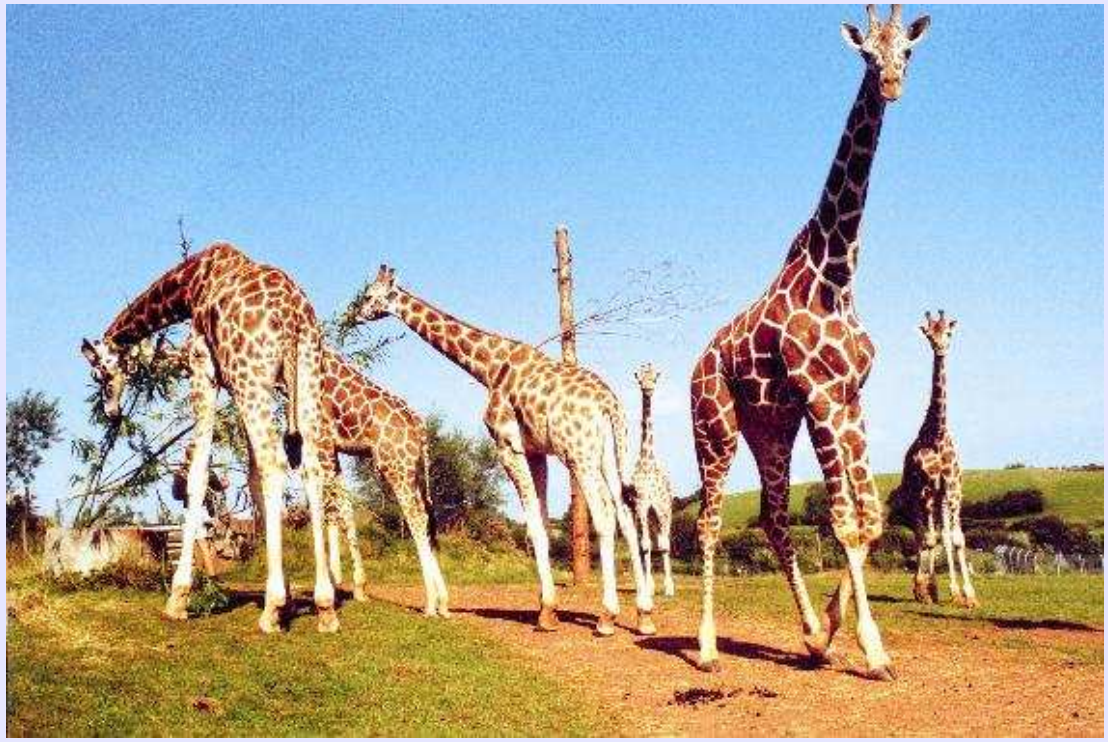
Cache-Oblivious String Dictionaries



Gerth Stølting Brodal
University of Aarhus

Rolf Fagerberg
University of Southern Denmark

SODA 2006, January 22-24, 2006, Miami, Florida

Computational Models

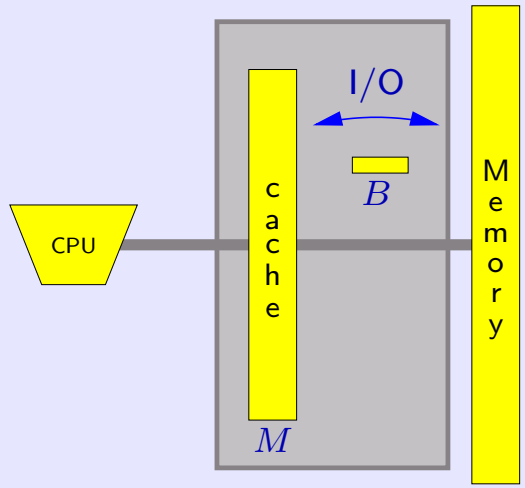
Computational Models

	RAM von Neumann 1946	I/O Model Aggarwal and Vitter 1988	Ideal Cache Model Frigo et al. 1999
Parameters	N	N M B	N M B
Complexity	# instructions	# I/Os	# I/Os
Properties	Simpel	Cache aware: M and B known	Cache oblivious: M and B unknown

N = problem size

M = memory size

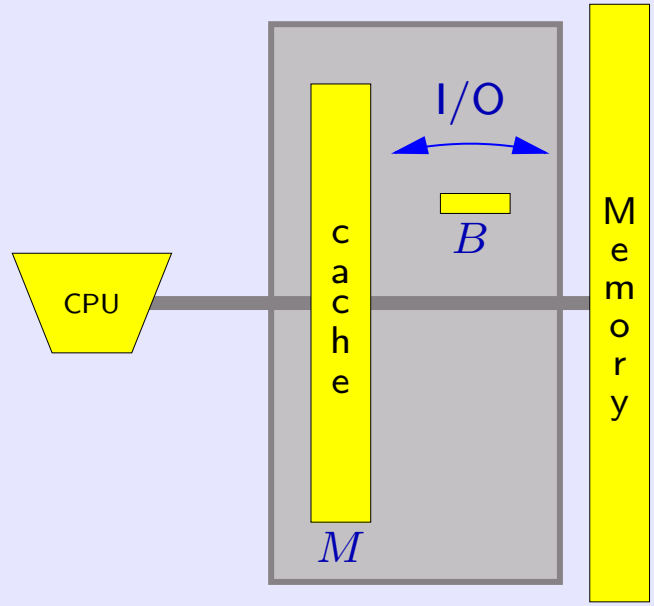
B = I/O block size



Ideal Cache Model — no parameters!?

Frigo, Leiserson, Prokop, Ramachandran 1999

- Program with only one memory
- Analyze in the I/O model for
- Optimal off-line cache replacement strategy arbitrary B and M

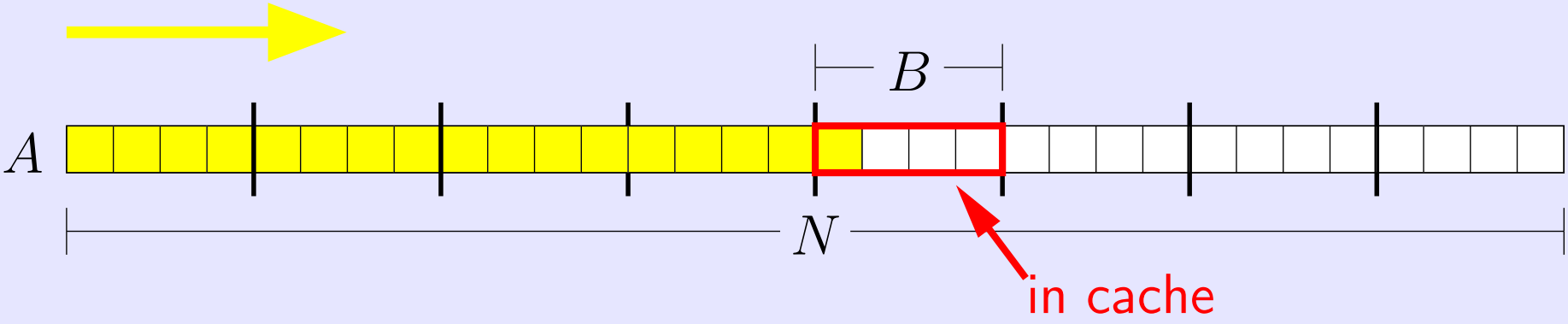


Advantages

- Optimal on arbitrary level $\Rightarrow$ optimal on all levels
- Portability, B and M not hard-wired into algorithm
- Dynamic changing M (and B)

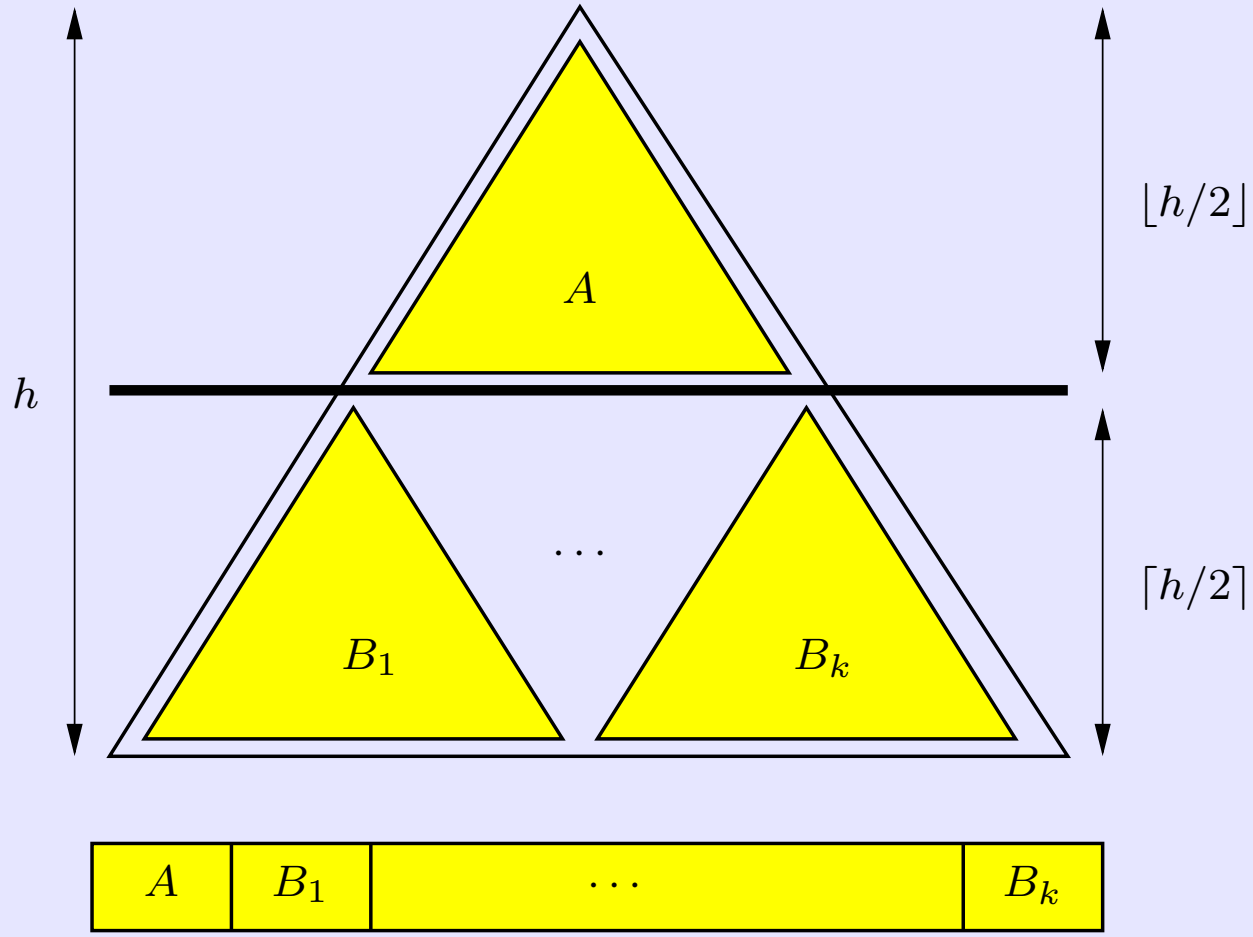
Cache-Oblivious Preliminaries

Cache-Oblivious Scanning



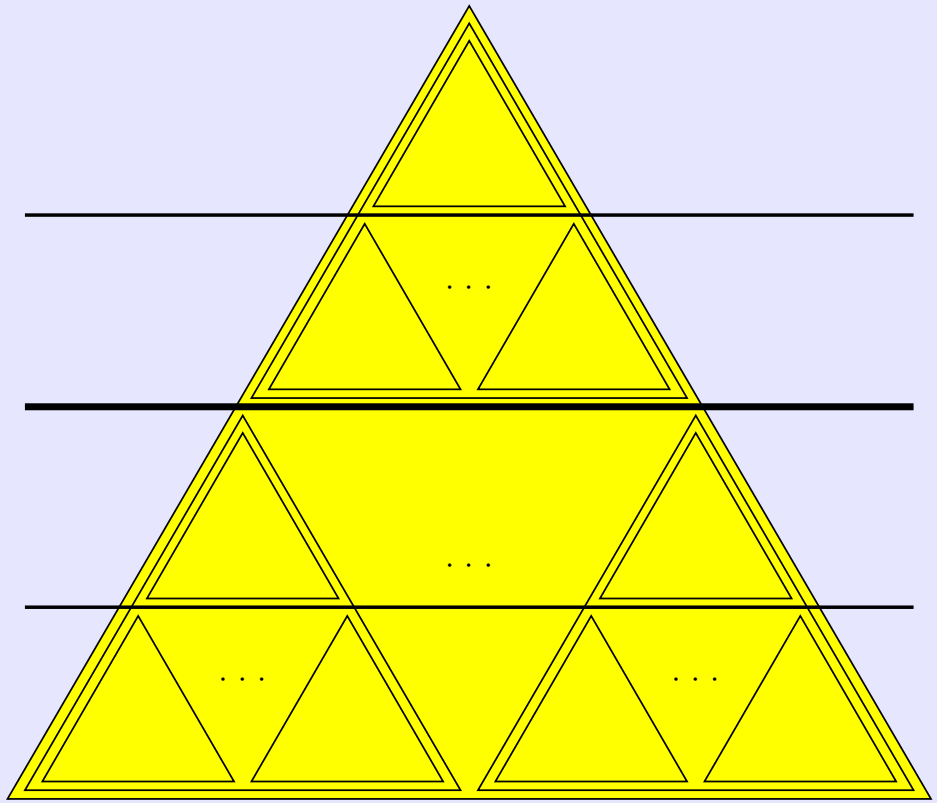
$$O\left(\frac{N}{B}\right) \text{ I/Os}$$

Static Cache-Oblivious B-Tree

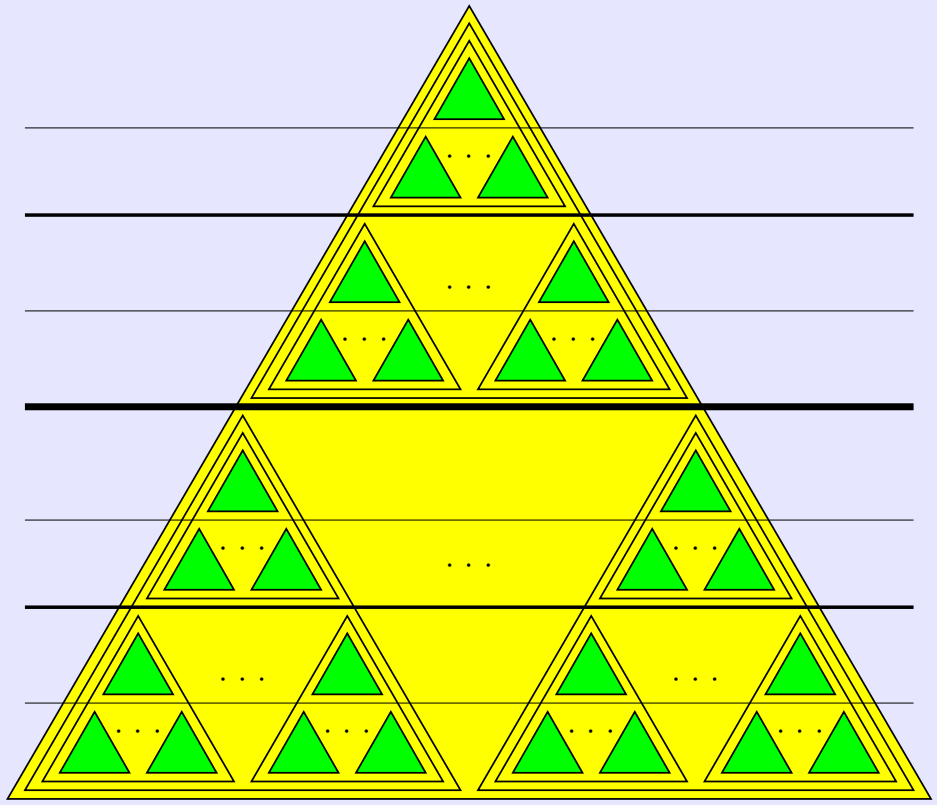


Recursive layout of binary tree $\equiv$ van Emde Boas layout

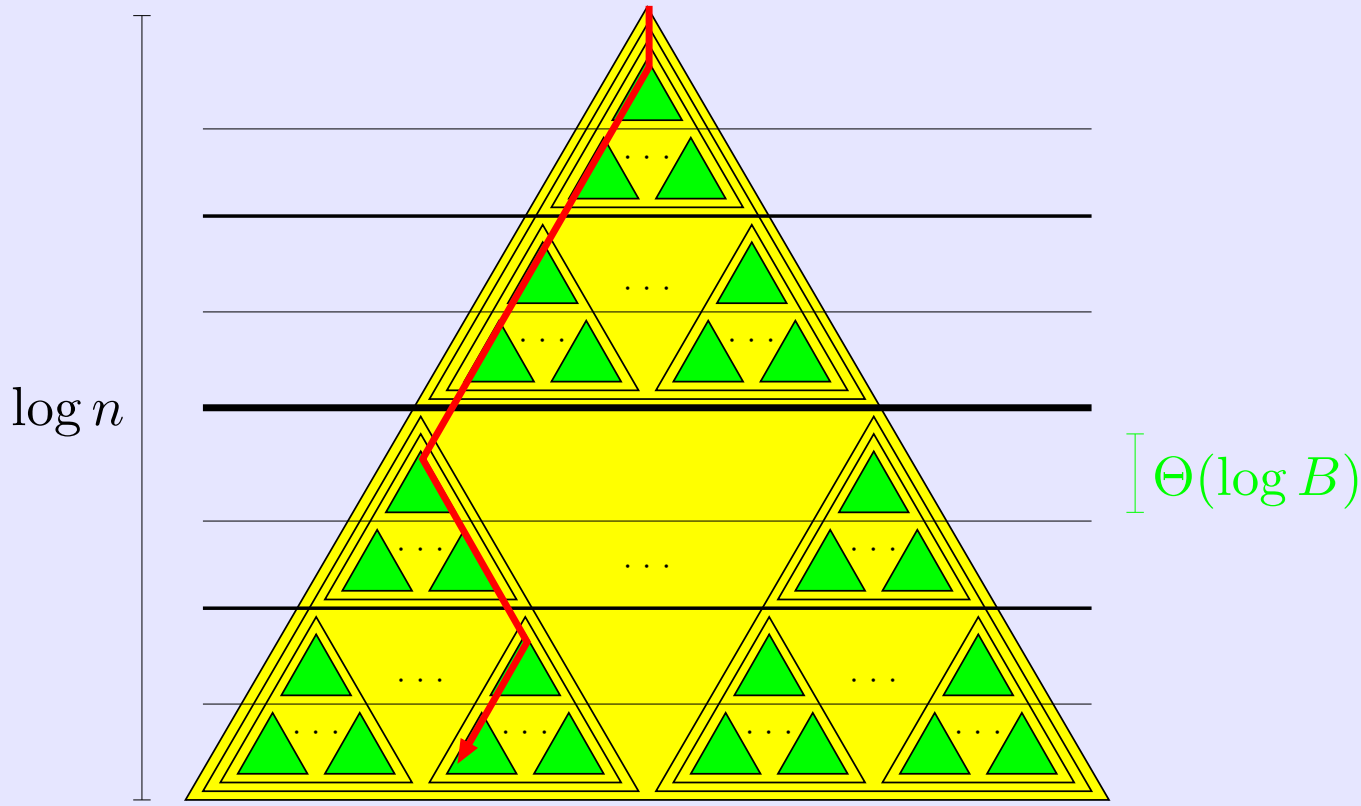
Static Cache-Oblivious B-Tree



Static Cache-Oblivious B-Tree



Static Cache-Oblivious B-Tree



Searches perform $O(\log_B N)$ I/Os

Summary Cache-Oblivious Tools

Scanning :	$O(N/B)$
B-tree searching :	$O(\log_B N)$
Sorting* :	$O\left(\frac{N}{B}\log_{M/B}\frac{N}{B}\right)$

* requires a tall cache assumption $M \geq B^{1+\varepsilon}$

Frigo, Leiserson, Prokop, Ramachandran 1999

Brodal and Fagerberg 2002, 2003

String Dictionaries

String Dictionaries

	RAM	I/O Model	Ideal Cache Model
Structure	Tries Morrison 1968	String B-trees Ferragina and Grossi 1999	This talk
Query	$O(\|P\|)$	$O(\|P\|/B + \log_B n)$	$O(\|P\|/B + \log_B n)$
Space	$O(N)$	$O(N)$	$O(N)$
	dynamic	dynamic	static, $M \geq B^{2+\varepsilon}$

n : number of strings

N : total length of strings

P : query string

The Trouble Starts...

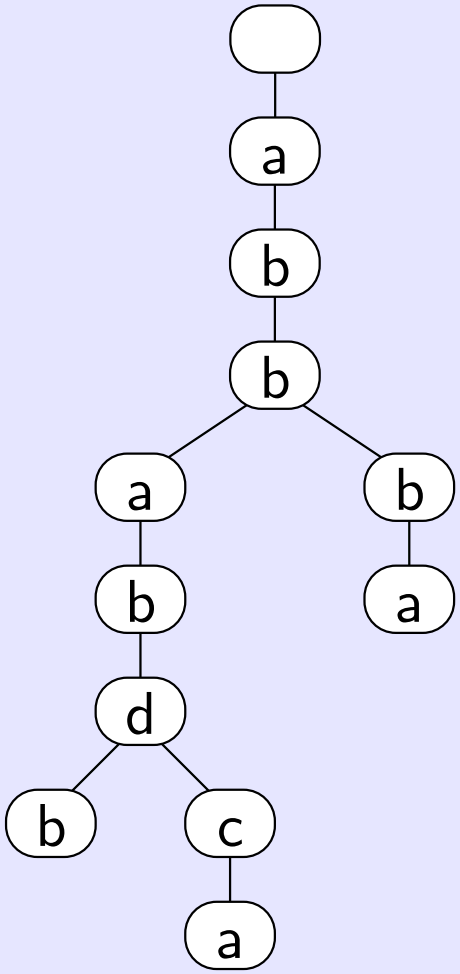
÷ Tries cannot be laid out in memory to support top-down searches in $O(\log_B N + |P|/B)$ I/Os Demaine et al. 2004

÷ Can construct suffix trees cache-obliviously using $O(\mathrm{Sort}(N))$ I/Os, but cannot search in it efficiently...

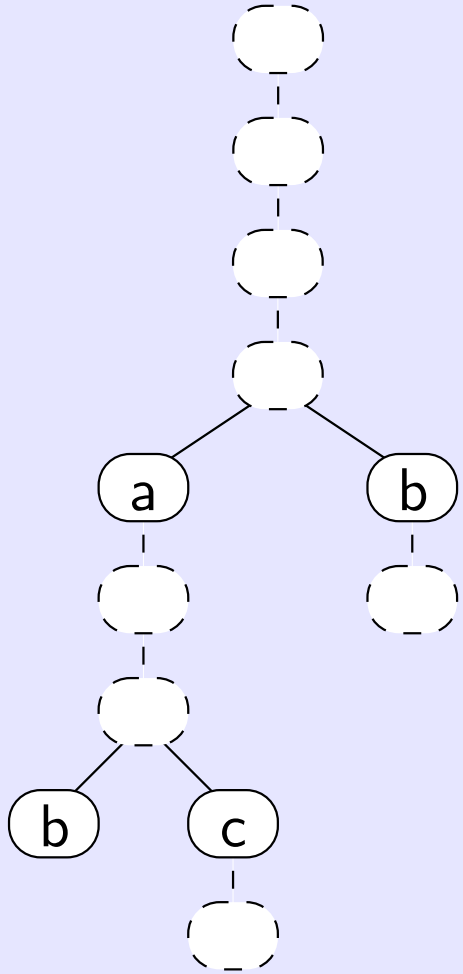
Farach et al. 2000

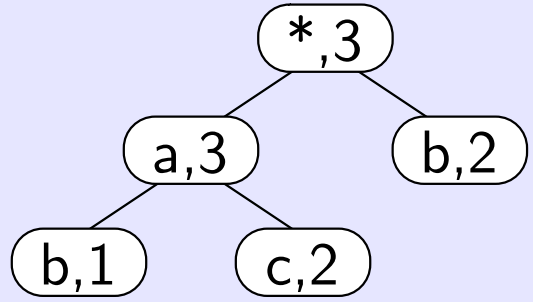
\+ Cache-aware string B-trees support searches in a set of strings in $O(\log_B n + |P|/B)$ I/Os Ferragina and Grossi 1999

Tries vs Blind Tries



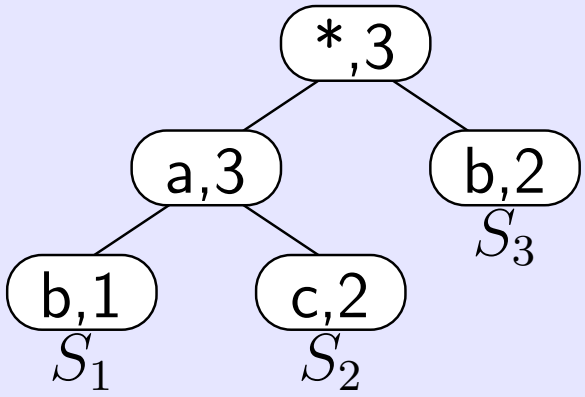
Trie

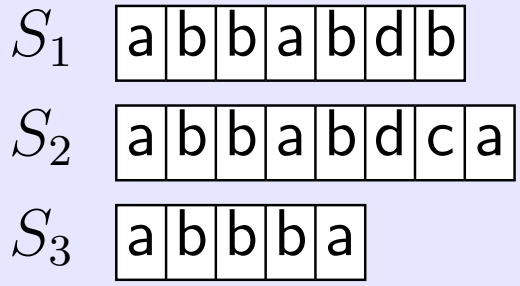


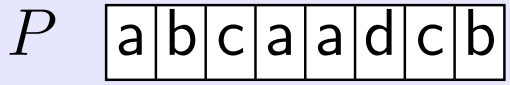


Blind trie

String Dictionary

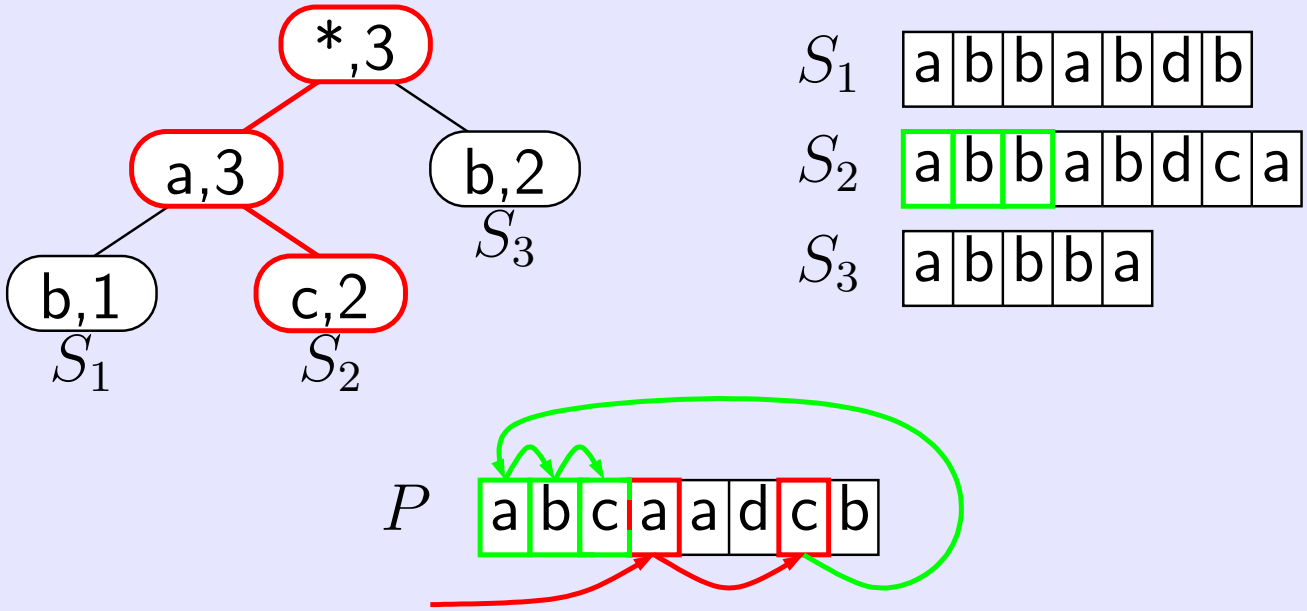






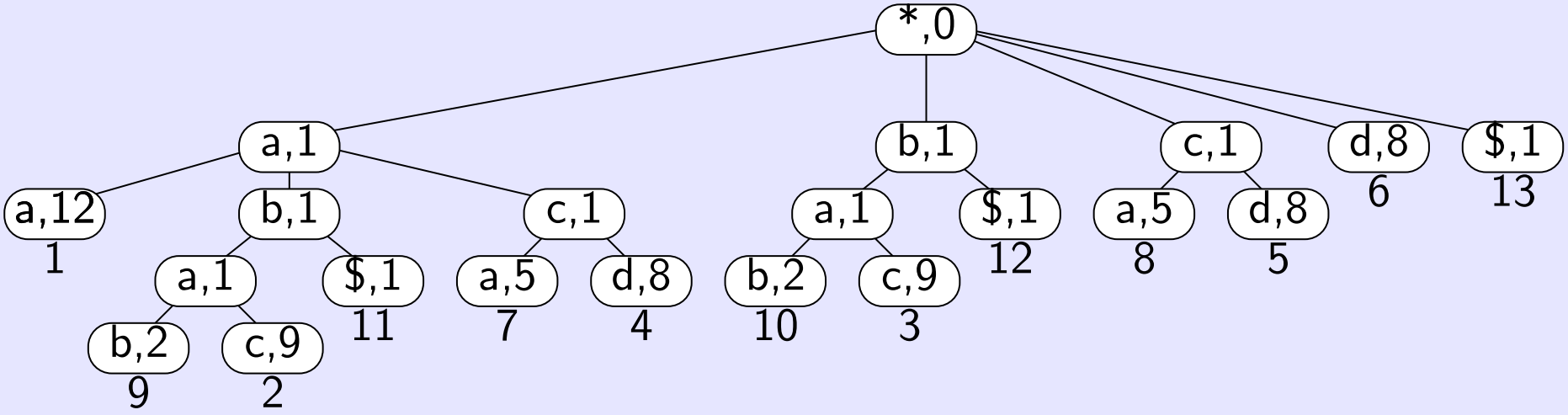
Queries: Search blind trie + Verify one string

String Dictionary



Queries: Search blind trie + Verify one string

Suffix Tree



T

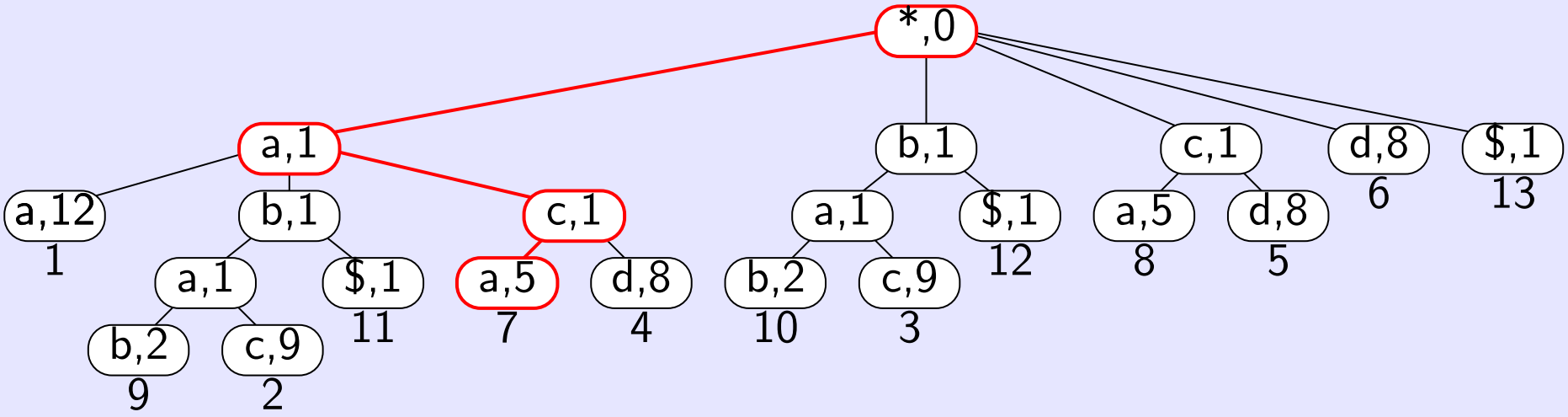
a	a	b	a	c	d	a	c	a	b	a	b	$
1	2	···			7		···					13

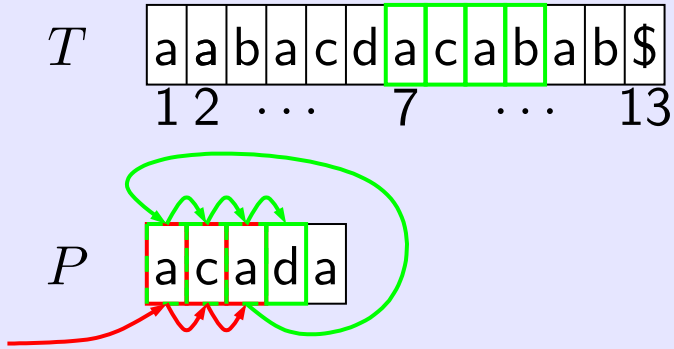
P

a	c	a	d	a

Queries: Search blind trie + Verify one suffix

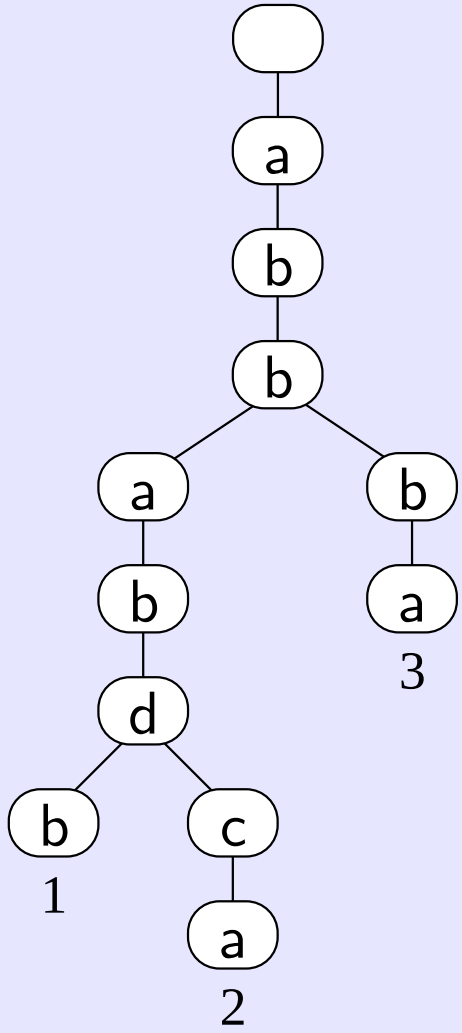
Suffix Tree

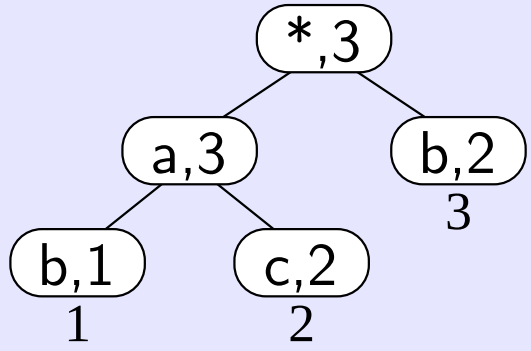




Queries: Search blind trie + Verify one suffix

Tries

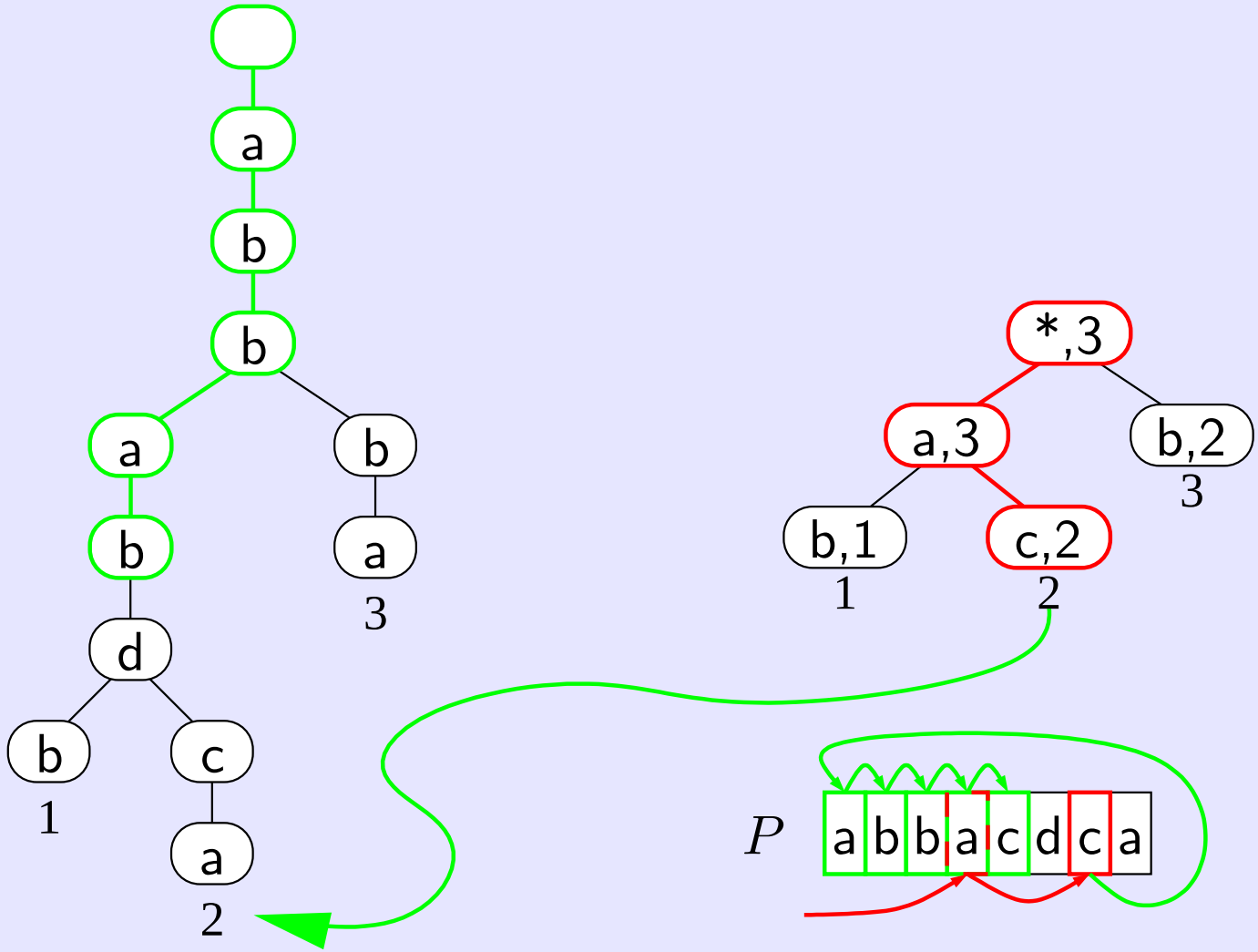




P | a | b | b | a | c | d | c | a |

Queries: Search blind trie + Verify prefix of one path

Tries

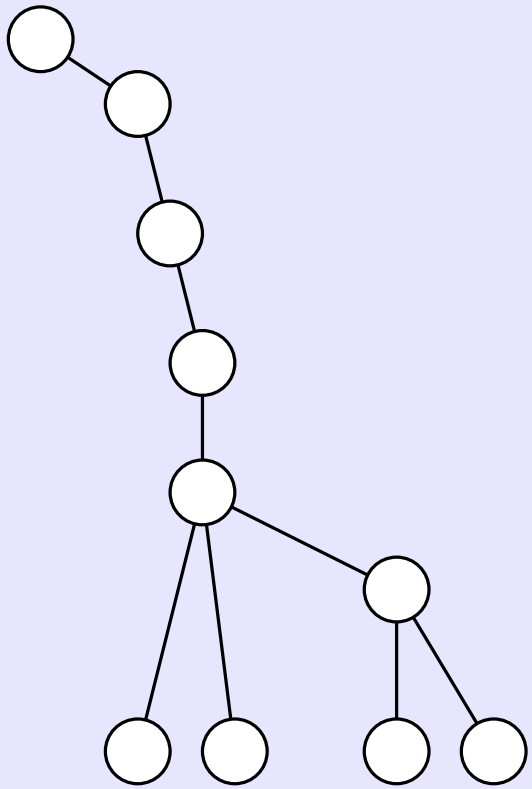


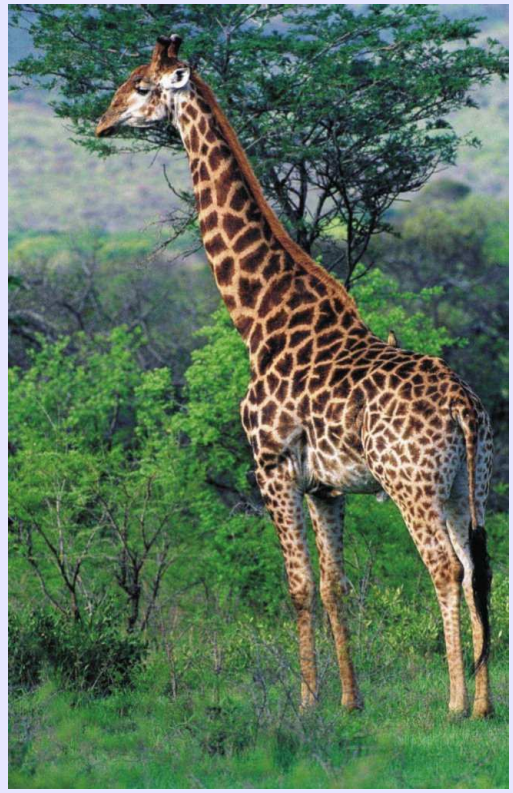
Queries: Search blind trie + Verify prefix of one path

Verifying a Prefix of a Path in a Tree

Verifying Paths in Giraffe Trees is Easy

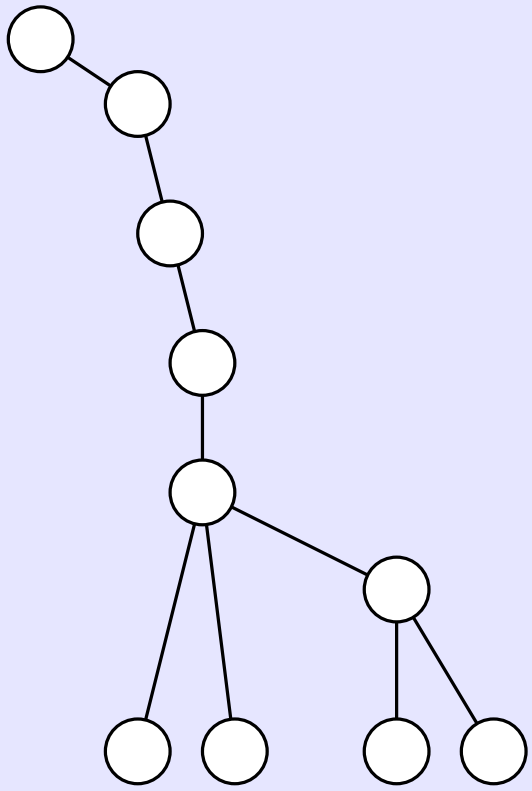
Definition A tree is a giraffe tree if all root-to-leaf paths share at least half of the nodes of the tree (long neck)

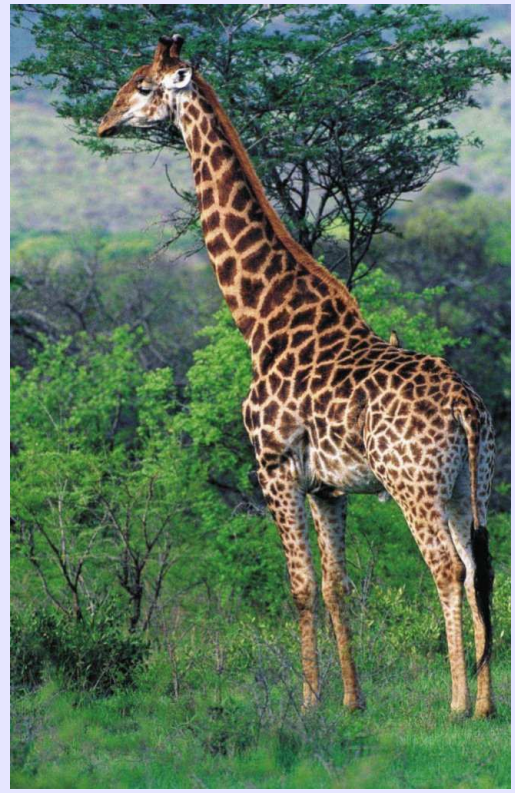




Verifying Paths in Giraffe Trees is Easy

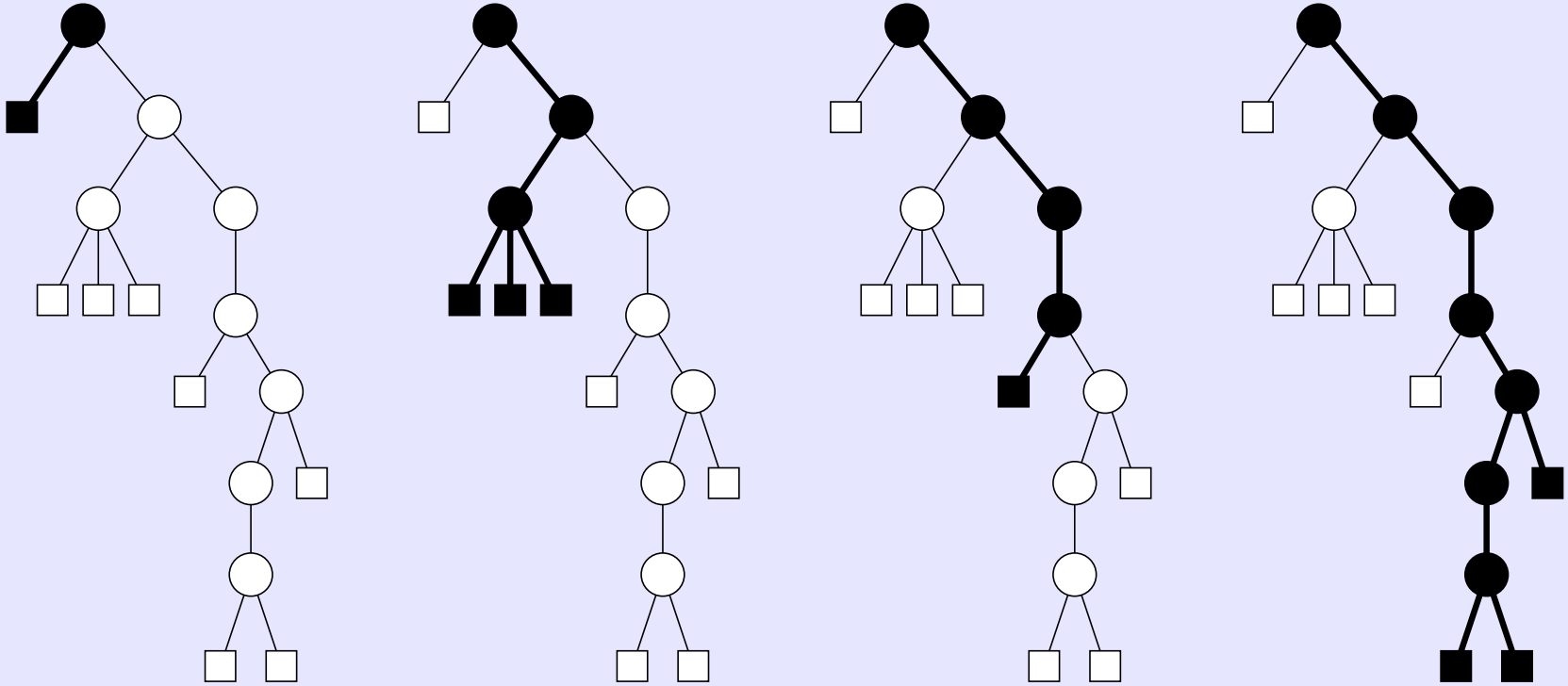
Definition A tree is a giraffe tree if all root-to-leaf paths share at least half of the nodes of the tree (long neck)





- A prefix of length p of a path in a giraffe tree using a BFS layout can be traversed in $O(p/B)$ I/Os

Giraffe Cover of a Tree



- Constructed left-to-right using $O(N/B)$ I/Os and space $O(N)$
- A prefix of length p of a path in a known giraffe in BFS layout can be traversed in $O(p/B)$ I/Os

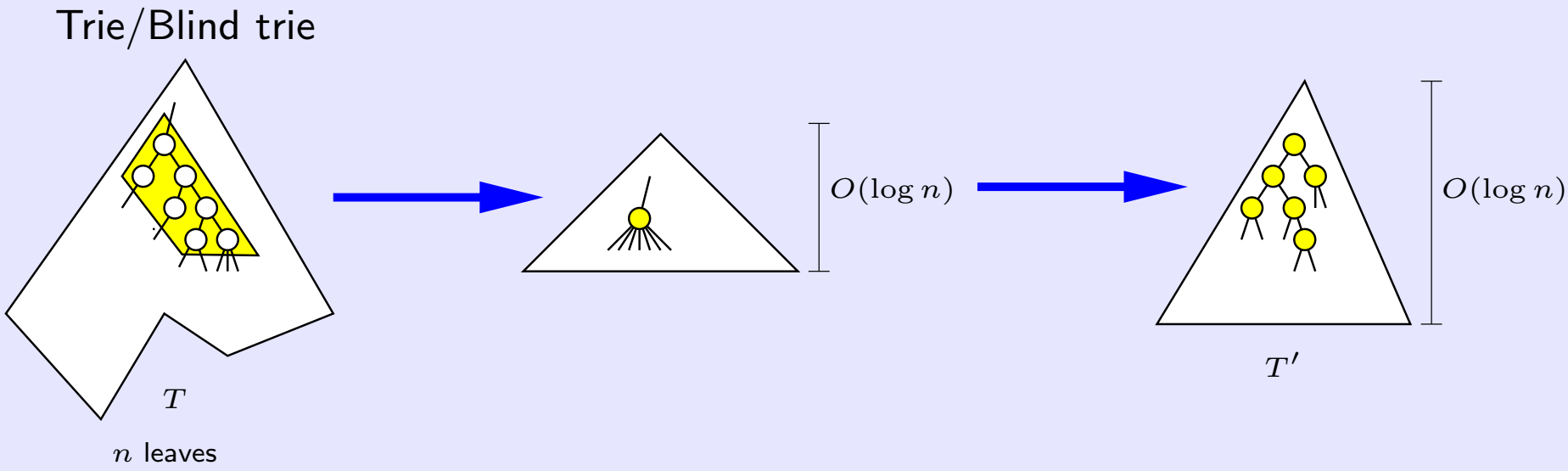
Summary so far...

String dictionary search
Suffix tree search
Trie search
} reduce to blind trie search

Query : Blind trie search + $O\left(1 + \frac{|P|}{B}\right)$ I/Os

Cache-Oblivious (Blind) Tries

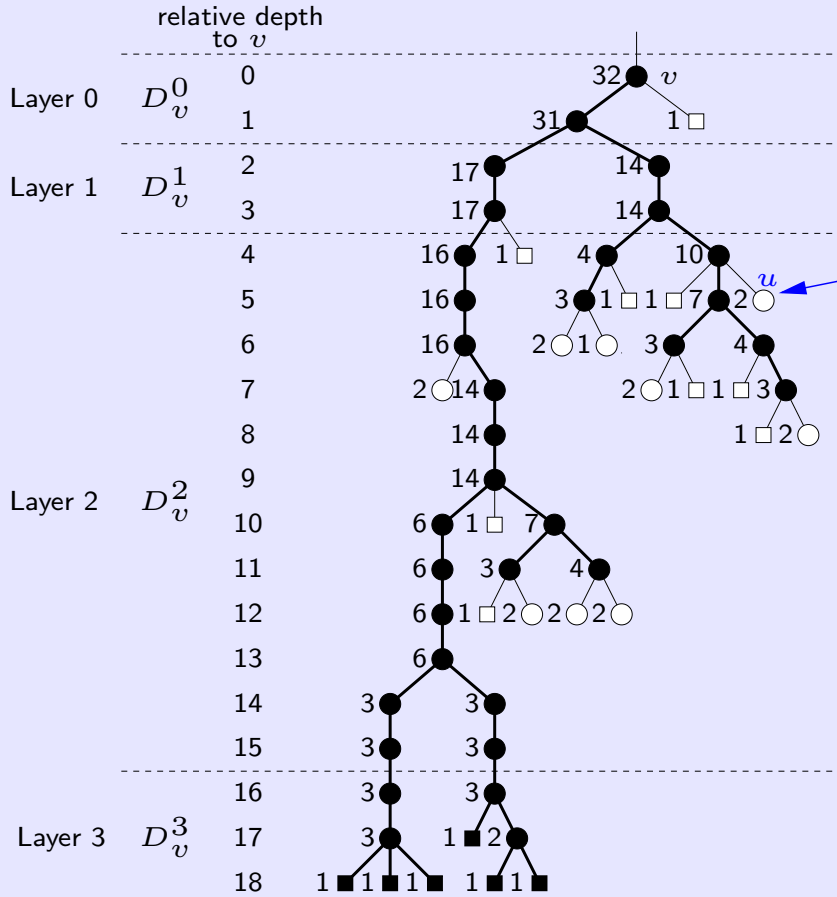
Cache-Oblivious (Blind) Tries



- Partition input trie T into components (generalization of heavy paths)
- T' = collapse components in T into high degree nodes and replace by weight balanced trees
- Apply van Emde Boas layout out to T'

Search: $O(\log_B n)$ I/O — ignoring searching inside components

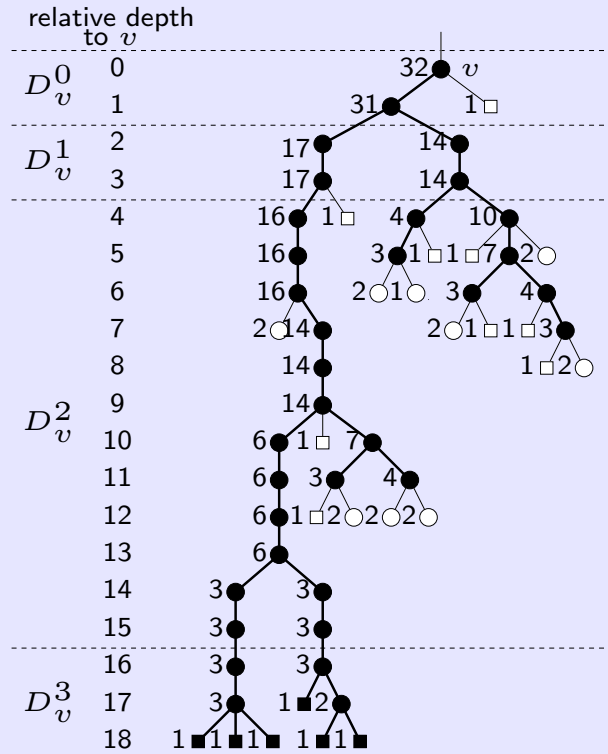
Decomposition into Components



$$
\begin{aligned}
D_v^0 &= \{u \in T_v \mid \mathrm{rank}(u) = \mathrm{rank}(v) \wedge \mathrm{depth}(u) - \mathrm{depth}(v) < 2^{2^0}\} \\
D_v^i &= \{u \in T_v \mid \mathrm{rank}(v) - \mathrm{rank}(u) < \varepsilon 2^i \\
&\qquad \wedge\ 2^{2^{i-1}} \leq \mathrm{depth}(u) - \mathrm{depth}(v) < 2^{2^i}\}
\end{aligned}
$$

Storing and Searching Components

- Store each layer D_v^i separately
- Make a giraf-decompostition of D_v^i
- For D_v^i have a blind trie
  of size $O(2^{\varepsilon 2^i})$ (using BFS layout)
  to select the right giraffe-tree
- Search: D_v^i search the blind trie + search in one giraffe-tree
- Distribute $D_v^0, D_v^1, D_v^2, \ldots$
  in the van Emde Boas layout of T'



- Analysis:
  - Search in blind trie for D_v^{i+1} (lookahead) dominated by the matched characters in D_v^i — requires $M \geq B^{2+\delta}$
  - Space in van Emde Boas layout for a subtree of size k becomes $O(k^3)$

Cache-Oblivious Tries

There exists a cache-oblivious trie supporting prefix queries in

$$O(\log_B |n| + |P|/B) \text{ I/Os},$$

where P is the query string, and n is the number of leaves in the trie.

It can be constructed in $O(\mathrm{Sort}(N))$ time, where N is the total number of characters in the input.

The space required is $O(N)$.

The structure assumes $M \geq B^{2+\delta}$.

Final Remaks

- Lookahead in the query string is crucial
  (both cache-aware and cache-oblivious)

- A giraffe cover is a simple construction allowing topdown path traversals in a tree using $O(|P|/B)$ I/Os

The End